Interpolation

Finite differences play an important role in numerical techniques, where tabulated values of the functions are available. for instance, consider a function $y = f(x)$. as x takes values $x_0, x_1, x_2, \ldots\ldots x_n$ let corresponding values of y be $y_0, y_1, y_2, \ldots\ldots y_n$ that is, for a given table of values (x_k, y_k), k=0, 1, 2,….,n the process of estimating the value of y, for any intermediate value of x is called interpolation .

In general, for interpolation of a tabulated function the concept of finite differences is important. The knowledge about various finite difference operators and their symbolic relations are very much needed to establish various interpolation formulae

❖ FINITE DIFFERENCE OPERATORS

a) Forward Differences

For given table of values (x_k,y_k), k=0, 1, 2,…..n with equally-spaced abscissas of a function $y=f(x)$,we define the forward difference operator Δ as follows:

$\Delta y_i = y_{i+1} - y_i$ i=0, 1, 2,…(n-1)

We write

$\Delta y_0 = y_1 - y_0$

$\Delta y_1 = y_2 - y_1$

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$\Delta y_{n-1} = y_n - y_{n-1}$

These difference are called first differences of the function y , Δ is called forward difference operator.

Similarly, the difference of the first differences are called second difference defined by

$\Delta^2 y_0 = \Delta y_1 - \Delta y_0$

$\Delta^2 y_1 = \Delta y_2 - \Delta y_1$

Thus, in general $\Delta^2 y_i = \Delta y_{i+1} - \Delta y_i$

Δ^2 is called the second difference operator. Thus, continuing we can define, rth difference of y, as

$\Delta^r y_i = \Delta^{r-1} y_{i+1} - \Delta^{r-1} y_i$

The above defined differences can be written down systematically by construction a difference table for value of (y_k. x_k) as shown bellow:

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$
x_0	y_0						
		Δy_0					
x_1	y_1		$\Delta^2 y_0$				
		Δy_1		$\Delta^3 y_0$			
x_2	y_2		$\Delta^2 y_1$		$\Delta^4 y_0$		
		Δy_2		$\Delta^3 y_1$		$\Delta^5 y_0$	
x_3	y_3		$\Delta^2 y_2$		$\Delta^4 y_3$		d^6y0
		Δy_3		$\Delta^3 y_2$		$\Delta^5 y_1$	
x_4	y_4		$\Delta^2 y_3$		$\Delta^4 y_2$		
		Δy_4		$\Delta^3 y_3$			
x_5	y_5		$\Delta^2 y_4$				
		Δy_5					
x_6	y_6						

This table is called forward difference table or diagonal difference table

Y0 is called the leading term, while the differences Δy_0, $\Delta^2 y_0$, $\Delta^3 y_0$,…..are called leading differences.

Example 1: construct a forward difference table for the following values of x and y

x= {0.1, 0.3, 0.5, .07, 0.9, 1.1, 1.3}

y={0.003, 0.067, 0.148, 0.248, 0.370, 0.518, 0.697}

solution

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$
0.1	0.003						
		0.064					
0.3	0.067		0.017				
		0.081		0.002			
0.5	0.148		0.019		0.001		
		0.100		0.003		0.00	
0.7	0.248		0.022		0.001		0.00
		0.122		0.004		0.00	
0.9	0.370		0.026		0.001		
		0.148		0.005			
1.1	0.518		31 0.013				
		0.179					
1.3	0.697						

1) Newton forward difference interpolation (Newton-Gregory forward difference interpolation formula)

Let y =*f*(x) be a function which takes values $f(x_0)$, $f(x_0+h)$, $f(x_0+2h)$,…. Corresponding to various equispaced values of x with spacing h, say x_0, x_0+h, x_0+2h .to evaluate the function *f*(x) for a value x_0+ph, where p is any real number

$$p = \frac{x - x0}{h}$$

$$f(x_0+ph)=[1+p\Delta+\frac{p(p-1)}{2!}\Delta^2+\frac{p(p-1)(p-2)}{3!}\Delta^3+\frac{p(p-1)(p-2)(p-3)}{4!}\Delta^4+\ldots\ldots]y_0$$

$$f(x)= y_0+p\Delta y_0+\frac{p(p-1)}{2!}\Delta^2 y_0+\frac{p(p-1)(p-2)}{3!}\Delta^3 y_0+\frac{p(p-1)(p-2)(p-3)}{4!}\Delta^4 y_0+$$

This formula is mainly used for interpolation the values of y near the beginning of a set of tabular values and for extrapolation values of y a short distance backward from y_0 .

