4 Structures

CHAPTER OUTLINE

4/1 Introduction

In Chapter 3 we studied the equilibrium of a single rigid body or a system of connected members treated as a single rigid body. We first drew a free-body diagram of the body showing all forces external to the isolated body and then we applied the force and moment equations of equilibrium. In Chapter 4 we focus on the determination of the forces internal to a structure, that is, forces of action and reaction between the connected members. An engineering structure is any connected system of members built to support or transfer forces and to safely withstand the loads applied to it. To determine the forces internal to an engineering structure, we must dismember the structure and analyze separate free-body diagrams of individual members or combinations of members. This analysis requires careful application of Newton's third law, which states that each action is accompanied by an equal and opposite reaction.

In Chapter 4 we analyze the internal forces acting in several types of structures, namely, trusses, frames, and machines. In this treatment we consider only *statically determinate* structures, which do not have more supporting constraints than are necessary to maintain an equilibrium configuration. Thus, as we have already seen, the equations of equilibrium are adequate to determine all unknown reactions.

The analysis of trusses, frames and machines, and beams under concentrated loads constitutes a straightforward application of the material developed in the previous two chapters. The basic procedure developed in Chapter 3 for isolating a body by constructing a correct free-body diagram is essential for the analysis of statically determinate structures.

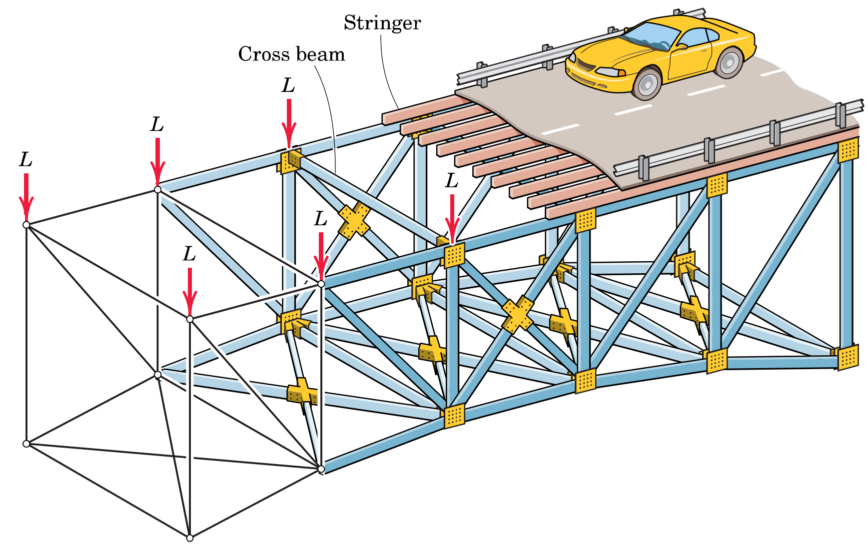


Figure 4/1

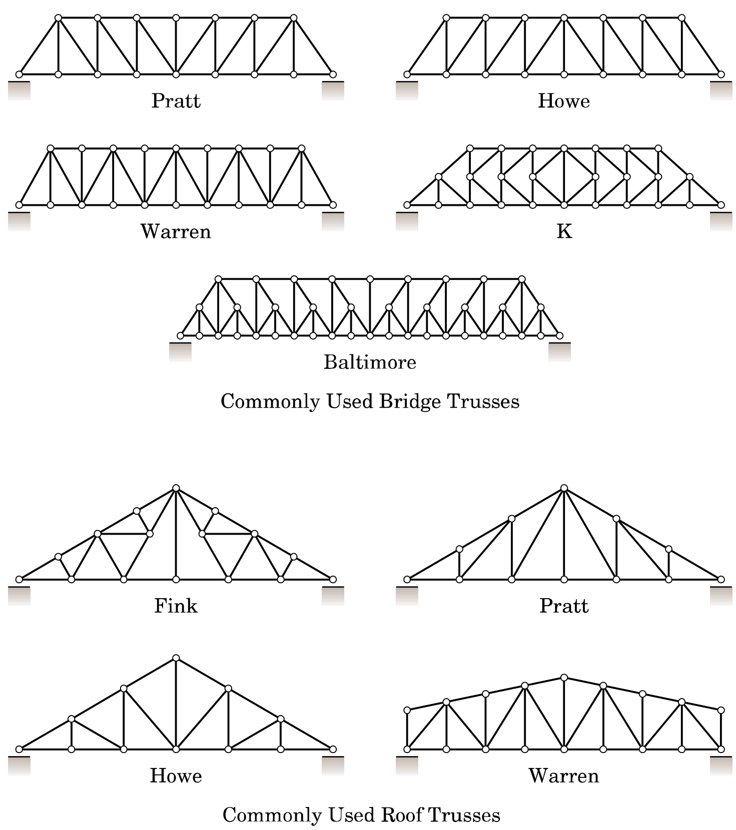


Figure 4/2

4/2 PLANE TRUSSES

