Flow of Fluids through Granular Beds and Packed Columns

The flow of fluids through beds composed of stationary granular particles is a frequent occurrence in the chemical industry and therefore expressions are needed to predict pressure drop across beds due to the resistance caused by the presence of the particles.

Darcy's law and permeability

The first experimental work on the subject was carried out by DARCYin 1830 in Dijon when he examined the rate of flow of water from the local fountains through beds of sand of various thicknesses. It was shown that the average velocity, as measured over the whole area of the bed, was directly proportional to the driving pressure and inversely proportional to the thickness of the bed. This relation, often termed Darcy's law, has subsequently been confirmed by a number of workers and can be written as follows:

$$u_c = K\frac{(-\Delta P)}{l} \tag{4.1}$$

where $-\Delta P$ is the pressure drop across the bed,
l is the thickness of the bed,
u_c is the average velocity of flow of the fluid, defined as $(1/A)(\mathrm{d}V/\mathrm{d}t)$,
A is the total cross sectional area of the bed,
V is the volume of fluid flowing in time t, and
K is a constant depending on the physical properties of the bed and fluid.

The linear relation between the rate of flow and the pressure difference leads one to suppose that the flow was streamline, because the Reynolds number for the flow through the pore spaces in a granular material is low, since both the velocity of the fluid and the width of the channels are normally small. The resistance to flow then arises mainly from viscous drag. Equation 4.1 can then be expressed as:

$$u_c = \frac{K(-\Delta P)}{l} = B\frac{(-\Delta P)}{\mu l} \tag{4.2}$$

where μ is the viscosity of the fluid and B is termed the permeability coefficient for the bed, and depends only on the properties of the bed.

Specific surface and voidage

The general structure of a bed of particles can often be characterised by the specific surface area of the bed $\boldsymbol{S_B}$ and the fractional voidage of the bed $\boldsymbol{e}$.

$\boldsymbol{S_B}$ is the surface area presented to the fluid per unit volume of bed when the particles are packed in a bed. Its units are (length)$^{-1}$.

e is the fraction of the volume of the bed not occupied by solid material and is termed the fractional voidage, or porosity. It is dimensionless.

Thus the fractional volume of the bed occupied by solid material is $(1 - e)$.

S is the specific surface area of the particles and is the surface area of a particle divided by its volume. Its units are again (length)$^{-1}$. For a sphere, for example:

$$S = \frac{\pi d^2}{\pi(d^3/6)} = \frac{6}{d} \tag{4.3}$$

It can be seen that S and S_B are not equal due to the voidage which is present when the particles are packed into a bed. If point contact occurs between particles so that only a very small fraction of surface area is lost by overlapping, then:

$$S_B = S(1 - e) \tag{4.4}$$

Streamline flow—Carman–Kozeny equation

$$u = \frac{d_t^2}{32\mu}\frac{(-\Delta P)}{l_t} \tag{4.5}$$

where: μ is the viscosity of the fluid,
u is the mean velocity of the fluid,
d_t is the diameter of the tube, and
l_t is the length of the tube.

If the free space in the bed is assumed to consist of a series of tortuous channels, equation 4.5 may be rewritten for flow through a bed as:

$$u_1 = \frac{d_m'^2}{K'\mu}\frac{(-\Delta P)}{l'} \tag{4.6}$$

where: d_m' is some equivalent diameter of the pore channels,
K' is a dimensionless constant whose value depends on the structure of the bed,
l' is the length of channel, and
u_1 is the average velocity through the pore channels.

In a cube of side X, the volume of free space is eX^3 so that the mean cross-sectional area for flow is the free volume divided by the height, or eX^2. The volume flowrate through this cube is $u_c X^2$, so that the average linear velocity through the pores, u_1, is given by:

$$u_1 = \frac{u_c X^2}{eX^2} = \frac{u_c}{e} \tag{4.7}$$

Although equation 4.7 is reasonably true for random packings, it does not apply to all regular packings. Thus with a bed of spheres arranged in cubic packing, $e = 0.476$, but the fractional free area varies continuously, from 0.215 in a plane across the diameters to 1.0 between successive layers.