Example

Evaluate *f*(15), given the following table of values:

x	10	15	20	30	40	50
y	46	?	66	81	93	101

Type text here

Solution: we may note that x=15 is very near to the beginning of the table. We use Newton's forward difference interpolation formula .the forward difference are calculate and tabulated as giving below :

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
10	46				
		20			
20	66		-5		
		15		2	
30	18		-3		-3
		12		-1	
40	93		-4		
		8			
50	101				

We have Newson's forward difference interpolation formula as

$$f(x_0+ph)= y_0+p\Delta y_0+\frac{p(p-1)}{2!}\Delta^2 y_0 +\frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 +\frac{p(p-1)(p-2)(p-3)}{4!}\Delta^4 y_0$$

$p=(x-x_0)/h$

$x_0=10$, $y0=46$, $\Delta y_0=20$, $\Delta^2 y_0=-5$, $\Delta^3 y_0=2$, $\Delta^4 y_0=-3$

$p=(15-10)/10=0.5$

$f(x_0+ph)= f(10+0.5*10)= f(15)$

$f(15) = 46 + 0.5*20 + \frac{0.5(0.5-1)}{2!}(-5) + \frac{0.5(0.5-1)(0.5-2)}{3!}(2) + \frac{0.5(0.5-1)(0.5-2)(0.5-3)}{4!}$ (-3)

f(15)=46+10+0.625+0.125+0.1172=56.8672

Example: find Newton forward difference interpolation polynomial for the following data:

x =	0.1	0.2	0.3	0.4	0.5
y=	1.4	1.56	1.76	2.00	2.28

Solution:

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
0.1	1.4				
		0.16			
0.2	1.56		0.04		
		0.2		0	
0.3	1.76		0.04		0
		0.24		0	
0.4	2		0.04		
		0.28			
0.5	2.28				

$f(x_0+ph) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0$

$x_0=0.1$, $y_0=1.4$, $\Delta y_0=0.16$, $\Delta^2 y_0=0.04$

$p = \frac{x - x0}{h} = \frac{x-0.1}{0.1} = 10x-1$

$f(x_0+ph) = f(0.1+(10x-1)0.1) = f(x)$

∴ $f(x) = 1.4 + (10x-1)(0.16) + \frac{(10x-1)(10x-1-1)}{2!}(0.04)$

$f(x) = y(x) = 2x^2 + x + 1.28$

b)Backward differences

For given table of values (x_k, y_k) with equally spaced abscissas, the first backward differences are usually expressed in terms of the backward difference operator ∇ as

$\nabla y_i = y_i - y_{i-1}$ i=n, (n-1), ….,1

Thus,

$\nabla y_1 = y_1 - y_0$

$\nabla y_2 = y_2 - y_1$

- - -
- - -

$\nabla y_n = y_n - y_{n-1}$

the difference of these differences are called second difference and they are denoted by

$\nabla^2 y_2, \nabla^2 y_3, \ldots\ldots \nabla^2 y_n$

$\nabla^2 y_2 = \nabla y_2 - \nabla y_1$

$\nabla^2 y_3 = \nabla y_3 - \nabla y_2$

Thus, in general the second backward differences given as

$\nabla^2 y_i = \nabla y_i - \nabla y_{i-1}$

$\nabla^k y_i = \nabla^{k-1} y_i - \nabla^{k-1} y_{i-1}$ i=n, (n-1),…k

These backward difference ca be systematically arranged for a table of values (x_k, y_k) , k=0, 1, 2,….6 as indicated in table:

