Electrical Engineering Fundamentals

second class

AC & DC

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2024-2025

Lecture 1: International System of Measurement

1.1 Systems of Units:

The most frequently used system of units is the one adopted by the National Bureau of Standards in 1964; it is used by all major professional engineering societies and is the language in which today's textbooks are written. This is the International System of Units (abbreviated in all languages), adopted by the General Conference SI on Weights and Measures in 1960. Modified several times since, the SI is built upon seven basic units: the meter, kilogram, second, ampere, kelvin, mole, and candela (see Table (1.1)). This is a "metric system," some form of candela which is now in common use in most countries of the world, although it is not yet widely used in the United States. Units for other quantities such as volume, force, energy, etc., are derived from these seven base units.

Table (1.1): SI Base Units.

Base Quantity	Name	Symbol
length	**meter**	**m**
mass	**kilogram**	**kg**
time	**second**	**s**
electric current	**ampere**	**A**
thermodynamic temperature	**kelvin**	**K**
amount of substance	**mole**	**mol**
luminous intensity	**candela**	**cd**

1.2 Significant Digits, Accuracy, and Rounding Off:

In general, there are two types of numbers: ***exact*** and ***approximate***. Exact numbers are precise to the exact number of digits presented, just as we know that there are 12 apples in a dozen and not 12.1. Throughout the lectures, the numbers that appear in the descriptions, diagrams, and examples are considered ***exact***, so that a battery of 100 V can be written as 100.0 V, 100.00 V, and so on, since it is 100 V at any level of precision. The additional zeros were not included for purposes of clarity. However, in the laboratory environment, where measurements are continually being taken and the level of accuracy can vary from one instrument to another, it is important to understand how to work with the results. Any reading obtained in the laboratory should be considered ***approximate***. The analog scales with their pointers may be difficult to read, and even though the digital meter provides only specific digits on its display, it is limited to the number of digits it can provide, leaving us to wonder about the less significant digits not appearing on the display.

When adding approximate numbers, it is important to be sure that the accuracy of the readings is consistent throughout. To add a quantity accurate only to the tenths place to a number accurate to the thousandths place will result in a total having accuracy only to the tenths place. One cannot expect the reading with the higher level of accuracy to improve the reading with only tenths-place accuracy.

In the addition or subtraction of approximate numbers, the entry with the lowest level of accuracy determines the format of the solution.

For the multiplication and division of approximate numbers, the result has the same number of significant digits as the number with the least number of significant digits.

For approximate numbers (and exact numbers, for that matter), there is often a need to ***round off*** the result; that is, you must decide on the appropriate level of accuracy and alter the result accordingly. The accepted procedure is simply to note the digit following the last to appear in the rounded-off form, add a 1 to the last digit if it is greater than or equal to 5, and leave it alone if it is less than 5. For example, $\mathbf{3.186} \cong \mathbf{3.19} \cong \mathbf{3.2}$, depending on the level of precision desired. The symbol $\cong$ means approximately equal to.

For instance, let us examine the following product:

$$(\mathbf{9.64})(\mathbf{0.4896}) = \mathbf{4.68504}$$

Clearly, we don't want to carry this level of accuracy through any further calculations in a particular example. Rather, using hundredths-place accuracy, we will write it as **4.69**.

The next calculation may be:

$$(\mathbf{4.69})(\mathbf{1.096}) = \mathbf{5.14024}$$

which to hundredths-place accuracy is **5.14**. However, if we had carried the original product to its full accuracy, we would have obtained:

$$(\mathbf{4.68504})(\mathbf{1.096}) = \mathbf{5.1348}$$

or, to hundredths-place accuracy, **5.13**.

Obviously, **5.13** is the more accurate solution, so there is a loss of accuracy using rounded-off results. However, as indicated above, these lectures will round off the final and intermediate results to hundredths place for clarity and ease of comparison.

Example 1: Perform the indicated operations with the following approximate numbers and round off to the appropriate level of accuracy.

a) $532.6 + 4.02 + 0.036 = 536.656 \cong \mathbf{536.7}$ (as determined by 536.6)

b) $0.04 + 0.003 + 0.0064 = 0.0494 \cong \mathbf{0.05}$ (as determined by 0.04)

Example 2: Round off the following numbers to the hundredths place.

a) 32.419 = **32.42**

b) 0.05328 = **0.05**

Example 3: Round off the result 5.8764 to

a) tenths-place accuracy.

b) hundredths-place accuracy.

c) thousandths-place accuracy.

Solution:

a) 5.9

b) 5.88

c) 5.876

1.3 Powers of Ten:

To ease the difficulty of mathematical operations with numbers of such varying size, powers of ten are usually employed. This notation takes full advantage of the mathematical properties of powers of ten. The notation used to represent numbers that are integer powers of ten is as follows:

$$\mathbf{1 = 10^{0}} \qquad \mathbf{\frac{1}{10} = 0.1 = 10^{-1}}$$

$$\mathbf{10 = 10^{1}} \qquad \mathbf{\frac{1}{100} = 0.01 = 10^{-2}}$$

$$\mathbf{100 = 10^{2}} \qquad \mathbf{\frac{1}{1000} = 0.001 = 10^{-3}}$$

$$\mathbf{1000 = 10^{3}} \qquad \mathbf{\frac{1}{10,000} = 0.0001 = 10^{-4}}$$

In particular, note that $10^{0} = 1$, and, in fact, any quantity to the zero power is, $(x^{0} = 1, 1000^{0} = 1 \; and \; so \; on)$. Numbers in the list ***greater than 1 are associated with positive powers of ten***, and numbers in the list ***less than 1 are associated with negative powers of ten***.

A quick method of determining the proper power of ten is to place a caret mark to the right of the numeral 1 wherever it may occur; then count from this point to the number of places to the right or left before arriving at the decimal point. Moving to the right indicates a positive power of ten, whereas moving to the left indicates a negative power. For example:

$$10,000.0 = 1\underbrace{0,000}_{1\;2\;3\;4}. = 10^{+4}$$

$$0.00001 = 0.\underbrace{0000 1}_{5\;4\;3\;2\;1} = 10^{-5}$$

Some important mathematical equations and relationships pertaining to powers of ten are listed below, along with a few examples. In each case, n and m can be any positive or negative real number.

$$\frac{1}{10^{n}} = 10^{-n} \qquad \frac{1}{10^{-n}} = 10^{n}$$

Equation above clearly reveals that shifting a power of ten from the denominator to the numerator, or the reverse, requires simply changing the sign of the power.

Example 4:

a) $\frac{1}{1000} = \frac{1}{10^{+3}} = \mathbf{10^{-3}}$

b) $\frac{1}{0.00001} = \frac{1}{10^{-5}} = \mathbf{10^{+5}}$

The product of powers of ten:

$$(10^n)(10^m) = (10)^{(n+m)}$$

Example 5:

a) $(1000)(10{,}000) = (10^3)(10^4) = (10)^{(3+4)} = \mathbf{10^7}$

b) $(0.00001)(100) = (10^{-5})(10^2) = (10)^{(-5+2)} = \mathbf{10^{-3}}$

The division of powers of ten:

$$\frac{10^n}{10^m} = (10)^{(n-m)}$$

Example 6:

a) $\frac{100{,}000}{100} = \frac{10^5}{10^2} = (10)^{(5-2)} = \mathbf{10^3}$

b) $\frac{1000}{0.0001} = \frac{10^3}{10^{-4}} = (10)^{(3-(-4))} = (10)^{(3+4)} = \mathbf{10^7}$

Note the use of parentheses in part (b) to ensure that the proper sign is established between operators.

The power of powers of ten:

$$(10^n)^m = 10^{nm}$$

Example 7:

a) $(100)^4 = (10^2)^4 = 10^{(2)(4)} = \mathbf{10^8}$

b) $(1000)^{-2} = (10^3)^{-2} = 10^{(3)(-2)} = \mathbf{10^{-6}}$

c) $(0.01)^{-3} = (10^{-2})^{-3} = 10^{(-2)(-3)} = \mathbf{10^6}$

1.4 Basic Arithmetic Operations:

1.4.1 Addition and Subtraction:

To perform addition or subtraction using powers of ten, the power of ten must be the same for each term; that is:

$$A \times 10^n \pm B \times 10^n = (A \pm B) \times 10^n$$

Example 8:

a) $6300 + 75{,}000 = (6.3)(1000) + (75)(1000)$

$$= 6.3 \times 10^3 + 75 \times 10^3$$
$$= (6.3 + 75) \times 10^3$$
$$= \mathbf{81.3 \times 10^3}$$

b) $0.00096 - 0.000086 = (96)(0.00001) - (8.6)(0.00001)$

$$= 96 \times 10^{-5} - 8.6 \times 10^{-5}$$
$$= (96 + 8.6) \times 10^{-5}$$
$$= \mathbf{87.4 \times 10^{-5}}$$

1.4.2 Multiplication:

In general,

$$(A \times 10^n)(B \times 10^m) = (A)(B) \times 10^{n+m}$$

revealing that the operations with the power of ten can be separated from the operation with the multipliers.

Example 9:

a) $(0.0002)(0.000007) = [(2)(0.0001)][(7)(0.000001)]$

$$= (2 \times 10^{-4})(7 \times 10^{-6})$$
$$= (2)(7) \times (10^{-4})(10^{-6})$$
$$= \mathbf{14 \times 10^{-10}}$$

b) $(340{,}000)(0.00061) = (3.4 \times 10^5)(61 \times 10^{-5})$

$$= (3.4)(61) \times (10^{-5})(10^{-5})$$
$$= 207.4 \times 10^{0}$$
$$= \mathbf{207.4}$$

1.4.3 Division:

In general,

$$\frac{A \times 10^{n}}{B \times 10^{m}} = \frac{A}{B} \times (10)^{(n-m)}$$

revealing again that the operations with the power of ten can be separated from the same operation with the multipliers.

Example 10:

a) $\frac{0.00047}{0.002} = \frac{47\times10^{-5}}{2\times10^{-3}} = \left(\frac{47}{2}\right) \times \left(\frac{10^{-5}}{10^{-3}}\right) = 23.5 \times 10^{-5-(-3)} = \mathbf{23.5 \times 10^{-2}}$

b) $\frac{690{,}000}{0.00000013} = \frac{69\times10^{4}}{13\times10^{-8}} = \left(\frac{69}{13}\right) \times \left(\frac{10^{4}}{10^{-8}}\right) = 5.31 \times 10^{4-(-8)} = \mathbf{5.31 \times 10^{12}}$

1.4.4 Powers:

In general,

$$(A \times 10^{n})^{m} = A^{m} \times 10^{nm}$$

which again permits the separation of the operation with the power of ten from the multiplier.

Example 11:

a) $(0.00003)^3 = (3 \times 10^{-5})^3 = (3)^3(10^{-5})^3 = \mathbf{27 \times 10^{-15}}$

b) $(90{,}800{,}000)^2 = (9.08 \times 10^{7})^2 = (9.08)^2(10^{7})^2 = \mathbf{82.45 \times 10^{14}}$

In particular, remember that the following operations are not the same. One is the product of two numbers in the power-of-ten format, while the other is a number in the power-of-ten format taken to a power.

As noted below, the results of each are quite different:

$$\mathbf{(10^3)(10^3) \neq (10^3)^3}$$
$$\mathbf{(10^3)(10^3) = 10^6 = 1,000,000}$$
$$\mathbf{(10^3)^3 = (10^3)(10^3)(10^3) = 10^9 = 1,000,000,000}$$

1.5 SI Prefixes:

The SI uses the decimal system to relate larger and smaller units to the basic unit, and it employs prefixes to signify the various powers of 10. A list of prefixes and their symbols is given in Table (1.2); the ones most commonly encountered in engineering are highlighted.

Table (1.2): SI Prefixes.

Multiplication Factors	SI Prefix	SI Symbol
$1\ 000\ 000\ 000\ 000\ 000\ 000 = 10^{18}$	exa	E
$1\ 000\ 000\ 000\ 000\ 000 = 10^{15}$	peta	P
$1\ 000\ 000\ 000\ 000 = 10^{12}$	tera	T
$1\ 000\ 000\ 000 = 10^{9}$	giga	G
$1\ 000\ 000 = 10^{6}$	mega	M
$1\ 000 = 10^{3}$	kilo	K
$0.001 = 10^{-3}$	milli	m
$0.000\ 001 = 10^{-6}$	micro	µ
$0.000\ 000\ 001 = 10^{-9}$	nano	n
$0.000\ 000\ 000\ 001 = 10^{-12}$	pico	p
$0.000\ 000\ 000\ 000\ 001 = 10^{-15}$	femto	f
$0.000\ 000\ 000\ 000\ 000\ 001 = 10^{-18}$	atto	a

Example 12:

a) $1{,}000{,}000 \text{ ohms} = 1 \times 10^{6} \text{ ohms}$

$$= \mathbf{1\ megaohms} = \mathbf{1\ M\Omega}$$

b) $100{,}000 \text{ meters} = 100 \times 10^{3} \text{ meters}$

$$= \mathbf{100\ kilometers} = \mathbf{100\ Km}$$

c) $0.0001 \text{ second} = 0.1 \times 10^{-3} \text{ second}$

$$= \mathbf{0.1\ millisecond = 0.1\ ms}$$

d) $0.000001 \text{ farad} = 1 \times 10^{-6} \text{ farad}$

$$= \mathbf{1\ microfarad = 1\ \mu F}$$

Here are a few examples with numbers that are not strictly powers of ten.

Example 13:

a) $41{,}200 \text{ m is equivalent to } 41.2 \times 10^{3} \text{ m} = 41.2 \text{ kilometers} = \mathbf{41.2\ Km}$

b) $0.00956 \text{ J is equivalent to } 9.56 \times 10^{-3} \text{ J} = 9.56 \text{ millijoules} = \mathbf{9.56\ mJ}$

c) $0.000768 \text{ s is equivalent to } 768 \times 10^{-6} \text{ s} = 768 \text{ microseconds} = \mathbf{768\ \mu s}$

d) $\frac{8400 \text{ m}}{0.06} = \frac{8.4\times10^{3} \text{ m}}{6\times10^{-2}} = \left(\frac{8.4}{6}\right) \times \left(\frac{10^{3}}{10^{-2}}\right) = 1.4 \times 10^{5} \text{ m}$

$$140 \times 10^{3} \text{ m} = 140 \text{ kilometers} = \mathbf{140\ Km}$$

e) $(0.0003)^{4} \text{ s} = (3 \times 10^{-4})^{4} \text{ s} = 81 \times 10^{-16} \text{ s} = 0.0081 \times 10^{-12} \text{ s}$

$$= 0.0081 \text{ picosecond} = \mathbf{0.0081\ ps}$$

1.6 Conversion Between Levels of Powers of Ten:

The procedure is best described by the following steps:

1) ***Replace the prefix by its corresponding power of ten.***
2) ***Rewrite the expression, and set it equal to an unknown multiplier and the new power of ten.***
3) ***Note the change in power of ten from the original to the new format.***

If it is an increase, move the decimal point of the original multiplier to the left (smaller value) by the same number. If it is a decrease, move the decimal point of the original multiplier to the right (larger value) by the same number.

For instance, if a meter measures kilohertz (kHz—a unit of measurement for the frequency of an ac waveform), it may be necessary to find the corresponding level in megahertz (MHz).

Example 14: Convert 20 KHz to megahertz.

Solution:

In the power-of-ten format:

$$20\ KHz = 20\ \times 10^{3}\ Hz$$

The conversion requires that we find the multiplying factor to appear in the space below:

$$20 \times 10^{3}\ \text{Hz} \Rightarrow \underline{\quad} \times 10^{6}\ \text{Hz}$$

Increase by 3 (power of ten); Decrease by 3 (multiplying factor)

Since the power of ten will be *increased* by a factor of *three*, the multiplying factor must be *decreased* by moving the decimal point *three* places to the left, as shown below:

$$\underbrace{020.}_{3} = 0.02$$

and $20\ \times 10^{3}\ \text{Hz} = 0.02\ \times 10^{6}\ \text{Hz} = \mathbf{0.02\ MHz}$

If time is measured in milliseconds (ms), it may be necessary to find the corresponding time in microseconds(ms) for a graphical plot.

Example 15: Convert 0.01 ms to microseconds.

Solution:

In the power-of-ten format:

$$0.01\ \text{ms} = 0.01\ \times 10^{-3}\ \text{s}$$

and

$$0.01 \times 10^{-3}\ \text{s} \Rightarrow \underline{\quad} \times 10^{-6}\ \text{s}$$

Decrease by 3 (power of ten); Increase by 3 (multiplying factor)

Since the power of ten will be *reduced* by a factor of three, the multiplying factor must be *increased* by moving the decimal point three places to the right, as follows:

$$0.\underbrace{010}_{3} = 10$$

and $0.01 \times 10^{-3}\ \text{s} = 10 \times 10^{-6}\ \text{s} = \mathbf{10\ \mu s}$

There is a tendency when comparing -3 to -6 to think that the power of ten has increased, but keep in mind when making your judgment about increasing or decreasing the magnitude of the multiplier that 10^{-6} is a great deal smaller than 10^{-3}.

Example 16: Convert 0.002 km to millimeters.

Solution:

Decrease by 6

$$0.002 \times 10^{3}\ \text{m} \Rightarrow \_\_\_ \times 10^{-3}\ \text{m}$$

Increase by 6

In this example we have to be very careful because the difference between +3 and −3 is a factor of 6, requiring that the multiplying factor be modified, as follows:

$$0.\underbrace{002000}_{6} = 2000$$

and $0.002 \times 10^{3}\ \text{m} = 2000 \times 10^{-3}\ \text{m} = \mathbf{2000\ mm}$

Lecture 2: Elementary Concepts and Definitions

2.1 Voltage:

If a total of 1 joule (J) of energy is used to move the negative charge of 1 coulomb (C), there is a difference of 1 volt (V) between the two points.

The defining equation is:

$$V = \frac{W}{Q}$$

$V =$ volts (V)
$W =$ joules (J)
$Q =$ coulombs (C)

Take particular note that the charge is measured in coulombs, the energy in joules, and the voltage in volts. The unit of measurement, volt, was chosen to honor the efforts of Alessandro Volta, who first demonstrated that a voltage could be established through chemical action

Through algebraic manipulations, we can define an equation to determine the energy required to move charge through a difference in voltage:

$$W = QV \quad \text{(joules, J)}$$

Finally, if we want to know how much charge was involved, we use:

$$Q = \frac{W}{V} \quad \text{(coulombs, C)}$$

Example 1: Find the voltage between two points if 60 J of energy are required to move a charge of 20 C between the two points.

Solution:

$$V = \frac{W}{Q} = \frac{60\,J}{20\,C} = \mathbf{3\,V}$$

Example 2: Determine the energy expended moving a charge of 50 µC between two points if the voltage between the points is 6 V.

Solution:

$$W = QV = (50 \times 10^{-6}\,C)(6\,V) = 300 \times 10^{-6} = \mathbf{300\,\mu J}$$

2.2 Current:

The applied voltage is the starting mechanism—the current is a reaction to the applied voltage.

The unit of current measurement, **ampere**, was chosen to honor the efforts of André Ampère in the study of electricity in motion.

Using the coulomb as the unit of charge, we can determine the current in amperes from the following equation:

$$I = \frac{Q}{t}$$

I = amperes (A)
Q = coulombs (C)
t = times (s)

The capital letter I was chosen from the French word for current, intensité. The SI abbreviation for each quantity in equation above is provided to the right of the equation.

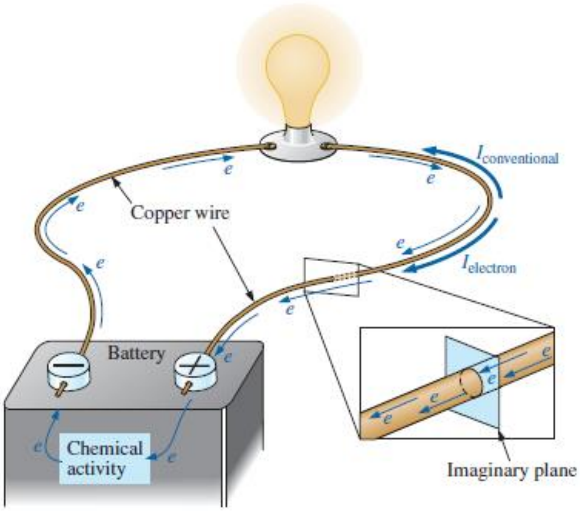
Through algebraic manipulations, the other two quantities can be determined, as follows:

$$Q = It \quad \text{(coulombs, C)}$$

and

$$t = \frac{Q}{I} \quad \text{(seconds, s)}$$

Example 3: The charge flowing through the imaginary surface in Figure below is 0.16 C every 64 ms. Determine the current in amperes.



Solution:

$$I = \frac{Q}{t} = \frac{0.16\ C}{64 \times 10^{-3}\ s} = \frac{160 \times 10^{-3}\ C}{64 \times 10^{-3}\ s} = \mathbf{2.5\ A}$$

Example 4: Determine how long it will take 4×10^{16} electrons to pass through the imaginary surface in the Figure mentioned in Example 3 if the current is 5 mA.

Solution: Determine the charge in coulombs:

$$Q = 4 \times 10^{16}\text{electrons}\left(\frac{1\ C}{6.242 \times 10^{18}\ \text{electrons}}\right) = 0.641 \times 10^{-2}\ C = \mathbf{6.41\ mC}$$

$$t = \frac{Q}{I} = \frac{6.41 \times 10^{-3}\ C}{5 \times 10^{-3}\ A} = \mathbf{1.28\ s}$$

In summary, therefore, ***the applied voltage (or potential difference) in an electrical/ electronic system is the "pressure" to set the system in motion, and the current is the reaction to that pressure.***

2.3 Resistance:

2.3.1 Circular Wires:

The resistance of any material is due primarily to four factors:

1) Material
2) Length
3) Cross-sectional area
4) Temperature of the material

The first three elements are related by the following basic equation for resistance:

$$R = \rho \frac{l}{A}$$

$\rho = \text{CM} - \frac{\Omega}{\text{ft}} \text{at T} = 20° \text{ C}$

$l = feet$

$\text{A} = \text{area in circular mils (CM)}$

with each component of the equation defined by Figure (2.1).

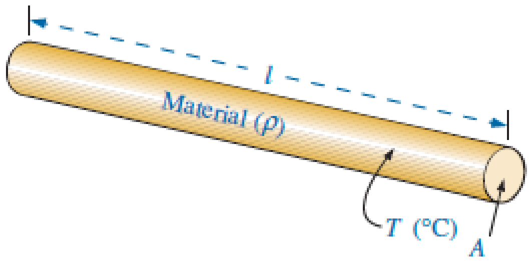


Figure (2.1): Factors affecting the resistance of a conductor.

The material is identified by a factor called the resistivity, which uses the Greek letter rho (ρ) as its symbol and is measured in $\text{CM} - \Omega/ft$. Its value at a temperature of 20° C (room temperature=68° F) is provided in Table (2.1) for a variety of common materials.

Table (2.1): Resistivity (ρ) of various materials.

Material	ρ CM − Ω/ft @ 20° C
Silver	9.9
Copper	**10.37**
Gold	14.7
Aluminum	17.0
Tungsten	33.0
Nickel	47.0
Iron	74.0
Constantan	295.0
Nichrome	600.0
Calorite	720.0
Carbon	21,000.0

Since the resistivity is in the numerator of equation $R = \rho \frac{l}{A}$, ***the higher the resistivity, the greater is the resistance of a conductor if all remaining parameters are the same*** (see Figure (2.2(a))).

Further, ***the longer the conductor, the greater is the resistance conductor if all remaining parameters are the same*** (see Figure (2.2(b))).

Finally, ***the greater the area of a conductor, the less is the resistance conductor if all remaining parameters are the same*** (see Figure (2.2(c))) because the area appears in the denominator of equation $R = \rho \frac{l}{A}$.

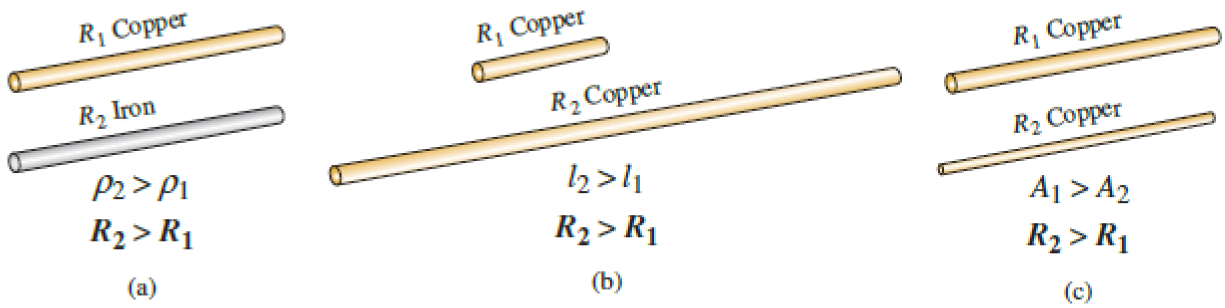


Figure (2.2): Cases in which $R_2 > R_1$. For each case, all remaining parameters that control the resistance level are the same.

Circular Mils (CM):

In equation $R = \rho \frac{l}{A}$, the area is measured in a quantity called circular mils (CM). It is the quantity used in most commercial wire tables, and thus it needs to be carefully defined. The mil is a unit of measurement for length and is related to the inch by:

$$1 \text{ mil} = \frac{1}{1000} \text{ in.}$$

or

$$1000 \text{ mil} = 1 \text{ in.}$$

a wire with a diameter of 1 mil has an area of 1 CM

The area of a circular wire in circular mils can be defined by the following equation:

$$A_{CM} = (d_{mils})^2$$

Example 5: What is the resistance of a 100 ft length of copper wire with a diameter of 0.020 in. at 20°C?

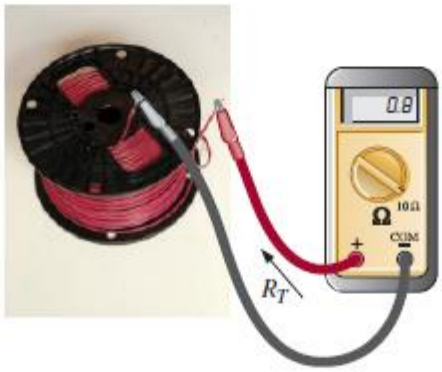
Solution: For copper wire:

$$\rho = 10.37\ \frac{\text{CM}-\Omega}{\text{ft}} \text{ and } 0.020 \text{ in.} = 20 \text{ mils}$$

$$A_{CM} = (d_{mils})^2 = (20 \text{ mils})^2 = 400\ CM$$

$$R = \rho\frac{l}{A} = \frac{(10.37\text{CM}-\Omega/\text{ft})(100\text{ft})}{400\ CM} = \mathbf{2.59}\ \Omega$$

Example 6: An undetermined number of feet of wire have been used from the 500′ spool of wire in Figure below. Find the length of the remaining copper wire if the diameter is 1/16 in. and the resistance is 0.8 Ω.



Solution:

$$\rho = 10.37\ \frac{\text{CM}-\Omega}{\text{ft}} \text{ and } \frac{1}{16}\text{in.} = 0.0625 \text{ in.} = 62.5 \text{ mils}$$

$$A_{CM} = (d_{mils})^2 = (62.5 \text{ mils})^2 = 3906.25\ CM$$

$$R = \rho\frac{l}{A} \Rightarrow l = \frac{RA}{\rho} = \frac{(0.8\Omega)(3906.25\ CM)}{10.37\ \frac{\text{CM}-\Omega}{\text{ft}}} = \frac{3125}{10.37} = \mathbf{301.35}\ \boldsymbol{ft}$$

2.3.2 Conductance:

By finding the reciprocal of the resistance of a material, we have a measure of how well the material conducts electricity. The quantity is called **conductance**, has the symbol G, and is measured in *siemens* (S). In equation form, conductance is:

$$G = \frac{1}{R} \qquad \text{(siemens, S)}$$

A resistance of 1 MΩ is equivalent to a conductance of 10^{-6} S, and a resistance of 10 Ω is equivalent to a conductance of 10^{-1} S. The larger the conductance, therefore, the less is the resistance and the greater is the conductivity.

In equation form, the conductance is determined by:

$$G = \frac{A}{\rho\, l}$$

indicating that increasing the area or decreasing either the length or the resistivity increases the conductance.

Example 7:

A) Determine the conductance of a 1 Ω, a 50 KΩ, and a 10 MΩ resistor.

B) How does the conductance level change with increase in resistance?

Solution:

A)

$$\mathbf{1}\ \Omega: G = \frac{1}{R} = \frac{1}{1\ \Omega} = \mathbf{1\ S}$$

$$\mathbf{50\ K}\Omega: G = \frac{1}{R} = \frac{1}{50 \times 10^3\ \Omega} = 0.02 \times 10^{-3} S = \mathbf{0.02\ mS}$$

$$\mathbf{10\ M}\Omega: G = \frac{1}{R} = \frac{1}{10 \times 10^6\ \Omega} = 0.1 \times 10^{-6} S = \mathbf{0.1\ \mu S}$$

B) The conductance level decreases rapidly with significant increase in resistance levels.

Example 8: What is the relative increase or decrease in conductivity of a conductor if the area is reduced by 30% and the length is increased by 40%? The resistivity is fixed.

Solution:

$$G_i = \frac{1}{R_i} = \frac{1}{\frac{\rho_i\, l_i}{A_i}} = \frac{A_i}{\rho_i\, l_i}$$

with the subscript i for the initial value. Using the subscript n for the new value, we obtain:

$$G_n = \frac{A_n}{\rho_n\, l_n} = \frac{0.70\, A_i}{\rho_i\,(1.4\, l_i)} = \frac{0.70}{1.4}\, G_i$$

and

$$G_n = \mathbf{0.5}\, \boldsymbol{G_i}$$

2.4 Ohm's Law:

A relationship was derived by the scientist Ohm; between the current, voltage, and resistance of the circuit. It says: "At a constant temperature, the current flowing through the circuit is directly proportional to the voltage and inversely proportional to the resistance".

$$I = \frac{V}{R}$$

And,

$$R = \frac{V}{I}$$

While,

$$V = I\, R$$

Example 9: Determine the current resulting from the application of a 9 V battery across a network with a resistance of 2.2 Ω.

Solution:

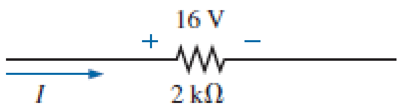
$$I = \frac{V}{R} = \frac{E}{R} = \frac{9\, V}{2.2\, \Omega} = \mathbf{4.09}\, \boldsymbol{A}$$

Example 10: Calculate the resistance of a 60 W bulb if a current of 500 mA results from an applied voltage of 120 V.

Solution:

$$R = \frac{V}{I} = \frac{E}{I} = \frac{120\, V}{500 \times 10^{-3}\, A} = \mathbf{240}\, \boldsymbol{\Omega}$$

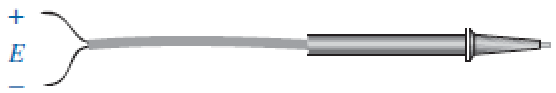
Example 11: Calculate the current through the 2 KΩ resistor in Figure below if the voltage drop across it is 16 V.

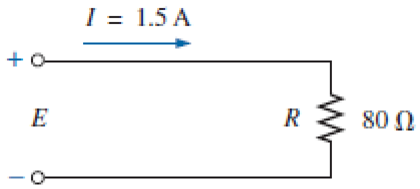


Solution:

$$I = \frac{V}{R} = \frac{16\ V}{2 \times 10^3\ \Omega} = \mathbf{8\ mA}$$

Example 12: Calculate the voltage that must be applied across the soldering iron in Figure below to establish a current of 1.5 A through the iron if its internal resistance is 80 Ω.





Solution:

$$E = V_R = I\ R = (1.5\ A)(80\ \Omega) = \mathbf{120\ V}$$

2.5 Power:

In general, ***the term power is applied to provide an indication of how much work (energy conversion) can be accomplished in a specified amount of time; that is, power is a rate of doing work.***

In equation form, power is determined by:

$$\boxed{P = \frac{W}{t}} \quad \text{(watts, W, or joules/second, J/s)}$$

with the **energy** (W) measured in joules and the time t in seconds.