For equation 4.6 to be generally useful, an expression is needed for d'_m, the equivalent diameter of the pore space. Kozeny[5,6] proposed that d'_m may be taken as:

$$d'_m = \frac{e}{S_B} = \frac{e}{S(1-e)} \tag{4.8}$$

where:

$$\frac{e}{S_B} = \frac{\text{volume of voids filled with fluid}}{\text{wetted surface area of the bed}}$$

$$= \frac{\text{cross-sectional area normal to flow}}{\text{wetted perimeter}}$$

The hydraulic mean diameter for such a flow passage has been shown in Volume 1, Chapter 3 to be:

$$4\left(\frac{\text{cross-sectional area}}{\text{wetted perimeter}}\right)$$

It is then seen that:

$$\frac{e}{S_B} = \frac{1}{4}\,(\text{hydraulic mean diameter})$$

Then taking $u_1 = u_c/e$ and $l' \propto l$, equation 4.6 becomes:

$$u_c = \frac{1}{K''}\frac{e^3}{S_B^2}\frac{1}{\mu}\frac{(-\Delta P)}{l}$$

$$= \frac{1}{K''}\frac{e^3}{S^2(1-e)^2}\frac{1}{\mu}\frac{(-\Delta P)}{l} \tag{4.9}$$

K'' is generally known as Kozeny's constant and a commonly accepted value for K'' is 5. As will be shown later, however, K'' is dependent on porosity, particle shape, and other factors. Comparison with equation 4.2 shows that B the permeability coefficient is given by:

$$B = \frac{1}{K''}\frac{e^3}{S^2(1-e)^2} \tag{4.10}$$

Inserting a value of 5 for K'' in equation 4.9:

$$u_c = \frac{1}{5}\frac{e^3}{(1-e)^2}\frac{-\Delta P}{S^2 \mu l} \tag{4.11}$$

For spheres: $S = 6/d$ and: (equation 4.3)

$$u_c = \frac{1}{180}\frac{e^3}{(1-e)^2}\frac{-\Delta P d^2}{\mu l} \tag{4.12}$$

$$= 0.0055\frac{e^3}{(1-e)^2}\frac{-\Delta P d^2}{\mu l} \tag{4.12a}$$

For non-spherical particles, the Sauter mean diameter d_s should be used in place of d. This is given in Chapter 1, equation 1.15.

Streamline and turbulent flow

The modified Reynolds number Re_1 is obtained by taking the same velocity and characteristic linear dimension d'_m as were used in deriving equation 4.9. Thus:

$$Re_1 = \frac{u_c}{e}\frac{e}{S(1-e)}\frac{\rho}{\mu}$$

$$= \frac{u_c \rho}{S(1-e)\mu} \tag{4.13}$$

The friction factor, which is plotted against the modified Reynolds number, is $R_1/\rho u_1^2$, where R_1 is the component of the drag force per unit area of particle surface in the direction of motion. R_1 can be related to the properties of the bed and pressure gradient as follows. Considering the forces acting on the fluid in a bed of unit cross-sectional area and thickness l, the volume of particles in the bed is $l(1-e)$ and therefore the total surface is $Sl(1-e)$. Thus the resistance force is $R_1 Sl(1-e)$. This force on the fluid must be equal to that produced by a pressure difference of ΔP across the bed. Then, since the free cross-section of fluid is equal to e:

$$(-\Delta P)e = R_1 Sl(1-e)$$

and

$$R_1 = \frac{e}{S(1-e)}\frac{(-\Delta P)}{l} \tag{4.14}$$

Thus

$$\frac{R_1}{\rho u_1^2} = \frac{e^3}{S(1-e)}\frac{(-\Delta P)}{l}\frac{1}{\rho u_c^2} \tag{4.15}$$

