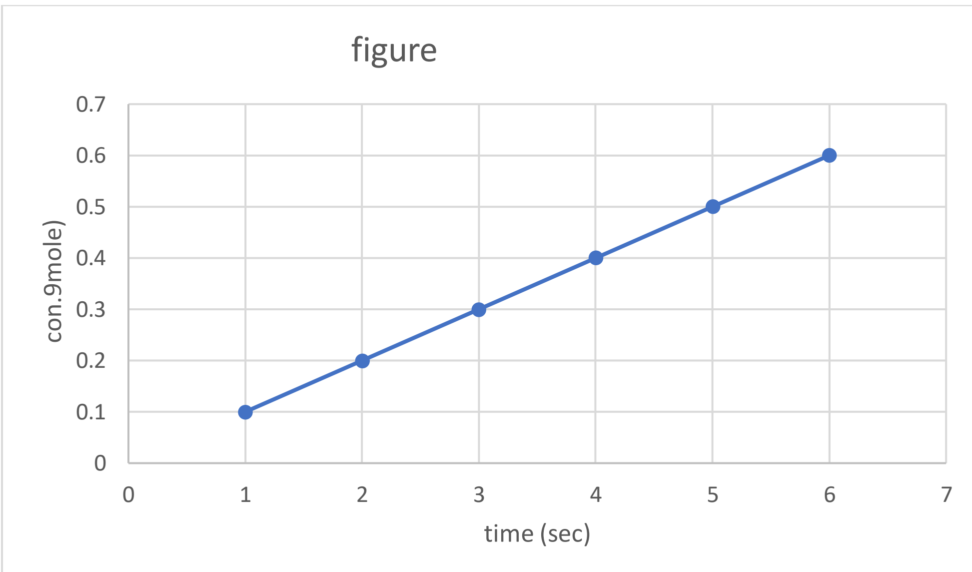
Curve fitting

In science and engineering often come across experimental two variables such as concentration and time, pressure and temperature. (in general x and y) can we find law or a relation connection them. we wish to find a relation connecting concentrations (mole) and time of relation (sec.)

The figure indicates, a linear the linear of form

$Y = mx + c$



In general, the problem of finding an equation of an approximating curve, which passes through as any points as possible is called curve fitting to find an equation of the curve of the pest fit .

1- Method of group average

Suppose we have N set of observation in an experiment, such as (x_1,y_1), (x_2,y_2),….,(x_n,y_n) assume that a straight line

$Y = mx + c$ …………….(1)

We required two equations in m and c to determine them. From the given data, when x=x1,we note that , the observed value of y is y1 . from assuming empirical law equation (1)the expected value of y is mx_1+c now We define the residual **e** as the difference between the observed value of y and expected value for the same x1 and write

Residual value = observed value – expected value

$e_1=y_1-(mx_1+c)$

$e_2 = y_2 - (mx_2 - c)$

these expressions clearly indicate that some of residuals may be positive and some of them may be negative the method of group averages states that the sum of all residual is zero ($\sum e=0$). we need two equation involving m and c to determine them, we divide the data into two group and assume ($\sum e=0$) is true for each group, each group contain equal number.

$$\sum_{i=1}^{j} ei = [y1 - (mx1 + c)] + [y2 - (mx2 + c)] + \cdots + [yj - (mxj + c)] = 0$$

And

$$\sum_{i=j+1}^{n} ei = [y(j+1) - (mx(j+1) + c)] + [y(j+2) - (mx(j+2) + c)] + \cdots + [yn - mxn + c] = 0$$

$(y_1+y_2+.....+y_j) - jc - m(x_1+x_2+....+x_j)=0$

$(y_{j+1}+y_{j+2}+.....+y_n) - (n-j)c - m(x_{j+1}+x_{j+2}+....+x_n)=0$

$(y_1+y_2+.....+y_j)/j = m(x_1+x_2+....+x_j)/j + c$

$(y_{j+1}+y_{j+2}+.....+y_n)/(n-j) = m(x_{j+1}+x_{j+2}+....+x_n)/(n-j) + c$

$$\frac{(y1 + y2 + \cdots .. + yj)}{j} = \frac{m(x1 + x2 + \cdots . + xj)}{j} + c$$

$$\frac{(y(j+1)+y(j+2)+\ldots+yn)}{(n-j)} = \frac{m(x(j+1)+x(j+2)+\cdots+xn)}{(n-j)} + c$$