The horsepower and watt are related in the following manner:

$$\boxed{1 \text{ horsepower} \cong 746 \text{ watts}}$$

The power delivered to, or absorbed by, an electrical device or system can be found in terms of the current and voltage, as follows:

$$P = \frac{W}{t} = \frac{Q\,V}{t} = V\,\frac{Q}{t}$$

But

$$I = \frac{Q}{t}$$

so that

$$\boxed{P = V\,I} \quad \text{(watts, W)}$$

By direct substitution of Ohm's law, the equation for power can be obtained in two other forms:

$$P = V\,I = V\left(\frac{V}{R}\right)$$

and

$$\boxed{P = \frac{V^2}{R}} \quad \text{(watts, W)}$$

or

$$P = V\,I = (I\,R)I$$

and

$$\boxed{P = I^2\,R} \quad \text{(watts, W)}$$

The result is that the power absorbed by the resistor in Figure (2.3) can be found directly, depending on the information available.

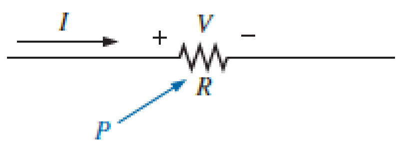
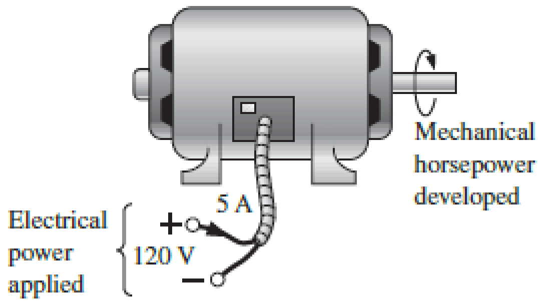


Figure (2.3): Defining the power to a resistive element.

The power supplied by a battery can be determined by simply inserting the supply voltage into equation $P = V\,I$ to produce:

$$\boxed{P = E\,I} \quad \text{(watts, W)}$$

Example 13: Find the power delivered to the dc motor of Figure below.



Solution:

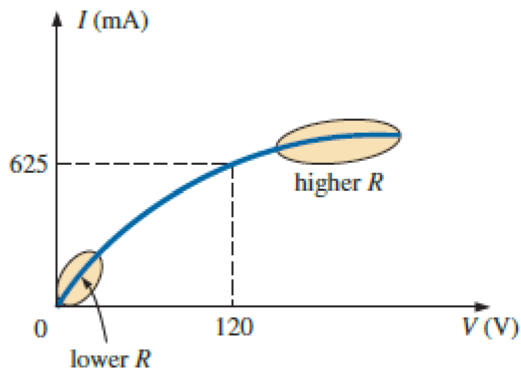
$$P = E\,I = (120\,V)(5\,A) = 600\,W = \mathbf{0.6\,KW}$$

Example 14: What is the power dissipated by a 5 Ω resistor if the current is 4 A?

Solution:

$$P = I^2\,R = (4\,A)^2(5\,\Omega) = \mathbf{80\,W}$$

Example 15: The I-V characteristics of a light bulb are provided in Figure below. Note the nonlinearity of the curve, indicating a wide range in resistance of the bulb with applied voltage. If the rated voltage is 120 V, find the wattage rating of the bulb. Also, calculate the resistance of the bulb under rated conditions.



Solution: At 120 V,

$$I = 625\,mA = 0.625\,A$$

and

$$P = V\,I = (120\,V)(0.625\,A) = \mathbf{75\,W}$$

At 120 V,

$$R = \frac{V}{I} = \frac{120\,V}{0.625\,A} = \mathbf{192\,\Omega}$$

Sometimes the power is given and the current or voltage must be determined. Through algebraic manipulations, an equation for each variable is derived, as follows:

$$P = I^2 R \Rightarrow I^2 = \frac{P}{R}$$

and

$$I = \sqrt{\frac{P}{R}} \quad \text{(amperes, A)}$$

$$P = \frac{V^2}{R} \Rightarrow V^2 = P R$$

and

$$V = \sqrt{P R} \quad \text{(volts, V)}$$

Example 16: Determine the current through a 5 KΩ resistor when the power dissipated by the element is 20 mW.

Solution:

$$I = \sqrt{\frac{P}{R}} = \sqrt{\frac{20 \times 10^{-3}\, W}{5 \times 10^{3}\, \Omega}} = \sqrt{4 \times 10^{-6}} = 2 \times 10^{-3} A = \boldsymbol{2\, mA}$$

2.6 Energy:

For power, which is the rate of doing work, to produce an energy conversion of any form, it must be *used over a period of time*. For example, a motor may have the horsepower to run a heavy load, but unless the motor is *used* over a period of time, there will be no energy conversion. In addition, the longer the motor is used to drive the load, the greater will be the energy expended.

The energy (W) lost or gained by any system is therefore determined by:

$$W = Pt \quad \text{(wattseconds, Ws, or joules)}$$

Since power is measured in watts (or joules per second) and time in seconds, the unit of energy is the *wattsecond* or *joule*. The wattsecond, however, is too small a quantity for most practical purposes, so the *watthour* (Wh) and *the kilowatthour* (kWh) are defined, as follows:

$$\text{Energy (Wh)} = \text{power (W)} \times \text{time(h)}$$

$$\text{Energy (KWh)} = \frac{\text{power (W)} \times \text{time(h)}}{1000}$$

Note that the energy in kilowatthours is simply the energy in watthours divided by 1000. To develop some sense for the kilowatthour energy level, consider that *1 KWh is the energy dissipated by a 100 W bulb in 10 h*.

Example 17: How much energy (in kilowatthours) is required to light a 60 W bulb continuously for 1 year (365 days)?

Solution:

$$W = \frac{Pt}{1000} = \frac{(60\ W)(24\ h/day)(365\ days)}{1000} = \frac{525{,}600\ Wh}{1000} = \mathbf{525.60\ KWh}$$

Example 18: How long can a 340 W plasma TV be on before it uses more than 4 KWh of energy?

Solution:

$$W = \frac{Pt}{1000} \Rightarrow t(hours) = \frac{(W)(1000)}{P} = \frac{(4\ KWh)(1000)}{340\ W} = \mathbf{11.76\ h}$$

2.7 Efficiency:

Conservation of energy requires that:

$$\textbf{Energy input} = \textbf{energy output} + \textbf{energy lost or stored by the system}$$

Dividing both sides of the relationship by t gives:

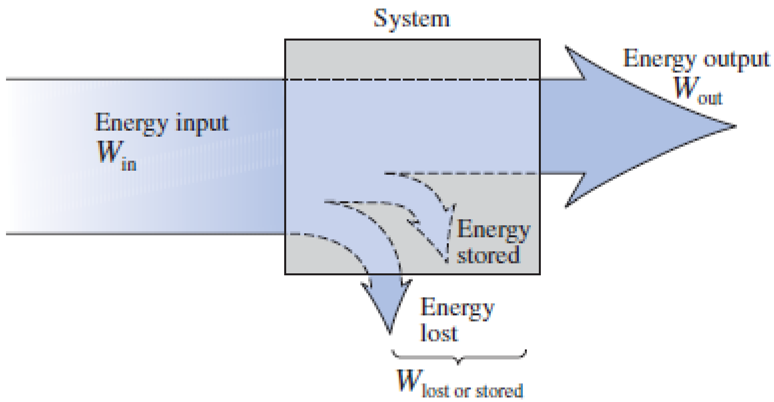


Figure (2.4): Energy flow through a system.

$$\frac{W_{in}}{t} = \frac{W_{out}}{t} + \frac{W_{lost\ or\ stored\ by\ the\ system}}{t}$$

Since $P = W/t$, we have the following:

$$\boxed{P_i = P_o + P_{lost\ or\ stored}} \quad \text{(watts, W)}$$

The efficiency (η) of the system is then determined by the following equation:

$$\text{Efficiency} = \frac{\text{power output}}{\text{power input}}$$

and

$$\boxed{\eta = \frac{P_o}{P_i}} \quad \text{(decimal number)}$$

where (the lowercase Greek letter *eta*) is a decimal number. Expressed as a percentage,

$$\boxed{\eta\% = \frac{P_o}{P_i} \times 100\%} \quad \text{(percent)}$$

In terms of the input and output energy, the efficiency in percent is given by:

$$\boxed{\eta\% = \frac{W_o}{W_i} \times 100\%} \quad \text{(percent)}$$

The maximum possible efficiency is 100%, which occurs when $P_o = P_i$, or when the power lost or stored in the system is zero. Obviously, the greater the internal losses of the system in generating the necessary output power or energy, the lower is the net efficiency.

Example 19: A 2 hp motor operates at an efficiency of 75%. What is the power input in watts? If the applied voltage is 220 V, what is the input current?

Solution:

$$\eta\% = \frac{P_o}{P_i} \times 100\%$$

$$75\% = \frac{(2\text{ hp})(746\text{ W/hp})}{\text{P}_\text{i}} \times 100\%$$

and

$$\text{P}_\text{i} = \frac{1492\,W}{0.75} = \mathbf{1989.33\,W}$$

$$\text{P}_\text{i} = E\,I$$

or

$$I = \frac{\text{P}_\text{i}}{E} = \frac{1989.33\,W}{220\,V} = \mathbf{9.04\,A}$$

Example 20: What is the output in horsepower of a motor with an efficiency of 80% and an input current of 8 A at 120 V?

Solution:

$$\eta\% = \frac{P_o}{P_i} \times 100\%$$

$$0.80 = \frac{P_o}{(120\,V)(8\,A)}$$

and

$$P_o = (0.80)(120\,V)(8\,A) = 768\,W$$

with

$$768\,W\left(\frac{1\,hp}{746\,W}\right) = \mathbf{1.03\,hp}$$

Example 21: If η = 0.85, determine the output energy level if the applied energy is 50 J.

Solution:

$$\eta = \frac{W_o}{W_i} \Rightarrow W_o = \eta\, W_i = (0.85)(50\,J) = \mathbf{42.5\,J}$$

Lecture 3: Series DC Resistive Circuits

3.1 Series Resistors:

First recognize that every fixed resistor has only two terminals to connect in a configuration—it is therefore referred to as a **two-terminal device**.

In Figure (3.1), one terminal of resistor R_2 is connected to resistor R_1 on one side, and the remaining terminal is connected to resistor R_3 on the other side, resulting in one, and only one, connection between adjoining resistors. When connected in this manner, the resistors have established a series connection.

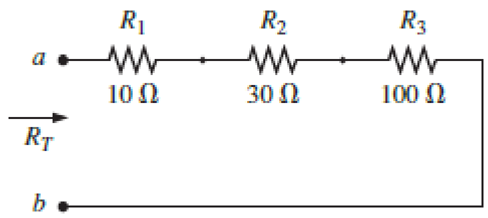


Figure (3.1): Series connection of resistors.

For resistors in series, ***the total resistance of a series configuration is the sum of the resistance levels.***

In equation form for any number (N) of resistors,

$$R_T = R_1 + R_2 + R_3 + R_4 + \cdots + R_N$$

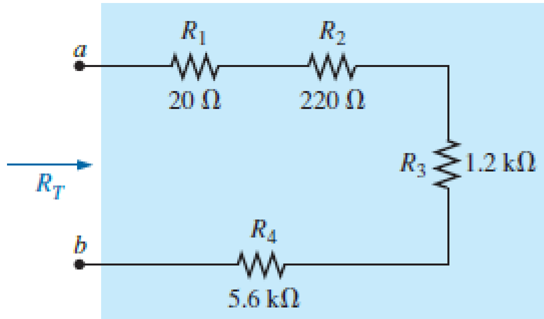
A result, ***the more resistors we add in series, the greater is the resistance, no matter what their value.***

Further, ***the largest resistor in a series combination will have the most impact on the total resistance.***

For the configuration in Figure (3.1), the total resistance is:

$$R_T = R_1 + R_2 + R_3 = 10\ \Omega + 30\ \Omega + 100\ \Omega = \mathbf{140}\ \Omega$$

Example 1: Determine the total resistance of the series connection in Figure below. Note that all the resistors appearing in this network are standard values.



Solution: Note in Figure above that even though resistor R_3 is on the vertical and resistor R_4 returns at the bottom to terminal b, all the resistors are in series since there are only two resistor leads at each connection point.

$$R_T = R_1 + R_2 + R_3 + R_4$$

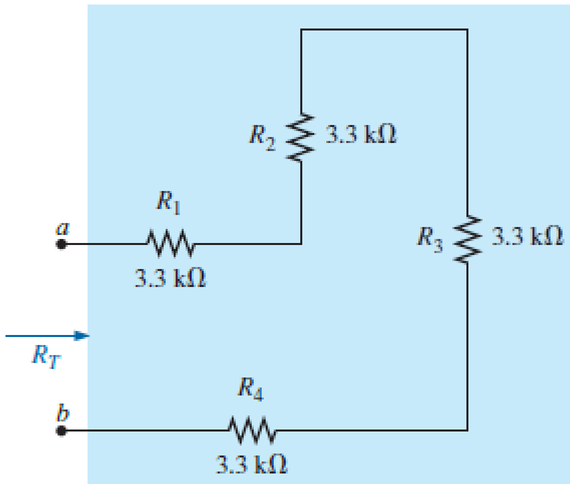
$$R_T = 20\ \Omega + 220\ \Omega + 1.2\ K\Omega + 5.6\ K\Omega = 7040\ \Omega = \mathbf{7.04\ K}\Omega$$

For the special case where resistors are the same value, equation $R_T = R_1 + R_2 + R_3 + R_4 + \cdots + R_N$ can be modified as follows:

$$\boxed{R_T = N\,R_N}$$

where N is the number of resistors in series of value R.

Example 2: Find the total resistance of the series resistors in Figure below. Again, recognize 3.3 KΩ as a standard value.



Solution:

$$R_T = N\,R_N = (4)(3.3\ K\Omega) = \mathbf{13.2\ K}\Omega$$

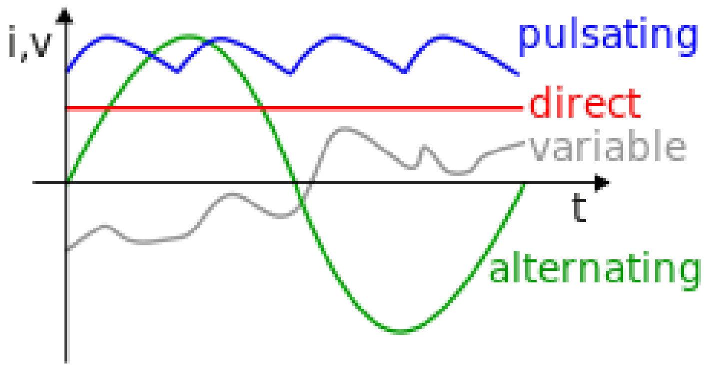
The total resistance of resistors in series is unaffected by the order in which they are connected.

3.2 Series Circuits:

Direct current (DC) is a type of electric current that has a fixed magnitude and direction. The net flow of DC electrons is the fixed direction from the negative terminal of the applied voltage to the positive terminal. On the other hand, alternating current has a magnitude and direction that changes over time.

Direct current may flow through a conductor such as a wire, but can also flow through semiconductors, insulators, or even through a vacuum as in electron or ion beams. The electric current flows in a constant direction, distinguishing it from alternating current (AC). The abbreviations AC and DC are often used to mean simply alternating and direct, as when they modify current or voltage.

In Figure below, the vertical axis shows current or voltage and the horizontal 't' axis measures time and shows the zero value.



If we now take an 8.4 V dc supply and connect it in series with the series resistors in Figure (3.1), we have the series circuit in Figure (3.2).

A circuit is any combination of elements that will result in a continuous flow of charge, or current, through the configuration.

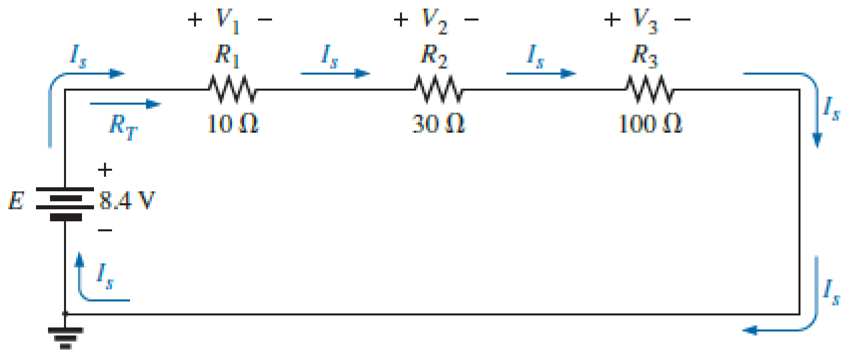


Figure (3.2): Schematic representation for a dc series circuit.

The direction of conventional current in a series dc circuit is such that it leaves the positive terminal of the supply and returns to the negative terminal, as shown in Figure (3.2).

The current is the same at every point in a series circuit.

Now that we have a complete circuit and current has been established, the level of current and the voltage across each resistor should be determined. To do this, return to Ohm's law and replace the resistance in the equation by the total resistance of the circuit. That is:

$$I_s = \frac{E}{R_T}$$

with the subscript s used to indicate source current.

It is important to realize that when a dc supply is connected, it does not "see" the individual connection of elements but simply the total resistance "seen" at the connection terminals, as shown in Figure (3.3(a)). The resulting current is:

$$I_s = \frac{E}{R_T} = \frac{8.4\ V}{140\ \Omega} = 0.06\ A = \mathbf{60\ mA}$$

The polarity of the voltage across a resistor is determined by the direction of the current.

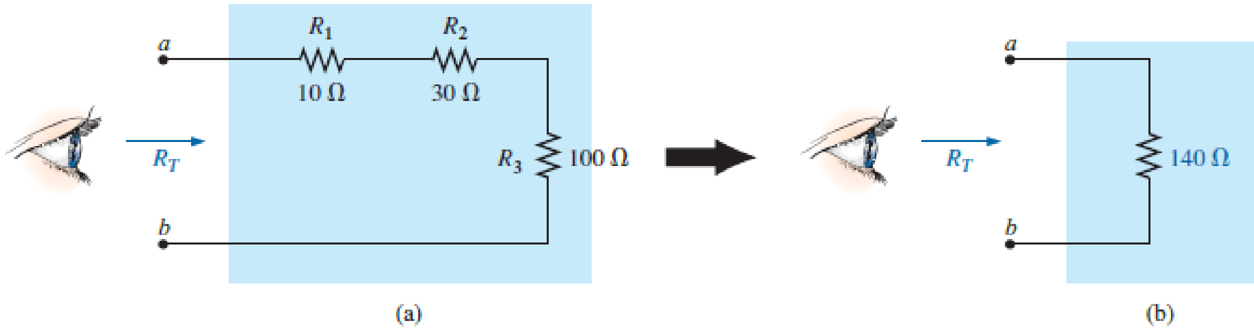


Figure (3.3): Resistance "seen" at the terminals of a series circuit.

The magnitude of the voltage drop across each resistor can then be found by applying Ohm's law using only the resistance of each resistor.

That is:

$$V_1 = I_1\,R_1$$
$$V_2 = I_2\,R_2$$
$$V_3 = I_3\,R_3$$

$$I_S = I_1 = I_2 = I_3$$

$$V_1 = I_1\,R_1 = I_s\,R_1 = (60\,mA)(10\,\Omega) = \mathbf{0.6\,V}$$

$$V_2 = I_2\,R_2 = I_s\,R_2 = (60\,mA)(30\,\Omega) = \mathbf{1.8\,V}$$

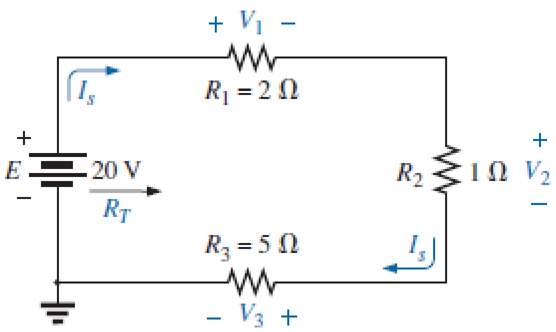
$$V_3 = I_3\,R_3 = I_s\,R_3 = (60\,mA)(100\,\Omega) = \mathbf{6.0\,V}$$

Example 3: For the series circuit in Figure below:

A) Find the total resistance R_T.

B) Calculate the resulting source current I_s.

C) Determine the voltage across each resistor.



Solution:

A)

$$R_T = R_1 + R_2 + R_3 = 2\,\Omega + 1\,\Omega + 5\,\Omega = \mathbf{8}\,\Omega$$

B)

$$I_s = \frac{E}{R_T} = \frac{20\,V}{8\,\Omega} = \mathbf{2.5\,A}$$

C)

$$V_1 = I_1\,R_1 = I_s\,R_1 = (2.5\,A)(2\,\Omega) = \mathbf{5\,V}$$

$$V_2 = I_2\,R_2 = I_s\,R_2 = (2.5\,A)(1\,\Omega) = \mathbf{2.5\,V}$$

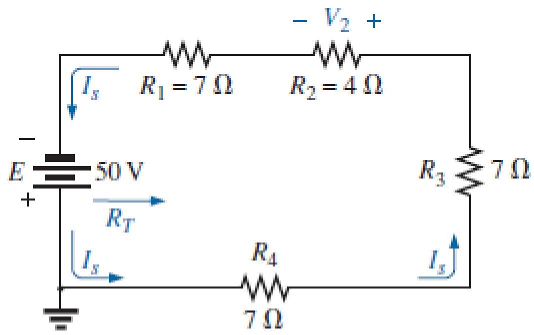
$$V_3 = I_3\,R_3 = I_s\,R_3 = (2.5\,A)(5\,\Omega) = \mathbf{12.5\,V}$$

Example 4: For the series circuit in Figure below:

A) Find the total resistance R_T.

B) Determine the source current I_s and indicate its direction on the circuit.

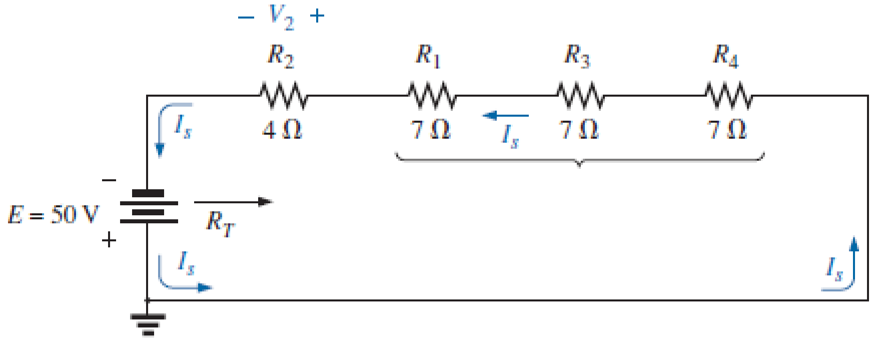
C) Find the voltage across resistor R_2 and indicate its polarity on the circuit.



Solution:

A) The elements of the circuit are rearranged as shown in Figure below.

$$R_T = R_2 + N\,R = 4\,\Omega + (3)(7\,\Omega) = 4\,\Omega + 21\,\Omega = \mathbf{25}\,\Omega$$



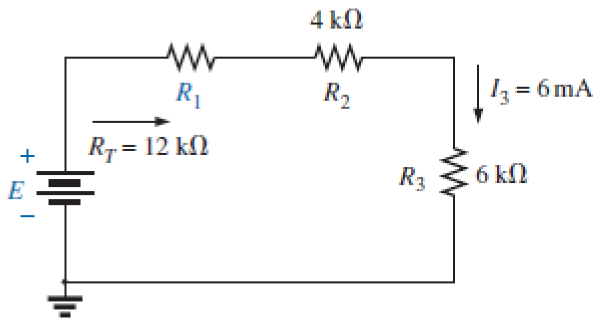
B) Note that because of the manner in which the dc supply was connected, the current now has a counterclockwise direction as shown in Figure above:

$$I_s = \frac{E}{R_T} = \frac{50\ V}{25\ \Omega} = \mathbf{2\ A}$$

C) The direction of the current will define the polarity for V_2 appearing in Figure above:

$$V_2 = I_2\ R_2 = I_s\ R_2 = (2\ A)(4\ \Omega) = \mathbf{8\ V}$$

Example 5: Given R_T and I_3, calculate R_1 and E for the circuit in Figure below.



Solution: Since we are given the total resistance, it seems natural to first write the equation for the total resistance and then insert what we know:

$$R_T = R_1 + R_2 + R_3$$

We find that there is only one unknown, and it can be determined with some simple mathematical manipulations. That is:

$$12\ K\Omega = R_1 + 4\ K\Omega + 6\ K\Omega = R_1 + 10\ K\Omega$$

and $12\ K\Omega - 10\ K\Omega = R_1$

so that: $R_1 = 2\ K\Omega$

The dc voltage can be determined directly from Ohm's law:

$$E = I_s\ R_T = I_3\ R_T = (6\ mA)(12\ K\Omega) = \mathbf{72\ V}$$

3.3 Power Distribution in a Series Circuit:

For any series circuit, such as that in Figure (3.4), ***the power applied by the dc supply must equal that dissipated by the resistive elements.***

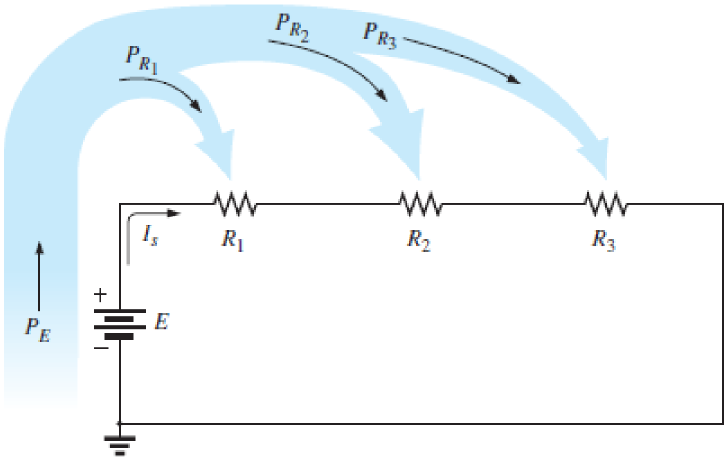


Figure (3.4): Power distribution in a series circuit.

In equation form:

$$P_E = P_{R_1} + P_{R_2} + P_{R_3}$$

The power delivered by the supply can be determined using:

$$P_E = E\, I_s \qquad \text{(watts, W)}$$

The power dissipated by the resistive elements can be determined by any of the following forms (shown for resistor R_1 only):

$$P_1 = V_1\, I_1 = I_1^2\, R_1 = \frac{V_1^2}{R_1} \qquad \text{(watts, W)}$$

Since the current is the same through series elements, you will find in the following examples that: ***In a series configuration, maximum power is delivered to the largest resistor.***

Example 6: For the series circuit in Figure below (all standard values):

A) Determine the total resistance R_T.

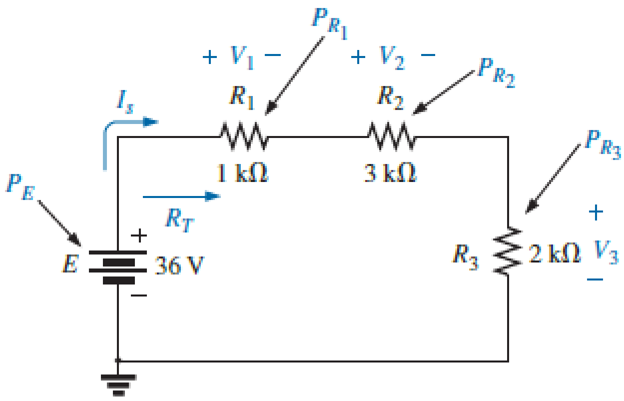
B) Calculate the current I_s.

C) Determine the voltage across each resistor.

D) Find the power supplied by the battery.

E) Determine the power dissipated by each resistor.

F) Comment on whether the total power supplied equals the total power dissipated.



Solution:

A)

$$R_T = R_1 + R_2 + R_3 = 1\ K\Omega + 3\ K\Omega + 2\ K\Omega = \mathbf{6\ K\Omega}$$

B)

$$I_s = \frac{E}{R_T} = \frac{36\ V}{6\ K\Omega} = \mathbf{6\ mA}$$

C)

$$V_1 = I_1\ R_1 = I_s\ R_1 = (6\ mA)(1\ K\Omega) = \mathbf{6\ V}$$

$$V_2 = I_2\ R_2 = I_s\ R_2 = (6\ mA)(3\ K\Omega) = \mathbf{18\ V}$$

$$V_3 = I_3\ R_3 = I_s\ R_3 = (6\ mA)(2\ K\Omega) = \mathbf{12\ V}$$

D)

$$P_E = E\ I_s = (36\ V)(6\ mA) = \mathbf{216\ mW}$$

E)

$$P_1 = V_1\ I_1 = (6\ V)(6\ mA) = \mathbf{36\ mW}$$

$$P_2 = I_2^2\ R_2 = (6\ mA)^2(3\ K\Omega) = \mathbf{108\ mW}$$

$$P_3 = \frac{V_3^2}{R_3} = \frac{(12\ V)^2}{2\ K\Omega} = \mathbf{72\ mW}$$

F)

$$P_E = P_{R_1} + P_{R_2} + P_{R_3} = 216\ mW$$

$$P_E = 36\ mW + 108\ mW + 72\ mW = \mathbf{216\ mW}\ \text{(checks)}$$

3.4 Voltage Source in Series:

Voltage sources can be connected in series, as shown in Figure (3.5), to increase or decrease the total voltage applied to a system. The net voltage is determined by summing the sources with the same polarity and subtracting the total of the sources with the opposite polarity. The net polarity is the polarity of the larger sum.

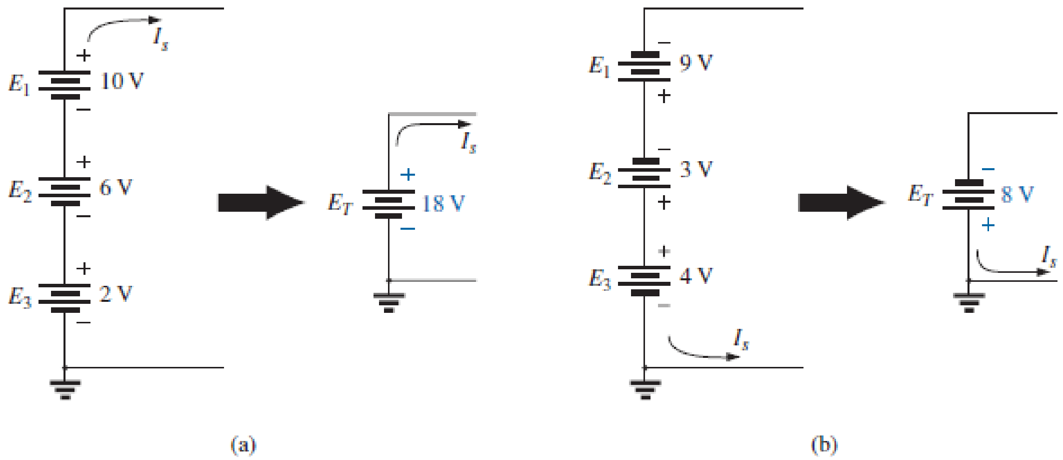


Figure (3.5): Reducing series dc voltage sources to a single source.

In Figure (3.5(a)), for example, the sources are all "pressuring" current to follow a clockwise path, so the net voltage is:

$$E_T = E_1 + E_2 + E_3 = 10\ V + 6\ V + 2\ V = \mathbf{18\ V}$$

In Figure (3.5(b)), however, the 4 V source is "pressuring" current in the clockwise direction while the other two are trying to establish current in the counterclockwise direction.

The net effect can be determined by finding the difference in applied voltage between those supplies "pressuring" current in one direction and the total in the other direction. In this case:

$$E_T = E_1 + E_2 - E_3 = 9\ V + 3\ V - 4\ V = \mathbf{8\ V}$$

with the polarity shown in the figure.

3.5 Kirchhoff's Voltage Law (KVL):

The algebraic sum of the potential rises and drops around a closed path (or closed loop) is zero.

In symbolic form it can be written as:

$$\sum_{\circlearrowright} V = 0$$ (Kirchhoff's voltage law in symbolic form)

where $\sum$ represents summation, $\circlearrowright$ the closed loop, and V the potential drops and rises. The term algebraic simply means paying attention to the signs that result in the equations as we add and subtract terms.

In Figure (3.6), as we proceed from point **d** to point **a** across the voltage source we move from a negative potential (the negative sign) to a positive potential (the positive sign), so a positive sign is given to the source voltage E. As we proceed from point a to point b, we encounter a positive sign followed by a negative sign, so a drop in potential has occurred, and a negative sign is applied. Continuing from b to c, we encounter another drop in potential, so another negative sign is applied. We then arrive back at the starting point d, and the resulting sum is set equal to zero as defined by equation above.

