Newtonian and non-Newtonian Fluids

- Newtonian fluids:
  Fluids which obey the Newton's law of viscosity are called as Newtonian fluids. Newton's law of viscosity is given by
- Non-Newtonian fluids:
  Fluids which do not obey the Newton's law of viscosity are called as non-Newtonian fluids. Generally non-Newtonian fluids are complex mixtures: slurries, pastes, gels, polymer solutions etc.

$$\tau_{xy} = \mu \frac{\partial v_x}{\partial y}$$

There is also one more - which is not real, it does not exist - known as the ideal fluid. This is a fluid which is assumed to have no viscosity. This is a useful concept when theoretical solutions are being considered - it does help achieve some practically useful solutions.

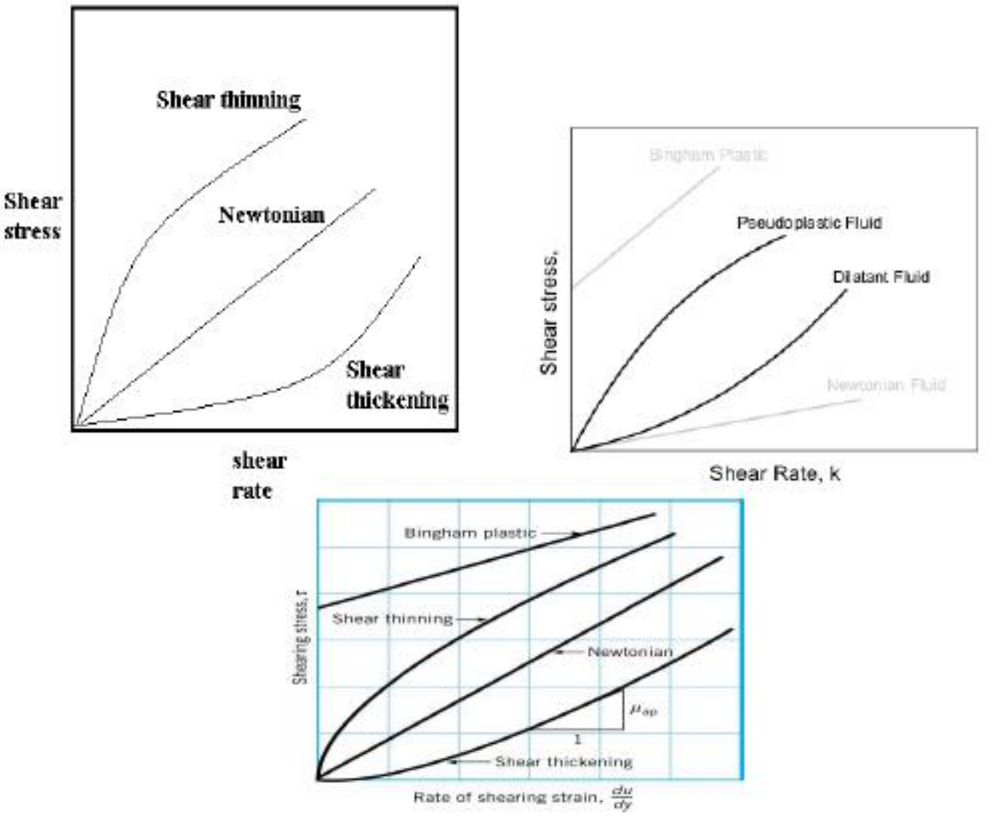

Fig. 19. Various Non-Newtonian Fluids

Power Consumption in Non-Newtonian Liquids

Newtonian fluid:

$$\tau_{xy} = \mu \frac{d\mathrm{v}_x}{\mathrm{dy}} = \mu \dot{\gamma} \qquad : \qquad \dot{\gamma} = \frac{d\mathrm{v}_x}{\mathrm{dy}}$$

Power Law Fluid:

$$\tau_{xy} = K \dot{\gamma}^n \qquad : \qquad \mu_a = \frac{\tau_{xy}}{\dot{\gamma}} = K \dot{\gamma}^{n-1}$$

When n<1, viscosity decreases with shear

When n>1, viscosity increases with shear

- Non-Newtonian liquids – viscosity varies with shear rate
- Use apparent viscosity, μ_a

$$N_{\mathrm{Re},n} = \frac{n D_a^2 \rho}{\mu_a}$$

$$\mu_a = K' \left(\frac{du}{dy} \right)_{av}^{n'-1}$$

- For a straight-blade turbine in pseudoplastic liquids

$$\left(\frac{du}{dy} \right)_{av} = 11n$$

Mixing of liquids

In general, agitators can be classified into the following two groups.