Carman found that when $R_1/\rho u_1^2$ was plotted against Re_1 using logarithmic coordinates, his data for the flow through randomly packed beds of solid particles could be correlated approximately by a single curve (curve A, Figure 4.1), whose general equation is:

$$\frac{R_1}{\rho u_1^2} = 5Re_1^{-1} + 0.4Re_1^{-0.1} \tag{4.16}$$

From equation 4.16 it can be seen that for values of Re_1 less than about 2, the second term is small and, approximately:

$$\frac{R_1}{\rho u_1^2} = 5Re_1^{-1} \tag{4.18}$$

Sawistowski[9] compared the results obtained for flow of fluids through beds of hollow packings (discussed later) and has noted that equation 4.16 gives a consistently low result for these materials. He proposed:

$$\frac{R_1}{\rho u_1^2} = 5Re_1^{-1} + Re_1^{-0.1} \tag{4.19}$$

For flow through ring packings which as described later are often used in industrial packed columns, Ergun[10] obtained a good semi-empirical correlation for pressure drop as follows:

$$\frac{-\Delta P}{l} = 150\frac{(1-e)^2}{e^3}\frac{\mu u_c}{d^2} + 1.75\frac{(1-e)}{e^3}\frac{\rho u_c^2}{d} \tag{4.20}$$

Writing $d = 6/S$ (from equation 4.3):

$$\frac{-\Delta P}{Sl\rho u_c^2}\frac{e^3}{1-e} = 4.17\frac{\mu S(1-e)}{\rho u_c} + 0.29$$

or:

$$\frac{R_1}{\rho u_1^2} = 4.17Re_1^{-1} + 0.29 \tag{4.21}$$

Dependence of K__ on bed structure

Carman[1] has shown that:

$$K'' = \left(\frac{l'}{l}\right)^2 \times K_0 \tag{4.22}$$

where (l'/l) is the tortuosity and is a measure of the fluid path length through the bed compared with the actual depth of the bed,

K_0 is a factor which depends on the shape of the cross-section of a channel through which fluid is passing.

K_0 is equal to 2.0, and for streamline flow through a rectangle where the ratio of the lengths of the sides is 10 : 1, K_0 = 2.65.

Wall effect. In a packed bed, the particles will not pack as closely in the region near the wall as in the centre of the bed, so that the actual resistance to flow in a bed of small diameter is less than it would be in an infinite container for the same flowrate per unit area of bed cross-section. A correction factor f_w for this effect has been determined experimentally by Coulson[15]. This takes the form:

$$f_w = \left(1 + \frac{1}{2}\frac{S_c}{S}\right)^2 \tag{4.23}$$

where S_c is the surface of the container per unit volume of bed.

Equation 4.9 then becomes:

$$u_c = \frac{1}{K''}\frac{e^3}{S^2(1-e)^2}\frac{1}{\mu}\frac{(-\Delta P)}{l}f_w \tag{4.24}$$

Equations 4.9 and 4.16, which involve e/S_B as a measure of the effective pore diameter, are developed from a relatively sound theoretical basis and are recommended for beds of small particles when they are nearly spherical in shape. The correction factor for wall effects, given by equation 4.23, should be included where appropriate. With larger particles which will frequently be far from spherical in shape, the correlations are not so reliable. As shown in Figure 4.1, deviations can occur for rings at higher values of Re_1. Efforts to correct for non-sphericity, though frequently useful, are not universally effective, and in such cases it will often be more rewarding to use correlations, such as equation 4.19, which are based on experimental data for large packings. The values of K'' shown on Figure 4.2 apply to equation 4.24.