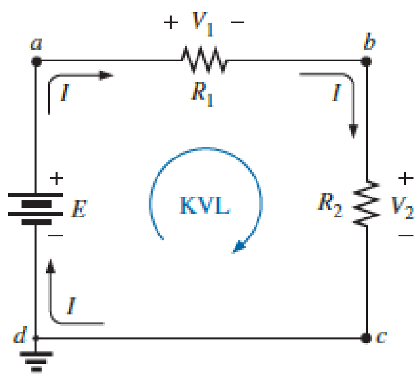


Figure (3.6): Applying Kirchhoff's voltage law to a series dc circuit.

Writing out the sequence with the voltages and the signs results in the following:

$$+E - V_1 - V_2 = 0$$

which can be rewritten as: $E = V_1 + V_2$

The result is particularly interesting because it tells us that ***the applied voltage of a series dc circuit will equal the sum of the voltage drops of the circuit.***

Kirchhoff's voltage law can also be written in the following form:

$$\sum_{\circlearrowright} V_{rises} = \sum_{\circlearrowright} V_{drops}$$

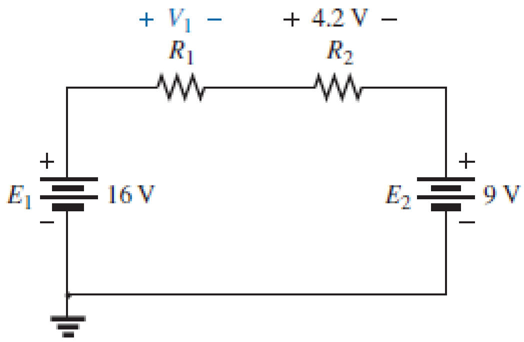
revealing that ***the sum of the voltage rises around a closed path will always equal the sum of the voltage drops.***

To demonstrate that the direction that you take around the loop has no effect on the results, let's take the counterclockwise path and compare results. The resulting sequence appears as:

$$-E + V_1 + V_2 = 0$$

yielding the same result of: $E = V_1 + V_2$

Example 7: Use Kirchhoff's voltage law to determine the unknown voltage for the circuit in Figure below.



Solution: When applying Kirchhoff's voltage law, be sure to concentrate on the polarities of the voltage rise or drop rather than on the type of element. In other words, do not treat a voltage drop across a resistive element differently from a voltage rise (or drop) across a source. If the polarity dictates that a drop has occurred, that is the important fact, not whether it is a resistive element or source.

Application of Kirchhoff's voltage law to the circuit in Figure above in the clockwise direction results in:

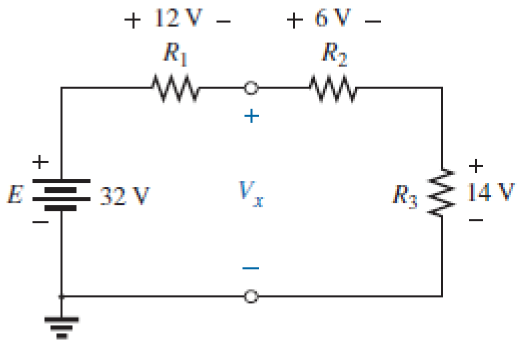
$$+E_1 - V_1 - V_2 - E_2 = 0$$

and

$$V_1 = E_1 - V_2 - E_2 = 16\ V - 4.2\ V - 9\ V = \mathbf{2.8\ V}$$

The result clearly indicates that you do not need to know the values of the resistors or the current to determine the unknown voltage. Sufficient information was carried by the other voltage levels to determine the unknown.

Example 8: Determine the unknown voltage for the circuit in Figure below.



Solution: In this case, the unknown voltage is not across a single resistive element but between two arbitrary points in the circuit. Simply apply Kirchhoff's voltage law around a path, including the source **or** resistor R_3. For the clockwise path, including the source, the resulting equation is the following:

$$+E - V_1 - V_x = 0$$

and

$$V_x = E - V_1 = 32\ V - 12\ V = \mathbf{20\ V}$$

For the clockwise path, including resistor R_3, the following results:

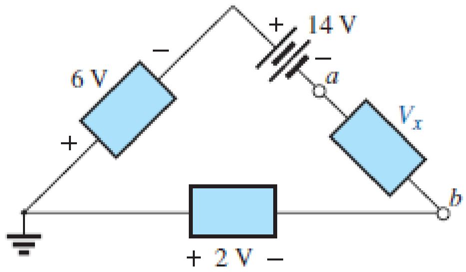
$$V_x - V_2 - V_3 = 0$$

and

$$V_x = V_2 + V_3 = 6\ V + 14\ V = \mathbf{20\ V}$$

providing exactly the same solution.

Example 9: Determine the voltage V_x for the circuit in Figure below. Note that the polarity of V_x was not provided.



Solution: For cases where the polarity is not included, simply make an assumption about the polarity, and apply Kirchhoff's voltage law as before. If the result has a positive sign, the assumed polarity was correct. If the result has a minus sign, the **magnitude is correct,** but the assumed polarity must be reversed. In this case, if we assume point *a* to be positive and point *b* to be negative, an application of Kirchhoff's voltage law in the clockwise direction results in:

$$-6\ V - 14\ V - V_x + 2\ V = 0$$

and

$$V_x = -6\ V - 14\ V + 2\ V = \mathbf{-18\ V}$$

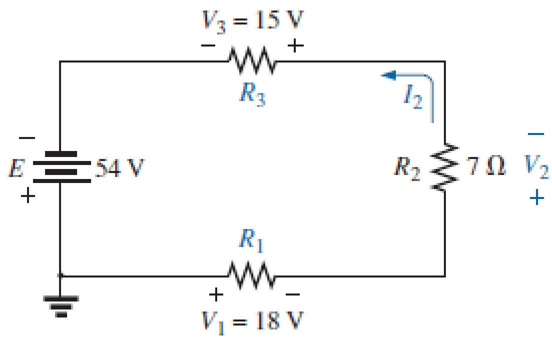
Since the result is negative, we know that point *a* should be negative and point *b* should be positive, but the magnitude of 18 V is correct.

Example 10: For the series circuit in Figure below.

A) Determine V_2 using Kirchhoff's voltage law.

B) Determine current I_2.

C) Find R_1 and R_3.



Solution:

A) Applying Kirchhoff's voltage law in the clockwise direction starting at the negative terminal of the supply results in:

$$-E + V_3 + V_2 + V_1 = 0$$

and $E = V_1 + V_2 + V_3$ (as expected)

so that: $V_2 = E - V_1 - V_3 = 54\,V - 18\,V - 15\,V = \mathbf{21\,V}$

B)

$$I_2 = \frac{V_2}{R_2} = \frac{21\,V}{7\,\Omega} = \mathbf{3\,A}$$

C) $I_S = I_1 = I_2 = I_3 = 3\,A$

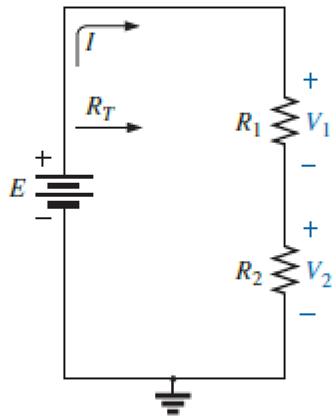
$$R_1 = \frac{V_1}{I_1} = \frac{18\,V}{3\,A} = \mathbf{6}\,\Omega$$

with

$$R_3 = \frac{V_3}{I_3} = \frac{15\,V}{3\,A} = \mathbf{5}\,\Omega$$

3.6 Voltage Divider Rule (VDR):

The voltage across a resistor in a series circuit is equal to the value of that resistor times the total applied voltage divided by the total resistance of the series configuration.



First, determine the total resistance, as follows:

$$R_T = R_1 + R_2$$

Then:

$$I_s = I_1 = I_2 = \frac{E}{R_T}$$

Apply Ohm's law to each resistor:

$$V_1 = I_1 R_1 = \left(\frac{E}{R_T}\right) R_1 = \frac{R_1\,E}{R_T}$$

$$V_2 = I_2 R_2 = \left(\frac{E}{R_T}\right) R_2 = \frac{R_2\,E}{R_T}$$

The resulting format for V_1 and V_2 is:

$$\boxed{V_x = \frac{R_x\,E}{R_T}} \qquad \text{(voltage divider rule)}$$

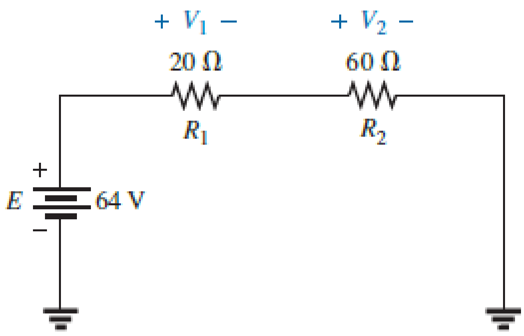
Where V_x is the voltage across the resistor R_x, E is the impressed voltage across the series elements, and R_T is the total resistance of the series circuit.

The voltage divider rule can be extended to the voltage across two or more series elements if the resistance in the numerator of equation $V_x = \frac{R_x\,E}{R_T}$ is expanded to include the total resistance of the series resistors across which the voltage is to be found (R'). That is:

$$\boxed{V' = \frac{R'\,E}{R_T}}$$

Example 11: For the series circuit in Figure below.

- **A)** Without making any calculations, how much larger would you expect the voltage across R_2 to be compared to that across R_1?
- **B)** Find the voltage V_1 using only the voltage divider rule.
- **C)** Using the conclusion of part (a), determine the voltage across R_2.
- **D)** Use the voltage divider rule to determine the voltage across R_2, and compare your answer to your conclusion in part (c).
- **E)** How does the sum of V_1 and V_2 compare to the applied voltage?



Solution:

A) Since resistor R_2 is three times R_1 and they are in series, it is expected that $V_2 = 3V_1$.

B)

$$V_1 = \frac{R_1\, E}{R_T} = 20\ \Omega\left(\frac{64\ V}{20\ \Omega + 60\ \Omega}\right) = 20\ \Omega\left(\frac{64\ V}{80\ \Omega}\right) = \mathbf{16\ V}$$

C)

$$V_2 = 3V_1 = (3)(16\ V) = \mathbf{48\ V}$$

D)

$$V_2 = \frac{R_2\, E}{R_T} = 60\ \Omega\left(\frac{64\ V}{20\ \Omega + 60\ \Omega}\right) = 60\ \Omega\left(\frac{64\ V}{80\ \Omega}\right) = \mathbf{48\ V}$$

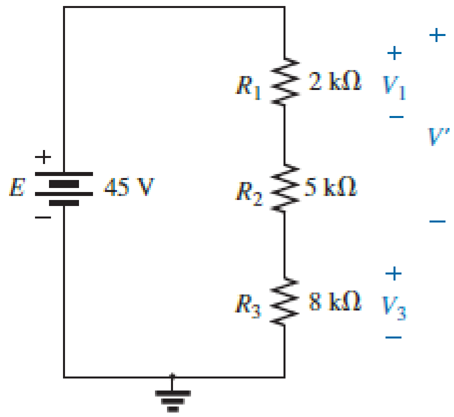
The results are an exact match.

E)

$$E = V_1 + V_2$$

$$64\ V = 16\ V + 48\ V = \mathbf{64\ V}\ \text{(checks)}$$

Example 12: Using the voltage divider rule, determine voltages V_1 and V_3 for the series circuit in Figure below.



Solution:

$$R_T = R_1 + R_2 + R_3 = 2\ K\Omega + 5\ K\Omega + 8\ K\Omega = 15\ K\Omega$$

$$V_1 = \frac{R_1\ E}{R_T} = 2\ K\Omega\left(\frac{45\ V}{15\ K\Omega}\right) = \mathbf{6\ V}$$

$$V_3 = \frac{R_3\ E}{R_T} = 8\ K\Omega\left(\frac{45\ V}{15\ K\Omega}\right) = \mathbf{24\ V}$$

Lecture 4: Parallel DC Resistive Circuits

4.1 Parallel Resistors:

The term parallel is used so often to describe a physical arrangement between two elements. In general, ***two elements, branches, or circuits are in parallel if they have two points in common.***

For instance, in Figure (4.1(a)), the two resistors are in parallel because they are connected at points **a** and **b**. If both ends were not connected as shown, the resistors would not be in parallel. In Figure (4.1(b)), resistors R_1 and R_2 are in parallel because they again have points **a** and **b** in common. R_1 is not in parallel with R_3 because they are connected at only one point (**b**). Further, R_1 and R_3 are not in series because a third connection appears at point **b**. The same can be said for resistors R_2 and R_3. In Figure (4.1(c)), resistors R_1 and R_2 are in series because they have only one point in common that is not connected elsewhere in the network. Resistors R_1 and R_3 are not in parallel because they have only point **a** in common. In addition, they are not in series because of the third connection to point **b**. The same can be said for resistors R_2 and R_3. In a broader context, it can be said that the series combination of resistors R_1 and R_2 is in parallel with resistor R_3. Furthermore, even though the discussion above was only for resistors, it can be applied to any two-terminal elements such as voltage sources and meters.

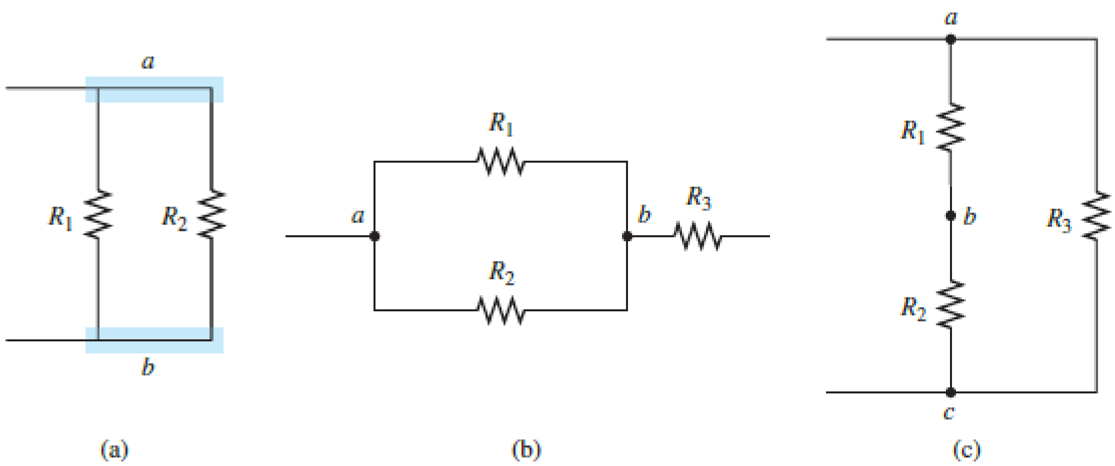


Figure (4.1): (a) Parallel resistors; (b) R_1 and R_2 are in parallel; (c) R_3 is in parallel with the series combination of R_1 and R_2.

For resistors in parallel as shown in Figure (4.2), the total resistance is determined from the following equation:

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_N}$$

For two parallel resistors, the total resistance is determined by:

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$$

Multiplying the top and bottom of each term of the right side of the equation by the other resistance results in:

$$\frac{1}{R_T} = \left(\frac{R_2}{R_2}\right)\frac{1}{R_1} + \left(\frac{R_1}{R_1}\right)\frac{1}{R_2} = \frac{R_2}{R_1\,R_2} + \frac{R_1}{R_1\,R_2} = \frac{R_2 + R_1}{R_1\,R_2}$$

and

$$R_T = \frac{R_1\,R_2}{R_1 + R_2}$$

Since $G = 1/R$, the equation can also be written in terms of conductance levels, as follows:

$$G_T = G_1 + G_2 + G_3 + \cdots + G_N \qquad \text{(siemens, S)}$$

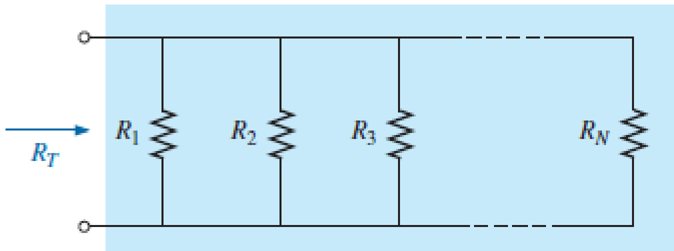


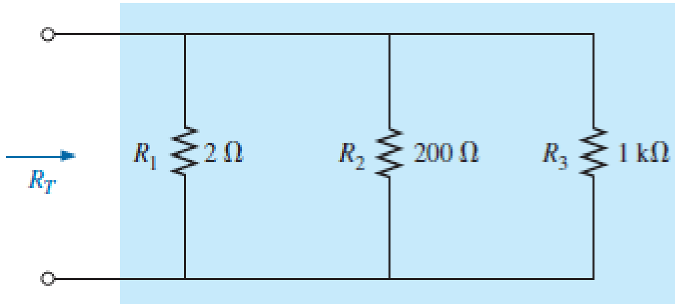
Figure (4.2): Parallel combination of resistors.

In general, however, when the total resistance is desired, the following format is applied:

$$R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_N}}$$

Example 1:

A) By inspection, which parallel element in Figure below has the least conductance? Determine the total conductance of the network and note whether your conclusion was verified.



B) Determine the total resistance from the results of part (a).

Solution:

A) Since the 1 KΩ resistor has the largest resistance and therefore the largest opposition to the flow of charge (level of conductivity), it will have the lowest level of conductance:

$$G_1 = \frac{1}{R_1} = \frac{1}{2\ \Omega} = \mathbf{0.5\ S}$$

$$G_2 = \frac{1}{R_2} = \frac{1}{200\ \Omega} = 0.005\ S = \mathbf{5\ mS}$$

$$G_3 = \frac{1}{R_3} = \frac{1}{1\ K\Omega} = \frac{1}{1000\ \Omega} = 0.001\ S = \mathbf{1\ mS}$$

$$G_T = G_1 + G_2 + G_3 = 0.5\ S + 5\ mS + 1\ mS = \mathbf{506\ mS}$$

Note the difference in conductance level between the 2 Ω (500 mS) and the 1 K Ω (1 mS) resistor.

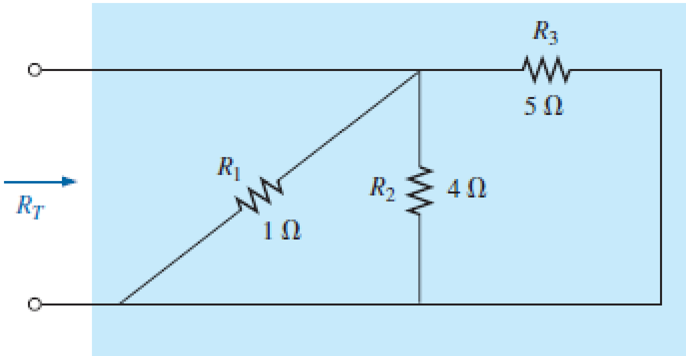
B)

$$R_T = \frac{1}{G_T} = \frac{1}{506\ mS} = \mathbf{1.976}\ \Omega$$

$$R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} = \frac{1}{\frac{1}{2\ \Omega} + \frac{1}{200\ \Omega} + \frac{1}{1\ K\Omega}}$$

$$R_T = \frac{1}{0.5\ S + 0.005\ S + 0.001\ S} = \mathbf{1.98}\ \Omega$$

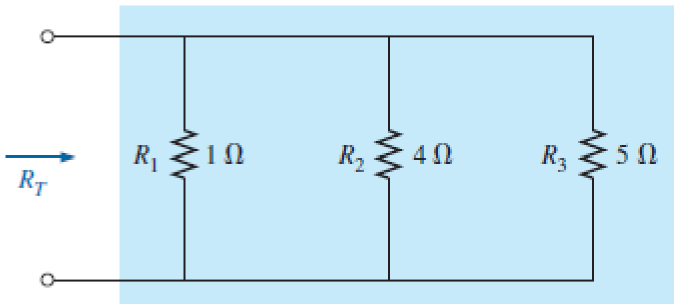
Example 2: Find the total resistance of the configuration in Figure below.



Solution: First the network is redrawn as shown in Figure below to clearly demonstrate that all the resistors are in parallel.

$$R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} = \frac{1}{\frac{1}{1\ \Omega} + \frac{1}{4\ \Omega} + \frac{1}{5\ \Omega}}$$

$$R_T = \frac{1}{1\ S + 0.25\ S + 0.2\ S} \cong \mathbf{0.69}\ \Omega$$



The total resistance of parallel resistors is always less than the value of the smallest resistor.

If the smallest resistance of a parallel combination is much smaller than that of the other parallel resistors, the total resistance will be very close to the smallest resistance value.

4.2 Parallel Circuits:

A parallel circuit can now be established by connecting a supply across a set of parallel resistors as shown in Figure (4.3). The positive terminal of the supply is directly connected to the top of each resistor, while the negative terminal is connected to the bottom of each resistor. Therefore, it should be quite clear that the applied voltage is the same across each resistor. In general, ***the voltage is always the same across parallel elements.***

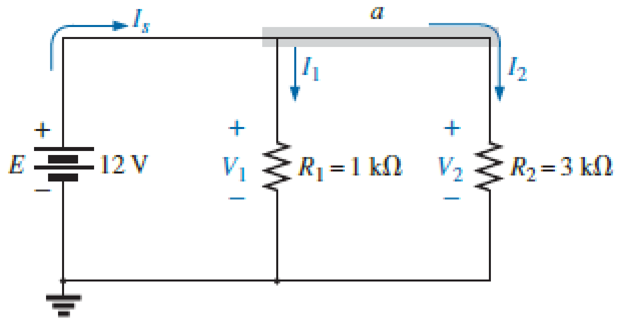


Figure (4.3): Parallel network.

For the voltages of the circuit in Figure (4.3), the result is that:

$$V_1 = V_2 = E$$

The parallel combination of elements reacts only to the total resistance of the circuit, as shown in Figure (4.4). The source current can then be determined using Ohm's law:

$$I_s = \frac{E}{R_T}$$

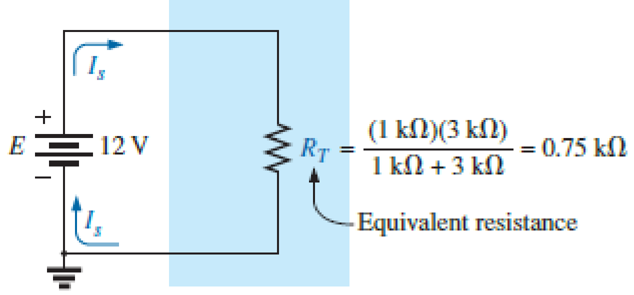


Figure (4.4): Replacing the parallel resistors in Figure (4.3) with the equivalent total resistance.

Since the voltage is the same across parallel elements, the current through each resistor can also be determined using Ohm's law. That is:

$$I_1 = \frac{V_1}{R_1} = \frac{E}{R_1} \qquad I_2 = \frac{V_2}{R_2} = \frac{E}{R_2}$$

And The relationship between the source current and the parallel resistor currents can be derived by simply taking the equation for the total resistance:

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$$

Multiplying both sides by the applied voltage gives:

$$E\left(\frac{1}{R_T}\right) = E\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

resulting in:

$$\frac{E}{R_T} = \frac{E}{R_1} + \frac{E}{R_2}$$

Then note that $E/R_1 = I_1$ and $E/R_2 = I_2$ to obtain:

$$I_s = I_1 + I_2$$

The result reveals a very important property of parallel circuits: ***For single-source parallel networks, the source current (I_s) is always equal to the sum of the individual branch currents.***

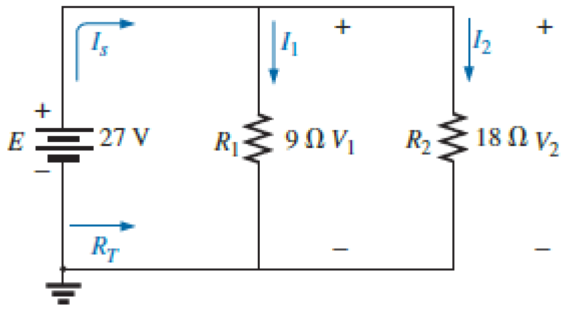
Example 3: For the parallel network in Figure below:

A) Find the total resistance.

B) Calculate the source current.

C) Determine the current through each parallel branch.

D) Show that equation $I_s = I_1 + I_2$ is satisfied.



Solution:

A)

$$R_T = \frac{R_1\,R_2}{R_1+R_2} = \frac{(9\,\Omega)(18\,\Omega)}{9\,\Omega+18\,\Omega} = \frac{162}{27}\Omega = \mathbf{6\,\Omega}$$

B) Applying Ohm's law gives:

$$I_s = \frac{E}{R_T} = \frac{27\,V}{6\,\Omega} = \mathbf{4.5\,A}$$

C) Applying Ohm's law gives:

$$I_1 = \frac{V_1}{R_1} = \frac{E}{R_1} = \frac{27\,V}{9\,\Omega} = \mathbf{3\,A}$$

$$I_2 = \frac{V_2}{R_2} = \frac{E}{R_2} = \frac{27\,V}{18\,\Omega} = \mathbf{1.5\,A}$$

D) Substituting values from parts (b) and (c) gives:

$$I_s = 4.5\,A = I_1 + I_2 = 3\,A + 1.5\,A = \mathbf{4.5\,A}\ \text{(checks)}$$

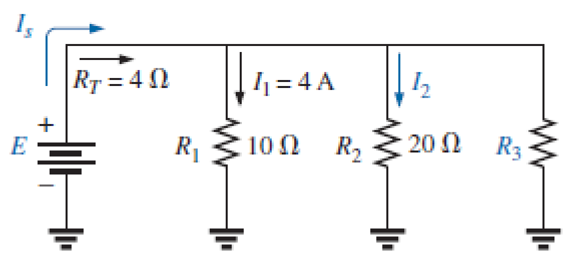
Example 4: Given the information provided in Figure below:

A) Determine R_3.

B) Find the applied voltage E.

C) Find the source current I_s.

D) Find I_2.



Solution:

A)

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Substituting gives:

$$\frac{1}{4\ \Omega} = \frac{1}{10\ \Omega} + \frac{1}{20\ \Omega} + \frac{1}{R_3}$$

so that:

$$0.25\ S = 0.1\ S + 0.05\ S + \frac{1}{R_3}$$

and

$$0.25\ S = 0.15\ S + \frac{1}{R_3}$$

with

$$\frac{1}{R_3} = 0.1\ S \Rightarrow R_3 = \frac{1}{0.1\ S} = \mathbf{10}\ \Omega$$

B) Using Ohm's law gives:

$$E = V_1 = I_1 R_1 = (4\ A)(10\ \Omega) = \mathbf{40\ V}$$

C)

$$I_s = \frac{E}{R_T} = \frac{40\ V}{4\ \Omega} = \mathbf{10\ A}$$

D) Applying Ohm's law gives:

$$I_2 = \frac{V_2}{R_2} = \frac{E}{R_2} = \frac{40\ V}{20\ \Omega} = \mathbf{2\ A}$$

4.3 Power Distribution in a Parallel Circuit:

In fact, ***for any network composed of resistive elements, the power applied by the source will equal that dissipated by the resistive elements.***

For the parallel circuit in Figure (4.5):

$$P_E = P_{R_1} + P_{R_2} + P_{R_3}$$

which is exactly the same as obtained for the series combination.

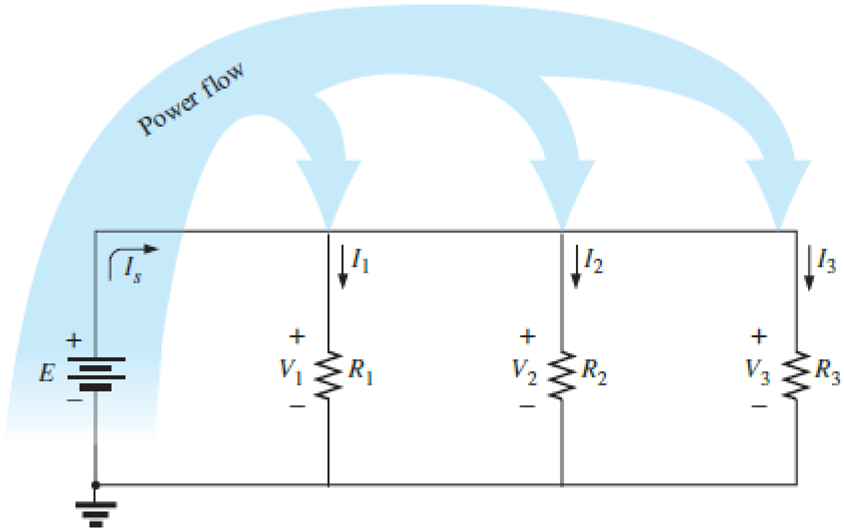


Figure (4.5): Power flow in a dc parallel network.

The power delivered by the source:

$$P_E = E\ I_s \quad \text{(watts, W)}$$

as is the equation for the power to each resistor (shown for R_1 only):

$$P_1 = V_1\ I_1 = I_1^2\ R_1 = \frac{V_1^2}{R_1} \quad \text{(watts, W)}$$

In the equation $P = V^2/R$, the voltage across each resistor in a parallel circuit will be the same. The only factor that changes is the resistance in the denominator of the equation. The result is that: ***In a parallel resistive network, the larger the resistor, the less is the power absorbed.***

Example 5: For the parallel network in Figure below (all standard values):

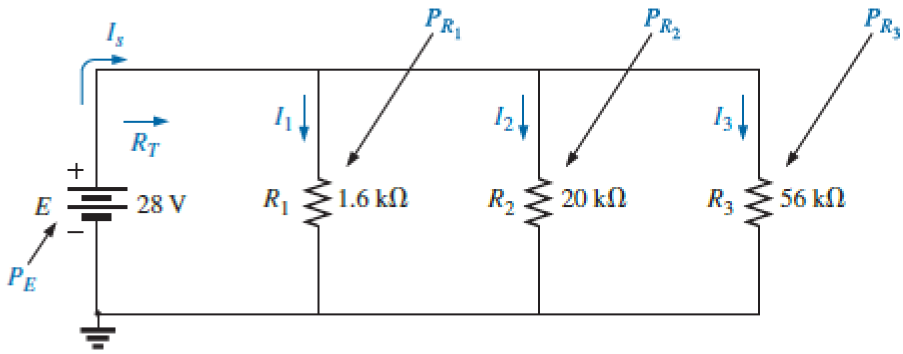
A) Determine the total resistance R_T.

B) Find the source current and the current through each resistor.

C) Calculate the power delivered by the source.

D) Determine the power absorbed by each parallel resistor.

E) Verify equation $P_E = P_{R_1} + P_{R_2} + P_{R_3}$.



Solution:

A) Without making a single calculation, it should now be apparent from previous examples that the total resistance is less than 1.6 kΩ and very close to this value because of the magnitude of the other resistance levels:

$$R_T = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}} = \frac{1}{\frac{1}{1.6\ K\Omega}+\frac{1}{20\ K\Omega}+\frac{1}{56\ K\Omega}}$$

$$R_T = \frac{1}{625\times10^{-6} + 50\times10^{-6} + 17.867\times10^{-6}} = \mathbf{1.44}\ \boldsymbol{k}\ \Omega$$

B) Applying Ohm's law gives:

$$I_s = \frac{E}{R_T} = \frac{28\ V}{1.44\ k\ \Omega} = \mathbf{19.44}\ \boldsymbol{mA}$$

Recalling that current always seeks the path of least resistance immediately tells us that the current through the 1.6 $K\Omega$ resistor will be the largest and the current through the 56 $K\Omega$ resistor the smallest.

Applying Ohm's law again gives:

$$I_1 = \frac{V_1}{R_1} = \frac{E}{R_1} = \frac{28\ V}{1.6\ K\Omega} = \mathbf{17.5}\ \boldsymbol{mA}$$

$$I_2 = \frac{V_2}{R_2} = \frac{E}{R_2} = \frac{28\ V}{20\ K\Omega} = \mathbf{1.4}\ \boldsymbol{mA}$$

$$I_3 = \frac{V_3}{R_3} = \frac{E}{R_3} = \frac{28\ V}{56\ K\Omega} = \mathbf{0.5\ mA}$$

C)

$$P_E = E\ I_s = (28\ V)(19.4\ mA) = \mathbf{543.2\ mW}$$

D) Applying each form of the power equation gives:

$$P_1 = V_1\ I_1 = E\ I_1 = (28\ V)(17.5\ mA) = \mathbf{490\ mW}$$

$$P_2 = I_2^2\ R_2 = (1.4\ mA)^2(20\ K\Omega) = \mathbf{39.2\ mW}$$

$$P_3 = \frac{V_3^2}{R_3} = \frac{(28\ V))^2}{56\ K\Omega} = \mathbf{14\ mW}$$

A review of the results clearly substantiates the fact that the larger the resistor, the less is the power absorbed.