(i) Agitators with a small blade area which rotate at high speeds. These include turbines and marine type propellers.

(ii) Agitators with a large blade area which rotate at low speeds. include anchors, paddles and helical screws.

The second group is more effective than the first in the mixing of high viscosity liquids.

Small blade high speed agitators

Small blade high speed agitators are used to mix low to medium viscosity liquids. Two of the most common types are the six-blade flat blade turbine and the marine type propeller shown in Figures 1 and 2 respectively.

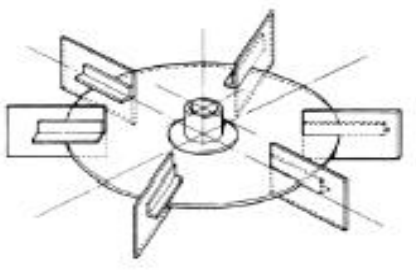

Fig. 1 Six-blade flat blade turbine

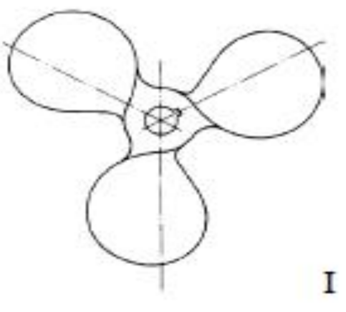

Fig. 2 Marine propeller

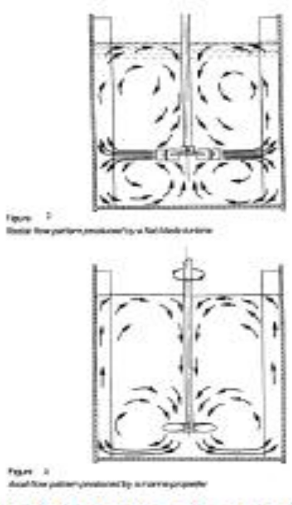

Fig. 3 Fluid flow for chemical engineers

Processing considerations sometimes necessitate deviations from the Agitator tip speeds *ilT* given by equation 3 are commonly used as a standard configuration.measure of the degree of agitation in a liquid mixing system.

$$U_T = \pi D_A N$$

Tip speed ranges for turbine agitators are recommended as follows:
2.5 to 3.3 m/s for low agitation
3.3 to 4.1 *m/s* for medium agitation
4.1 to 5.6 m/s for high agitation

Dimensionless groups for mixing :
In the design of liquid **mixing** systems the following dimensionless groups are of importance.

The power number

$$Po = \frac{P_A}{\rho N^3 D_A^5}$$

The Reynolds number for mixing Re_M represents the ratio of the applied to the opposing viscous drag forces.

$$Re_M = \frac{\rho N D_A^2}{\mu}$$

The Froude number for mixing *FYM* represents the ratio of the applied to the opposing gravitational forces.

$$Fr_M = \frac{N^2 D_A}{g}$$

The Weber number for **mixing** *WeM* represents the ratio of the applied to the opposing surface tension forces.

$$We_M = \frac{\rho N^2 D_A^3}{\sigma}$$

The power number Po can be related to the Reynolds number for mixing Re_M, and the Froude number for mixing Fr_M , by the equation

$$Po = C Re_M^x Fr_M^y$$

where *C* is an overall dimensionless shape factor which represents the geometry of the system.
Equation can also be written in the form

Example

Calculate the theoretical power for a six-blade flat blade turbine agitator with diameter ***DA*** = **3** m running at a speed of ***N*** = 0.2 reds in a tank system conforming to the standard tank configuration. The liquid in the tank has a dynamic viscosity μ = 1 Pa s and a density of p = 1000 kg/m^3.

Solution

$$Re_M = \frac{\rho N D_A^2}{\mu}$$

Substituting the given values

$$Re_M = \frac{(1000\ \mathrm{kg/m^3})(0.2\ \mathrm{rev/s})(9.0\ \mathrm{m^2})}{1.0\ \mathrm{Pa\ s}}$$

$$Re_M = 1800$$

From the graph of ϕ against Re_M in Figure 5.8

$$\phi = Po = 4.5$$

The theoretical power for mixing is

$$P_A = Po\rho N^3 D_A^5$$

$$= (4.5)(1000\ \mathrm{kg/m^3})(0.008\ \mathrm{rev^3/s^3})(243\ \mathrm{m^5})$$

$$= \underline{8748\ \mathrm{W}}$$

Example 2

Calculate the theoretical power for a six-blade flat blade turbine agitator with diameter *DA* = 0.1 m running at *N* = 16 reds in a tank system without baffles but otherwise conforming to the standard tank configuration illustrated in Figure The liquid in the tank has a dynamic viscosity of *ul* = 0.08Pas and a density of *p* = 900kg/m3. For this configuration *α* = 1.0 and *β* = 40.0.