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$	$\nabla^5 y$	$\nabla^6 y$
x_0	y_0						
		∇y_1					
x_1	y_1		$\nabla^2 y_2$				
		∇y_2		$\nabla^3 y_3$			
x_2	y_2		$\nabla^2 y_3$		$\nabla^4 y_4$		
		∇y_3		$\nabla^3 y_4$		$\nabla^5 y_5$	
x_3	y_3		$\nabla^2 y_4$		$\nabla^4 y_5$		$\nabla^6 y_6$
		∇y_4		$\nabla^3 y_5$		$\nabla^5 y_6$	
x_4	y_4		$\nabla^2 y_5$		$\nabla^4 y_6$		
		∇y_5		$\nabla^3 y_6$			
x_5	y_5		$\nabla^2 y_6$				
		∇y_6					
x_6	y_6						

2) Newton's backward difference interpolation formula

$$f(x) = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n + \frac{p(p+1)(p+2)(p+3)}{4!}\nabla^4 y_n + \ldots + \frac{p(p+1)(p+2)+\cdots+(p+n-1)}{4!}\nabla^n y_n$$

$$p = (x - x_n)/h$$

Example

Evaluate $f(7.5)$, given the following table of values:

x	1	2	3	4	5	6	7	8
y	1	8	27	64	125	216	343	512

Solution: we may note that x=7.5 is at the end of table use Newtons backward interpolation formula. We shall first construct the backward difference table for the given data.

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$	$\nabla^5 y$	$\nabla^6 y$
1	1						
		7					
2	8		12				
		19		6			
3	27		18		0		
		37		6		0	
4	64		24		0		0
		61		6		0	
5	125		30		0		
		91		6			
6	216		36				
		127		6			
7	343		42				
		169					
8	512						

$f(x) = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n + \frac{p(p+1)(p+2)(p+3)}{4!}\nabla^4 y_n$

$p = \frac{x - xn}{h} = \frac{7.5-8}{1} = -0.5$

and $yn = 512$, $\nabla^2 y_n = 42$, $\nabla^3 y_n = 6$

$f(7.5) = 512 + (-0.5)(169) + \frac{-0.5(-0.5+1)}{2*1}(42) + \frac{-0.5(-0.5+1)(-0.5+2)}{3*2*1}(6) + \frac{p(p+1)(p+2)(p+3)}{4!}(0)$

$f(7.5) = 421.875$

Example: the sales in particular department store for the last five years is given in the following table:

Year	1974	1676	1978	1980	1982
Sales	40	43	48	52	57

Estimate the sales for the year 1979

Solution

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
1974	40				
		3			
1976	43		2		
		5		-3	
1978	48		-1		5
		4		2	
1980	52		1		
		5			
1982	57				

$$f(1979)= y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n + \frac{p(p+1)(p+2)(p+3)}{4!}\nabla^4 y_n$$

$$p=\frac{x-xn}{h} = \frac{1979-1982}{2} = -1.5$$

and $y_n = 57$, $\nabla^2 y_n = 1$, $\nabla^3 y_n = 2$, $\nabla^4 y_n = 5$

$$f(1979)= 57 + (-1.5)(5) + \frac{-1.5(-1.5+1)}{2*1}(1) + \frac{-0.5(-1.5+1)(-1.5+2)}{3*2*1}(2) + \frac{-1.5(-1.5+1)(-1.5+2)(-1.5+3)}{4*3*2*1}(5)$$

$$f(1979) = y(1979) = 50.1172$$

Divided difference

When the function values are given at non-equispaced point, we have already developed the Lagrange's interpolation formula for interpolation ,now we shall interduce the concept of divided difference and then develop newton's divided difference interpolation formula, whose accuracy is same as that of Lagrange's formula, but have the advantage of being computationally economical in the sense that it involves less number of arithmetic operations .let us assume that the function y=(x) is known for several values of x,(x_i, y_i) for i=0(1)n .the divided differences of orders 0,1,2,…n are defined recursively as follows :

$$y\,[x_0] = y(x_0) = y_0$$

similarly, the higher order divided differences are defined in terms of lower order divided difference by the relations of the form

$$y\,[x_0, x_1, x_2] = \frac{y[x1,x2]-y[x0,x1]}{x2-x0}$$

while

$$y\,[x_0, x_1, \ldots, x_n] = \frac{y[x1,x2,\ldots\ldots,xn]-y[x0,x1,\ldots xn-1]}{xn-x0}$$

the standard format of the divided differences are display in table :