E)

$$P_E = P_{R_1} + P_{R_2} + P_{R_3}$$

$$\mathbf{543.2\ mW} = 490\ mW + 39.2\ mW + 14\ mW = \mathbf{543.2\ mW}\ \text{(checks)}$$

4.4 Kirchhoff's Current Law (KCL):

The algebraic sum of the currents entering and leaving a junction (or region) of a network is zero.

The law can also be stated in the following way: ***The sum of the currents entering a junction (or region) of a network must equal the sum of the currents leaving the same junction (or region).***

In equation form, the above statement can be written as follows:

$$\sum I_i = \sum I_o$$

with I_i representing the current entering, or "in," and I_o representing the current leaving, or "out."

In Figure (4.6), for example, the shaded area can enclose an entire system or a complex network, or it can simply provide a connection point (junction) for the displayed currents. In each case, the current entering must equal that leaving, as required by equation above:

$$\sum I_i = \sum I_o$$

$$I_1 + I_4 = I_2 + I_3$$

$$4\ A + 8\ A = 2\ A + 10\ A$$

$$\mathbf{12\ A = 12\ A}\ \text{(checks)}$$

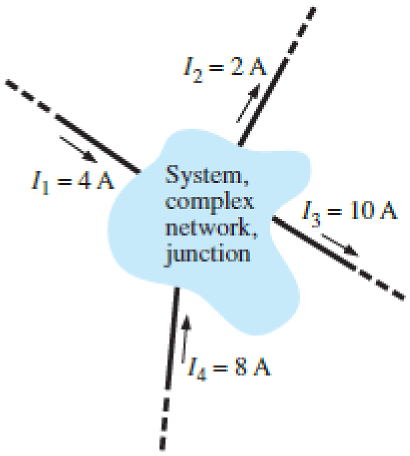
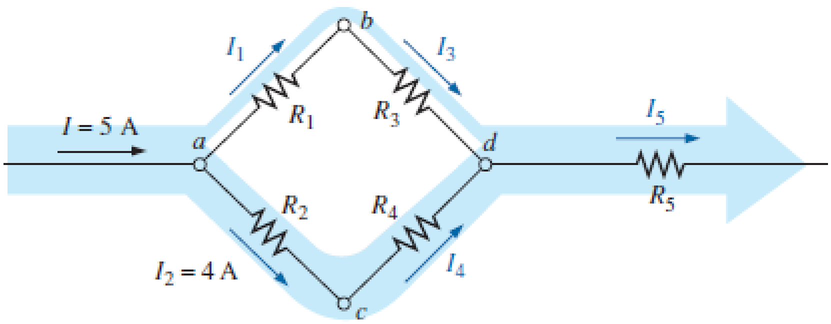


Figure (4.6): Introducing Kirchhoff's current law.

Example 6: Determine currents I_1, I_3, I_4, and I_5 for the network in Figure below.



Solution: In this configuration, four nodes are defined. Nodes *a* and *c* have only one unknown current at the junction, so Kirchhoff's current law can be applied at either junction.

At node *a*:

$$\sum I_i = \sum I_o$$

$$I = I_1 + I_2$$

$$5\ A = I_1 + 4\ A$$

and $I_1 = 5\,A - 4\,A = \mathbf{1\,A}$

At node *c:*

$$\sum I_i = \sum I_o$$

$$I_2 = I_4$$

and $I_4 = I_2 = \mathbf{4\,A}$

Using the above results at the other junctions results in the following.

At node *b:*

$$\sum I_i = \sum I_o$$

$$I_1 = I_3$$

and $I_3 = I_1 = \mathbf{1\,A}$

At node *d:*

$$\sum I_i = \sum I_o$$

$$I_3 + I_4 = I_5$$

$$1\,A + 4\,A = I_5 = \mathbf{5\,A}$$

If we enclose the entire network, we find that the current entering from the far left is I = 5 A, while the current leaving from the far right is $I_5 = 5\,A$. The two must be equal since the net current entering any system must equal the net current leaving.

4.5 Current Divider Rule (CDR):

The current divider rule:

$$I_x = \frac{R_T}{R_x} I_T$$

states that ***the current through any branch of a parallel resistive network is equal to the total resistance of the parallel network divided by the resistance of the resistor of interest and multiplied by the total current entering the parallel configuration.***

In Figure (4.7(a)), the current I_T (using the subscript T to indicate the total entering current) splits between the N parallel resistors and then gathers itself together again at the bottom of the configuration. In Figure (4.7(b)), the parallel combination of resistors has been replaced by a single resistor equal to the total resistance of the parallel combination as determined in the previous sections.

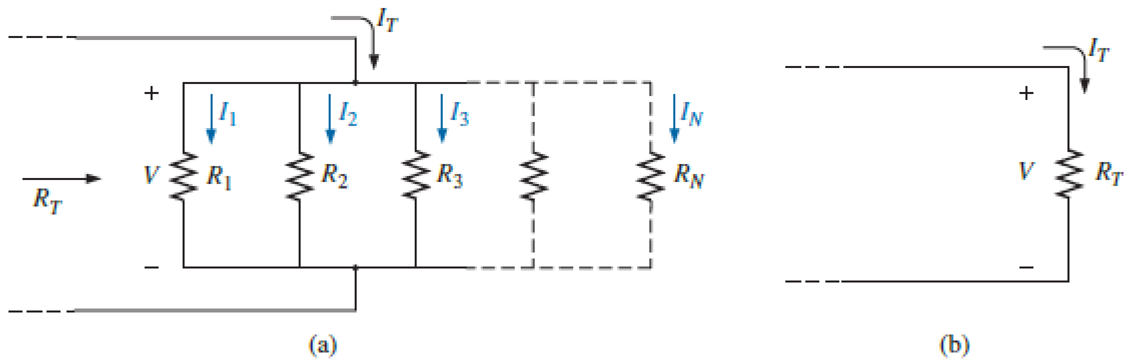


Figure (4.7): Deriving the current divider rule: (a) parallel network of N parallel resistors; (b) reduced equivalent of part (a).

The current I_T can then be determined using Ohm's law:

$$I_T = \frac{V}{R_T}$$

Since the voltage V is the same across parallel elements, the following is true:

$$V = I_1\,R_1 = I_2\,R_2 = I_3\,R_3 = \cdots = I_x\,R_x$$

where the product $I_x\,R_x$ refers to any combination in the series.

Substituting for V in the above equation for I_T, we have:

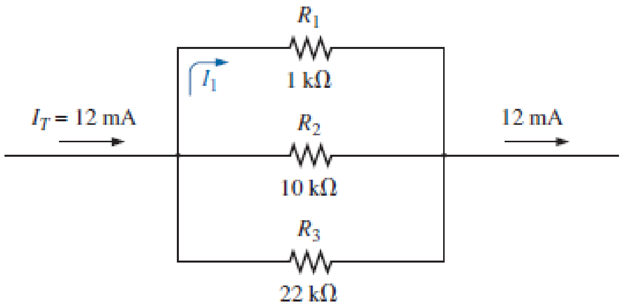
$$I_T = \frac{I_x\,R_x}{R_T}$$

Solving for I_x, the final result is the current divider rule:

$$I_x = \frac{R_T}{R_x} I_T$$

Since R_T and I_T are constants, for a particular configuration the larger the value of R_x (in the denominator), the smaller is the value of I_x for that branch, confirming the fact that current always seeks the path of least resistance.

Example 7: For the parallel network in Figure below, determine current I_1.



Solution:

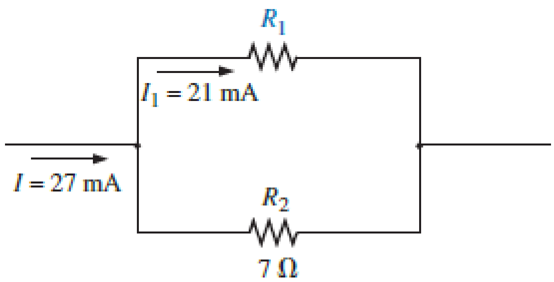
$$R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} = \frac{1}{\frac{1}{1\ K\Omega} + \frac{1}{10\ K\Omega} + \frac{1}{22\ K\Omega}}$$

$$R_T = \frac{1}{1 \times 10^{-3} + 100 \times 10^{-6} + 45.46 \times 10^{-6}} = \mathbf{873.01}\ \Omega$$

$$I_1 = \frac{R_T}{R_1} I_T = \frac{(873.01\ \Omega)}{(1\ K\Omega)}(12\ mA) = (0.873)(12\ mA) = \mathbf{10.48\ mA}$$

with the smallest parallel resistor receives the majority of the current.

Example 8: Determine resistor R_1 in Figure below to implement the division of current shown.



Solution: For the approach using the current divider rule:

$$I_1 = \frac{R_2}{R_1 + R_2} I_T$$

$$21\ mA = \frac{7\ \Omega}{R_1 + 7\ \Omega}(27\ mA)$$

$$(R_1 + 7\ \Omega)(21\ mA) = (7\ \Omega)(27\ mA)$$

$$(21\ mA)R1 + 147\ mV = 189\ mV$$

$$(21\ mA)R1 = 189\ mV - 147\ mV = 42\ mV$$

and

$$R1 = \frac{42\ mV}{21\ mA} = \mathbf{2}\ \Omega$$

4.6 Voltage Sources in Parallel:

Because the voltage is the same across parallel elements, ***voltage sources can be placed in parallel only if they have the same voltage.***

The primary reason for placing two or more batteries or supplies in parallel is to increase the current rating above that of a single supply. For example, in Figure (4.8), two ideal batteries of 12 V have been placed in parallel. The total source current using Kirchhoff's current law is now the sum of the rated currents of each supply. The resulting power available will be twice that of a single supply if the rated supply current of each is the same. That is:

with $I_1 = I_2 = I$

then $P_T = E(I_1 + I_2) = E(I + I) = E(2I) = 2(E\ I) = 2P_{(one\ supply)}$

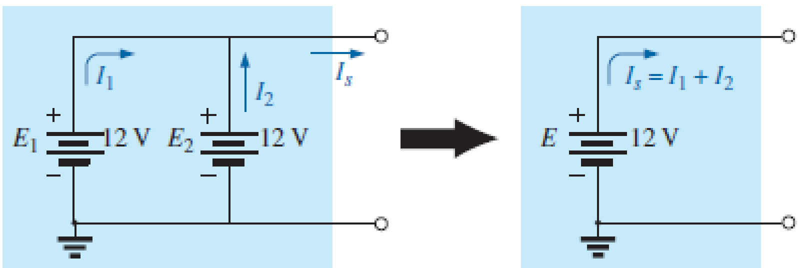


Figure (4.8): Demonstrating the effect of placing two ideal supplies of the same voltage in parallel.

If for some reason two batteries of different voltages are placed in parallel, both will become ineffective or damaged because the battery with the larger voltage will rapidly discharge through the battery with the smaller terminal voltage.

In general, ***it is always recommended that when you are replacing batteries in series or parallel, replace all the batteries.***

A fresh battery placed in parallel with an older battery probably has a higher terminal voltage and immediately starts discharging through the older battery. In addition, the available current is less for the older battery, resulting in a higher-than-rated current drain from the newer battery when a load is applied.

4.7 Series – Parallel Circuits:

A series-parallel configuration is one that is formed by a combination of series and parallel elements. A complex configuration is one in which none of the elements are in series or parallel.

The network of Figure (4.9) is redrawn as Figure (4.10(a)). For this discussion, let us assume that voltage V_4 is desired. First combine the series resistors R_3 and R_4 to form an equivalent resistor R' as shown in Figure (4.10(b)). Resistors R_2 and R' are then in parallel and can be combined to establish an equivalent resistor R'_T as shown in Figure (4.10(c)). Resistors R_1 and R'_T are then in series and can be combined to establish the total resistance of the network as shown in Figure (4.10(d)). The **reduction phase** of the analysis is now complete. The network cannot be put in a simpler form. We can now proceed with the **return phase** whereby we work our way back to the desired voltage V_4. Due to the resulting series configuration, the source current is also the current through R_1 and R'_T. The voltage across R'_T (and therefore across R_2) can be determined using Ohm's law as shown in Figure (4.10(e)). Finally, the desired voltage V_4 can be determined by an application of the voltage divider rule as shown in Figure (4.10(f)).

The reduce and return approach has now been introduced. This process enables you to reduce the network to its simplest form across the source and then determine the source current. In the return phase, you use the resulting source current to work back to the desired unknown. For most single-source series-parallel networks, the above approach provides a viable option toward the solution. In some cases, shortcuts can be applied that save some time and energy.

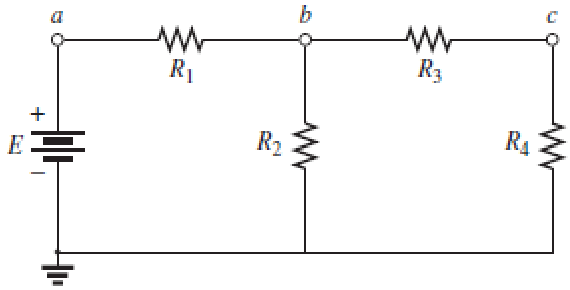


Figure (4.9): Series-parallel dc network.

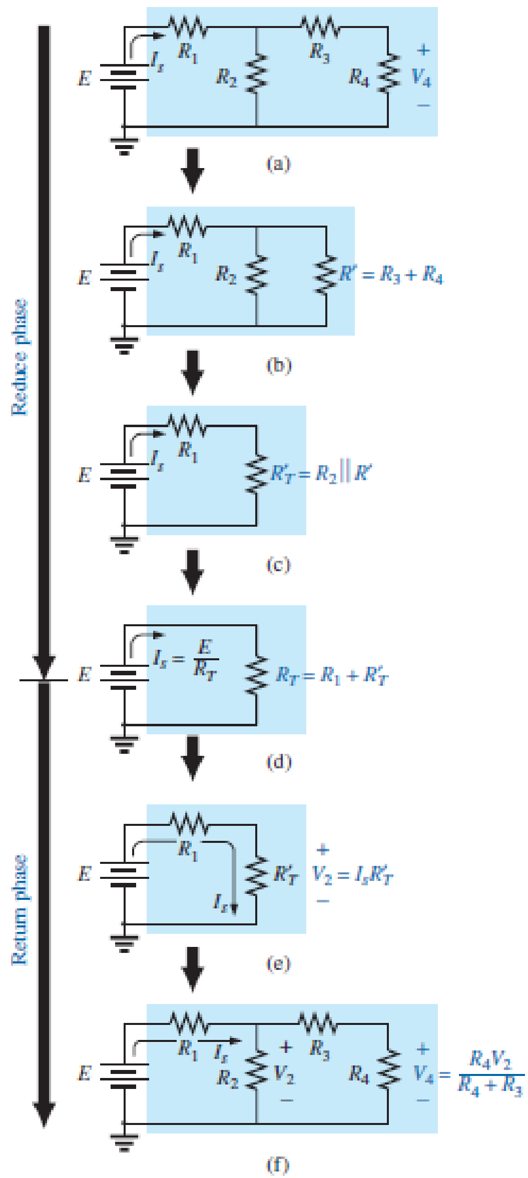
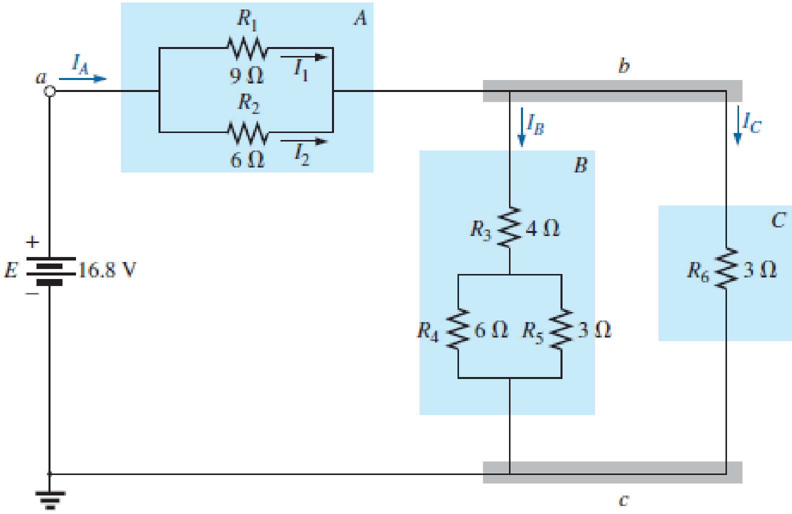


Figure (4.10): Introducing the reduce and return approach.

Example 9: Determine all the currents and voltages of the network in Figure below.



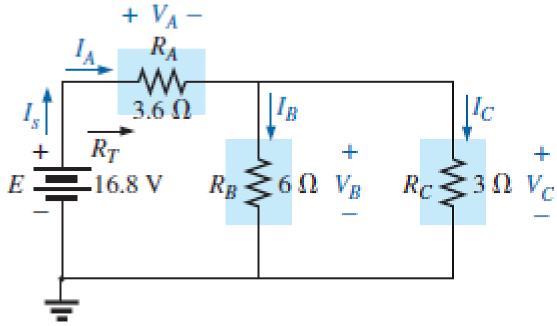
Solution:

$$R_A = R_1 \| R_2 = \frac{R_1 R_2}{R_1 + R_2} = \frac{(9\ \Omega)(6\ \Omega)}{9\ \Omega + 6\ \Omega} = \frac{54\ \Omega}{15} = 3.6\ \Omega$$

$$R_B = R_3 + R_4 \| R_5 = R_3 + \frac{R_4 R_5}{R_4 + R_5} = 4\ \Omega + \frac{(6\ \Omega)(3\ \Omega)}{6\ \Omega + 3\ \Omega} = 4\ \Omega + 2\ \Omega = 6\ \Omega$$

$$R_C = 3\ \Omega$$

The network in Figure above can then be redrawn in reduced form, as shown in Figure below.



$$R_T = R_A + R_B \| R_C = R_A + \frac{R_B R_C}{R_B + R_C} = 3.6\ \Omega + \frac{(6\ \Omega)(3\ \Omega)}{6\ \Omega + 3\ \Omega} = 3.6\ \Omega + 2\ \Omega = \mathbf{5.6}\ \Omega$$

$$I_s = \frac{E}{R_T} = \frac{16.8\ V}{5.6\ \Omega} = \mathbf{3\ A}$$

$$I_A = I_s = \mathbf{3\ A}$$

Applying the current divider rule yields:

$$I_B = \frac{R_C I_A}{R_C + R_B} = \frac{(3\ \Omega)(3\ A)}{3\ \Omega + 6\ \Omega} = \frac{9\ A}{9} = \mathbf{1\ A}$$

By Kirchhoff’s current law:

$$I_C = I_A - I_B = 3\ A - 1\ A = \mathbf{2\ A}$$

By Ohm’s law:

$$V_A = I_A\, R_A = (3\ A)(3.6\ \Omega)\ =\ \mathbf{10.8\ V}$$

$$V_B = I_B\, R_B = V_C = I_C\, R_C = (2\ A)(3\ \Omega)\ =\ \mathbf{6\ V}$$

Returning to the original network and applying the current divider rule gives:

$$I_1 = \frac{R_2\, I_A}{R_2 + R_1} = \frac{(6\ \Omega)(3\ A)}{6\ \Omega + 9\ \Omega} = \frac{18\ A}{15} = \mathbf{1.2\ A}$$

By Kirchhoff’s current law: $I_2 = I_A - I_1 = 3\ A - 1.2\ A = \mathbf{1.8\ A}$

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2023-2024

Lecture 5: Capacitors and Inductors

5.1 Capacitance:

In Figure (5.1), for example, two parallel plates of a material such as aluminum (the most commonly used metal in the construction of capacitors) have been connected through a switch and a resistor to a battery. If the parallel plates are initially uncharged and the switch is left open, no net positive or negative charge exists on either plate. The instant the switch is closed, however, electrons are drawn from the upper plate through the resistor to the positive terminal of the battery. There will be a surge of current at first, limited in magnitude by the resistance present. The level of flow then declines. Electrons are being repelled by the negative terminal through the lower conductor to the bottom plate at the same rate they are being drawn to the positive terminal. This transfer of electrons continues until the potential difference across the parallel plates is exactly equal to the battery voltage.

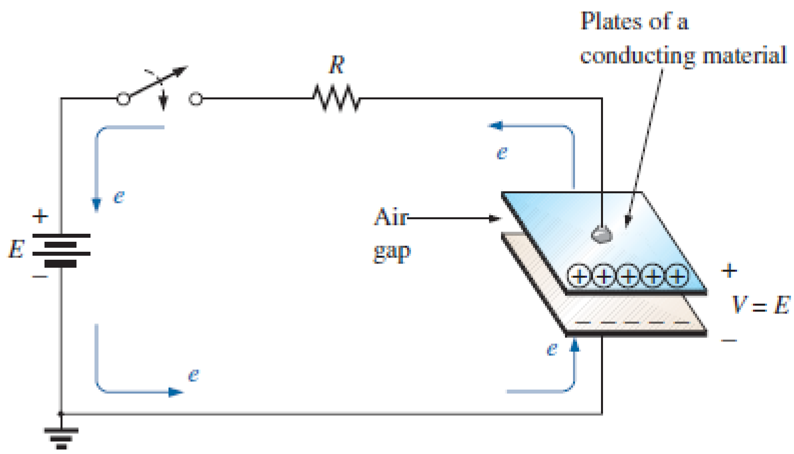


Figure (5.1): Fundamental charging circuit.

This element, constructed simply of two conducting surfaces separated by the air gap, is called a capacitor.

Capacitance is a measure of a capacitor's ability to store charge on its plates—in other words, its storage capacity.

In addition, ***the higher the capacitance of a capacitor, the greater is the amount of charge stored on the plates for the same applied voltage.***

The unit of measure applied to capacitors is the farad (F), named after an English scientist, Michael Faraday, who did extensive research in the field. In particular, ***a capacitor has a capacitance of 1 F if 1 C of charge (6.242×10^{18} electrons) is deposited on the plates by a potential difference of 1 V across its plates.***

The farad, however, is generally too large a measure of capacitance for most practical applications, so the microfarad (10^{-6}) or picofarad (10^{-12}) are more commonly encountered.

The relationship connecting the applied voltage, the charge on the plates, and the capacitance level is defined by the following equation:

$$C = \frac{Q}{V}$$

$\mathrm{C} = \text{farads (F)}$
$Q = \text{coulombs (C)}$
$V = \text{volts (V)}$

Equation above reveals that for the same voltage (V), the greater the charge (Q) on the plates (in the numerator of the equation), the higher is the capacitance level (C).

If we write the equation in the form:

$$Q = CV \quad \text{(coulombs, C)}$$

it becomes obvious through the product relationship that the higher the capacitance (C) or applied voltage (V), the greater is the charge on the plates.

Example 1:

A) If 82.4×10^{14} electrons are deposited on the negative plate of a capacitor by an applied voltage of 60 V, find the capacitance of the capacitor.

B) If 40 V are applied across a 470 μF capacitor, find the charge on the plates.

Solution:

A) First, find the number of coulombs of charge, as follows:

$$82.4 \times 10^{14}\ electrons \left(\frac{1\ C}{6.242 \times 10^{18}\ electrons}\right) = 1.32\ mC$$

and then:

$$C = \frac{Q}{V} = \frac{1.32\ mC}{60\ V} = \mathbf{22\ \mu F}$$

B)

$$Q = CV = (470\ \mu F)(40\ V) = \mathbf{18.8\ mC}$$

5.2 Capacitor Construction:

Different materials placed between the plates establish different amounts of additional charge on the plates. All, however, must be insulators and must have the ability to set up an electric field within the structure. A list of common materials appears in Table (5.1) **using air as the reference level of 1**. All of these materials are referred to as **dielectrics**, the "di" for *opposing*, and the "electric" from *electric field*. The symbol ϵ_r in Table (5.1) is called the **relative permittivity** (or **dielectric constant**). The term **permittivity** is applied as a measure of how easily a material "permits" the establishment of an electric field in the material. The relative permittivity compares the permittivity of a material to that of air.

Table (5.1): Relative permittivity (dielectric constant) ϵ_r of various dielectrics.

Dielectric	ϵ_r (Average Values)
Vacuum	1.0
Air	1.0006
Teflon®	2.0
Paper, paraffined	2.5
Rubber	3.0
Polystyrene	3.0
Oil	4.0
Mica	5.0
Porcelain	6.0
Bakelite®	7.0
Aluminum oxide	7
Glass	7.5
Tantalum oxide	30
Ceramics	20–7500
Barium-strontium titanite (ceramic)	7500.0

Defining ϵ_o as the permittivity of air, we define the relative permittivity of a material with a permittivity ϵ by: (dimensionless)

$$\epsilon_r = \frac{\epsilon}{\epsilon_o}$$

Permittivity does have the units of farads/meter (F/m) and is 8.85×10^{-12} F/m for air. Although the relative permittivity for the air we breathe is listed as 1.006, a value of 1 is normally used for the relative permittivity of air.

Larger plates permit an increased area for the storage of charge, so the area of the plates should be in the numerator of the defining equation. *The smaller the distance between the plates*, the larger is the capacitance, so this factor should appear in the denominator of the equation. Finally, since *higher levels of permittivity* result in higher levels of capacitance, the factor ϵ should appear in the numerator of the defining equation.

The result is the following general equation for capacitance:

$$C = \epsilon \frac{A}{d}$$

$\mathrm{C} = \text{farads (F)}$
ϵ=permittivity (F/m)
$A = m^2$
$d = \mathrm{m}$

If we substitute the permittivity of the material, we obtain the following equation for the capacitance:

$$C = \epsilon_o \epsilon_r \frac{A}{d} \quad \text{(farads, F)}$$

or if we substitute the known value for the permittivity of air, we obtain the following useful equation:

$$C = 8.85 \times 10^{-12} \epsilon_r \frac{A}{d} \quad \text{(farads, F)}$$

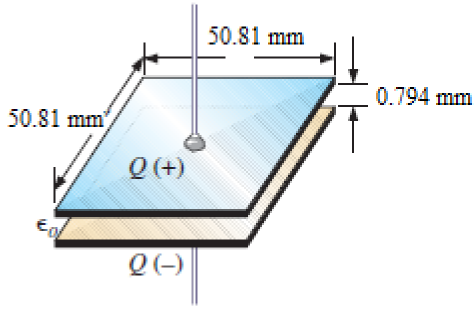
It is important to note in equation above that the area of the plates (actually the area of only one plate) is in meters squared (m^2); (the distance between the plates is measured in meters; and the numerical value of ϵ_r is simply taken from Table (5.1).

You should also be aware that most capacitors are in the µF, nF, or pF range, not the 1 F or greater range. A 1 F capacitor can be as large as a typical flashlight, requiring that the housing for the system be quite large. Most capacitors in electronic systems are the size of a thumbnail or smaller.

Example 2: For the capacitor in Figure below:

A) Find the capacitance.

B) Find the charge on each plate if 48 V are applied across the plates.



Solution:

A)

$$A = (50.81 \times 10^{-3}\, m)(50.81 \times 10^{-3}\, m) = 2581 \times 10^{-6}\, m^2 = 2.581 \times 10^{-3}\, m^2$$

$$C = 8.85 \times 10^{-12} \epsilon_r \frac{A}{d} = 8.85 \times 10^{-12}(1)\frac{2.581 \times 10^{-3}\, m^2}{0.794\, mm} = \mathbf{28.8\, pF}$$

B) The charge on the plates is determined by:

$$Q = CV = (28.8\, pF)(48\, V) = \mathbf{1.38\, nC}$$

5.3 Transients in Capacitive Networks:

5.3.1 The Charging Phase:

The charging phase—***the phase during which charge is deposited on the plates***—can be described by reviewing the response of the simple series circuit in Figure (5.2).

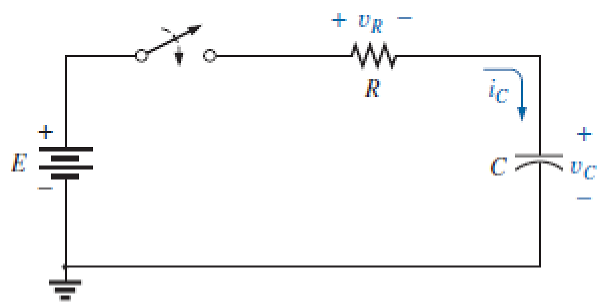


Figure (5.2): Basic R-C charging network.

The voltage across the capacitor will be zero volts, as shown in Figure (5.3) below t=0 s, the current through the circuit will be zero ampere. Since the voltage across the plates is directly related to the charge on the plates by $V = Q/C$, a plot of the voltage across the capacitor will have the same shape as a plot of the charge on the plates over time. The voltage across the capacitor is zero volts when the switch is closed ($t = 0\, s$). It then builds up very quickly at first since charge is being deposited at a very high rate of speed. As time passes, the charge is deposited at a slower rate, and the change in voltage drops off. The voltage continues to grow, but at a much slower rate. Eventually, as the voltage across the plates approaches the applied voltage, the charging rate is very slow, until finally the voltage across the plates is equal to the applied voltage—the transient phase has passed.

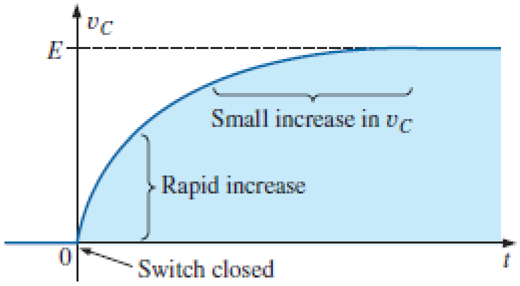


Figure (5.3): v_C during the charging phase.

The waveform in Figure (5.3) from beginning to end can be described using the mathematical function e^{-x}.

If we substitute zero for x, we obtain e^{-0}, which by definition is 1, as shown in Table (5.2) and on the plot in Figure (5.4). Table (5.2) reveals that **as *x* increases, the function e^{-x} decreases in magnitude until it is very close to zero after *x* = 5**. As noted in Table (5.2), the exponential factor $e^1 = e = 2.71828$.

A plot of $1 - e^{-x}$ is also provided in Figure (5.4) since it is a component of the voltage v_C in Figure (5.3). When e^{-x} is 1, $1 - e^{-x}$ is zero, as shown in Figure (5.4), and when e^{-x} decreases in magnitude, $1 - e^{-x}$ approaches 1, as shown in the same figure.

Table (5.2): Selected values of e^{-x}.

$x = 0$	$e^{-x} = e^{-0} = \frac{1}{e^0} = \frac{1}{1} = 1$
$x = 1$	$e^{-1} = \frac{1}{e} = \frac{1}{2.71828\ldots} = 0.3679$
$x = 2$	$e^{-2} = \frac{1}{e^2} = 0.1353$
$x = 5$	$e^{-5} = \frac{1}{e^5} = 0.00674$
$x = 10$	$e^{-10} = \frac{1}{e^{10}} = 0.0000454$
$x = 100$	$e^{-100} = \frac{1}{e^{100}} = 3.72 \times 10^{-44}$

We simply place the exponential in the proper mathematical form, as follows:

$$v_C = E\left(1 - e^{-t/\tau}\right) \quad \text{charging} \qquad \text{(volts, V)}$$

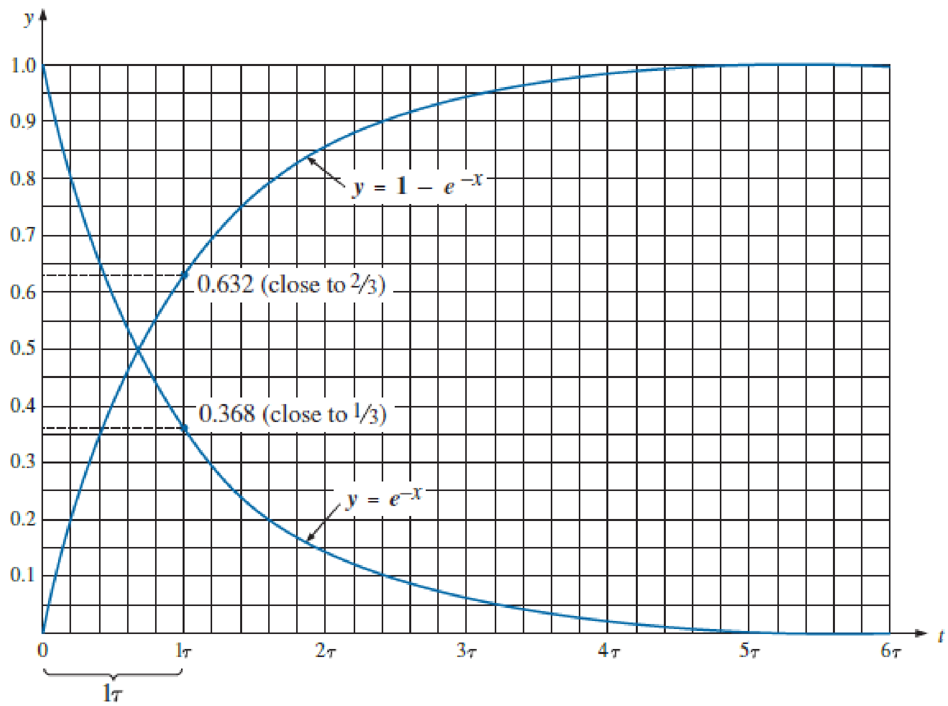


Figure (5.4): Universal time constant chart.