A framework composed of members joined at their ends to form a rigid structure is called a *truss*. Bridges, roof supports, derricks, and other such structures are common examples of trusses. Structural members commonly used are I-beams, channels, angles, bars, and special shapes which are fastened together at their ends by welding, riveted connections, or large bolts or pins. When the members of the truss lie essentially in a single plane, the truss is called a *plane truss*.

For bridges and similar structures, plane trusses are commonly utilized in pairs with one truss assembly placed on each side of the structure. A section of a typical bridge structure is shown in Fig. 4/1. The combined weight of the roadway and vehicles is transferred to the longitudinal stringers, then to the cross beams, and finally, with the weights of the stringers and cross beams accounted for, to the upper joints of the two plane trusses which form the vertical sides of the structure. A simplified model of the truss structure is indicated at the left side of the illustration; the forces L represent the joint loadings.

Several examples of commonly used trusses which can be analyzed as plane trusses are shown in Fig. 4/2.

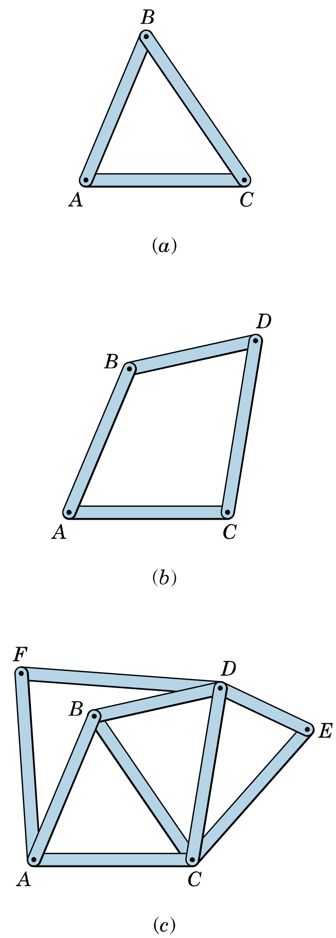


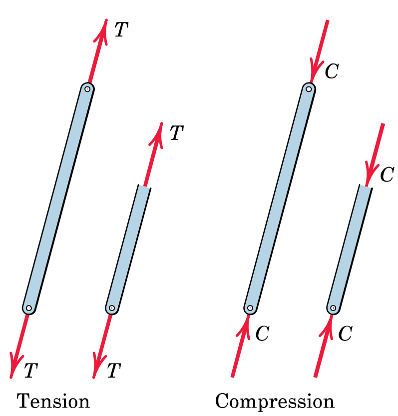
Figure 4/3

Simple Trusses

The basic element of a plane truss is the triangle. Three bars joined by pins at their ends, Fig. 4/3*a*, constitute a rigid frame. The term *rigid* is used to mean noncollapsible and also to mean that deformation of the members due to induced internal strains is negligible. On the other hand, four or more bars pin-jointed to form a polygon of as many sides constitute a nonrigid frame. We can make the nonrigid frame in Fig. 4/3*b* rigid, or stable, by adding a diagonal bar joining A and D or B and C and thereby forming two triangles. We can extend the structure by adding additional units of two end-connected bars, such as DE and CE or AF and DF, Fig. 4/3*c*, which are pinned to two fixed joints. In this way the entire structure will remain rigid.