X	y(x)	1st order	2nd order	3rd order	4th order
x_0	y_0				
		$y[x_1, x_0]$			
x_1	y_1		$y[x_0, x_1, x_2]$		
		$y[x_2, x_1]$		$y[x_0, x_1, x_2, x_3]$	
x_2	y_2		$y[x_1, x_2, x_3]$		$y[x_0, x_1, x_2, x_3, x_4]$
		$y[x_3, x_2]$		$y[x_1, x_2, x_3, x_4]$	
x_3	y_3		$y[x_2, x_3, x_4]$		
		$y[x_4, x_3]$			
x_4	y_4				

We can easily verify that that the divided difference is a symmetric function of its arguments. that is,

$y\,[x_1, x_0] = y\,[x_0, x_1] = \frac{y0}{x0-x1} + \frac{y1}{x1-x0}$

now,

$y\,[x_0, x_1, x_2] = \frac{y\,[x1,\ x2]-y\,[x0,\ x1]}{x2-x0} = \frac{1}{x2-x0}\left[\frac{y2-y1}{x2-x1} - \frac{y1-y0}{x1-x0}\right]$

therefor,

$y\,[x_0, x_1, x_2] = \frac{y\,0}{(x0-x1)(x0-x2)} + \frac{y\,1}{(x1-x0)(x1-x2)} + \frac{y\,2}{(x2-x0)(x2-x1)}$

which is a symmetric form, hence suggests the general result as

$$y\,[x_0,\ldots.., x_k] = \frac{y0}{(x0-x1)\ldots\ldots..(x0-xk)} + \frac{y1}{(x1-x0)\ldots\ldots..(x0-xk)}$$

$$+\ldots..+\frac{yk}{(xk-x0)\ldots\ldots..(xk-x(k-1))} = \sum_{i=0}^{k} \frac{yi}{\prod_{i=1}^{k}(xi-xi)}$$

Example1: construct a divided difference table for the following data:

x	0	1	2	4
y	1	1	2	5

Solution: the divided difference table for given data is constructed as following;

X	y(x)	1^{st} order	2^{nd} order	3^{rd} order
x_0 =0	y_0 =1			
		$y[x_0, x_1]$		
x_1 =1	y_1 =1		$y[x_0, x_1, x_2]$	
		$y[x_1, x_2]$		$y[x_0, x_1, x_2, x_3]$
x_2 =2	y			
$_2$ =2			$y[x_1, x_2, x_3]$	
		$y[x_2, x_3]$		
x_3 =4	y_3 =5			

$$y[x_1, x_0] = y[x_0, x_1] = \frac{y0}{x0-x1} + \frac{y1}{x1-x0} = \frac{1}{(0-1)} + \frac{1}{(1-0)} = 0$$

$$y[x_1, x_2] = y[x_2, x_1] = \frac{y1}{x1-x2} + \frac{y2}{x2-x1} = \frac{1}{(1-2)} + \frac{2}{(2-1)} = 1$$

$$y[x_2, x_3] = y[x_3, x_2] = \frac{y2}{x2-x3} + \frac{y3}{x3-x2} = \frac{2}{(2-4)} + \frac{5}{(4-2)} = 1.5$$

$$y[x_0, x_1, x_2] = \frac{y\,0}{(x0-x1)(x0-x2)} + \frac{y\,1}{(x1-x0)(x1-x2)} + \frac{y\,2}{(x2-x0)(x2-x1)}$$

$$y[x_0, x_1, x_2] = \frac{1}{(0-1)(0-2)} + \frac{1}{(1-0)(1-2)} + \frac{2}{(2-0)(2-1)} = 0.5$$