Note that the voltage v_C is written in *lowercase (not capital) italic* to point out that it is a function that will change with time—it is not a constant. The exponent of the exponential function is no longer just x, but now is time (t) divided by a constant τ, the Greek letter *tau*. The quantity τ is defined by:

$$\boxed{\tau = RC} \qquad \text{(time, s)}$$

The factor τ, called the **time constant** of the network, has the units of time, as shown below using some of the basic equations introduced earlier:

$$\tau = RC = \left(\frac{V}{I}\right)\left(\frac{Q}{V}\right) = \left(\frac{V}{Q/t}\right)\left(\frac{Q}{V}\right) = t \text{ (seconds)}$$

If we substitute $t = 0\ s$, we find that:

$$e^{-t/\tau} = e^{-0/\tau} = e^{-0} = \frac{1}{e^0} = \frac{1}{1} = 1$$

and

$$v_C = E\left(1 - e^{-t/\tau}\right) = E(1 - 1) = \mathbf{0\ V}$$

as appearing in the plot in Figure (5.5).

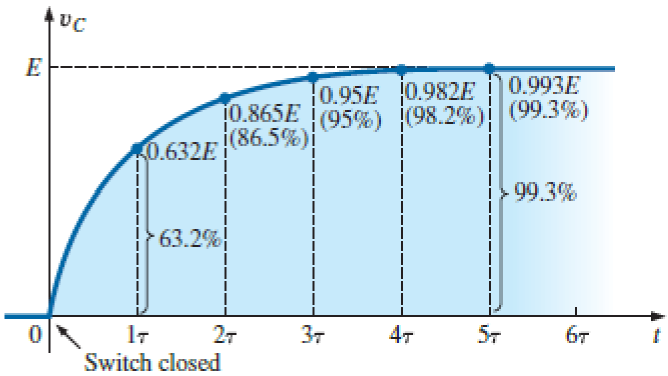


Figure (5.5): Plotting the equation $v_C = E\left(1 - e^{-t/\tau}\right)$ versus time (t).

It is important to realize at this point that the plot in Figure (5.5) is not against simply time but against τ, the time constant of the network. If we want to know the voltage across the plates after one time constant, we simply plug $t = 1\tau$.

$$e^{-t/\tau} = e^{-\tau/\tau} = e^{-1} \cong 0.368$$

and

$$v_C = E\left(1 - e^{-t/\tau}\right) = E(1 - 0.368) = \mathbf{0.632\ E}$$

as shown in Figure (5.5).

At $t = 2\tau$

$$e^{-t/\tau} = e^{-2\tau/\tau} = e^{-2} \cong 0.135$$

and

$$v_C = E\left(1 - e^{-t/\tau}\right) = E(1 - 0.135) \cong \mathbf{0.865\ E}$$

as shown in Figure (5.5).

As the number of time constants increases, the voltage across the capacitor does indeed approach the applied voltage.

At $t = 5\tau$

$$e^{-t/\tau} = e^{-5\tau/\tau} = e^{-5} \cong 0.007$$

and

$$v_C = E\left(1 - e^{-t/\tau}\right) = E(1 - 0.007) = \mathbf{0.993\ E}$$

The voltage across a capacitor in a dc network is essentially equal to the applied voltage after five time constants of the charging phase have passed.

Or, in more general terms, ***the transient or charging phase of a capacitor has essentially ended after five time constants.***

When the switch is first closed, the flow of charge or current jumps very quickly to a value limited by the applied voltage and the circuit resistance, as shown in Figure (5.6). The rate of deposit, and hence the current, then decreases quite rapidly, until eventually charge is not being deposited on the plates and the current drops to zero amperes.

The equation for the current is:

$$i_C = \frac{E}{R} e^{-t/\tau} \quad \text{charging} \qquad \text{(Ampers, A)}$$

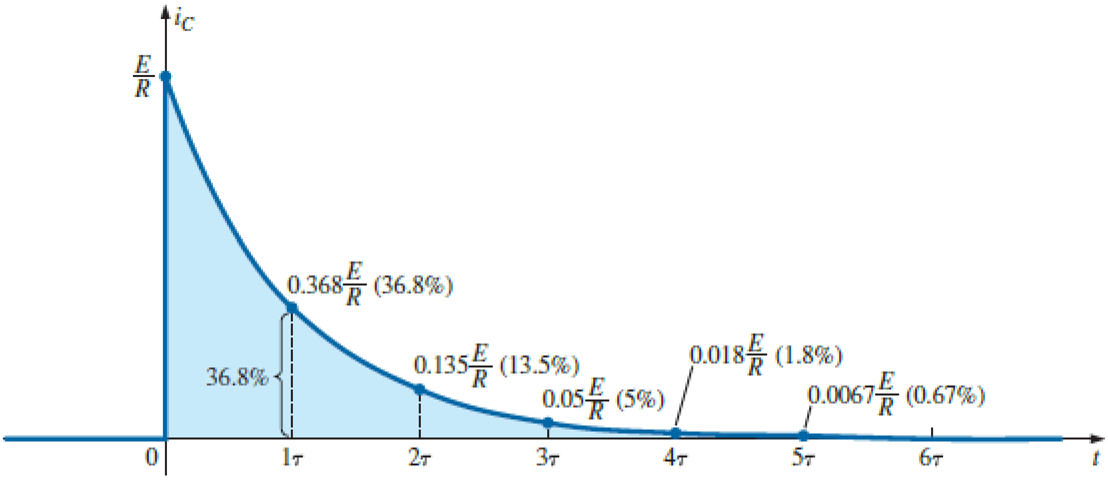


Figure (5.6): Plotting the equation $i_C = \frac{E}{R}e^{-t/\tau}$ versus time (t).

At $t = 0\ s$,

$$e^{-t/\tau} = e^{-0/\tau} = e^{-0} = \frac{1}{e^0} = \frac{1}{1} = 1$$

and

$$i_C = \frac{E}{R}e^{-t/\tau} = \frac{E}{R}(1) = \boldsymbol{\frac{E}{R}}$$

At $t = 1\tau$,

$$e^{-t/\tau} = e^{-\tau/\tau} = e^{-1} \cong 0.368$$

and

$$i_C = \frac{E}{R}e^{-t/\tau} = \frac{E}{R}(0.368) = \boldsymbol{0.368\frac{E}{R}}$$

In general, ***the current of a capacitive dc network is essentially zero amperes after five time constants of the charging phase have passed.***

It is also important to recognize that ***during the charging phase, the major change in voltage and current occurs during the first time constant.***

A capacitor can be replaced by an open-circuit equivalent once the charging phase in a dc network has passed. While, a capacitor has the characteristics of a short-circuit equivalent at the instant the switch is closed in an uncharged series R-C circuit.

Since the resistor and the capacitor in Figure (5.2) are in series, the current through the resistor is the same as that associated with the capacitor. The voltage across the resistor can be determined by using Ohm's law in the following manner:

$$v_R = i_R R = i_C R$$

so that:

$$v_R = \left(\frac{E}{R} e^{-t/\tau}\right) R$$

and

$$\boxed{v_R = E e^{-t/\tau}}_{\text{charging}} \quad \text{(volts, V)}$$

A plot of the voltage as shown in Figure (5.7) has the same shape as that for the current because they are related by the constant *R*.

Kirchhoff's voltage law is applicable at any instant of time for any type of voltage in any type of network.

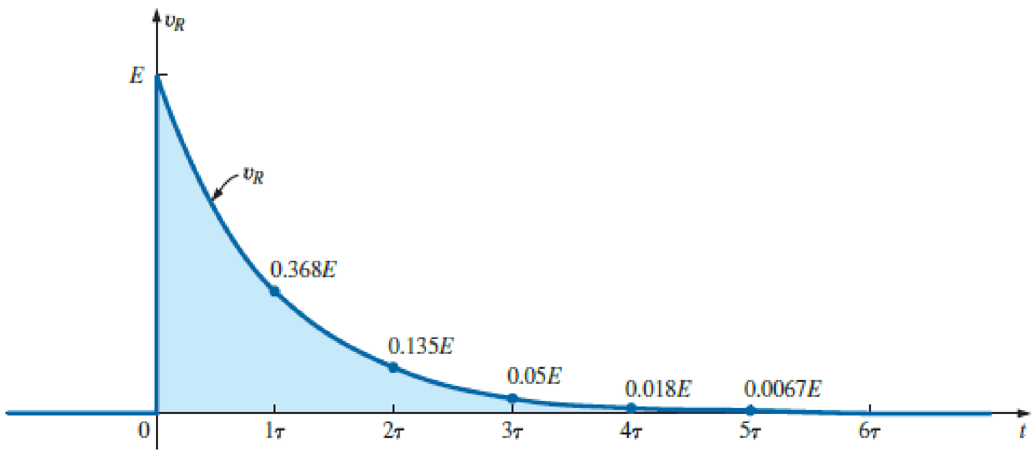


Figure (5.7): Plotting the equation $v_R = Ee^{-t/\tau}$ versus time (t).

Example 3: For the circuit in Figure below:

A) Find the mathematical expression for the transient behavior of v_C, i_C, and v_R if the switch is closed at $t = 0\ s$.

B) Plot the waveform of v_C versus the time constant of the network.

C) Plot the waveform of v_C versus time.

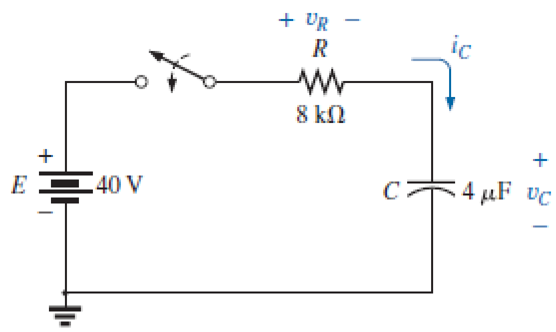
D) Plot the waveforms of i_C and v_R versus the time constant of the network.

E) What is the value of v_C at $t = 20\ ms$?

F) On a practical basis, how much time must pass before we can assume that the charging phase has passed?

G) When the charging phase has passed, how much charge is sitting on the plates?

H) If the capacitor has a leakage resistance of 10,000 MΩ, what is the initial leakage current? Once the capacitor is separated from the circuit, how long will it take to totally discharge, assuming a linear (unchanging) discharge rate?



Solution:

A) The time constant of the network is:

$$\tau = RC = (8\ K\Omega)(4\ \mu F) = 32\ ms$$

resulting in the following mathematical equations:

$$v_C = E\left(1 - e^{-t/\tau}\right) = \mathbf{40\ V\left(1 - e^{-t/32\ ms}\right)}$$

$$i_C = \frac{E}{R}e^{-\frac{t}{\tau}} = \frac{40\ V}{8\ K\Omega}e^{-t/32\ ms} = \mathbf{5\ mA\ e^{-t/32\ ms}}$$

$$v_R = Ee^{-\frac{t}{\tau}} = \mathbf{40\ V\ e^{-t/32\ ms}}$$

B) The resulting plot appears in Figure (5.8).

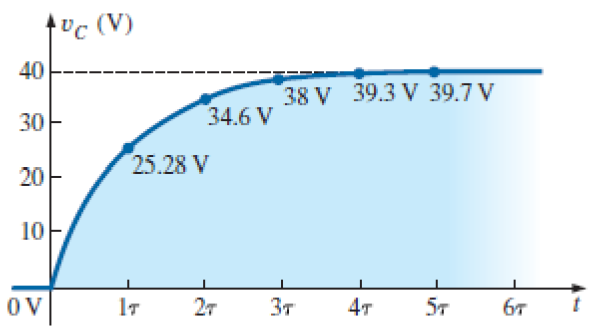


Figure (5.8): v_C versus time for the charging network.

C) The horizontal scale will now be against time rather than time constants, as shown in Figure (5.9). The plot points in Figure (5.9) were taken from Figure (5.8).

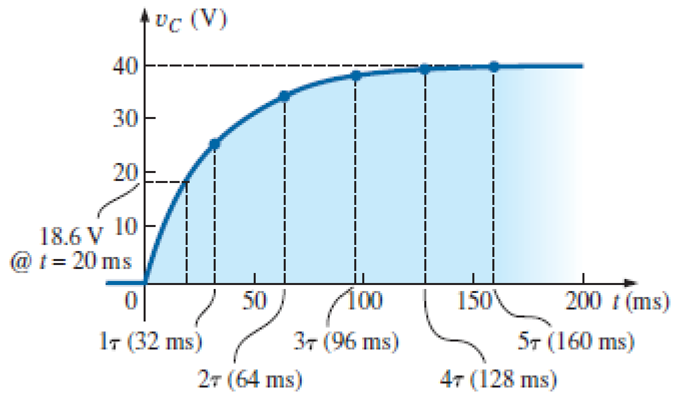
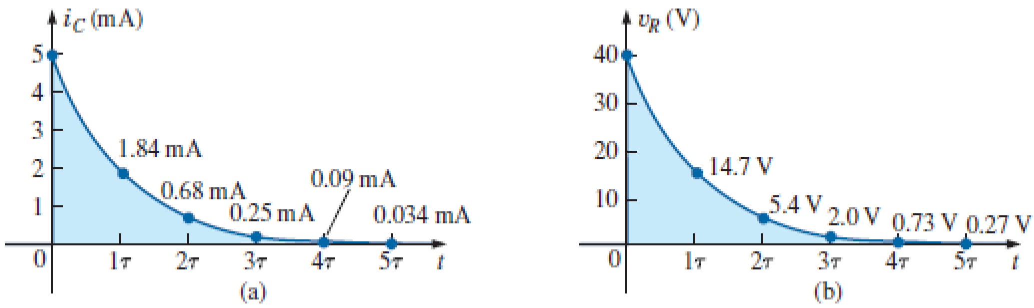


Figure (5.9): Plotting the waveform versus time (t).

D) Both plots appear in Figure below.



E) Substituting the time t = 20 ms results in the following for the exponential part of the equation:

$$e^{-\frac{t}{\tau}} = e^{-\frac{20\,ms}{32\,ms}} = e^{-0.625} = 0.535 \quad \text{(using a calculator)}$$

so that:

$$v_C = 40\,V\left(1 - e^{-\frac{t}{32}ms}\right) = 40\,V(1 - 0.535) = (40\,V)(0.465)$$

$$v_C = \mathbf{18.6\,V} \quad \textbf{(as verified by Figure (5.9))}$$

F) Assuming a full charge in five time constants results in:

$$5\tau = 5(32\,ms) = \mathbf{160\,ms} = \mathbf{0.16\,s}$$

G)

$$Q = CV = (4\,\mu F)(40\,V) = \mathbf{160\,\mu C}$$

H) Using Ohm's law gives:

$$I_{leakage} = \frac{40\ V}{10{,}000\ M\Omega} = 4\ nA$$

Finally, the basic equation $I = Q/t$ results in:

$$t = \frac{Q}{I} = \frac{160\ \mu C}{4\ nA} = (40{,}000\ s)\left(\frac{1\ min}{60\ s}\right)\left(\frac{1\ h}{60\ min}\right) = \mathbf{11.11\ h}$$

5.3.2 The Discharging Phase:

In Figure (5.10(a)), a second contact for the switch was added to the circuit in Figure (5.2) to permit a controlled discharge of the capacitor. With the switch in position 1, we have the charging network described in the last section. Following the full charging phase, if we move the switch to position 2, the voltage across the capacitor appears directly across the resistor to establish a discharge current. Initially, the current jumps to a relatively high value; then it begins to drop. It drops with time because charge is leaving the plates of the capacitor, which in turn reduces the voltage across the capacitor and thereby the voltage across the resistor and the resulting current.

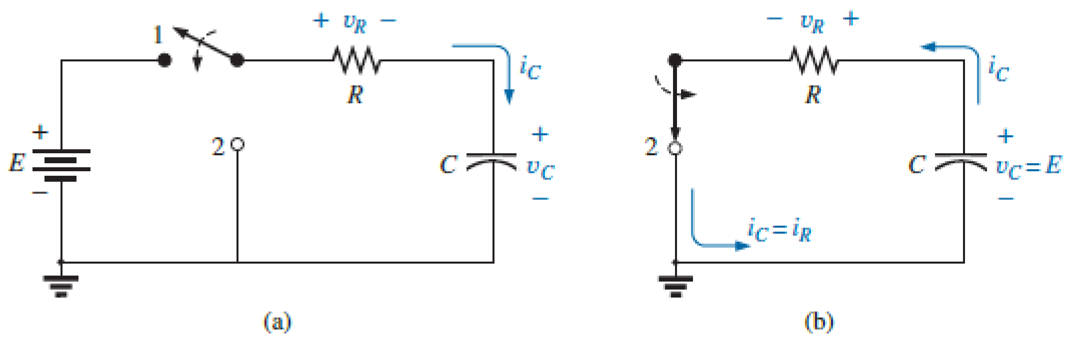


Figure (5.10): (a) Charging network; (b) discharging configuration.

Note that current i_C has now reversed direction as shown in Figure (5.10(b)). As shown in parts (a) and (b) in Figure (5.10), the voltage across the capacitor does not reverse polarity, but the current reverses direction.

For the voltage across the capacitor that is decreasing with time, the mathematical expression is:

$$v_C = Ee^{-t/\tau} \quad \text{discharging} \qquad \text{(volts, V)}$$

For this circuit, the time constant τ is defined by the same equation as used for the charging phase. That is:

$$\tau = RC \quad \text{discharging}$$

Since the current decreases with time, it will have a similar format:

$$i_C = \frac{E}{R} e^{-t/\tau} \quad \text{discharging}$$

For the configuration in Figure (5.10(b)), since $v_R = v_C$ (in parallel), the equation for the voltage has the same format:

$$v_R = E e^{-t/\tau} \quad \text{discharging}$$

The complete discharge will occur, for all practical purposes, in five time constants. If the switch is moved between terminals 1 and 2 every five time constants, the wave shapes in Figure (5.11) will result for v_C, i_C, and v_R.

For each curve, the current directions and voltage polarities are as defined by the configurations in Figure (5.10).

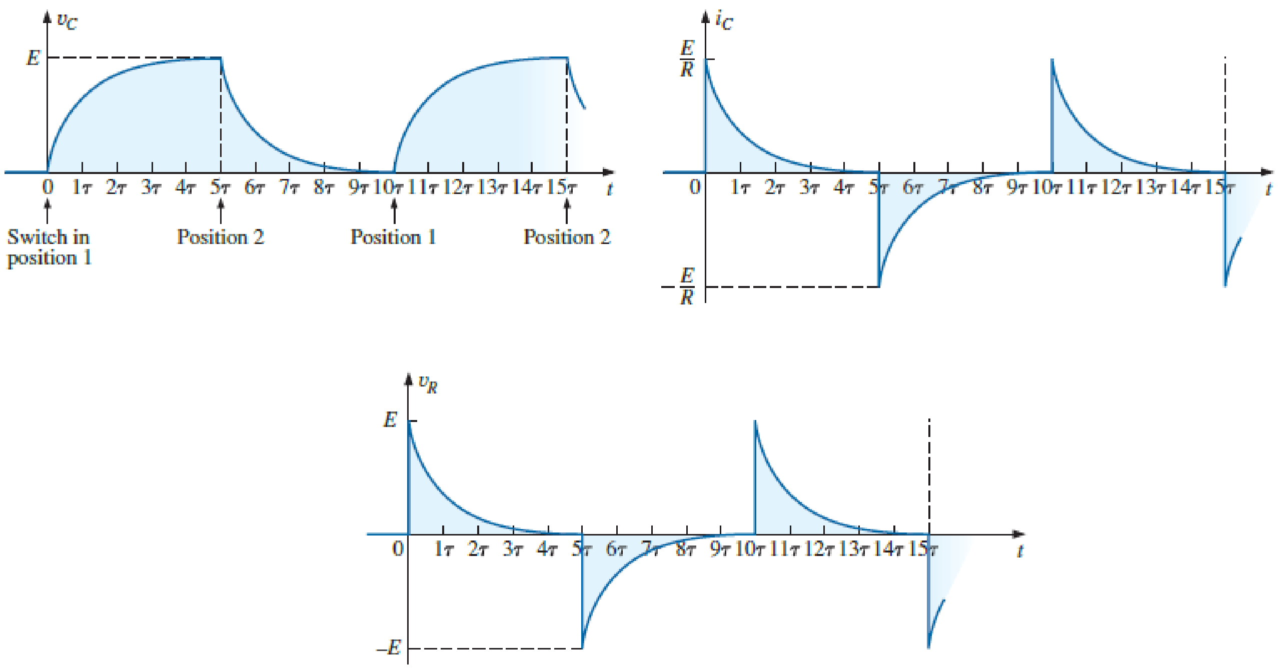


Figure (5.11): v_C, i_C, and v_R for 5τ switching between contacts in Figure (5.10(a)).

Example 4: Using the values in Example 3, plot the waveforms for v_C and i_C resulting from switching between contacts 1 and 2 in Figure (5.10) every five time constants.

Solution: The time constant is the same for the charging and discharging phases. That is:

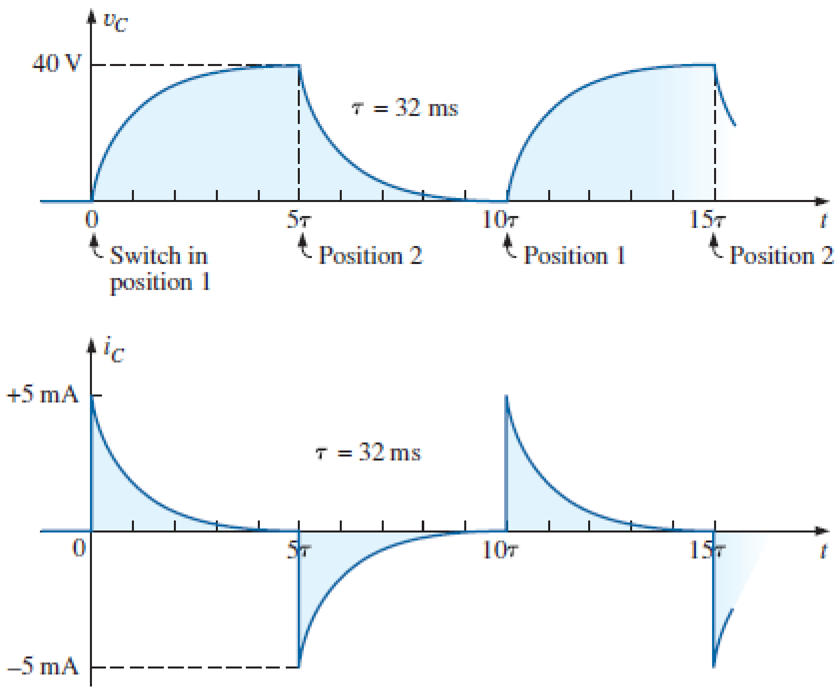
$$\tau = RC = (8\ K\Omega)(4\ \mu F) = 32\ ms$$

For the discharge phase, the equations are:

$$v_C = Ee^{-t/\tau} = \mathbf{40\ V\ e^{-t/32\ ms}}$$

$$i_C = \frac{E}{R}e^{-t/\tau} = \frac{40\ V}{8\ K\Omega}e^{-t/32\ ms} = \mathbf{-5\ mA\ e^{-t/32\ ms}}$$

A continuous plot for the charging and discharging phases appears in Figure below.



5.4 Capacitors in Series and in Parallel:

Capacitors, like resistors, can be placed in series and in parallel. Increasing levels of capacitance can be obtained by placing capacitors in parallel, while decreasing levels can be obtained by placing capacitors in series.

For capacitors in series, the charge is the same on each capacitor (Figure (15.12)):

$$Q_T = Q_1 = Q_2 = Q_3$$

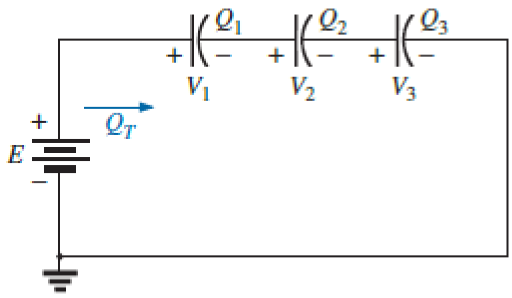


Figure (15.12): Series capacitors.

Applying Kirchhoff's voltage law around the closed loop gives:

$$E = V_1 + V_2 + V_3$$

However,

$$V = \frac{Q}{C}$$

so that:

$$\frac{Q_T}{C_T} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2} + \frac{Q_3}{C_3}$$

Using $Q_T = Q_1 = Q_2 = Q_3$ and dividing both sides by Q yields:

$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

which is similar to the manner in which we found the total resistance of a parallel resistive circuit. The total capacitance of two capacitors in series is:

$$C_T = \frac{C_1\, C_2}{C_1 + C_2}$$

For capacitors in parallel, as shown in Figure (5.13), the voltage is the same across each capacitor, and the total charge is the sum of that on each capacitor:

$$Q_T = Q_1 + Q_2 + Q_3$$

However,

$$Q = CV$$

Therefore,

$$C_T\, E = C_1 V_1 + C_2 V_2 + C_3 V_3$$

but

$$E = V_1 = V_2 = V_3$$

Thus,

$$C_T = C_1 + C_2 + C_3$$

which is similar to the manner in which the total resistance of a series circuit is found.

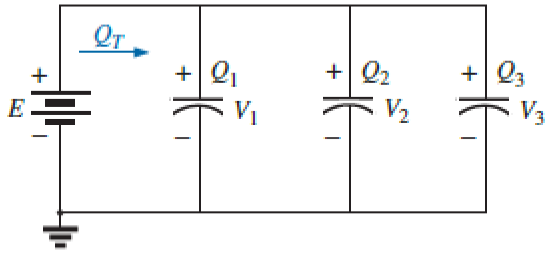


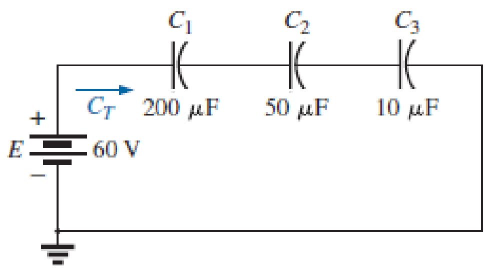
Figure (5.13): Parallel capacitors.

Example 5: For the circuit in Figure below:

A) Find the total capacitance.

B) Determine the charge on each plate.

C) Find the voltage across each capacitor.



Solution:

A)

$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{200 \times 10^{-6}\ F} + \frac{1}{50 \times 10^{-6}\ F} + \frac{1}{10 \times 10^{-6}\ F}$$

$$= 0.005 \times 10^6 + 0.02 \times 10^6 + 0.1 \times 10^6 = 0.125 \times 10^6$$

and

$$C_T = \frac{1}{0.125 \times 10^6} = \mathbf{8}\ \mu\boldsymbol{F}$$

B)

$$Q_T = Q_1 = Q_2 = Q_3 = C_T E = (8 \times 10^{-6}\ F)\,(60\ V) = \mathbf{480}\ \mu\boldsymbol{C}$$

C)

$$V_1 = \frac{Q_1}{C_1} = \frac{480 \times 10^{-6}\ C}{200 \times 10^{-6}\ F} = \mathbf{2.4\ V}$$

$$V_2 = \frac{Q_2}{C_2} = \frac{480 \times 10^{-6}\ C}{50 \times 10^{-6}\ F} = \mathbf{9.6\ V}$$

$$V_3 = \frac{Q_3}{C_3} = \frac{480 \times 10^{-6}\ C}{10 \times 10^{-6}\ F} = \mathbf{48.0\ V}$$

and

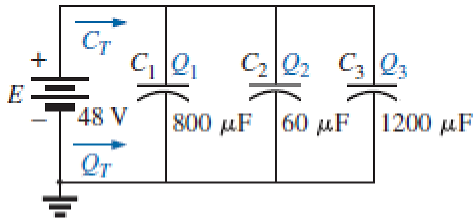
$$E = V_1 + V_2 + V_3 = 2.4\ V + 9.6\ V + 48.0\ V = \mathbf{60\ V}\ (\mathbf{checks})$$

Example 6: For the network in Figure below:

A) Find the total capacitance.

B) Determine the charge on each plate.

C) Find the total charge.



Solution:

A)

$$C_T = C_1 + C_2 + C_3 = 800\ \mu F + 60\ \mu F + 1200\ \mu F = \mathbf{2060\ \mu F}$$

B)

$$Q_1 = C_1E = (800 \times 10^{-6}\ F)(48\ V) = \mathbf{38.4\ mC}$$

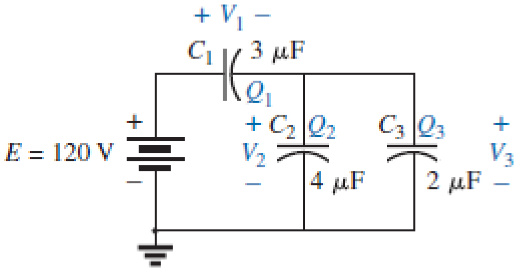
$$Q_2 = C_2E = (60 \times 10^{-6}\ F)(48\ V) = \mathbf{2.88\ mC}$$

$$Q_3 = C_3E = (1200 \times 10^{-6}\ F)(48\ V) = \mathbf{57.6\ mC}$$

C)

$$Q_T = Q_1 + Q_2 + Q_3 = 38.4\ mC + 2.88\ mC + 57.6\ mC = \mathbf{98.88\ mC}$$

Example 7: Find the voltage across and the charge on each capacitor for the network in Figure below.



Solution:

$$C'_T = C_2 + C_3 = 4\ \mu F + 2\ \mu F = 6\ \mu F$$

$$C_T = \frac{C_1\, C'_T}{C_1 + C'_T} = \frac{(3\ \mu F)(6\ \mu F)}{3\ \mu F + 6\ \mu F} = 2\ \mu F$$

$$Q_T = C_T E = (2 \times 10^{-6}\ F)(120\ V) = \mathbf{240\ \mu C}$$

An equivalent circuit (Figure below) has:

$$Q_T = Q_1 = Q'_T$$

and, therefore,

$$Q_1 = \mathbf{240\ \mu C}$$

and

$$V_1 = \frac{Q_1}{C_1} = \frac{240 \times 10^{-6}\ C}{3 \times 10^{-6}\ F} = \mathbf{80\ V}$$

$$Q'_T = \mathbf{240\ \mu C}$$

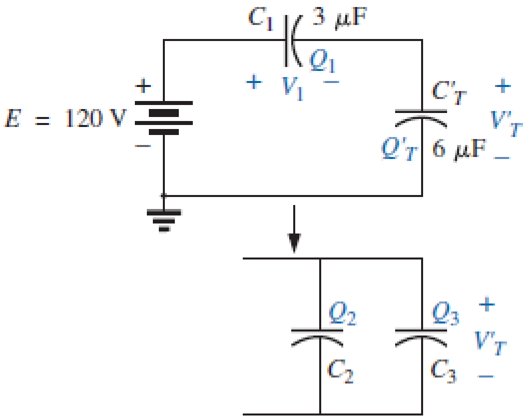
Therefore,

$$V'_T = \frac{Q'_T}{C'_T} = \frac{240\ \mu C}{6\ \mu F} = \mathbf{40\ V}$$

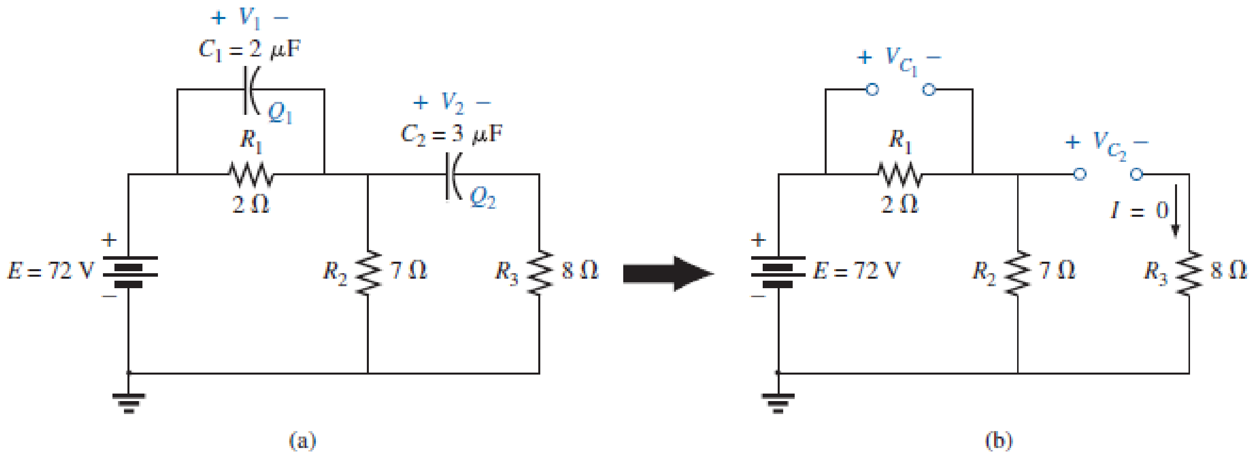
and

$$Q_2 = C_2 V'_T = (4 \times 10^{-6}\ F)(40\ V) = \mathbf{160\ \mu C}$$

$$Q_3 = C_3 V'_T = (2 \times 10^{-6}\ F)(40\ V) = \mathbf{80\ \mu C}$$



Example 8: Find the voltage across and the charge on each capacitor of the network in Figure below (a) after each has charged up to its final value.