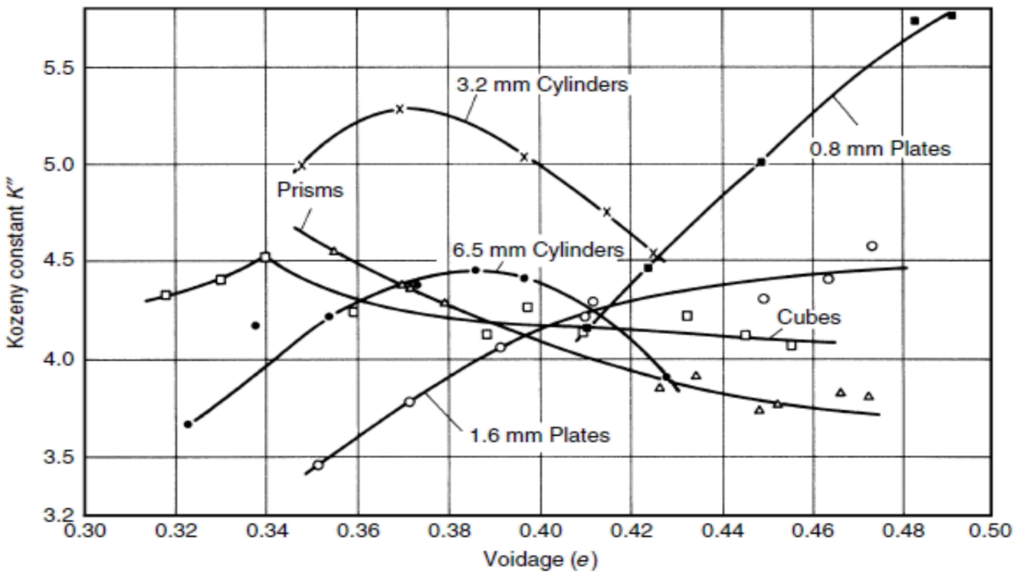

Figure 4.2. Variation of Kozeny's constant K'' with voidage for various shapes

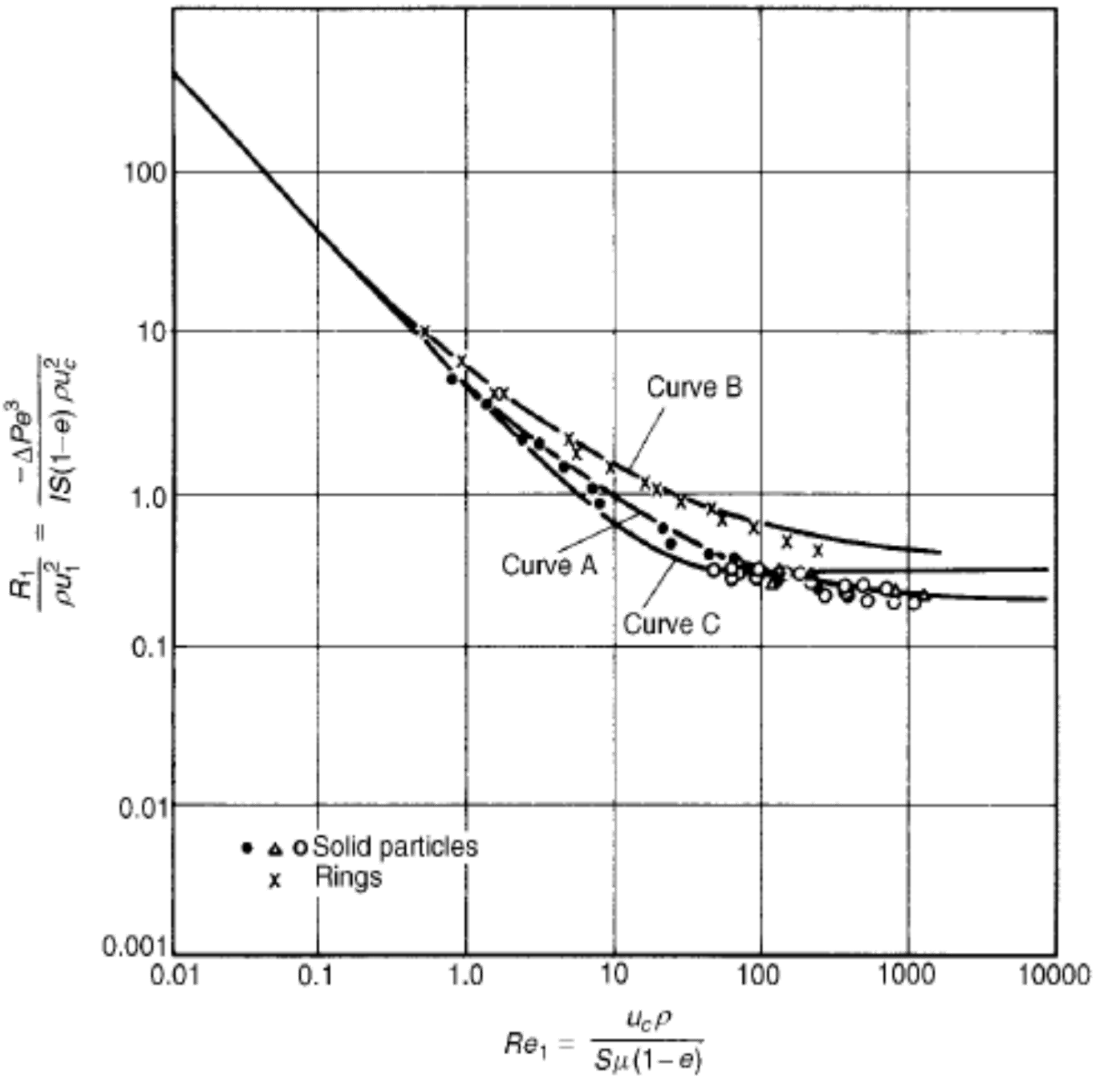

Figure 4.1. Carman's graph of $R_1/\rho u_1^2$ against Re_1

PACKED COLUMNS

packed towers are used for bringing two phases in contact with one another and there will be strong interaction between the fluids. Normally one of the fluids will preferentially wet the packing and will flow as a film over its surface; the second fluid then passes through the remaining volume of the column. With gas (or vapour)–liquid systems, the liquid will normally be the wetting fluid and the gas or vapour will rise through the column making close contact with the down-flowing liquid and having little direct contact with the packing elements. An example of the liquid–gas system is an absorption process where a soluble gas is scrubbed from a mixture of gases by means of a liquid, as shown in Figure below In a packed column used for distillation, the more volatile component of, say, a binary mixture is progressively transferred to the vapour phase and the less volatile condenses out in the liquid. Packed columns have also been used extensively for liquid–liquid extraction processes where a solute is transferred from one solvent to another.