Structures built from a basic triangle in the manner described are known as *simple trusses*. When more members are present than are needed to prevent collapse, the truss is statically indeterminate. A statically indeterminate truss cannot be analyzed by the equations of equilibrium alone. Additional members or supports which are not necessary for maintaining the equilibrium configuration are called *redundant*.

To design a truss we must first determine the forces in the various members and then select appropriate sizes and structural shapes to withstand the forces. Several assumptions are made in the force analysis of simple trusses. First, we assume all members to be *two-force members*. A two-force member is one in equilibrium under the action of two forces only, as defined in general terms with Fig. 3/4 in Art. 3/3. Each member of a truss is normally a straight link joining the two points of application of force. The two forces are applied at the ends of the member and are necessarily equal, opposite, and *collinear* for equilibrium.



Two-Force Members

Figure 4/4

The member may be in tension or compression, as shown in Fig. 4/4. When we represent the equilibrium of a portion of a two-force member, the tension T or compression C acting on the cut section is the same for all sections. We assume here that the weight of the member is small compared with the force it supports. If it is not, or if we must account for the small effect of the weight, we can replace the weight W of the member by two forces, each $W/2$ if the member is uniform, with one force acting at each end of the member. These forces, in effect, are treated as loads externally applied to the pin connections. Accounting for the weight of a member in this way gives the correct result for the average tension or compression along the member but will not account for the effect of bending of the member.

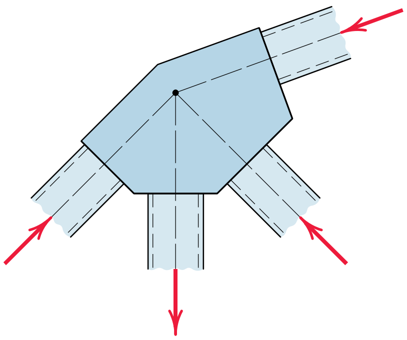


Figure 4/5

Truss Connections and Supports

When welded or riveted connections are used to join structural members, we may usually assume that the connection is a pin joint if the centerlines of the members are concurrent at the joint as in Fig. 4/5.

We also assume in the analysis of simple trusses that all external forces are applied at the pin connections. This condition is satisfied in most trusses. In bridge trusses the deck is usually laid on cross beams which are supported at the joints, as shown in Fig. 4/1.

For large trusses, a roller, rocker, or some kind of slip joint is used at one of the supports to provide for expansion and contraction due to temperature changes and for deformation from applied loads. Trusses and frames in which no such provision is made are statically indeterminate, as explained in Art. 3/3. Figure 3/1 shows examples of such joints.

Two methods for the force analysis of simple trusses will be given. Each method will be explained for the simple truss shown in Fig. 4/6*a*. The free-body diagram of the truss as a whole is shown in Fig. 4/6*b*. The external reactions are usually determined first, by applying the equilibrium equations to the truss as a whole. Then the force analysis of the remainder of the truss is performed.

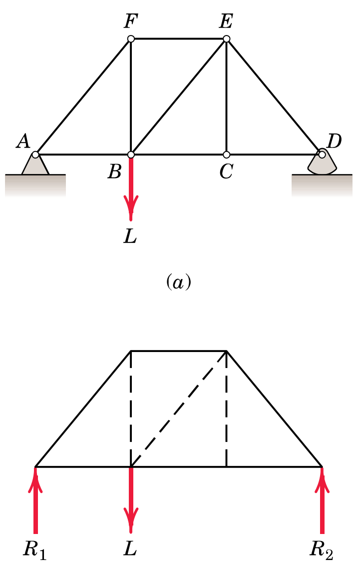


Figure 4/6

4/3 Method of Joints

This method for finding the forces in the members of a truss consists of satisfying the conditions of equilibrium for the forces acting on the connecting pin of each joint. The method therefore deals with the equilibrium of concurrent forces, and only two independent equilibrium equations are involved.