$$y[x_1, x_2, x_3] = \frac{y\,1}{(x1-x2)(x1-x3)} + \frac{y\,2}{(x2-x1)(x2-x3)} + \frac{y\,3}{(x3-x1)(x3-x2)}$$

$$y[x_1, x_2, x_3] = \frac{1}{(1-2)(1-4)} + \frac{2}{(2-1)(2-4)} + \frac{5}{(4-1)(4-2)} = -1/6$$

$$y[x_0, x_1, x_2, x_3] = \frac{y0}{(x0-x1)(x0-x2)(x0-x3)}$$

$$\frac{y1}{(x1-x0)(x1-x2)(x1-x3)} + \frac{y2}{(x2-x0)(x2-x1)(x2-x3)} + \frac{y3}{(x3-x0)(x3-x1)(x3-x2)}$$

$$\frac{1}{(0-1)(0-2)(0-4)} + \frac{1}{(1-0)(1-2)(1-4)} + \frac{2}{(2-0)(2-1)(2-4)} + \frac{5}{(4-0)(4-1)(4-2)}$$

$$= \frac{-1}{8} + \frac{1}{3} + \frac{2}{-4} + \frac{5}{24} = -1/12$$

X	y(x)	1st order	2nd order	3rd order
0	1			
		$y[x_0, x_1]$ =0		
1	1		$y[x_0, x_1, x_2]$ =0.5	
		$y[x_1, x_2]$ =1		$y[x_0, x_1, x_2, x_3]$ =-1/12
2	2		$y[x_1, x_2, x_3]$ =1/6	
		$y[x_2, x_3]$ = 1.5		
4	5			
+				

3) Newton divided difference interpolation

Newton's divided difference interpolation formula can be written as

$y=f(x) = y_0 + (x-x_0)\, y[x_0,x_1] + (x-x_0)(x-x_1)\, y[x_0,x_1,x_2] + (x-x_0)(x-x_1)(x-x_2)\, y[x_0,x_1,x_2,x_3] + \ldots\ldots\ldots\ldots\ldots + +(x-x_0)(x-x_1)\ldots\ldots(x-x_{n-1})\, y[x_0,x_1,\ldots,x_n]$

Example2: using Newton divided difference formula, find the quadratic equation for the following data. Hence find $y(2)$.

X	0	1	4
Y	2	1	4

Solution: the divided table for the given data is constructed as follows:

x	y(x)	1st order	2nd order
0	2		
		$y[x_0, x_1] = -1$	
1	1		$y[x_0, x_1, x_2] = 0.5$
		$y[x_1, x_2] = 1$	
4	4		

Now, using Newton divided difference formula, we have:

$y=f(x) = y_0 + (x-x_0)\, y[x_0,x_1] + (x-x_0)(x-x_1)\, y[x_0,x_1,x_2]$

$y(x) = 2 + (x-0)(-1) + (x-0)(x-1)(0.5)$

$y(x) = 0.5(x^2-3x+4)$

Hence, $y(2) = 0.5(4-6+4)$

$y(2)=1$

Example 3:find the interpolating polynomial by Newtons divided difference formula for the data in Example 1,

Solution: from Example 1 we have

X	y(x)	1st order	2nd order	3rd order
0	1			
		$y[x_0, x_1] = 0$		
1	1		$y[x_0, x_1, x_2] = 0.5$	
		$y[x_1, x_2] = 1$		$y[x_0, x_1, x_2, x_3] = -1/12$
2	2		$y[x_1, x_2, x_3] = 1/6$	
		$y[x_2, x_3] = 1.5$		
4	5			

$f(x) = y_0 + (x-x_0)\, y[x_0,x_1] + (x-x_0)(x-x_1)\, y[x_0,x_1,x_2] + (x-x_0)(x-x_1)(x-x_2)\, y[x_0,x_1,x_2,x_3]$

$f(x) = 1 + (x-0)(0) + (x-0)(x-1)(0.5) + (x-0)(x-1)(x-2)(-1/12)$

$f(x) = \frac{-1}{12}x^3 + \frac{3}{4}x^2 - \frac{2}{3}x + 1$

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4) LAGRANGES INTERPOLATION FORMUA