In order to obtain a good rate of transfer per unit volume of the tower, a packing is selected which will promote a high interfacial area between the two phases and a high degree of turbulence in the fluids. Usually increased area and turbulence are achieved at the expense of increased capital cost and/or pressure drop, and a balance must be made between these factors when arriving at an economic design.

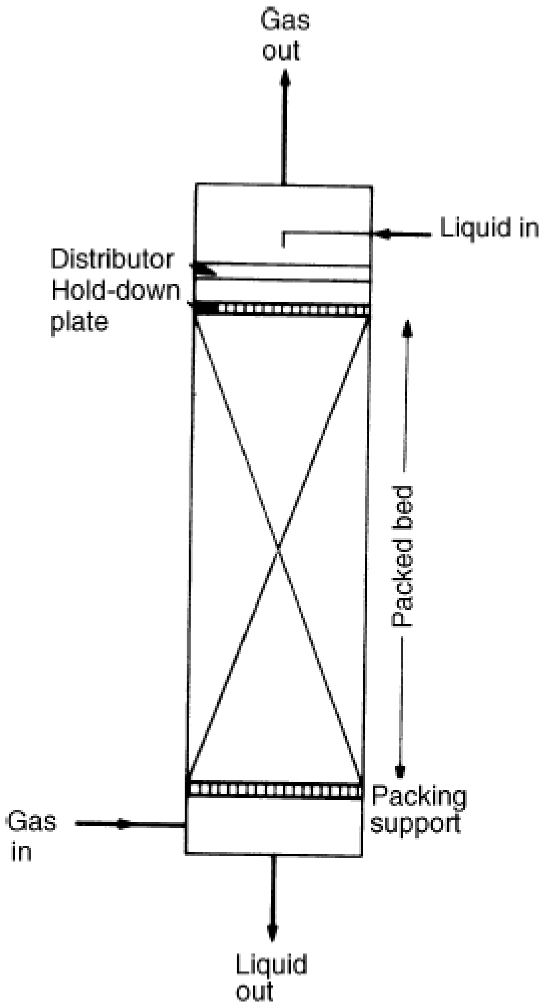

Figure 4.9. Packed absorption column

The packing

The packing should be of as uniform size as possible so as to produce a bed of uniform characteristics with a desired voidage. The most commonly used packings are Raschig rings, Pall rings, Lessing rings, and Berl saddles. Newer packings include Nutter rings, Intalox and Intalox metal saddles, Hy-Pak, and Mini rings and, because of their high performance characteristics and low pressure drop, these packings now account for a large share of the market. Commonly used packing elements are illustrated in Figure 4.13. Most of these packings are available in a widenrange of materials such as ceramics, metals, glass, plastics, carbon, and sometimes rubber. Ceramic packings are resistant to corrosion and comparatively cheap, but are heavy and may require a stronger packing support and foundations. The smaller metal rings are also available made from wire mesh, and these give much-improved transfer characteristics in small columns.