Calculations:

The Reynolds number for mixing is given by

$$Re_M = \frac{\rho N D_A^2}{\mu}$$

$$= \frac{(900\ \mathrm{kg/m^3})(16\ \mathrm{rev/s})(0.01\ \mathrm{m^2})}{0.08\ \mathrm{Pa\ s}}$$

$$= 1800$$

$$P_A = \phi\rho N^3 D_A^5 \left(\frac{N^2 D_A}{g}\right)^y$$

Now

$$y = \frac{\alpha - \log Re_M}{\beta}$$

with

$$\alpha = 1.0 \text{ and } \beta = 40.0$$

and

$$\log 1800 = 3.2553$$

Therefore

$$y = \frac{-2.2553}{40} = -0.05638$$

Substituting known values

$$N^2 D_A \quad (256\ \text{rev}^2/\text{s}^2)(0.1\ \text{m})$$

Substituting known values

$$\frac{N^2 D_A}{g} = \frac{(256\ \text{rev}^2/\text{s}^2)(0.1\ \text{m})}{9.81\ \text{m/s}^2}$$

$$= 2.610$$

So

$$\left(\frac{N^2 D_A}{g}\right)^y = 2.610^{(-0.05638)} = 0.9479$$

Therefore

$$P_A = (2.2)(900\ \text{kg/m}^3)(4096\ \text{rev}^3/\text{s}^3)(0.00001\ \text{m}^5)\ (0.9479)$$

$$= \underline{76.88\ \text{W}}$$

Agitator Scale-Up

- Scale-up the laboratory-size or pilot-size agitation system to a full-scale unit.
- Scale-up procedure:

1. Calculate the scale-up ratio R. Assuming that the original vessel is a standard cylinder with $D_{T1} = H_1$, the volume is:

$$V_1 = \left(\frac{\pi D_{T1}^2}{4}\right)(H_1) = \left(\frac{\pi D_{T1}^3}{4}\right)$$

The ratio of the volume is:

$$\frac{V_2}{V_1} = \left(\frac{\pi D_{T2}^2 / 4}{\pi D_{T1}^2 / 4}\right)(H_1) = \left(\frac{D_{T2}^3}{D_{T1}^3}\right)$$

The scale-up ratio is then:

$$R = \left(\frac{V_2}{V_1}\right)^{1/3} = \left(\frac{D_{T2}}{D_{T1}}\right)$$

2. Using this value of R, apply it to all of the dimensions to calculate the new dimensions. For Example,

$$D_{a2} = RD_{a1}, \quad J_2 = RJ_1\ldots$$

3. Determine the agitator speed N_2, to be used to duplicate the small scale results using N_1. The equation is:

$$N_2 = N_1\left(\frac{1}{R}\right)^n = N_1\left(\frac{D_{T1}}{D_{T2}}\right)^n$$

Where $n = 1$ for equal liquid motion, $n = 3/4$ for equal suspension solids and $n = 2/3$ for equal rates of mass transfer (which equivalent to equal power per unit volume, $P_1V_1 = P_2V_2$). This value of n is based on empirical and theoretical considerations.

4. Knowing N_2, the power required can be determined using power number equation and figures.

Example

An existing agitation system is the same as given in Example 1a for a flat-blade turbine with a disk and six blades. The given conditions and sizes are D_{T1} = 1.83 m, D_{a1} = 0.61 m, W_1 = 0.122 m, J_1 = *0.15 m*, N_1 = 90/60 = 1.50 rev/s, ρ = 929 kg/m^3 and μ = 0.01 Pa.s. It is desired to scale up these results for a vessel whose volume is 3.0 times as large. Do this for the following two process objectives:

a) Where equal rate of mass transfer is desired.
b) Where equal liquid motion is needed.

Solution

Since $H_1 = D_{T1} = 1.83$ m,

the original tank volume,

Volume $V_2 = 3.0\ (4.813) = 14.44\ m^3$.