Solution: See Figure above (b). We have:

$$V_{C_2} = \frac{(7\,\Omega)(72\,V)}{7\,\Omega + 2\,\Omega} = \mathbf{56\,V}$$

$$V_{C_1} = \frac{(2\,\Omega)(72\,V)}{2\,\Omega + 7\,\Omega} = \mathbf{16\,V}$$

$$Q_1 = C_1 V_{C_1} = (2 \times 10^{-6}\,F)(16\,V) = \mathbf{32\,\mu C}$$

$$Q_2 = C_2 V_{C_2} = (3 \times 10^{-6}\,F)(56\,V) = \mathbf{168\,\mu C}$$

5.5 Energy Stored by a Capacitor:

An ideal capacitor does not dissipate any of the energy supplied to it. It stores the energy in the form of an electric field between the conducting surfaces. A plot of the voltage, current, and power to a capacitor during the charging phase is shown in

Figure (5.14). The power curve can be obtained by finding the product of the voltage and current at selected instants of time and connecting the points obtained. The energy stored is represented by the shaded area under the power curve. Using calculus, we can determine the area under the curve:

$$W_C = \frac{1}{2}CE^2$$

In general,

$$\boxed{W_C = \frac{1}{2}CV^2} \quad \text{(J)}$$

where V is the steady-state voltage across the capacitor.

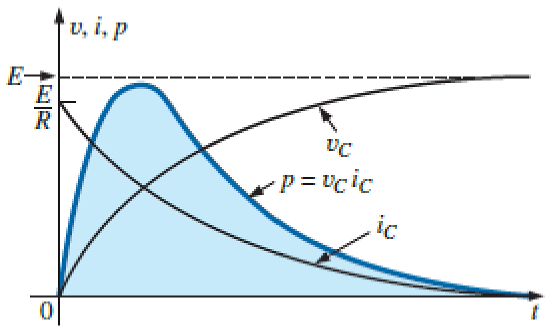


Figure (5.14): Plotting the power to a capacitive element during the transient phase.

Example 9: For the network in Figure (a) of Example 8, determine the energy stored by each capacitor.

Solution: For C_1:

$$W_C = \frac{1}{2}CV^2 = \frac{1}{2}(2 \times 10^{-6}\,F)\,(16\,V)^2 = (1 \times 10^{-6}\,F)(256) = \mathbf{256}\ \mu \boldsymbol{J}$$

For C_1:

$$W_C = \frac{1}{2}CV^2 = \frac{1}{2}(3 \times 10^{-6}\,F)\,(56\,V)^2 = (1.5 \times 10^{-6}\,F)(3136) = \mathbf{4704}\ \mu \boldsymbol{J}$$

Due to the squared term, the energy stored increases rapidly with increasing voltages.

5.6 Inductance:

The inductor exhibits its true characteristics only when a change in voltage or current is made in the network.

Sending a current through a coil of wire, with or without a core, establishes a magnetic field through and surrounding the unit. This component, of rather simple construction (see Figure (5.15)), is called an **inductor** (often referred to as a **coil**). Its **inductance** level determines the strength of the magnetic field around the coil due to an applied current. The higher the inductance level, the greater is the strength of the magnetic field. In total, therefore, ***inductors are designed to set up a strong magnetic field linking the unit, whereas capacitors are designed to set up a strong electric field between the plates.*** Inductance is measured in **henries (H),** after the American physicist Joseph Henry. However, just as the farad is too large a unit for most applications, most inductors are of the millihenry (mH) or microhenry (µH) range.

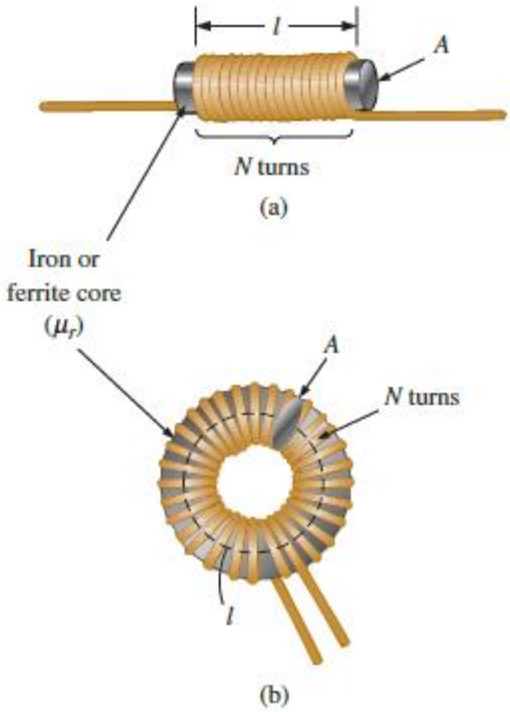


Figure (5.15): Defining the parameters

For inductors, ***1 henry is the inductance level that will establish a voltage of 1 volt across the coil due to a change in current of 1 A/s through the coil.***

5.7 Inductor Construction:

The level of inductance is sensitive to the area within the coil, the length of the unit, and the permeability of the core material. It is also sensitive to the number of turns of wire in the coil as dictated by the following equation and defined in Figure (5.15) for two of the most popular shapes:

$$L = \frac{\mu N^2 A}{l}$$

μ = permeability (Wb/Am)
N = number of turns (t)
$A = m^2$
$l = m$
L = henries (H)

First note that since the turns are squared in the equation, the number of turns is a big factor. However, also keep in mind that the more turns, the bigger is the unit. If the wire is made too thin to get more windings on the core, the rated current of the inductor is limited. Since higher levels of permeability result in higher levels of magnetic flux, permeability should, and does, appear in the numerator of the equation. Increasing the area of the core or decreasing the length also increases the inductance level.

Another factor that affects the magnetic field strength is the type of core used. Materials in which magnetic flux lines can readily be set up are said to be **magnetic** and to have a high **permeability**. Again, note the similarity with the word "permit" used to describe permittivity for the dielectrics of capacitors. Similarly, the permeability (represented by the Greek letter mu, μ) of a material is a measure of the ease with which magnetic flux lines can be established in the material. Just as there is a specific value for the permittivity of air, there is a specific number associated with the permeability of air:

$$\mu_o = 4\pi \times 10^{-7}\ Wb/Am$$

The ratio of the permeability of a material to that of free space is called its **relative permeability**; that is:

$$\mu_r = \frac{\mu}{\mu_o}$$

Substituting $\mu = \mu_r \mu_o$ for the permeability results in the following equation, which is very similar to the equation for the capacitance of a capacitor:

$$L = \frac{\mu_r \mu_o N^2 A}{l}$$

with $\mu_o = 4\pi \times 10^{-7}\ Wb/Am$

and

$$L = 4\pi \times 10^{-7}\ \frac{\mu_r N^2 A}{l} \quad \text{(Henries, H)}$$

If we break out the relative permeability as:

$$L = \mu_r \left(\frac{\mu_o N^2 A}{l}\right)$$

we obtain the following useful equation:

$$L = \mu_r\ L_o$$

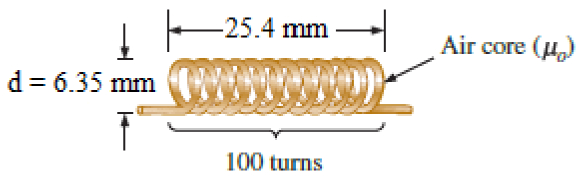
which is very similar to the equation $C = \epsilon_r\ C_o$. Equation above states the following: ***The inductance of an inductor with a ferromagnetic core is μ_r times the inductance obtained with an air core.***

Although $L = \frac{\mu N^2 A}{l}$ is approximate at best, the equations for the inductance of a wide variety of coils can be found in reference handbooks. Most of the equations are mathematically more complex than $L = \frac{\mu N^2 A}{l}$, but the impact of each factor is the same in each equation.

Example 10: For the air-core coil in Figure below:

A) Find the inductance.

B) Find the inductance if a metallic core with $\mu_r = 2000$ is inserted in the coil.



Solution:

A)

$$A = \frac{\pi d^2}{4} = \frac{\pi (6.35\ mm)^2}{4} = 31.67\ \mu m^2$$

$$L = 4\pi \times 10^{-7}\ \frac{\mu_r N^2 A}{l} = 4\pi \times 10^{-7}\ \frac{(1)(100\ t)^2 (31.67\ \mu m^2)}{25.4\ mm} = \mathbf{15.68}\ \mu \boldsymbol{H}$$

B)

$$L = \mu_r\, L_o = (2000)(15.68\ \mu H)\ =\ \mathbf{31.36\ mH}$$

Example 11: In Figure below, if each inductor in the left column is changed to the type appearing in the right column, find the new inductance level. For each change, assume that the other factors remain the same.

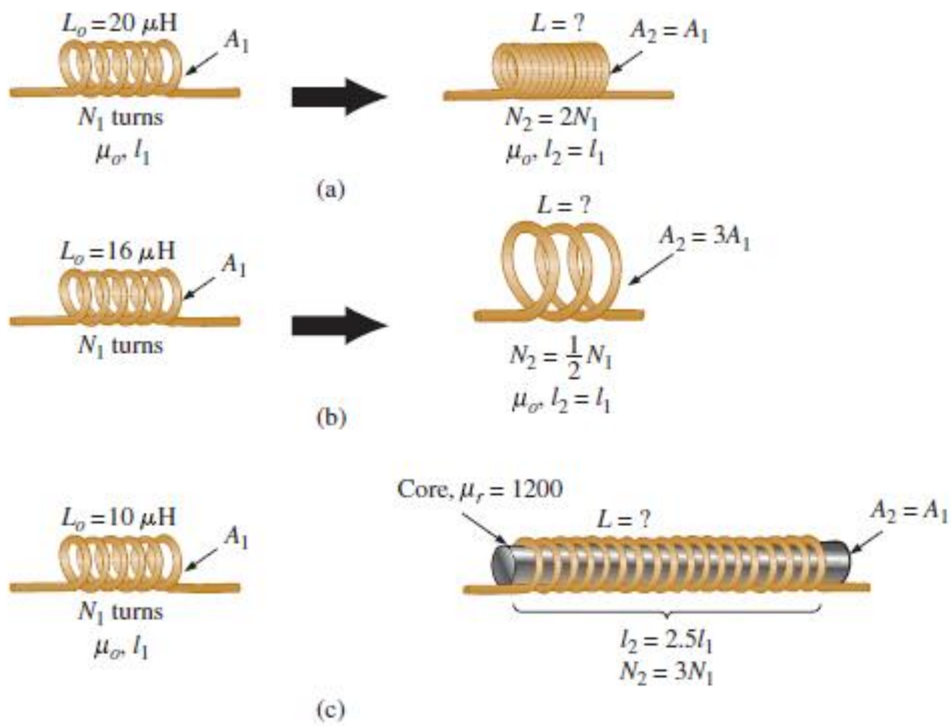
Solution:

a. The only change was the number of turns, but it is a squared factor, resulting in:

$$L_o = \frac{\mu_r \mu_o N_1^2 A_1}{l_1} \quad and \quad L = \frac{\mu_r \mu_o N_2^2 A_2}{l_2} = \frac{\mu_r \mu_o (2N_1)^2 A_1}{l_1}$$

$$\frac{L}{L_o} = \frac{\frac{\mu_r \mu_o (2N_1)^2 A_1}{l_1}}{\frac{\mu_r \mu_o N_1^2 A_1}{l_1}} = 2^2$$

$$\therefore L = (2)^2\, L_o = (2)^2 (20\ \mu H)\ =\ \mathbf{80\ mH}$$



b. In this case, the area is three times the original size, and the number of turns is 1/2. Since the area is in the numerator, it increases the inductance by a factor of three. The drop in the number of turns reduces the inductance by a factor of $(1/2)^2 = 1/4$. Therefore,

$$L_o = \frac{\mu_r \mu_o N_1^2 A}{l_1} \quad and \quad L = \frac{\mu_r \mu_o N_2^2 A_2}{l_2} = \frac{\mu_r \mu_o \left(\frac{1}{2} N_1\right)^2 (3A_1)}{l_1}$$

$$\frac{L}{L_o} = \frac{\dfrac{\mu_r \mu_o \left(\frac{1}{2} N_1\right)^2 (3A_1)}{l_1}}{\dfrac{\mu_r \mu_o N_1^2 A_1}{l_1}} = \left(\frac{1}{2}\right)^2 \times 3 = \frac{3}{4}$$

$$\therefore L = \frac{3}{4} L_o = \left(\frac{3}{4}\right)(16\ \mu H) = \mathbf{12\ \mu H}$$

c. Both μ and the number of turns have increased, although the increase in the number of turns is squared. The increased length reduces the inductance. Therefore,

$$L_o = \frac{\mu_r \mu_o N_1^2 A}{l_1} = \frac{(1)\mu_o N_1^2 A}{l_1} \quad and \quad L = \frac{\mu_r \mu_o N_2^2 A_2}{l_2} = \frac{(1200)\mu_o (3N_1)^2 A_1}{2.5 l_1}$$

$$\frac{L}{L_o} = \frac{\dfrac{(1200)\mu_o(3N_1)^2A_1}{2.5l_1}}{\dfrac{\mu_oN_1^2A_1}{l_1}} = \frac{(1200)(3)^2}{2.5}$$

$$\therefore L = \frac{(1200)(3)^2}{2.5}L_o = \left(\frac{(1200)(3)^2}{2.5}\right)(10\ \mu H)\ = \mathbf{43.2\ mH}$$

5.8 Induced Voltage v_L:

5.8.1 Faraday's Law:

Faraday's law of electromagnetic induction, is one of the most important in this field because it enables us to establish ac and dc voltages with a generator. If we move a conductor (any material with conductor characteristics through a magnetic field so that it cuts magnetic lines of flux as shown in Figure (5.16), a voltage is induced across the conductor that can be measured with a sensitive voltmeter. That's all it takes, and, in fact, the faster you move the conductor through the magnetic flux, the greater is the induced voltage. The same effect can be produced if you hold the conductor still and move the magnetic field across the conductor. Note that the direction in which you move the conductor through the field determines the polarity of the induced voltage. Also, if you move the conductor through the field at right angles to the magnetic flux, you generate the maximum induced voltage. Moving the conductor parallel with the magnetic flux lines results in an induced voltage of zero volts since magnetic lines of flux are not crossed.

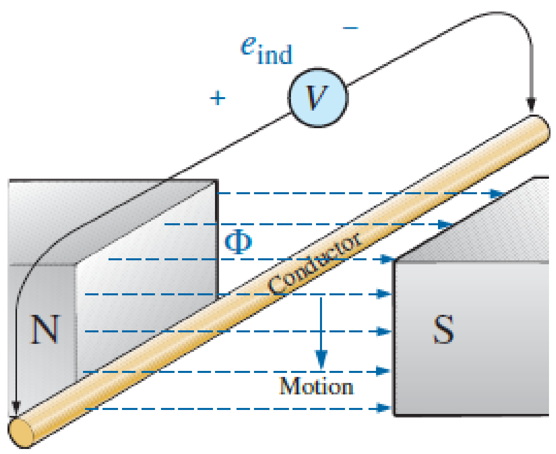


Figure (5.16): Generating an induced voltage by moving a conductor through a magnetic field.

If we now go a step further and move a coil of N turns through the magnetic field as shown in Figure (5.17), a voltage will be induced across the coil as determined by **Faraday's law:**

$$e = N\frac{d\phi}{dt} \quad \text{(volts, V)}$$

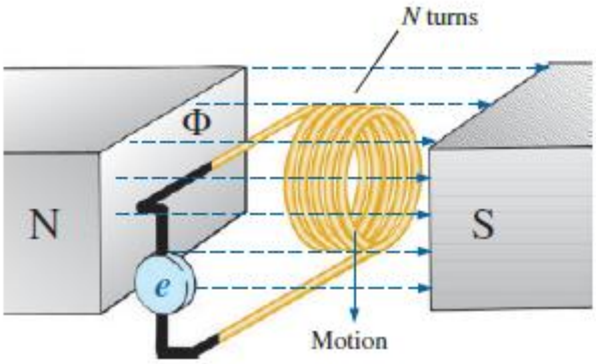


Figure (5.17): Demonstrating Faraday's law.

The greater the number of turns or the faster the coil is moved through the magnetic flux pattern, the greater is the induced voltage. The term $d\phi/dt$ is the differential change in magnetic flux through the coil at a particular instant in time. If the magnetic flux passing through a coil remains constant—no matter how strong the magnetic field—the term will be zero, and the induced voltage zero volts. It doesn't matter whether the changing flux is due to moving the magnetic field or moving the coil in the vicinity of a magnetic field: The only requirement is that the flux linking (passing through) the coil changes with time. Before the coil passes through the magnetic poles, the induced voltage is zero because there are no magnetic flux lines passing through the coil. As the coil enters the flux pattern, the number of flux lines cut per instant of time increases until it peaks at the center of the poles. The induced voltage then decreases with time as it leaves the magnetic field.

5.8.2 Lenz's Law:

The inductor in Figure (5.18) is simply an extended version of the coil in Figure (5.17). We found that the magnetic flux linking the coil of N turns with a current I has the distribution shown in Figure (5.18). If the current through the coil increases in magnitude, the flux linking the coil also increases. We just learned through Faraday's law, however, that a coil in the vicinity of a changing magnetic flux will have a voltage induced across it. The result is that a voltage is induced across the coil in Figure (5.18) due to the *change in current through the coil.*

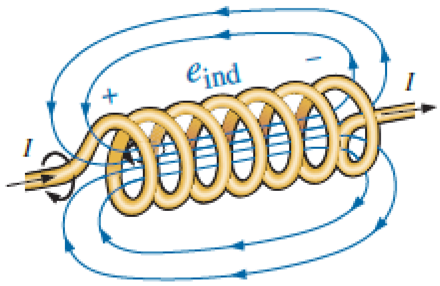


Figure (5.18): Demonstrating the effect of Lenz's law.

It is very important to note in Figure (5.18) that the polarity of the induced voltage across the coil is such that it opposes the increasing level of current in the coil. In other words, the changing current through the coil induces a voltage across the coil that is opposing the applied voltage that establishes the increase in current in the first place. The quicker the change in current through the coil, the greater is the opposing induced voltage to squelch the attempt of the current to increase in magnitude. The "choking" action of the coil is the reason inductors or coils are often referred to as **chokes.** This effect is a result of an important law referred to as **Lenz's law,** which states that ***an induced effect is always such as to oppose the cause that produced it.***

The inductance of a coil is also a measure of the change in flux linking the coil due to a change in current through the coil. That is,

$$L = N\frac{d\phi}{di_L} \quad \text{(henries, H)}$$

The equation reveals that the greater the number of turns or the greater the change in flux linking the coil due to a particular change in current, the greater is the level of inductance. In other words, coils with smaller levels of inductance generate smaller changes in flux linking the coil for the same change in current through the coil. If the inductance level is very small, there will be almost no change in flux linking the coil, and the induced voltage across the coil will be very small.

$$e = N\frac{d\phi}{dt} = \left(N\frac{d\phi}{di_L}\right)\left(\frac{di_L}{dt}\right)$$

and substitute $L = N\frac{d\phi}{di_L}$, we obtain:

$$e_L = L\frac{di_L}{dt} \quad \text{(volts, V)}$$

which relates the voltage across a coil to the number of turns of the coil and the change in current through the coil.

When induced effects are used in the generation of voltages such as those from dc or ac generators, the symbol e is applied to the induced voltage. However, in network analysis, the voltage induced across an inductor will always have a polarity that opposes the applied voltage (like the voltage across a resistor). Therefore, the following notation is used for the induced voltage across an inductor:

$$v_L = L\frac{di_L}{dt} \quad \text{(volts, V)}$$

The equation clearly states that ***the larger the inductance and/or the more rapid the change in current through a coil, the larger will be the induced voltage across the coil.***

If the current through the coil fails to change with time, the induced voltage across the coil will be zero. We will find in the next section that for dc applications, when the transient phase has passed, $di_L/dt = 0$, and the induced voltage across the coil is:

$$v_L = L\frac{di_L}{dt} = L(0) = 0\,V$$

5.9 R-L Transient:

5.9.1 The Storage Phase:

The circuit in Figure (5.19) is used to describe the storage phase.

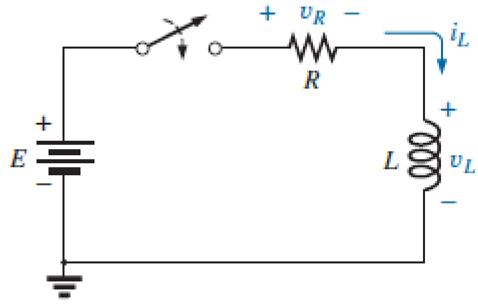
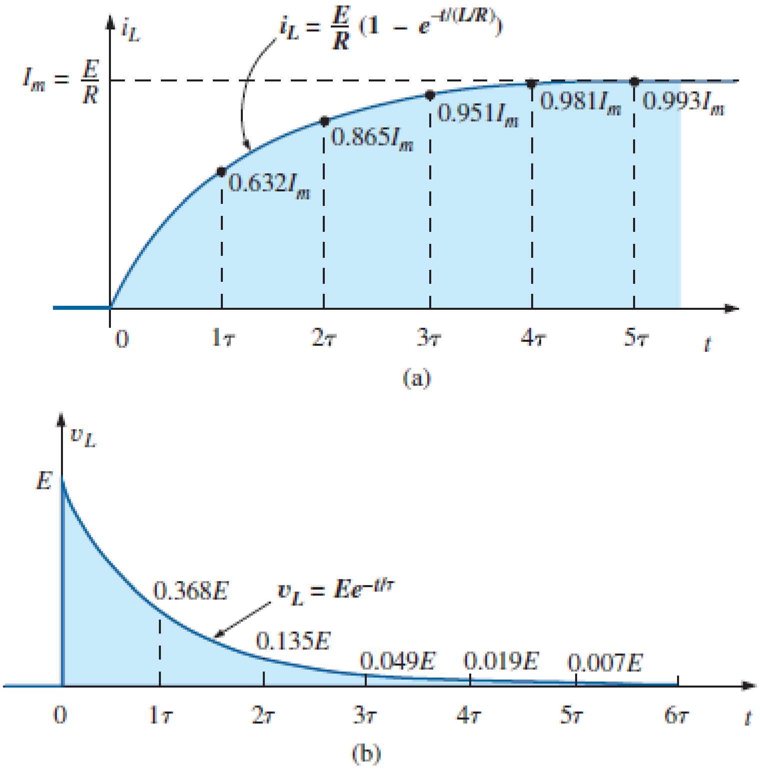


Figure (5.19): Basic R-L transient network.

For inductors, energy is stored in the form of a magnetic field linking the coil. ***The current cannot change instantaneously in an inductive network.***

1) **The current through the coil:**

At the instant the switch is closed, the choking action of the coil prevents an instantaneous change in current through the coil, resulting in $i_L = 0\ A$, as shown in Figure (5.20(a)). Initially, the current increases very rapidly and then at a much slower rate as it approaches its steady-state value determined by the parameters of the network (E/R).



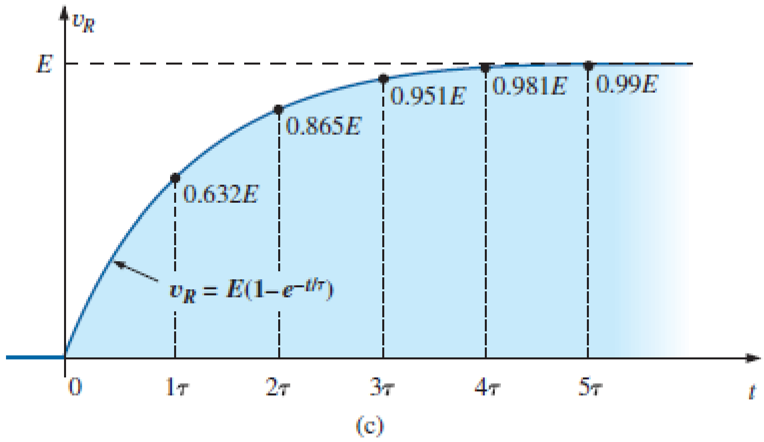


Figure (5.20): i_L, v_L, and v_R for the circuit in Figure (5.19) following the closing of the switch.

The equation for the transient response of the current through an inductor is:

$$i_L = \frac{E}{R}(1 - e^{-\frac{t}{\tau}}) \qquad \text{(Ampers, A)}$$

with the time constant now defined by:

$$\tau = \frac{L}{R} \qquad \text{(seconds, s)}$$

Our experience with the factor $(1 - e^{-\frac{t}{\tau}})$ verifies the level of 63.2% for the inductor current after one time constant, 86.5% after two time constants, and so on. If we keep R constant and increase L, the ratio L/R increases, and the rise time of 5τ increases as shown in Figure (5.21) for increasing levels of L. The change in transient response is expected because the higher the inductance level, the greater is the choking action on the changing current level, and the longer it will take to reach steady-state conditions.

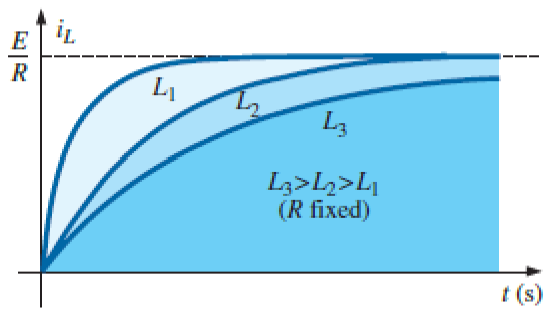


Figure (5.21): Effect of L on the shape of the i_L storage waveform.

2) The voltage across the resistor:

The absence of a current through the coil and circuit at the instant the switch is closed results in zero volts across the resistor as determined by $v_R = i_R R = i_L R = (0\ A)R = 0\ V$, as shown in Figure (5.20(c)).

As the current increases very rapidly, as shown in Figure (5.20(a)) and then at a much slower rate as it approaches its steady-state value determined by the parameters of the network (E/R). The voltage across the resistor rises at the same rate because $v_R = i_R R = i_L R$.

The equation for the voltage across the resistor is:

$$v_R = E(1 - e^{-\frac{t}{\tau}}) \qquad \text{(volts, V)}$$

3) The voltage across the coil:

Applying Kirchhoff's voltage law around the closed loop results in E volts across the coil at the instant the switch is closed, as shown in Figure (5.20(b)). We find for the inductive network of Figure (5.19) that $v_L(0-) = 0\ V$ and $v_L(0+) = E$ volts.

Since the voltage across the coil is sensitive to the rate of change of current through the coil, the voltage will be at or near its maximum value early in the storage phase. Finally, when the current reaches its steady-state value of E/R amperes, the current through the coil ceases to change, and the voltage across the coil drops to zero volts. At any instant of time, the voltage across the coil can be determined using Kirchhoff's voltage law in the following manner: $v_L = E - v_R$.

The equation for the voltage across the coil is:

$$v_L = Ee^{-\frac{t}{\tau}} \qquad \text{(volts, V)}$$

The inductor takes on the characteristics of an open circuit (see Figure (5.22)) at the instant the switch is closed.

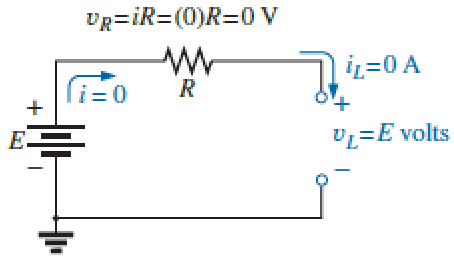


Figure (5.22): Circuit in Figure (5.19) the instant the switch is closed.

The inductor takes on the characteristics of a short circuit (see Figure (5.23)) when steady-state conditions have been established.

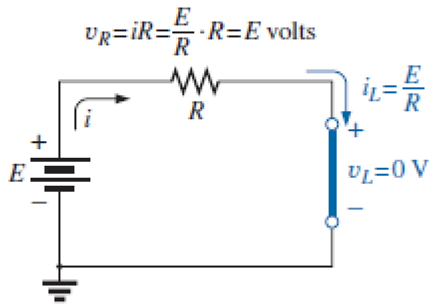
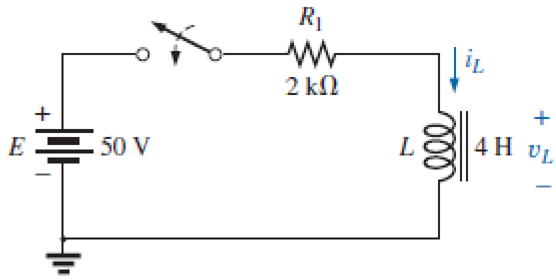


Figure (5.23): Circuit in Figure (5.19) under steady-state conditions.

Example 12: Find the mathematical expressions for the transient behavior of i_L and v_L for the circuit in Figure below if the switch is closed at $t = 0\ s$. Sketch the resulting curves.



Solution: First, we determine the time constant:

$$\tau = \frac{L}{R_1} = \frac{4\ H}{2\ K\Omega} = 2\ ms$$

Then the maximum or steady-state current is:

$$I_m = \frac{E}{R_1} = \frac{50\ V}{2\ K\Omega} = 25 \times 10^{-3}\ A = 25\ mA$$

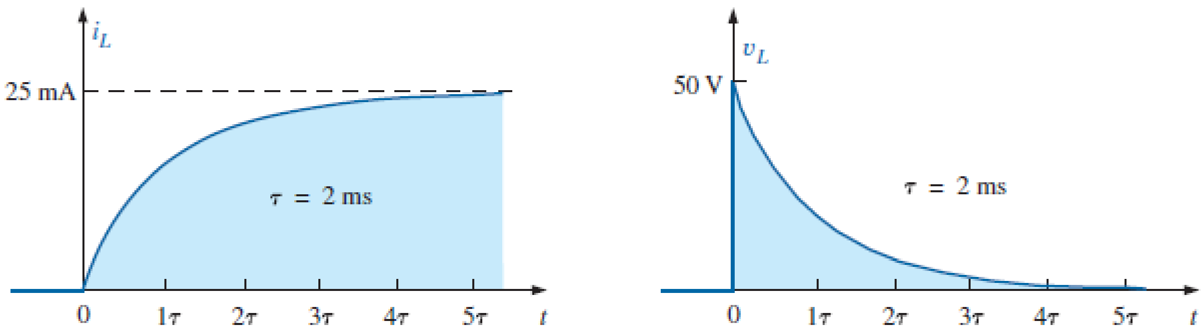
Substituting into $i_L = \frac{E}{R_1}(1 - e^{-\frac{t}{\tau}})$ gives:

$$\boldsymbol{i_L = 25\,mA(1 - e^{-\frac{t}{2\,ms}})}$$

Using $v_L = Ee^{-\frac{t}{\tau}}$ gives:

$$\boldsymbol{v_L = 50\,V\,e^{-\frac{t}{2\,ms}}}$$

The resulting waveforms appear in Figure below.