We begin the analysis with any joint where at least one known load exists and where not more than two unknown forces are present. The solution may be started with the pin at the left end. Its free-body diagram is shown in Fig. 4/7. With the joints indicated by letters, we usually designate the force in each member by the two letters defining the ends of the member. The proper directions of the forces should be evident by inspection for this simple case. The free-body diagrams of portions of members AF and AB are also shown to clearly indicate the mechanism of the action and reaction. The member AB actually makes contact on the left side of the pin, although the force AB is drawn from the right side and is shown acting away from the pin. Thus, if we consistently draw the force arrows on the *same* side of the pin as the member, then tension (such as AB) will always be indicated by an arrow *away*

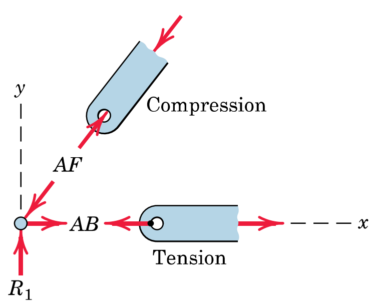


Figure 4/7

from the pin, and compression (such as AF) will always be indicated by an arrow *toward* the pin. The magnitude of AF is obtained from the equation $\Sigma F_y = 0$ and AB is then found from $\Sigma F_x = 0$.

Joint F may be analyzed next, since it now contains only two unknowns, EF and BF. Proceeding to the next joint having no more than two unknowns, we subsequently analyze joints B, C, E, and D in that order. Figure 4/8 shows the free-body diagram of each joint and its corresponding force polygon, which represents graphically the two equilibrium conditions $\Sigma F_x = 0$ and $\Sigma F_y = 0$. The numbers indicate the order in which the joints are analyzed. We note that, when joint D is finally reached, the computed reaction R_2 must be in equilibrium with the forces in members CD and ED, which were determined previously from the two neighboring joints. This requirement provides a check on the correctness of our work. Note that isolation of joint C shows that the force in CE is zero when the equation $\Sigma F_y = 0$ is applied. The force in

This New York City bridge structure suggests that members of a simple truss need not be straight.