Following the steps in the scale-up procedure, and using Eq.

$$R = \left(\frac{V_2}{V_1}\right)^{1/3} = \left(\frac{14.44}{4.813}\right)^{1/3}$$

The dimensions of the larger agitation system are as follows:

$D_{T2} = RD_{T1} = 1.442\ (1.83) = 2.64$ m, $D_{a2} = 1.442\ (0.61) = 0.880$ m, $W_2 = 1.442\ (0.122) = 0.176$ m and $J_2 = 1.442\ (0.15) = 0.216$ m.

For part (a), for equal mass transfer, $n = 2/3$ in Eq. (3.4-10):

$$N_2 = N_1\left(\frac{1}{R}\right)^{2/3} = (1.50)\left(\frac{1}{1.442}\right)^{2/3} = 1.175\text{rev/s}(70.5\text{rpm})$$

$$N'_{\mathrm{Re}} = \frac{D_a^2 N \rho}{\mu} = \frac{(0.880)^2(1.175)(929)}{0.01} = 8.453\times10^4$$

- Refer to Figure 3.4-5, Curve 1 and N_{Re} = 8.453 x 10^4, gives N_p = 5.0

$$P_1 = N_p \rho N_1^3 D_{a1}^5 = (5)(929)(1.5)^3(0.61)^5$$

$$P_1 = 1324\,\mathrm{J/s} = 1.324\,\mathrm{kW}$$

$$P_2 = N_p \rho N_2^3 D_{a2}^5 = (5)(929)(1.175)^3(0.880)^5$$

$$P_2 = 3977\,\mathrm{J/s} = 3.977\,\mathrm{kW}$$

- The power per unit volume is

$$\frac{P_1}{V_1} = \frac{1.324}{4.813} = 0.2752\,\mathrm{kW/m^3}$$

$$\frac{P_2}{V_2} = \frac{3.977}{14.44} = 0.2752\,\mathrm{kW/m^3}$$

- The value of 0.2752 kW/m^3 is somewhat lower than the approximate guidelines of 0.8 to 2.0 for mass transfer.
- For part (b), for equal liquid motion, n = 1.0

$$N_2 = N_1\left(\frac{1}{R}\right)^{1.0} = (1.50)\left(\frac{1}{1.442}\right)^{1.0} = 1.040\,\mathrm{rev/s}$$

$$P_2 = N_p \rho N_2^3 D_{a2}^5 = (5)(929)(1.040)^3(0.880)^5$$

$$P_2 = 2757 = 2.757\,\mathrm{kW}$$

$$\frac{P_2}{V_2} = \frac{2.757}{14.44} = 0.1909\,\mathrm{kW/m^3}$$

Heat Transfer in Agitated Vessel

- Often it is necessary to cool or heat the contents of the vessel during agitation.
- This is usually done by heat-transfer surfaces, which may be in the form of:
  1) cooling or heating jackets in the wall of the vessel
  2) coils of pipe immersed in the liquid.