Example: use the method of group average and find a curve of the form $y = mx^n$ that fits the following data

X	10	20	30	40	50	60	70	80
Y	1.06	1.33	1.52	1.68	1.81	1.91	2.01	2

Solution: required curve is of the form

$y = mx^n$

Taking logarithms

$\log y = \log_{10} m + n \log_{10} x$

$Y = C + nX$

$\log_{10} y = Y$, $\log_{10} m = C$, $\log_{10} x = X$

Group 1

x	**y**	**Y=** $\log_{10} y$	**X=** $\log_{10} x$
10	1.06	0.0253	1
20	1.33	0.1239	1.3010
30	1.52	0.1818	1.4771
40	1.68	0.2253	1.6021
		∑Y=0.5563	**∑X=5.3802**

Group 2

x	**y**	**Y=** $\log_{10} x$	**X=** $\log_{10} x$
50	1.81	0.2577	1.699
60	1.91	0.2810	1.7782
70	2.01	0.3032	1.8451
80	2.11	0.3243	1.9031
		∑Y=1.1662	**∑X=7.2254**

$$\frac{(Y1+Y2+Y3+Y4)}{4}=\frac{n(X1+X2+X3+X4)}{4}+c$$

$$\frac{(Y5+Y6+Y7+Y8)}{4}=\frac{n(X5+X6+X7+X8)}{4}+c$$

(0.5563)/4 = n(5.3802)/4 +c

(1.1662)/4 = n(7.2254)/4 +c

4c + 5.38n – 0.5563 =0

4c + 7.2254n – 1.1662 =0

n= 0.3305

c= -0.3055

log m=c

$$\frac{\ln m}{\ln 10}=c$$

Ln m =2.3025(-0.3055)

m= $e^{-0.7034}$

m= 0.5

$y = mx^n$

$y = 0.5x^{0.3305}$

Example: using the method of group averages fit a curve of the form $y=\frac{x}{a+bx}$

For the following data:

X	8	10	15	20	30	40
Y	13	14	15.3	16.3	17.2	17.8

Solution: let the required fit is

$$y = \frac{x}{a+bx}$$

$$\frac{1}{y} = \frac{a+bx}{x}$$

$$\frac{1}{y} = \frac{a}{x} + b$$

$$Y = \frac{1}{y}$$

$$X = \frac{1}{x}$$

Y=aX +b

Group 1

x	**y**	$Y = \frac{1}{y}$	$X = \frac{1}{x}$
8	13	0.0769	0.125
10	14	0.0714	0.1
15	15.4	0.0649	0.0667
		∑Y=0.2917	**∑X=0.2132**

Group 2

x	**y**	$Y = \frac{1}{y}$	$X = \frac{a}{x}$
20	16.3	0.0614	0.05
30	17.2	0.0581	0.030
40	17.8	0.0562	0.025
		∑Y=0.1757	**∑X=0.1088**

$$\frac{(Y1 + Y2 + Y3)}{3} = \frac{n(X1 + X2 + X3)}{3} + c$$

$$\frac{(Y4 + Y5 + Y6)}{3} = \frac{n(X4 + X5 + X6)}{3} + c$$

(0.2132)/3 = a(0.2917)/3 + b

(0.1757)/3=a(0.1083)/3 + b

3b + 0.2917a – 0.2132 =0

3b4c + 0.1083a – 0.1757 =0

a= 0.2045

b= 0.0512

$$y = \frac{x}{a + bx}$$

$$y = \frac{x}{0.2045 + 0.0512x}$$

2-the least squares method