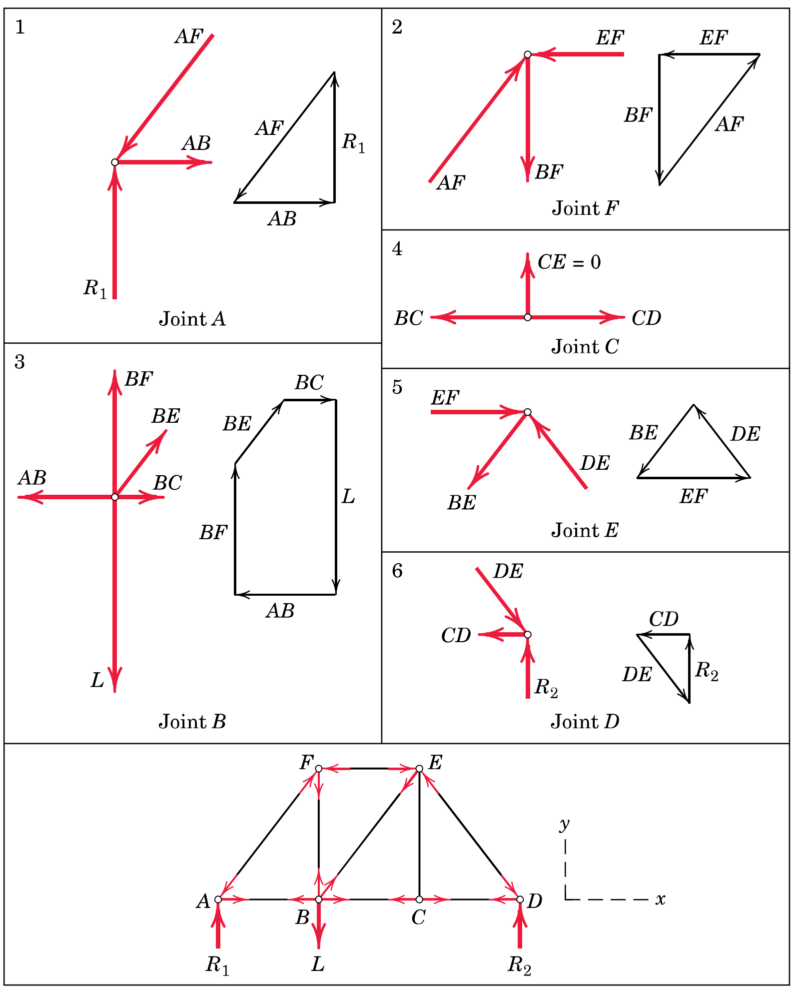


Figure 4/8

this member would not be zero, of course, if an external vertical load were applied at C.

It is often convenient to indicate the tension T and compression C of the various members directly on the original truss diagram by drawing arrows away from the pins for tension and toward the pins for compression. This designation is illustrated at the bottom of Fig. 4/8.

Sometimes we cannot initially assign the correct direction of one or both of the unknown forces acting on a given pin. If so, we may make an arbitrary assignment. A negative computed force value indicates that the initially assumed direction is incorrect.

Internal and External Redundancy

If a plane truss has more external supports than are necessary to ensure a stable equilibrium configuration, the truss as a whole is statically indeterminate, and the extra supports constitute *external* redundancy. If a truss has more internal members than are necessary to prevent collapse when the truss is removed from its supports, then the extra members constitute *internal* redundancy and the truss is again statically indeterminate.

For a truss which is statically determinate externally, there is a definite relation between the number of its members and the number of its joints necessary for internal stability without redundancy. Because we can specify the equilibrium of each joint by two scalar force equations, there are in all $2j$ such equations for a truss with j joints. For the entire truss composed of m two-force members and having the maximum of three unknown support reactions, there are in all $m + 3$ unknowns (m tension or compression forces and three reactions). Thus, for any plane truss, the equation $m + 3 = 2j$ will be satisfied if the truss is statically determinate internally.

A *simple* plane truss, formed by starting with a triangle and adding two new members to locate each new joint with respect to the existing structure, satisfies the relation automatically. The condition holds for the initial triangle, where $m = j = 3$, and m increases by 2 for each added joint while j increases by 1. Some other (nonsimple) statically determinate trusses, such as the K-truss in Fig. 4/2, are arranged differently, but can be seen to satisfy the same relation.

Harbour Bridge in Sydney, Australia

This equation is a necessary condition for stability but it is not a sufficient condition, since one or more of the m members can be arranged in such a way as not to contribute to a stable configuration of the entire truss. If $m + 3 > 2j$, there are more members than independent equations, and the truss is statically indeterminate internally with redundant members present. If $m + 3 < 2j$, there is a deficiency of internal members, and the truss is unstable and will collapse under load.