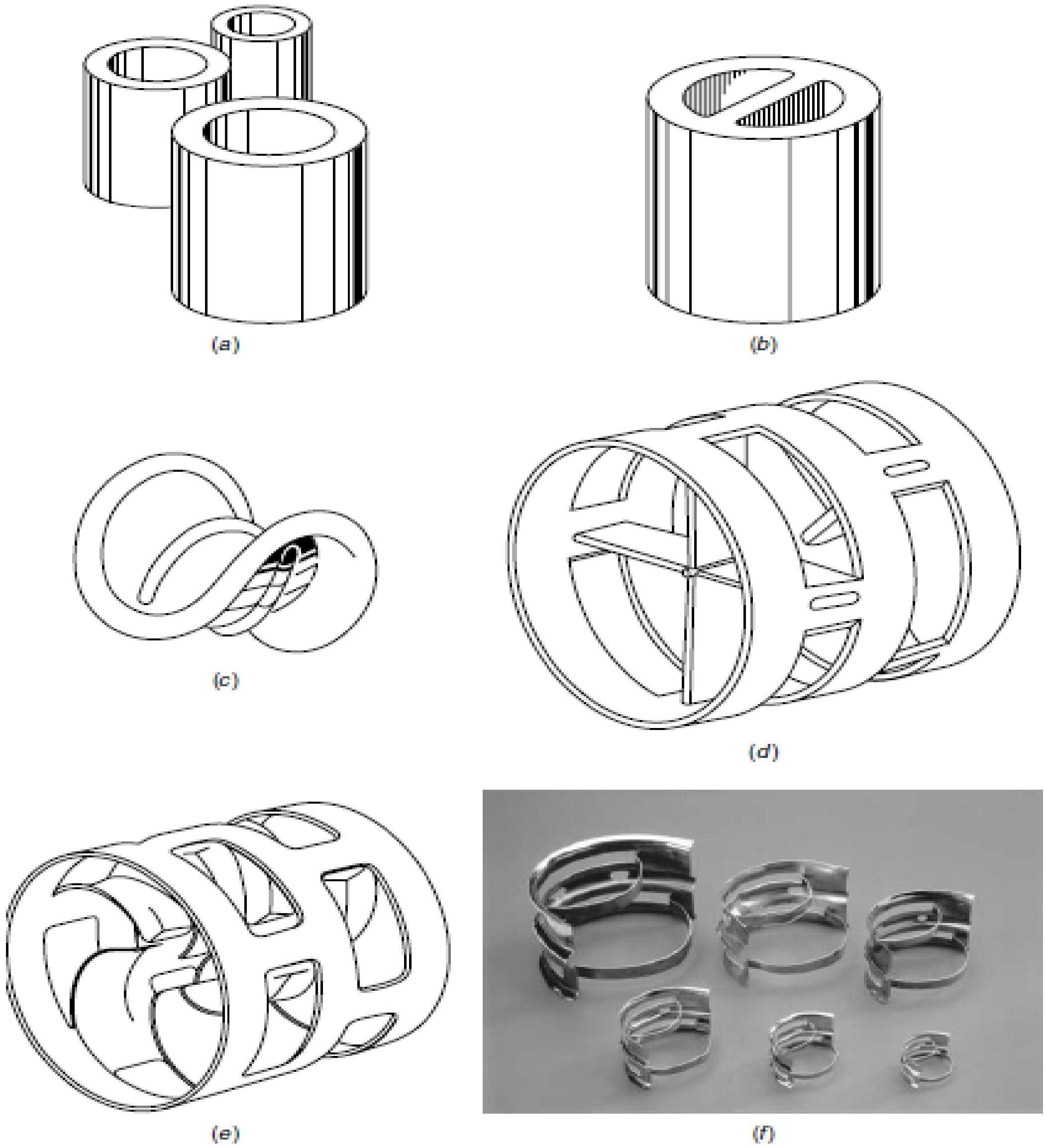

Figure 4.13. (a) Ceramic Raschig rings; (b) Ceramic Lessing ring; (c) Ceramic Berl saddle; (d) Pall ring (plastic); (e) Pall ring (metal); (f) Metal Nutter rings; (g) Plastic Nutter ring