Newtons interpolation formula developed in the earlier section can be used only when the values of independent variable x are equally spaced. Also, the differences of y must ultimately become small. If the values of the x are not given at equidistance intervals then we have the (LaGrange interpolation formula).

$$y(x) = \frac{(x-x_1)(x-x_2)\ldots(x-x_n)}{(x_0-x_1)(x_0-x_2)\ldots(x_0-x_n)}\, y_0 + \frac{(x-x_0)(x-x_2)\ldots(x-x_n)}{(x_1-x_0)(x_1-x_2)\ldots(x_1-x_n)}\, y_1 +$$

$$\frac{(x-x_0)(x-x_1)(x-x_3)\ldots(x-x_n)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)\ldots(x_2-x_n)}\, y_2 + \ldots + \frac{(x-x_0)(x-x_1)\ldots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\ldots(x_n-x_{n-1})}\, y_n$$

This equation is Lagrange's interpolation formula. This formula can be used whether the values of x are equally spaced or not.

Example 1: given the following data, evaluate f(3) using Lagrange's interpolation polynomial.

x	1	2	5
y	1	4	10

Solution: $x_0 = 1$ $x_1 = 2$ $x_2 = 5$

$y_0 = 1$ $y_1 = 4$ $y_2 = 10$

$$y(x) = \frac{(x-x1)(x-x2)}{(x0-x1)(x0-x2)} y_0 + \frac{(x-x0)(x-x2)}{(x1-x0)(x1-x2)} y_1 + \frac{(x-x0)(x-x1)}{(x2-x0)(x2-x1)} y_2$$

$$y(3) = \frac{(3-2)(3-5)}{(1-2)(1-5)}(1) + \frac{(3-1)(3-5)}{(2-1)(2-5)}(4) + \frac{(3-1)(3-2)}{(5-1)(5-2)}(10)$$

y(3) = 6.4999

Example2: find Lagrange's interpolation polynomial fitting the points y(1)=- 3, y(3)=0, y(4) =30, y(6) =132

Solution; the given data can be arranged as follows:

X	1	3	5	6
Y	-3	0	30	132

$X_0 = 1$ $x_1 = 3$ $x_2 = 5$ $x_3 = 6$

$Y_0 = -3$ $y_1 = 0$ $y_2 = 30$ $y_3 = 132$

$$\mathbf{y(x)} = \frac{(x-x1)(x-x2)(x-x3)}{(x0-x1)(x0-x2)(x0-x3)} y_0 + \frac{(x-x0)(x-x2)(x-x3)}{(x1-x0)(x1-x2)(x1-x3)} y_1 + \frac{(x-x0)(x-x1)(x-x3)}{(x2-x0)(x2-x1)(x2-x3)} y_2 + \frac{(x-x0)(x-x1)(x-x2)}{(x3-x0)(x3-x1)(x3-x2)} y_3$$

$$\mathbf{y\ (x) =} \frac{(x-3)(x-4)(x-6)}{(1-3)(1-4)(1-6)}(-3) + \frac{(x-1)(x-4)(x-6)}{(3-1)(3-4)(3-6)}\mathbf{(0)} +$$

$$\frac{(x-1)(x-3)(x-6)}{(5-1)(5-3)(5-6)}(30) + \frac{(x-1)(x-3)(x-5)}{(6-1)(6-3)(6-5)}(132)$$

$=[(x^3-13x^2+54x-72)/-30]\ (-3)+[(\ x^3-11x^2+34x-24)/6](0)+[\ x^3-10x^2+27x-18)/-6](30)+[\ x^3-8x^2+19x-12)/30](132)$

On simplification, we get

$Y(x)= 1/10(-5x^3+135x^2-460x+300) =1/2(-x^3+27x^2-92x+60)$

Which is the required lagranges interpolation polynomial. Now,y(5)=75.

H.W given the following data, evaluate f(3) using Lagrange's interpolation polynomial.

x	0	1	2	4
y	1	1	2	5