Special Conditions

We often encounter several special conditions in the analysis of trusses. When two collinear members are under compression, as indicated in Fig. 4/9a, it is necessary to add a third member to maintain

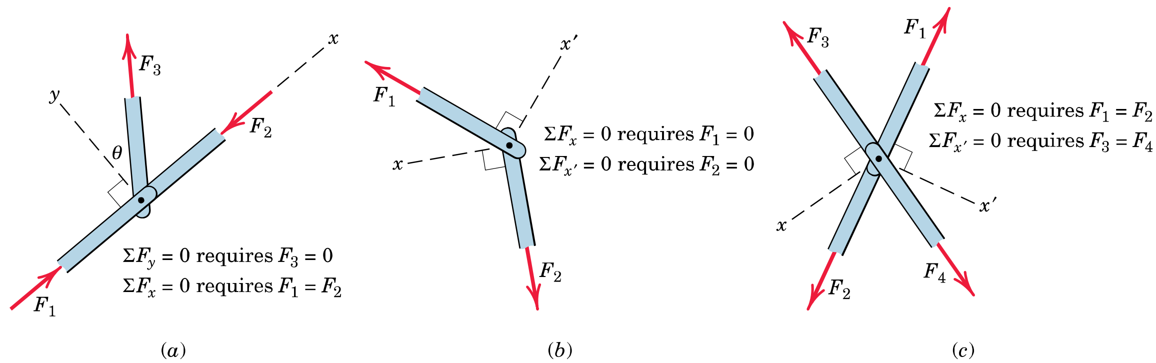


Figure 4/9

alignment of the two members and prevent buckling. We see from a force summation in the y-direction that the force F_3 in the third member must be zero and from the x-direction that $F_1 = F_2$. This conclusion holds regardless of the angle θ and holds also if the collinear members are in tension. If an external force with a component in the y-direction were applied to the joint, then F_3 would no longer be zero.

When two noncollinear members are joined as shown in Fig. 4/9*b*, then in the absence of an externally applied load at this joint, the forces in both members must be zero, as we can see from the two force summations.

When two pairs of collinear members are joined as shown in Fig. 4/9*c*, the forces in each pair must be equal and opposite. This conclusion follows from the force summations indicated in the figure.

Truss panels are frequently cross-braced as shown in Fig. 4/10*a*. Such a panel is statically indeterminate if each brace can support either tension or compression. However, when the braces are flexible members incapable of supporting compression, as are cables, then only the tension member acts and we can disregard the other member. It is usually evident from the asymmetry of the loading how the panel will deflect. If the deflection is as indicated in Fig. 4/10*b*, then member *AB* should be retained and *CD* disregarded. When this choice cannot be made by inspection, we may arbitrarily select the member to be retained. If the assumed tension turns out to be positive upon calculation, then the choice was correct. If the assumed tension force turns out to be negative, then the opposite member must be retained and the calculation redone.

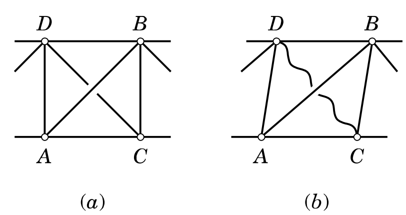


Figure 4/10

We can avoid simultaneous solution of the equilibrium equations for two unknown forces at a joint by a careful choice of reference axes. Thus, for the joint indicated schematically in Fig. 4/11 where L is known and F_1 and F_2 are unknown, a force summation in the x-direction eliminates reference to F_1 and a force summation in the x'-direction eliminates reference to F_2. When the angles involved are not easily found, then a simultaneous solution of the equations using one set of reference directions for both unknowns may be preferable.

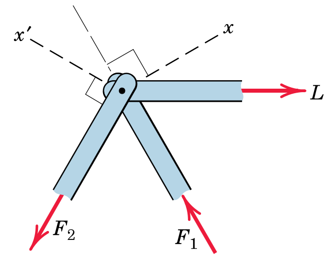
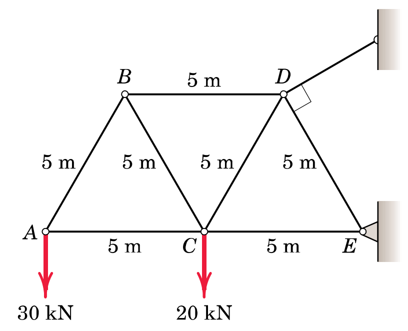
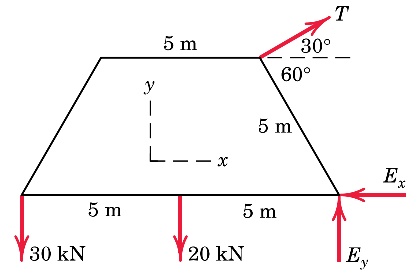


Figure 4/11

Sample Problem 4/1

Compute the force in each member of the loaded cantilever truss by the method of joints.