Figure 4.14. Visco Coolflo 3 extended surface, cooling tower packing

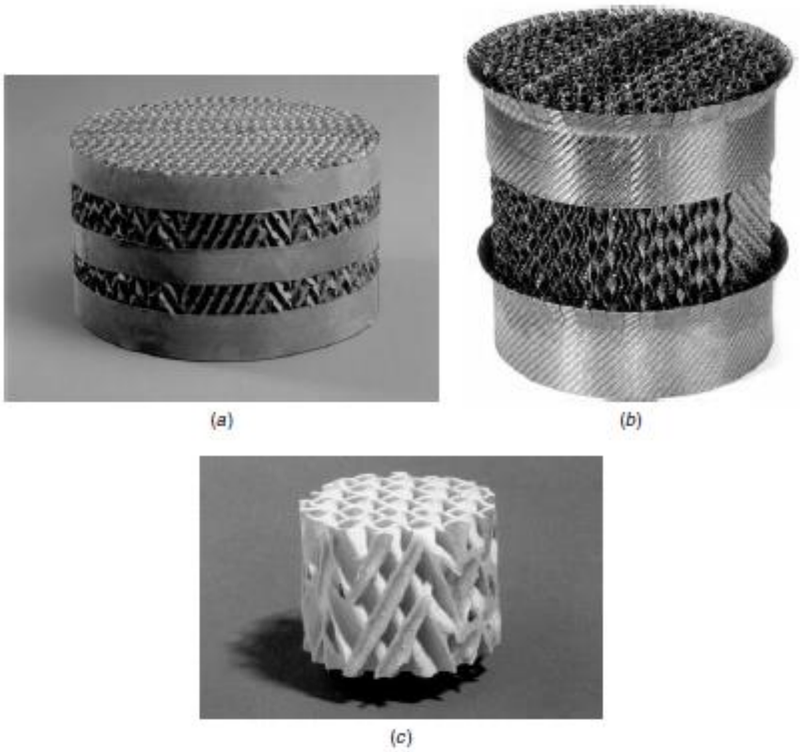

Figure 4.15. Structured packings (*a*) metal gauze (*b*) carbon (*c*) corrosion-resistant plastic

Fluid flow in packed columns

In the majority of cases the gas flow is turbulent and the general form of the relation between the drop in pressure ΔP and the volumetric gas flowrate per unit area of column u_G is shown on curve A of Figure 4.16.

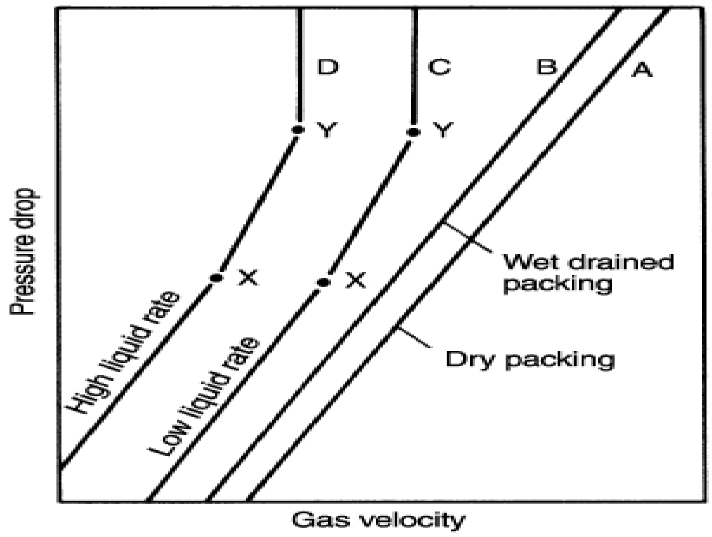

Figure 4.16. Pressure drops in wet packings (logarithmic axes)

Loading and flooding points

Although the loading and flooding points have been shown on Figure 4.16, there is no completely generalised expression for calculating the onset of loading, although one of the following semi-empirical correlations will often be adequate. Morris and Jackson[57] gave their results in the form of plots of $\psi(u_G/u_L)$ at the loading rate for various wetting rates L_W (m^3/s m). u_G and u_L are average gas and liquid velocities based on the empty column and $\psi = (\sqrt{(\rho_G/\rho_A)}$ is a gas density correction factor, where ρ_A is the density of air at 293 K.