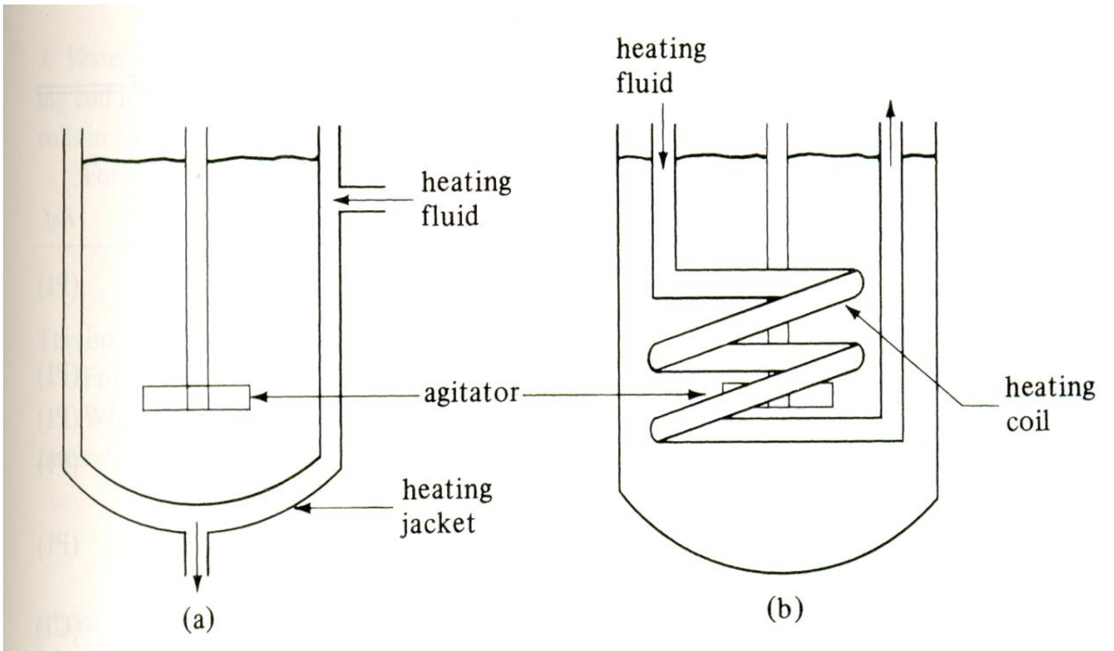

FIGURE 4.13-1. *Heat transfer in agitated vessels: (a) vessel with heating jacket, (b) vessel with heating coils.*

<u>Vessel with heating jacket</u>

- When heating, the fluid entering is often steam, which condenses inside the jacket and leaves at the bottom.
- The vessel is equipped with an agitator and in most cases also with baffles (not shown).

- Correlations for the heat-transfer coefficient from the agitated Newtonian liquid inside the vessel to the jacket walls of the vessel have the following form:

$$\frac{hD_t}{k} = a\left(\frac{D_a^2 N\rho}{\mu}\right)^b\left(\frac{c_p\mu}{k}\right)^{1/3}\left(\frac{\mu}{\mu_w}\right)^m$$

1. Paddle agitator with no baffles (C5, U1)

$$a = 0.36, \quad b = \tfrac{2}{3}, \quad m = 0.21, \quad N'_{Re} = 300 \text{ to } 3 \times 10^5$$

2. Flat-blade turbine agitator with no baffles (B4)

$$a = 0.54, \quad b = \tfrac{2}{3}, \quad m = 0.14, \quad N'_{Re} = 30 \text{ to } 3 \times 10^5$$

3. Flat-blade turbine agitator with baffles (B4, B5)

$$a = 0.74, \quad b = \tfrac{2}{3}, \quad m = 0.14, \quad N'_{Re} = 500 \text{ to } 3 \times 10^5$$

4. Anchor agitator with no baffles (U1)

$$a = 1.0, \quad b = \tfrac{1}{2}, \quad m = 0.18, \quad N'_{Re} = 10 \text{ to } 300$$

$$a = 0.36, \quad b = \tfrac{2}{3}, \quad m = 0.18, \quad N'_{Re} = 300 \text{ to } 4 \times 10^4$$

5. Helical-ribbon agitator with no baffles (G4)

$$a = 0.633, \quad b = \tfrac{1}{2}, \quad m = 0.18, \quad N'_{Re} = 8 \text{ to } 10^5$$

TABLE 4.13-1. *Typical Overall Heat-Transfer Coefficients in Jacketed Vessels*

Fluid in Jacket	*Fluid in Vessel*	*Wall Material*	*Agitation*	*U* btu/($h \cdot ft^2 \cdot °F$)	*U* W/($m^2 \cdot K$)	*Ref.*
Steam	Water	Copper	None	150	852	(P1)
			Simple stirring	250	1420	
Steam	Paste	Cast iron	Double scrapers	125	710	(P1)
Steam	Boiling water	Copper	None	250	1420	(P1)
Steam	Milk	Enameled cast iron	None	200	1135	(P1)
			Stirring	300	1700	
Hot water	Cold water	Enameled cast iron	None	70	398	(P1)
Steam	Tomato purée	Metal	Agitation	30	170	(C1)

Vessel with heating coil

- Correlations for the heat-transfer coefficient to the outside surface of the coils in agitated vessel have the following form:

  a) for a paddle agitator

$$\frac{hD_t}{k} = 0.87\left(\frac{D_a^2 N\rho}{\mu}\right)^{0.65}\left(\frac{c_p\mu}{k}\right)^{1/3}\left(\frac{\mu}{\mu_w}\right)^{0.14}$$

  b) for vertical baffle tube with a flat-blade turbine:

$$\frac{hD_o}{k} = 0.09\left(\frac{D_a^2 N\rho}{\mu}\right)^{0.65}\left(\frac{c_p\mu}{k}\right)^{1/3}\left(\frac{D_a}{D_t}\right)^{1/3}\left(\frac{2}{n_b}\right)^{0.2}\left(\frac{\mu}{\mu_f}\right)^{0.4}$$

D_o is outside diameter of the coil tube (in m), n_b is number of vertical baffle tubes and μ_f is the viscosity of the mean film temperature