Solution. If it were not desired to calculate the external reactions at D and E, the analysis for a cantilever truss could begin with the joint at the loaded end. However, this truss will be analyzed completely, so the first step will be to compute the external forces at D and E from the free-body diagram of the truss as a whole. The equations of equilibrium give

$[\Sigma M_E = 0]$ $\quad 5T - 20(5) - 30(10) = 0 \quad$ $T = 80$ kN

$[\Sigma F_x = 0]$ $\quad 80 \cos 30° - E_x = 0 \quad$ $E_x = 69.3$ kN

$[\Sigma F_y = 0]$ $\quad 80 \sin 30° + E_y - 20 - 30 = 0 \quad$ $E_y = 10$ kN

Next we draw free-body diagrams showing the forces acting on each of the connecting pins. The correctness of the assigned directions of the forces is verified when each joint is considered in sequence. There should be no question about the correct direction of the forces on joint A. Equilibrium requires

$[\Sigma F_y = 0]$ $\quad 0.866AB - 30 = 0 \quad$ $AB = 34.6$ kN T *Ans.*

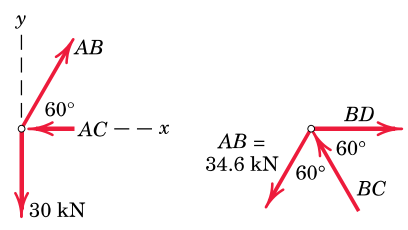
$[\Sigma F_x = 0]$ $\quad AC - 0.5(34.6) = 0 \quad$ $AC = 17.32$ kN C *Ans.*

① where T stands for tension and C stands for compression.

Joint B must be analyzed next, since there are more than two unknown forces on joint C. The force BC must provide an upward component, in which case BD must balance the force to the left. Again the forces are obtained from

$[\Sigma F_y = 0]$ $\quad 0.866BC - 0.866(34.6) = 0 \quad$ $BC = 34.6$ kN C *Ans.*

$[\Sigma F_x = 0]$ $\quad BD - 2(0.5)(34.6) = 0 \quad$ $BD = 34.6$ kN T *Ans.*



Joint A Joint B

Joint C now contains only two unknowns, and these are found in the same way as before:

$[\Sigma F_y = 0]$ $\quad 0.866CD - 0.866(34.6) - 20 = 0$

$CD = 57.7$ kN T *Ans.*

$[\Sigma F_x = 0]$ $\quad CE - 17.32 - 0.5(34.6) - 0.5(57.7) = 0$

$CE = 63.5$ kN C *Ans.*

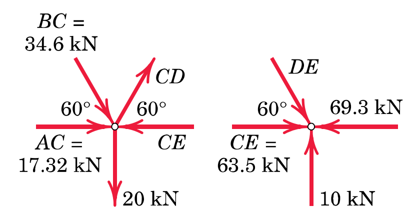
Finally, from joint E there results

$[\Sigma F_y = 0]$ $\quad 0.866DE = 10 \quad$ $DE = 11.55$ kN C *Ans.*

and the equation $\Sigma F_x = 0$ checks.

Helpful Hint

① It should be stressed that the tension/compression designation refers to the *member*, not the joint. Note that we draw the force arrow on the same side of the joint as the member which exerts the force. In this way tension (arrow away from the joint) is distinguished from compression (arrow toward the joint).



Joint C Joint E