5.9.2 The Release Phase:

If the series R-L circuit in Figure (5.24) reaches steady-state conditions and the switch is quickly opened, a spark will occur across the contacts due to the rapid change in current from a maximum of E/R to zero amperes. The change in current di/dt of the equation $v_L = L(di/dt)$ establishes a high v_L voltage across the coil that, in conjunction with the applied voltage E, appears across the points of the switch.

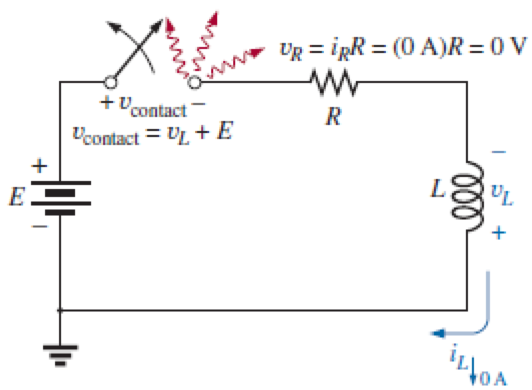


Figure (5.24): Demonstrating the effect of opening a switch in series with an inductor with a steady-state current.

If opening the switch to move it to another position causes such a rapid discharge in stored energy. The solution is to use a network like that in Figure (5.25(a)). When the switch is closed, the voltage across resistor R_2 is E volts, and the R-L branch responds in the same manner as described above, with the same wave forms and levels. A Thévenin network of E in parallel with R_2 results in the source as shown in Figure (5.25(b)) since R_2 will be shorted out by the short-circuit replacement of the voltage source E when the Thévenin resistance is determined.

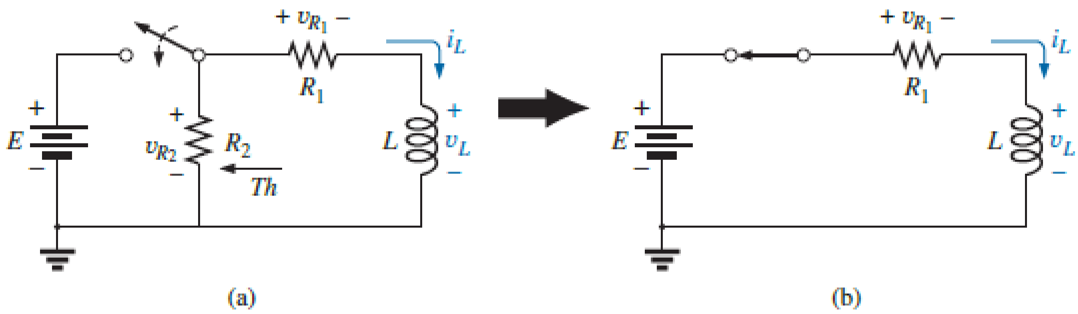


Figure (5.25): Initiating the storage phase for an inductor by closing the switch.

After the storage phase has passed and steady-state conditions are established, the switch can be opened without the sparking effect or rapid discharge due to resistor R_2, which provides a complete path for the current i_L. In fact, for clarity the discharge path is isolated in Figure (5.26). The voltage v_L across the inductor reverses polarity and has a magnitude determined by: $v_L = -(v_{R_1} + v_{R_2})$

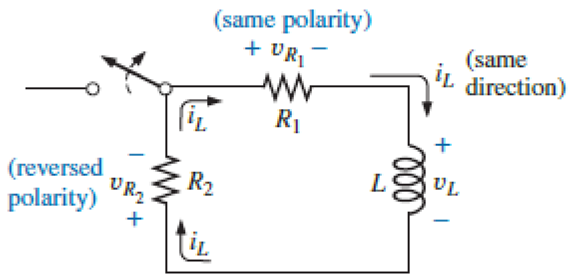


Figure (5.26): Network in Figure (5.25) the instant the switch is opened.

Recall that the voltage across an inductor can change instantaneously but the current cannot. The result is that the current i_L must maintain the same direction and magnitude, as shown in Figure (5.26). Therefore, the instant after the switch is opened, i_L is still $I_m = E/R_1$, and:

$$v_L = -(v_{R_1} + v_{R_2}) = -(i_1R_1 + i_2R_2) = -\,i_L(R_1 + R_2) = -\frac{E}{R_1}(R_1 + R_2)$$

$$v_L = -E\left(\frac{R_1}{R_1} + \frac{R_2}{R_1}\right)$$

and

$$v_L = -\left(1 + \frac{R_2}{R_1}\right)E \quad \text{(switch opened)}$$

which is bigger than E volts by the ratio R_2/R_1.

As an inductor releases its stored energy, the voltage across the coil decays to zero in the following manner:

$$v_L = -V_i\, e^{-t/\tau'}$$

with

$$V_i = -\left(1 + \frac{R_2}{R_1}\right)E$$

and

$$\tau' = \frac{L}{R_T} = \frac{L}{R_1 + R_2}$$

The current decays from a maximum of $I_m = E/R_1$ to zero.

and

$$i_L = \frac{E}{R_1} e^{-t/\tau'}$$

with

$$\tau' = \frac{L}{R_1 + R_2}$$

The mathematical expression for the voltage across either resistor can then be determined using Ohm's law:

$$v_{R_1} = i_1 R_1 = i_L R_1 = \frac{E}{R_1} R_1 e^{-t/\tau'}$$

and

$$v_{R_1} = E e^{-t/\tau'}$$

The voltage v_{R_1} has the same polarity as during the storage phase since the current i_L has the same direction. The voltage v_{R_2} is expressed as follows using the defined polarity of Figure (5.25):

$$v_{R_2} = -i_2R_2 = -i_LR_2 = -\frac{E}{R_1}R_2e^{-t/\tau'}$$

and

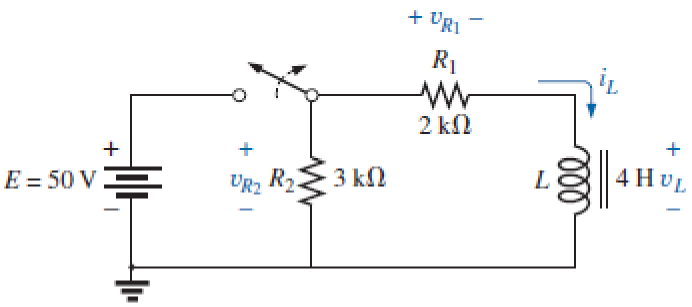
$$v_{R_2} = -\frac{R_2}{R_1}Ee^{-t/\tau'}$$

Example 13: Resistor R_2 was added to the network in Figure of Example 12 as shown in Figure below.

A) Find the mathematical expressions for i_L, v_L, v_{R_1}, and v_{R_2} for five time constants of the storage phase.

B) Find the mathematical expressions for i_L, v_L, v_{R_1}, and v_{R_2} if the switch is opened after five time constants of the storage phase.

C) Sketch the waveforms for each voltage and current for both phases covered by this example. Use the defined polarities in Figure below.



Solution:

A) From Example 12:

Substituting into $i_L = \frac{E}{R_1}(1 - e^{-\frac{t}{\tau}})$ gives:

$$i_L = \mathbf{25\ mA(1 - e^{-\frac{t}{2\ ms}})}$$

Using $v_L = Ee^{-\frac{t}{\tau}}$ gives:

$$v_L = \mathbf{50\ V\ e^{-\frac{t}{2\ ms}}}$$

$$v_{R_1} = i_1R_1 = i_LR_1 = \left[\frac{E}{R_1}\left(1 - e^{-t/\tau'}\right)\right]R_1 = E\left(1 - e^{-t/\tau'}\right)$$

and

$$v_{R_1} = \mathbf{50\,V}\left(\mathbf{1 - e^{-t/2\,ms}}\right)$$

$$v_{R_2} = E = \mathbf{50\,V}$$

B)

$$\tau' = \frac{L}{R_1 + R_2} = \frac{4\,H}{2\,K\Omega + 3\,K\Omega} = \frac{4\,H}{5\ \times 10^3\ \Omega} = 0.8 \times 10^{-3}\,s\ =\ 0.8\,ms$$

By $\left[v_L = -\left(1 + \frac{R_2}{R_1}\right)E\right]$ and $v_L = -V_i\,e^{-t/\tau'}$:

$$V_i = \left(1 + \frac{R_2}{R_1}\right)E = \left(1 + \frac{3\,K\Omega}{2\,K\Omega}\right)(50\,V) = 125\,V$$

and

$$v_L = -V_i\,e^{-\frac{t}{\tau'}} = \mathbf{-125\,V\,e^{-\frac{t}{-0.8ms}}}$$

By $i_L = \frac{E}{R_1}e^{-t/\tau'}$:

$$I_m = \frac{E}{R_1} = \frac{50\,V}{2\,K\Omega} = 25\,mA$$

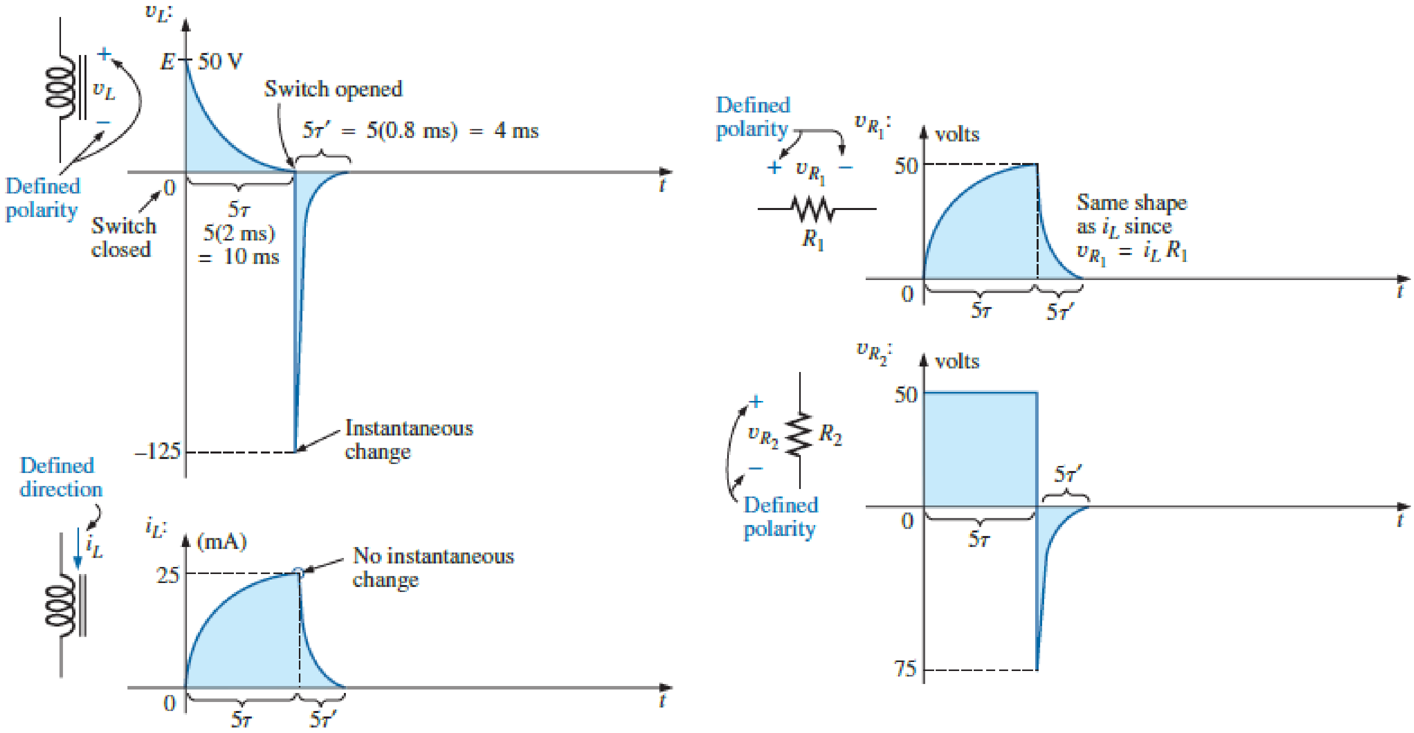
and

$$i_L = I_m\,e^{-\frac{t}{\tau'}} = \mathbf{25\,mA\,e^{-\frac{t}{0.8ms}}}$$

$$v_{R_1} = E e^{-t/\tau'} = \mathbf{50\,V\,e^{-\frac{t}{0.8ms}}}$$

$$v_{R_2} = -\frac{R_2}{R_1}Ee^{-\frac{t}{\tau'}} = -\frac{3\,K\Omega}{2\,K\Omega}(50\,V\,)e^{-\frac{t}{0.8ms}} = \mathbf{-75\,Ve^{-\frac{t}{0.8ms}}}$$

C) See Figure below.



5.10 Inductors in Series and in Parallel:

Inductors, like resistors and capacitors, can be placed in series or in parallel. Increasing levels of inductance can be obtained by placing inductors in series, while decreasing levels can be obtained by placing inductors in parallel.

For inductors in series, the total inductance is found in the same manner as the total resistance of resistors in series (Figure (5.27)):

$$L_T = L_1 + L_2 + L_3 + \cdots + L_N$$

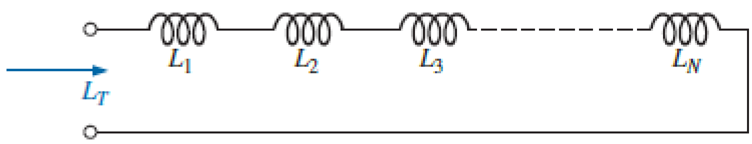


Figure (5.27): Inductors in series.

For inductors in parallel, the total inductance is found in the same manner as the total resistance of resistors in parallel (Figure (5.28)):

$$\frac{1}{L_T} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \cdots + \frac{1}{L_N}$$

For two inductors in parallel:

$$L_T = \frac{L_1 L_2}{L_1 + L_2}$$

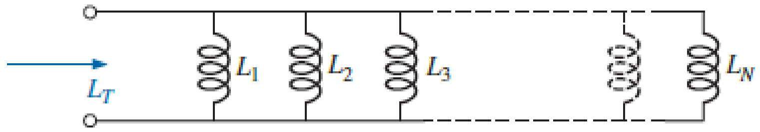
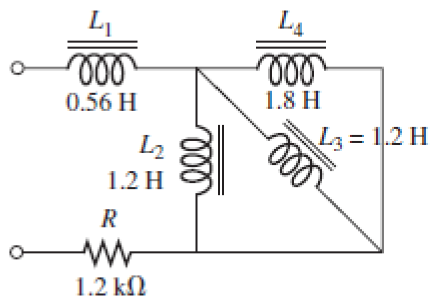


Figure (5.28): Inductors in parallel.

Example 14: Reduce the network in Figure below to its simplest form.



Solution: Inductors L_2 and L_3 are equal in value and they are in parallel, resulting in an equivalent parallel value of:

$$L'_T = \frac{L}{N} = \frac{1.2\ H}{2} = 0.6\ H$$

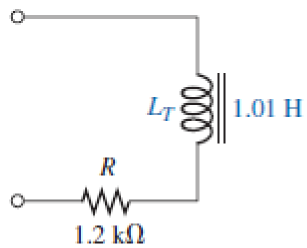
The resulting 0.6 H is then in parallel with the 1.8 H inductor, and:

$$L^n_T = \frac{L'_T\ L_4}{L'_T + L_4} = \frac{(0.6\ H)(1.8\ H)}{0.6\ H\ +\ 1.8\ H} = 0.45\ H$$

Inductor L_1 is then in series with the equivalent parallel value, and:

$$L_T = L_1 + L^n_T = 0.56\ H\ +\ 0.45\ H\ =\ \mathbf{1.01}\ \boldsymbol{H}$$

The reduced equivalent network appears in Figure below.



5.11 Steady-State Conditions:

For all practical purposes, an ideal (ignoring internal resistance and stray capacitances) inductor can be replaced by a short-circuit equivalent once steady-state conditions have been established. For all practical purposes, our assumption is that steady-state conditions have been established after five time constants of the storage or release phase have passed. The term steady state implies that the voltage and current levels have reached their final resting value and will no longer change unless a change is made in the applied voltage or circuit configuration.

For the circuit in Figure (5.29(a)), if we assume that steady-state conditions have been established, the inductor can be removed and replaced by a short-circuit equivalent as shown in Figure (5.29(b)). The short-circuit equivalent shorts out the 3 Ω resistor, and current I_1 is determined by:

$$I_1 = \frac{E}{R_1} = \frac{10\ V}{2\ \Omega} = 5\ A$$

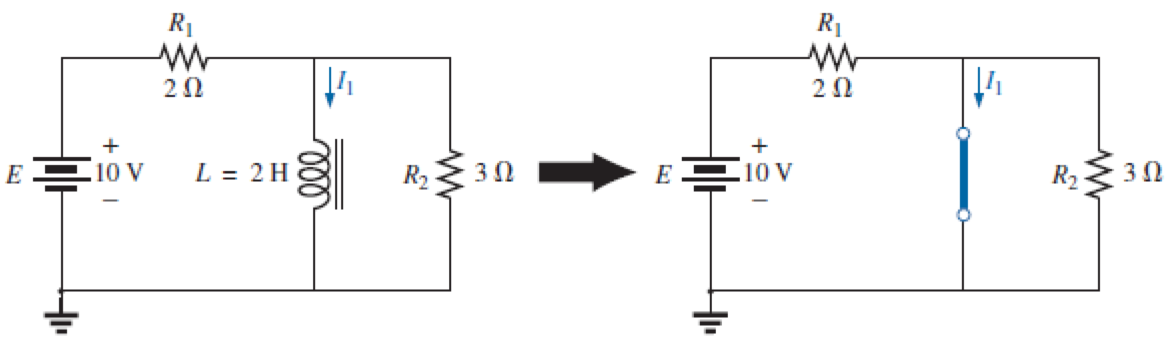


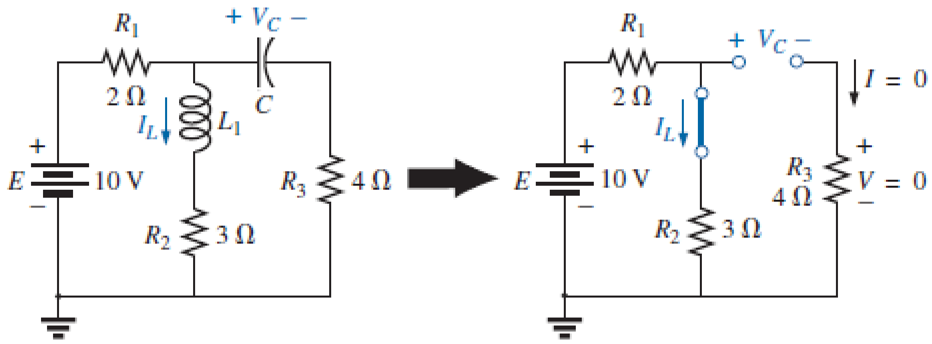
Figure (5.29): Substituting the short-circuit equivalent for the inductor for $t > 5\tau$.

Example 15: Find the current I_L and the voltage V_C for the network in Figure below.

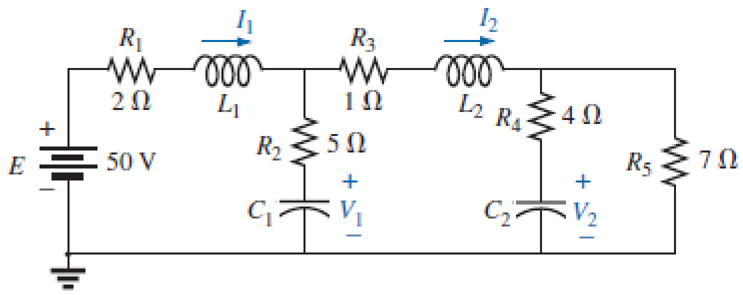
Solution:

$$I_L = \frac{E}{R_1 + R_2} = \frac{10\ V}{2\ \Omega + 3\ \Omega} = \frac{10\ V}{5\ \Omega} = \mathbf{2\ A}$$

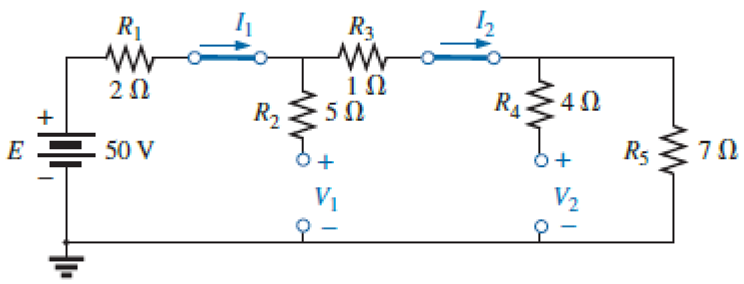
$$V_C = \frac{R_2\ E}{R_1 + R_2} = \frac{(3\ \Omega)(10\ V)}{2\ \Omega + 3\ \Omega} = \frac{30\ V}{5} = \mathbf{6\ V}$$



Example 16: Find currents I_1 and I_2 and voltages V_1 and V_2 for the network in Figure below.



Solution: Note Figure below.



$$I_1 = I_2 = \frac{E}{R_1 + R_3 + R_5} = \frac{50\ V}{2\ \Omega +\ 1\ \Omega +\ 7\Omega} = \frac{50\ V}{10\ \Omega} = \mathbf{5\ A}$$

$$V_2 = I_2 R_5 = (5\ A)(7\ \Omega) = \mathbf{35\ V}$$

Applying the voltage divider rule yields:

$$V_1 = \frac{(R_3 + R_5)E}{R_1 + R_3 + R_5} = \frac{(\ 1\ \Omega +\ 7\Omega)(50\ V)}{2\ \Omega +\ 1\ \Omega +\ 7\Omega} = \frac{(\ 8\ \Omega)(50\ V)}{10\ \Omega} = \mathbf{40\ V}$$

5.12 Energy Stored by an Inductor:

The ideal inductor, like the ideal capacitor, does not dissipate the electrical energy supplied to it. It stores the energy in the form of a magnetic field. A plot of the voltage, current, and power to an inductor is shown in Figure (5.30) during the buildup of the magnetic field surrounding the inductor. The energy stored is represented by the shaded area under the power curve. Using calculus, we can show that the evaluation of the area under the curve yields:

$$W_{stored} = \frac{1}{2} L I_m^2 \quad \text{(joules, J)}$$

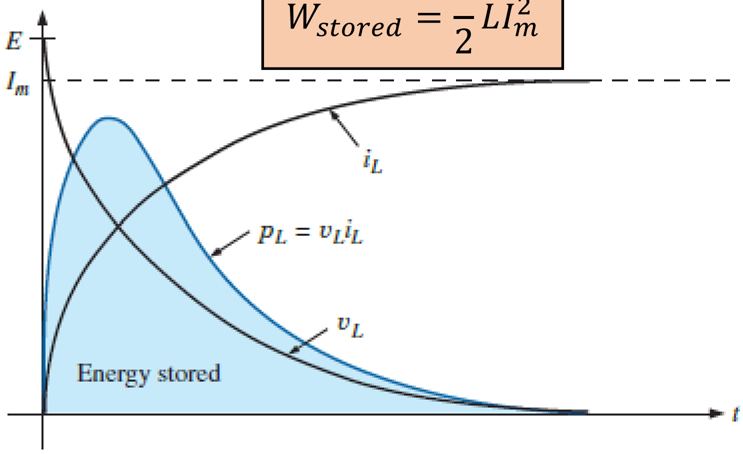
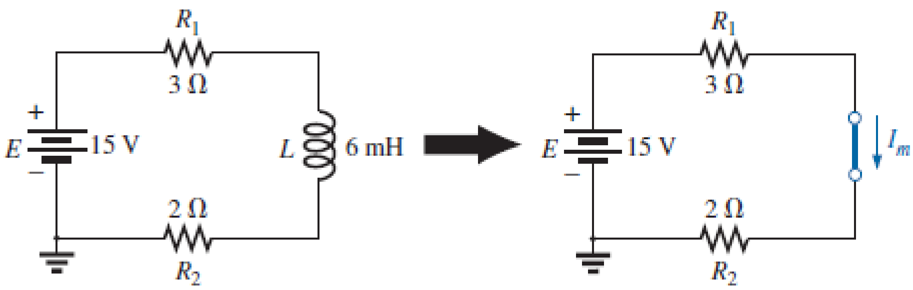


Figure (5.30): The power curve for an inductive element under transient conditions.

Example 17: Find the energy stored by the inductor in the circuit in Figure below when the current through it has reached its final value.



Solution:

$$I_m = \frac{E}{R_1 + R_2} = \frac{15\ V}{3\ \Omega + 2\ \Omega} = \frac{15\ V}{5\ \Omega} = 3\ A$$

$$W_{stored} = \frac{1}{2} L I_m^2 = \frac{1}{2}(6 \times 10^{-3} H)(3\ A)^2 = \frac{54}{2} 10^{-3}\ J = \mathbf{27\ mJ}$$

Lecture 6: Sinusoidal Alternating Waveforms

6.1 Alternating Waveforms:

In dc networks, the currents or voltages are fixed in magnitude except for transient effects. We now turn our attention to the analysis of networks in which the magnitude of the source varies in a set manner. Of particular interest is the time-varying voltage that is commercially available in large quantities and is commonly called the ac voltage. (The letters ac are an abbreviation for alternating current). To be absolutely rigorous, the terminology ac voltage or ac current is not sufficient to describe the type of signal we will be analyzing. Each waveform in Figure (6.1) is an alternating waveform available from commercial suppliers. The term alternating indicates only that the waveform alternates between two prescribed levels in a set time sequence. To be absolutely correct, the term sinusoidal, square-wave, or triangular must also be applied.

The pattern of particular interest is the sinusoidal ac voltage in Figure (6.1). Since this type of signal is encountered in the vast majority of instances, the abbreviated phrases ac voltage and ac current are commonly applied without confusion. For the other patterns in Figure (6.1), the descriptive term is always present, but frequently the ac abbreviation is dropped, resulting in the designation square-wave or triangular waveforms.

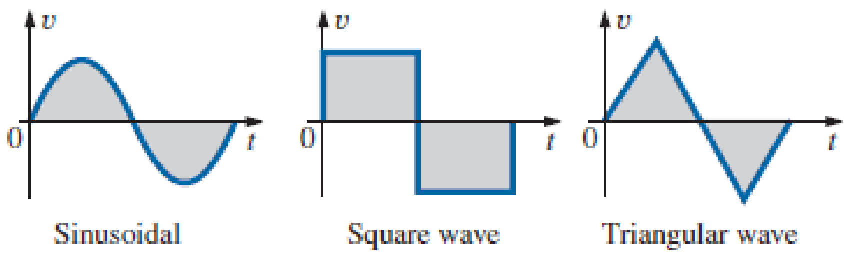


Figure (6.1): Alternating waveforms.

The important reasons for concentrating on the sinusoidal ac voltage it is the voltage generated by utilities throughout the world and its application throughout electrical, electronic, communication, and industrial systems. The waveform itself has a number of characteristics that result in a unique response when it is applied to basic electrical elements.

6.2 Definitions of Alternating Waveforms:

The sinusoidal waveform in Figure (6.2) with its additional notation will now be used as a model in defining a few basic terms. These terms, however, can be applied to any alternating waveform. It is important to remember, that the vertical scaling is in volts or amperes and the horizontal scaling is in units of time.

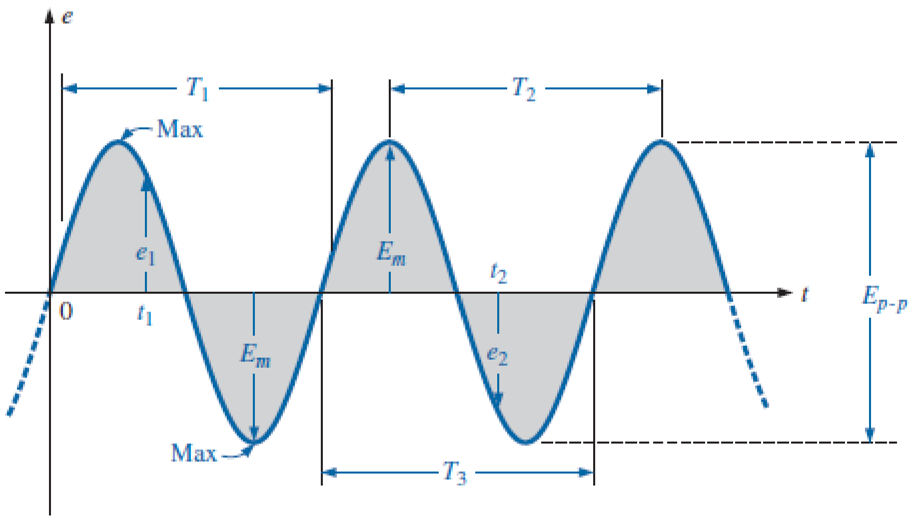


Figure (6.2): Important parameters for a sinusoidal voltage.

- **Waveform:** The path traced by a quantity, such as the voltage in Figure (6.2), plotted as a function of some variable, such as time (as above), position, degrees, radians, temperature, and so on.
- **Instantaneous value:** The magnitude of a waveform at any instant of time; denoted by lowercase letters (e_1, e_2 in Figure (6.2)).
- **Peak amplitude:** The maximum value of a waveform as measured from its average, or mean, value, denoted by uppercase letters [such as E_m (Figure (6.2)) for sources of voltage and V_m for the voltage drop across a load]. For the waveform in Figure (6.2), the average value is zero volts, and E_m is as defined by the figure.

- **Peak value:** The maximum instantaneous value of a function as measured from the zero volt level. For the waveform in Figure (6.2), the peak amplitude and peak value are the same since the average value of the function is zero volts.
- **Peak-to-peak value:** Denoted by E_{P-P} or V_{P-P} (as shown in Figure (6.2)), the full voltage between positive and negative peaks of the waveform, that is, the sum of the magnitude of the positive and negative peaks.
- **Periodic waveform:** A waveform that continually repeats itself after the same time interval. The waveform in Figure (6.2) is a periodic waveform.
- **Period (T):** The time of a periodic waveform.
- **Cycle:** The portion of a waveform contained in one period of time. The cycles within T_1, T_2, and T_3 in Figure (6.2) may appear differently in Figure (6.3), but they are all bounded by one period of time and therefore satisfy the definition of a cycle.

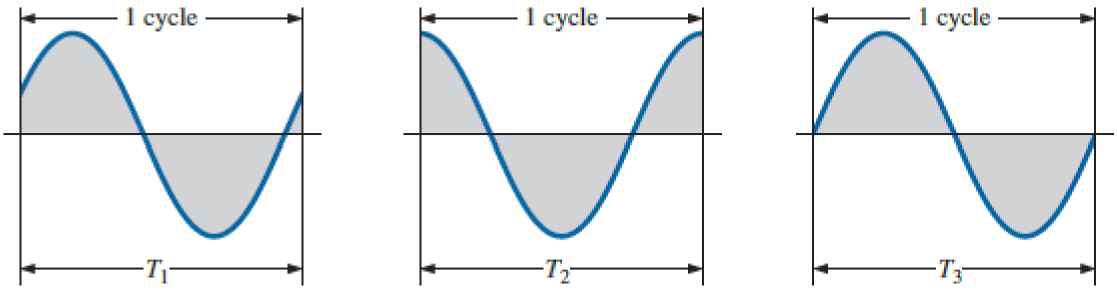


Figure (6.3): Defining the cycle and period of a sinusoidal waveform.

- **Frequency (f):** The number of cycles that occur in 1 s. The frequency of the waveform in Figure (6.4(a)) is 1 cycle per second, and for Figure (6.4(b)), $2\frac{1}{2}$ cycles (2.5 cycles) per second. If a waveform of similar shape had a period of 0.5 s [Figure (6.4(c))], the frequency would be 2 cycles per second.

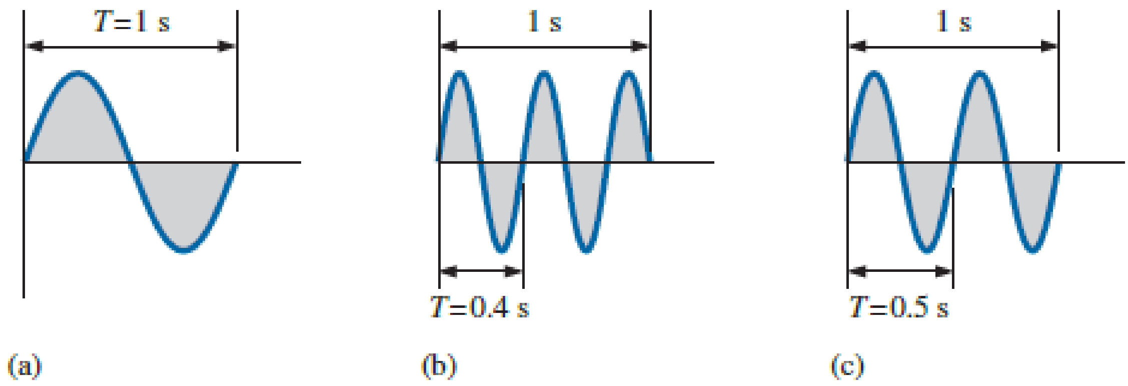


Figure (6.4): Demonstrating the effect of a changing frequency on the period of a sinusoidal waveform.