A useful graphical correlation for flooding rates was first presented by Sherwood *et al.*[60] and later developed by Lobo *et al.*[61] for random-dumped packings, as shown in Figure 4.17 in which:

$$\left(\frac{u_G^2 S_B}{ge^3}\right)\left(\frac{\rho_G}{\rho_L}\right)\left(\frac{\mu_L}{\mu_w}\right)^{0.2} \text{ is plotted against } \frac{L'}{G'}\sqrt{\left(\frac{\rho_G}{\rho_L}\right)}$$

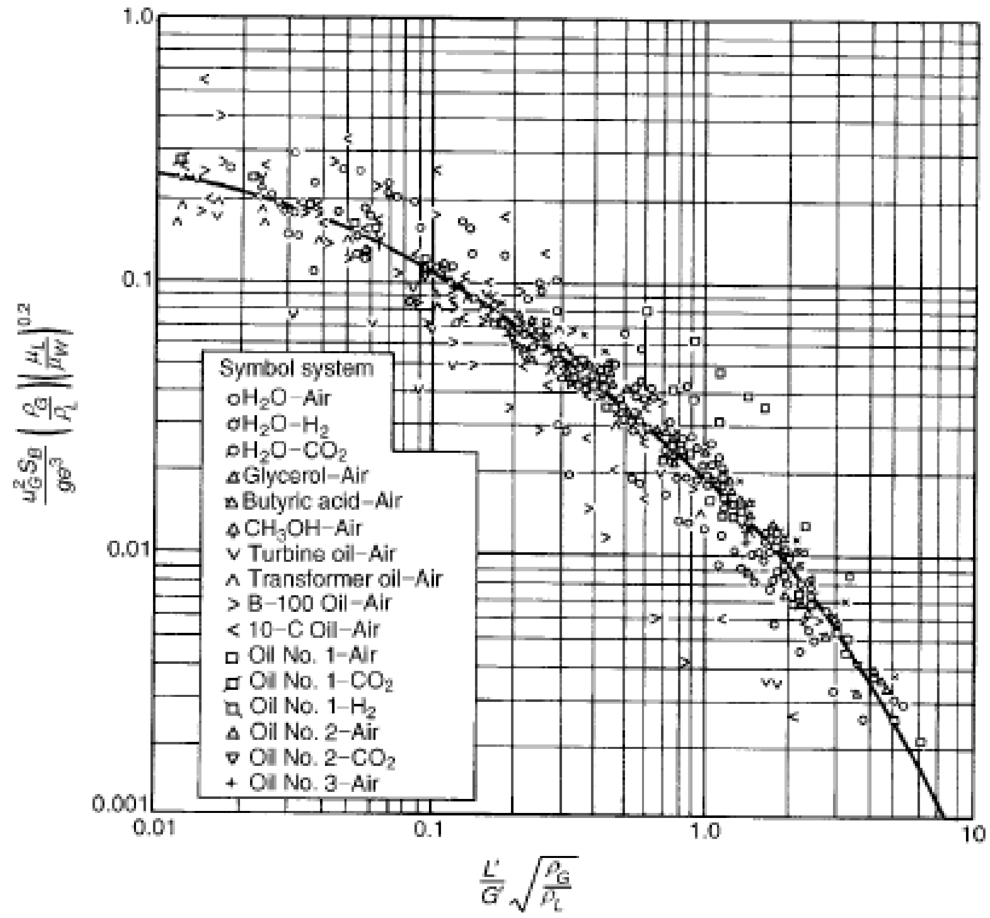

Figure 4.17. Generalised correlation for flooding rates in packed towers[61]