The unit of measure for frequency is the hertz (Hz), where:

$$1\ hertz\ (Hz) = 1 \text{ cycle per second (cps)}$$

The unit hertz is derived from the surname of Heinrich Rudolph Hertz, who did original research in the area of alternating currents and voltages and their effect on the basic R, L, and C elements. The frequency standard for North America is 60 Hz, whereas for Europe it is predominantly 50 Hz.

Since the frequency is inversely related to the period—that is, as one increases, the other decreases by an equal amount—the two can be related by the following equation:

$$f = \frac{1}{T} \qquad f = \text{Hz}, \quad T = \text{second (s)}$$

or

$$T = \frac{1}{f}$$

Example 1: For the sinusoidal waveform in Figure below:

A) What is the peak value?

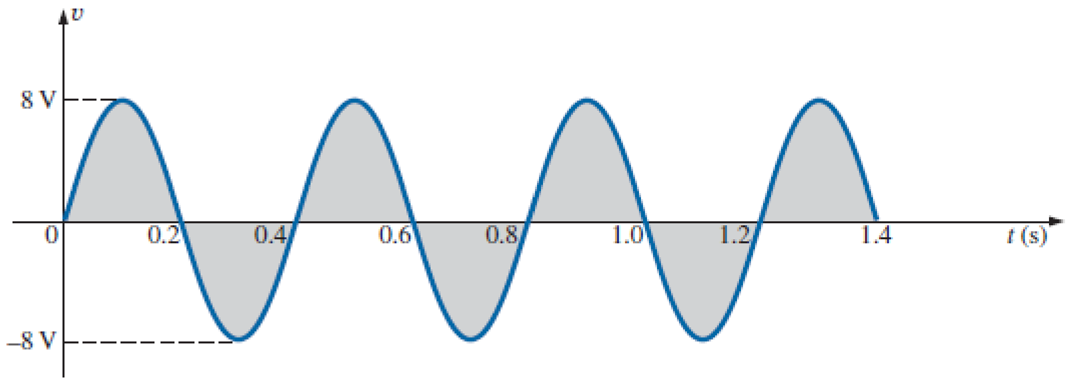
B) What is the instantaneous value at 0.3 s and 0.6 s?

C) What is the peak-to-peak value of the waveform?

D) What is the period of the waveform?

E) How many cycles are shown?

F) What is the frequency of the waveform?



Solution:

A) 8 V.

B) At 0.3 s, **_8 V;** at 0.6 s, **0 V.**

C) 16 V.

D) 0.4 s.

E) 3.5 cycles.

F) 2.5 cps, or **2.5 Hz.**

Example 2: Find the period of periodic waveform with a frequency of:

A) 60 Hz.

B) 1000 Hz.

Solution:

A)

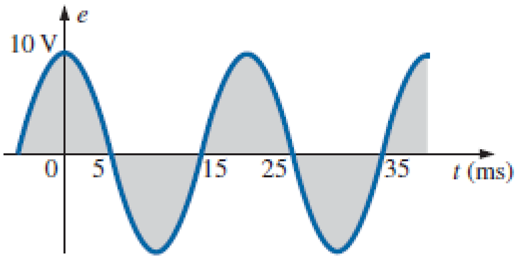
$$T = \frac{1}{f} = \frac{1}{60\ Hz} \cong 0.01667\ s\ or\ \mathbf{16.67\ ms}$$

(a recurring value since 60 Hz is so prevalent)

B)

$$T = \frac{1}{f} = \frac{1}{1000\ Hz} = \mathbf{10^{-3} s = 1\ ms}$$

Example 3: Determine the frequency of the waveform in Figure below.



Solution: From the figure, T = (25 ms - 5 ms) or (35 ms - 15 ms) = 20 ms, and

$$f = \frac{1}{T} = \frac{1}{2 \times 10^{-3} s} = \mathbf{50\ Hz}$$

6.3 Defined Polarities and Direction:

For a period of time, a voltage has one polarity, while for the next equal period it reverses. To take care of this problem, a positive sign is applied if the voltage is above the axis, as shown in Figure (6.5(a)). For a current source, the direction in the symbol corresponds with the positive region of the waveform, as shown in Figure (6.5(b)).

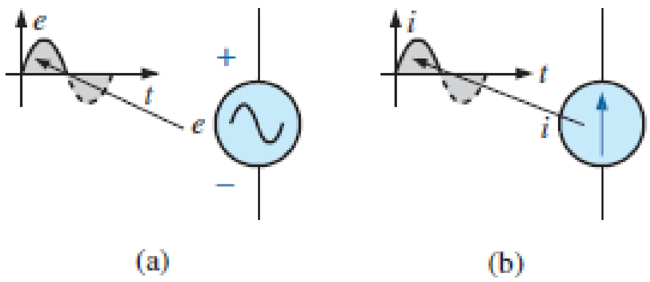


Figure (6.5): (a) Sinusoidal ac voltage sources; (b) sinusoidal current sources.

For any quantity that will not change with time, an uppercase letter such as V or I is used. For expressions that are time dependent or that represent a particular instant of time, a lowercase letter such as e or i is used.

6.4 The sinusoidal Waveform:

The sinusoidal waveform is the only alternating waveform whose shape is unaffected by the response characteristics of R, L, and C elements.

In other words, if the voltage across (or current through) a resistor, inductor, or capacitor is sinusoidal in nature, the resulting current (or voltage, respectively) for each will also have sinusoidal characteristics, as shown in Figure (6.6). If any other alternating waveform such as a square wave or a triangular wave were applied, such would not be the case.

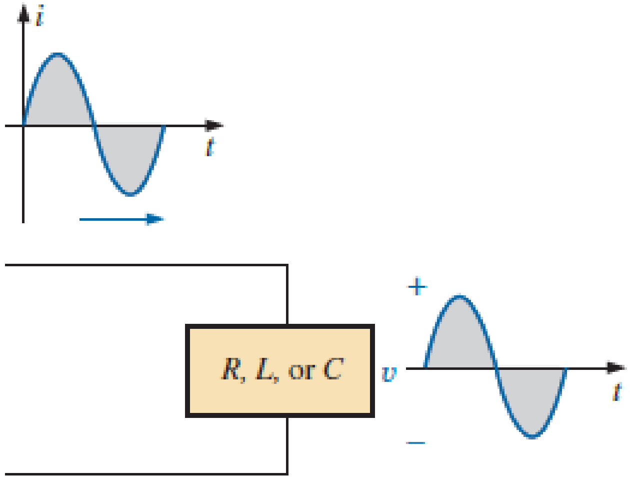


Figure (6.6): The sine wave is the only alternating waveform whose shape is not altered by the response characteristics of a pure resistor, inductor, or capacitor.

The unit of measurement for the horizontal axis can be time (as appearing in the figures thus far), degrees, or radians. The term radian can be defined as follows: If we mark off a portion of the circumference of a circle by a length equal to the radius of the circle, as shown in Figure (6.7), the angle resulting is called 1 radian. The result is:

$$1\,rad = 57.296° \cong 57.3°$$

Where 57.3° is the usual approximation applied.

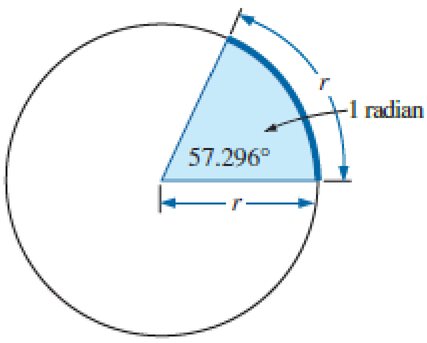


Figure (6.7): Defining the radian.

One full circle has 2π radians, as shown in Figure (6.8). That is:

$$2\pi\,rad = 360°$$

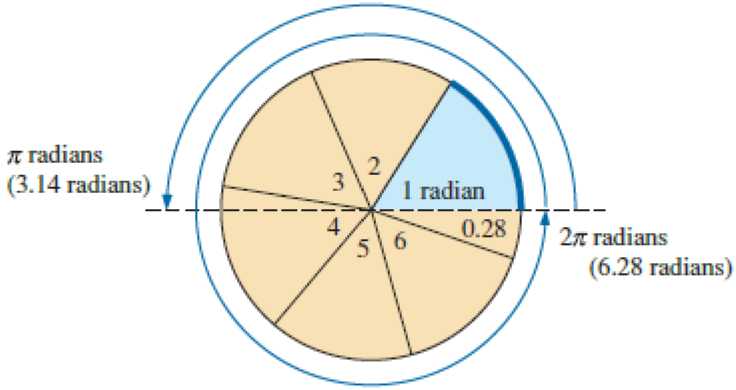


Figure (6.8): There are 2π radians in one full circle of 360°.

A number of electrical formulas contain a multiplier of π. For this reason, it is sometimes preferable to measure angles in radians rather than in degrees.

The quantity π is the ratio of the circumference of a circle to its diameter.

$$\pi = 3.14159\ 26535\ 89793\ 23846\ 26433\ \ldots$$

Although the approximation $\pi \cong 3.14$ is often applied.

The units of measurement Degrees and Radians, are related as shown in Figure (6.8). The conversions equations between the two are the following:

$$Radians = \left(\frac{\pi}{180°}\right) \times (degrees)$$

$$Degrees = \left(\frac{180°}{\pi}\right) \times (radians)$$

Applying these equations, we find:

$$\mathbf{90°}:\ \ Radians = \left(\frac{\pi}{180°}\right) \times (90°) = \boldsymbol{\frac{\pi}{2}\ rad}$$

$$\mathbf{30°}:\ \ Radians = \left(\frac{\pi}{180°}\right) \times (30°) = \boldsymbol{\frac{\pi}{6}\ rad}$$

$$\boldsymbol{\frac{\pi}{3}\ rad}:\ Degrees = \left(\frac{180°}{\pi}\right) \times \left(\frac{\pi}{3}\right) = \mathbf{60°}$$

$$\boldsymbol{\frac{3\pi}{2}\ rad}:\ Degrees = \left(\frac{180°}{\pi}\right) \times \left(\frac{3\pi}{2}\right) = \mathbf{270°}$$

The velocity with which the radius vector rotates about the center, called the angular velocity, can be determined from the following equation:

$$Angular\ velocity = \frac{distance\ (degrees\ or\ radians)}{time\ (seconds)}$$

Substituting into equation above and assigning the lowercase Greek letter $omega$ (ω) to the angular velocity, we have:

$$\omega = \frac{\alpha}{t}$$

and

$$\alpha = \omega t$$

Since ω is typically provided in radians per second, the angle α is usually in radians.

The time required to complete one revolution is equal to the period (T) of the sinusoidal waveform. The radians subtended in this time interval are 2π. Substituting, we have:

$$\omega = \frac{2\pi}{T} \quad \text{(rad/s)}$$

The frequency of the generated waveform is inversely related to the period of the waveform; that is, $f = 1/T$. Thus:

$$\omega = 2\pi f \quad \text{(rad/s)}$$

Example 4: Determine the angular velocity of a sine wave having a frequency of 60 Hz.

Solution:

$$\omega = 2\pi f = (2\pi)(60\ Hz) \cong \mathbf{377\ rad/s}$$

(a recurring value due to 60 Hz predominance).

Example 5: Given $\omega = 200\ rad/s$, determine how long it will take the sinusoidal waveform to pass through an angle of 90°.

Solution: $\alpha = \omega t$, and

$$t = \frac{\alpha}{\omega}$$

However, a must be substituted as $\pi/2$ $(= 90°)$ since ω is in radians per second:

$$t = \frac{\alpha}{\omega} = \frac{\pi/2\ rad}{200\ rad/s} = \frac{\pi}{400} s = \mathbf{7.85\ ms}$$

Example 6: Find the angle through which a sinusoidal waveform of 60 Hz will pass in a period of 5 ms.

Solution: $\alpha = \omega t$, or

$$\alpha = 2\pi f t = (2\pi)(60\ Hz)(5 \times 10^{-3}\ s) \ = \ \mathbf{1.89\ rad}$$

If not careful, you might be tempted to interpret the answer as 1.885° However,

$$\alpha(°) = \frac{180°}{\pi\ rad}(1.89\ \text{rad}) \ = \ \mathbf{108.3°}$$

6.5 General Format for the Sinusoidal Voltage or Current:

The basic mathematical format for the sinusoidal waveform is:

$$A_m \sin \alpha$$

Where A_m is the peak value of the waveform and α is the unit of measure for the horizontal axis, as shown in Figure (6.9).

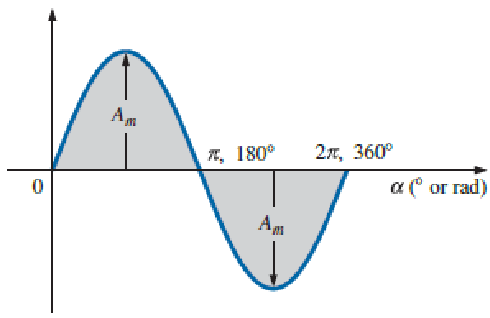


Figure (6.9): Basic sinusoidal function.

Due to equation $\alpha = \omega t$, the general format of a sine wave can also be written:

$$A_m \sin \omega t$$

with ωt as the horizontal unit of measure.

For electrical quantities such as current and voltage, the general format is:

$$i = I_m \sin \omega t = I_m \sin \alpha$$

$$e = E_m \sin \omega t = E_m \sin \alpha$$

where the capital letters with the subscript m represent the amplitude, and the lowercase letters i and e represent the instantaneous value of current and voltage, respectively, at any time t.

The angle at which a particular voltage level is attained can be determined by rearranging the equation:

$$e = E_m \sin \alpha$$

in the following manner:

$$\sin \alpha = \frac{e}{E_m}$$

which can be written:

$$\alpha = \sin^{-1} \frac{e}{E_m}$$

Similarly, for a particular current level:

$$\alpha = \sin^{-1} \frac{i}{I_m}$$

Example 7: Given $e = 5 \sin \alpha$, determine e at $\alpha = 40°$ and $\alpha = 0.8\pi$.

Solution: For $\alpha = 40°$,

$$e = 5 \sin 40° = 5(0.6428) = \mathbf{3.21}\ \boldsymbol{V}$$

For $\alpha = 0.8\pi$

$$\alpha(°) = \frac{180°}{\pi\ rad}(0.8\pi) = \mathbf{144°}$$

$$e = 5 \sin 144° = 5(0.5878) = \mathbf{2.94}\ \boldsymbol{V}$$

Example 8:

A) Determine the angle at which the magnitude of the sinusoidal function $v = 10 \sin 377t$ is 4 V.

B) Determine the time at which the magnitude is attained.

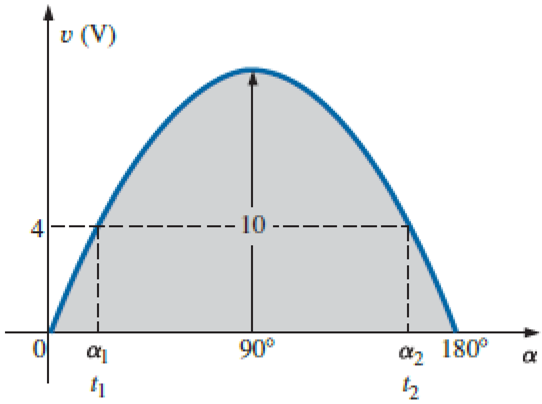
Solution:

A)

$$\alpha_1 = \sin^{-1}\frac{v}{E_m} = \sin^{-1}\frac{4\,V}{10\,V} = \sin^{-1}0.4 = \mathbf{23.578°}$$

However, Figure below reveals that the magnitude of 4 V (positive) will be attained at two points between 0° and 180°. The second intersection is determined by:

$$\alpha_2 = 180° - 23.578° = \mathbf{156.42°}$$



In general, therefore, keep in mind that equations $\alpha = \sin^{-1}\frac{e}{E_m}$ and $\alpha = \sin^{-1}\frac{i}{I_m}$ will provide an angle with a magnitude between 0° and 90°.

B) $\alpha = \omega t$, and $t = \alpha/\omega$. However, a must be in radians. Thus:

$$\alpha(rad) = \left(\frac{\pi}{180°}\right) \times (23.578°) = 0.412\,rad$$

and

$$t_1 = \frac{\alpha}{\omega} = \frac{0.412\,rad}{377\,rad/s} = \mathbf{1.09\,ms}$$

For the second intersection:

$$\alpha(rad) = \left(\frac{\pi}{180°}\right) \times (156.422°) = 2.73\,rad$$

$$t_2 = \frac{\alpha}{\omega} = \frac{2.73\,rad}{377\,rad/s} = \mathbf{7.24\,ms}$$

Example 9: Given $i = 6 \times 10^{-3} \sin 1000t$, determine i at $t = 2\ ms$.

Solution:

$$\alpha = \omega t = (1000\ rad/s)(2 \times 10^{-3}\ s) = 2\ rad$$

$$\alpha(°) = \left(\frac{180°}{\pi}\right) \times (2\ rad) = 114.59°$$

$$i = (6 \times 10^{-3})(\sin 114.59°) = (6\ mA)(0.9093) = \mathbf{5.46\ mA}$$

6.6 Phase Relations:

Thus far, we have considered only sine waves that have maxima at $\pi/2$ and $3\pi/2$, with a zero value at 0, π, and 2π, as shown in Figure (6.10). If the waveform is shifted to the right or left of 0°, the expression becomes:

$$A_m \sin(\omega t \pm \theta)$$

where θ is the angle in degrees or radians that the waveform has been shifted.

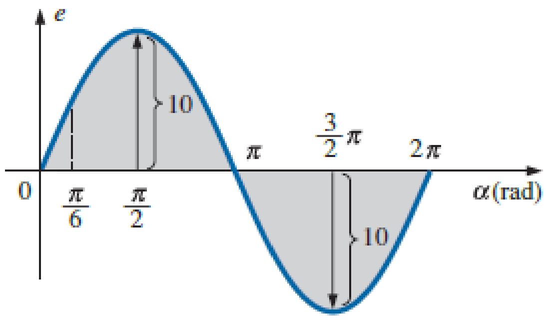


Figure (6.10): Horizontal axis in radians.

If the waveform passes through the horizontal axis with a *positive-going* (increasing with time) *slope before* 0°, as shown in Figure (6.11), the expression is:

$$A_m \sin(\omega t + \theta)$$

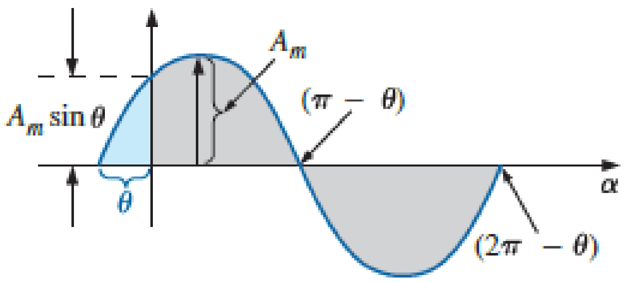


Figure (6.11): Defining the phase shift for a sinusoidal function that crosses the horizontal axis with a positive slope before 0°.

Finally, at $\omega t = \alpha = 0°$, the magnitude is $A_m \sin(-\theta)$, which, by a trigonometric identity, is $-A_m \sin(\theta)$.

If the waveform crosses the horizontal axis with a positive-going slope 90° ($\pi/2$) sooner, as shown in Figure (6.12), it is called a *cosine wave*; that is:

$$\sin(\omega t + 90°) = \sin\left(\omega t + \frac{\pi}{2}\right) = \cos \omega t$$

or

$$\sin \omega t = \cos(\omega t - 90°) = \cos\left(\omega t - \frac{\pi}{2}\right)$$

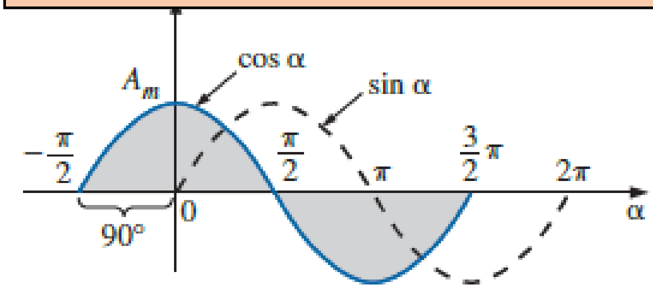


Figure (6.12): Phase relationship between a sine wave and a cosine wave.

The terms **leading** and **lagging** are used to indicate the relationship between two sinusoidal waveforms of the *same frequency* plotted on the same set of axes. In Figure (6.12), the cosine curve is said to *lead* the sine curve by 90°, and the sine curve is said to *lag* the cosine curve by 90°.

The 90° is referred to as the phase angle between the two waveforms. In language commonly applied, the waveforms are *out of phase* by 90°.

Note that the phase angle between the two waveforms is measured between those two points on the horizontal axis through which each passes with the *same slope*. If both waveforms cross the axis at the same point with the same slope, they are in *phase*.

Example 10: What is the phase relationship between the sinusoidal waveforms of each of the following sets?

A) $v = 10\sin(\omega t + 30°)$

$i = 5\sin(\omega t + 70°)$

B) $i = 15\sin(\omega t + 60°)$

$v = 10\sin(\omega t - 20°)$

C) $i = 2\cos(\omega t + 10°)$

$v = 3\sin(\omega t - 10°)$

D) $i = \sin(\omega t + 30°)$

$v = 2\sin(\omega t + 10°)$

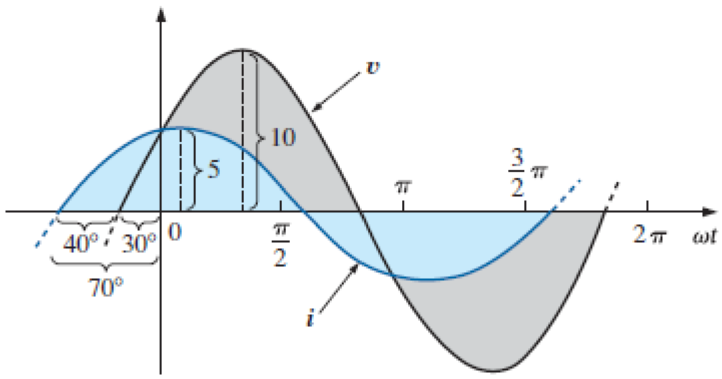
E) $v = -2\,\mathrm{scos}(\omega t - 60°)$

$i = 3\sin(\omega t - 150°)$

Solution:

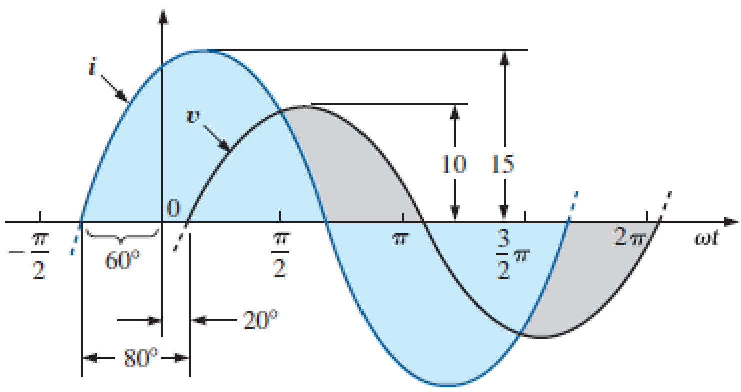
A) See Figure below.

$$\boldsymbol{i\ leads\ v\ by\ 40°, or\ v\ lags\ i\ by\ 40°.}$$



B) See Figure below.

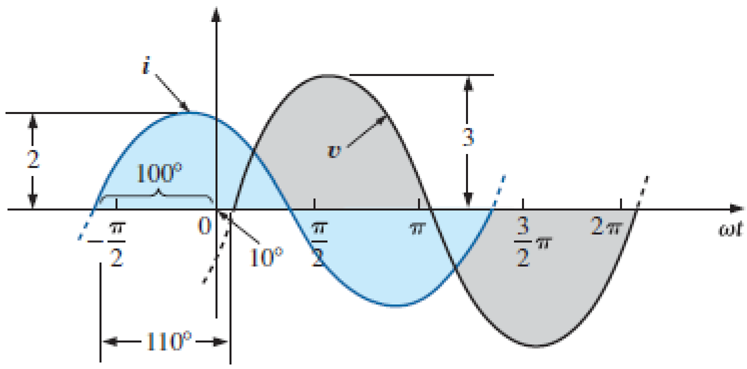
$$\boldsymbol{i\ leads\ v\ by\ 80°, or\ v\ lags\ i\ by\ 80°.}$$



C) c. See Figure below.

$$i = 2\cos(\omega t + 10°) = 2\sin(\omega t + 10° + 90°) = 2\sin(\omega t + 100°)$$

$$\boldsymbol{i\ leads\ v\ by\ 110°, or\ v\ lags\ i\ by\ 110°.}$$



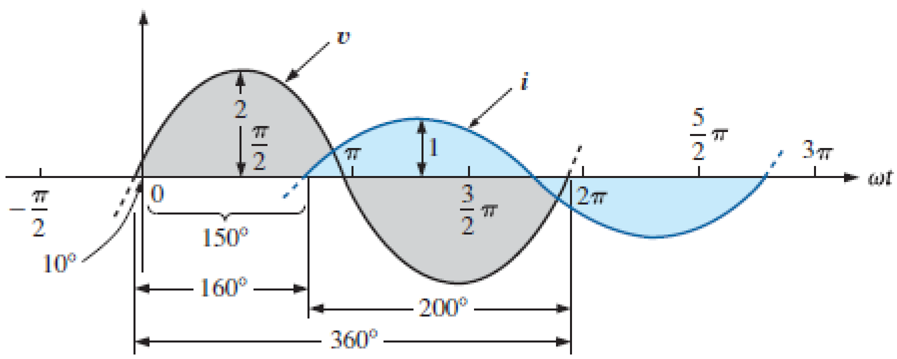
D) See Figure below.

$$-\sin(\omega t + 30°) = \sin(\omega t + 30° \overset{\text{Note}}{-} 180°)$$
$$= \sin(\omega t - 150°)$$

$$\boldsymbol{v\ leads\ i\ by\ 160°, or\ i\ lags\ v\ by\ 160°.}$$

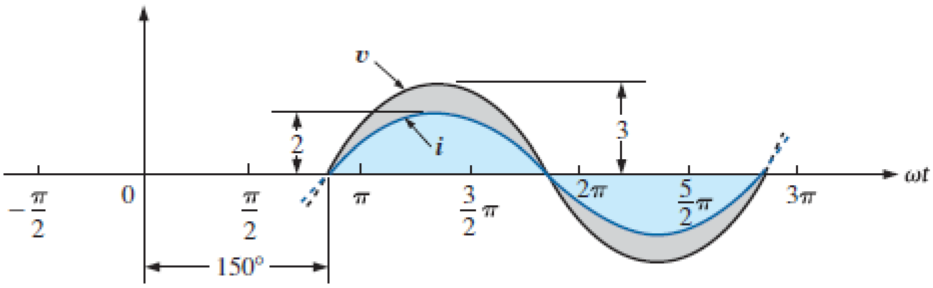
$$-\sin(\omega t + 30°) = \sin(\omega t + 30° \overset{\text{Note}}{+} 180°)$$
$$= \sin(\omega t + 210°)$$

$$\boldsymbol{i\ leads\ v\ by\ 200°, or\ v\ lags\ i\ by\ 200°.}$$



E) See Figure below.

$$i = -2\cos(\omega t - 60°) = 2\cos(\omega t - 60° \overset{\text{By choice}}{-} 180°)$$
$$= 2\cos(\omega t - 240°)$$



However, $\cos\alpha = sin\ (\alpha\ +\ 90°)$

so that $2\ cos\ (\omega t - 240°) = 2\ sin\ (\omega t - 240° + 90°) = 2\ sin(\omega t - 150°)$

$$\boldsymbol{v\ and\ i\ are\ in\ phase.}$$

6.7 Average Value:

After traveling a considerable distance by car, some drivers like to calculate their average speed for the entire trip. This is usually done by dividing the miles traveled by the hours required to drive that distance. For example, if a person traveled 225 mi in 5 h, the average speed was 225 mi/5 h, or 45 mi/h. This same distance may have been traveled at various speeds for various intervals of time, as shown in Figure (6.13).

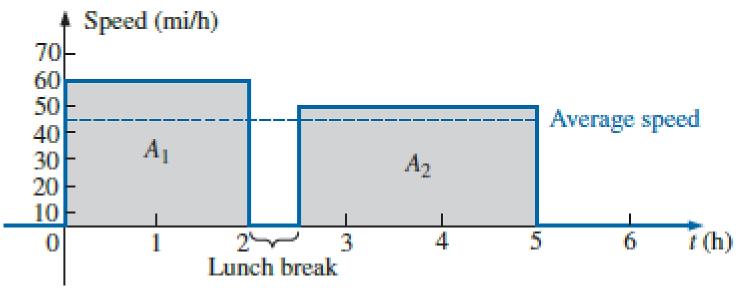


Figure (6.13): Plotting speed versus time for an automobile excursion.

By finding the total area under the curve for the 5 h and then dividing the area by 5 h (the total time for the trip), we obtain the same result of 45 mi/h; that is,

$$Average\ speed = \frac{area\ under\ curve}{length\ of\ curve}$$

$$Average\ speed = \frac{A_1 + A_2}{5\ h} = \frac{\left(60\frac{mi}{h}\right)(2\ h) + \left(50\frac{mi}{h}\right)(2.5\ h)}{5\ h}$$

$$Average\ speed = \frac{225}{5} mi/h = \boldsymbol{45\ mi/s}$$

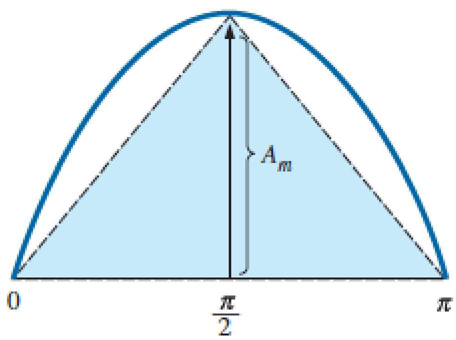
Equation above can be extended to include any variable quantity, such as current or voltage, if we let G denote the average value, as follows:

$$G\ (average\ value) = \frac{algebraic\ sum\ of\ areas}{length\ of\ curve}$$

The *algebraic* sum of the areas must be determined since some area contributions are from below the horizontal axis. Areas above the axis are assigned a positive sign and those below it a negative sign. A positive average value is then above the axis, and a negative value is below it.

The average value of *any* current or voltage is the value indicated on a dc meter. In other words, over a complete cycle, the average value is the equivalent dc value. In the analysis of electronic circuits to be considered in the courses of electronics, both dc and ac sources of voltage will be applied to the same network. You will then need to know or determine the dc (or average value) and ac components of the voltage or current in various parts of the system. ***The area of the positive (or negative) pulse of a sine wave is 2Am.***

The procedure of calculus that gives the exact solution $2A_m$ is known as *integration*.



Finding the area under the positive pulse of a sine wave using integration, we have:

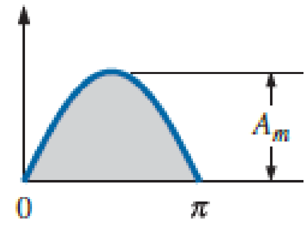
$$Area = \int_0^{\pi} A_m \sin\alpha\ d\alpha$$

where $\int$ is the sign of integration, 0 and π are the limits of integration, $A_m \sin\alpha$ is the function to be integrated, and $d\alpha$ indicates that we are integrating with respect to α.

Integrating, (for demonstrating only) we obtain:

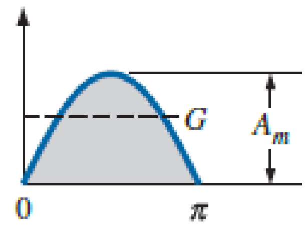
$$Area = A_m[-\cos\alpha]_0^{\pi} = -A_m(\cos\pi - \cos 0°) = -A_m\big(-1-(-1)\big) = -A_m(-2)$$

$$\boxed{Area = 2\,A_m}$$

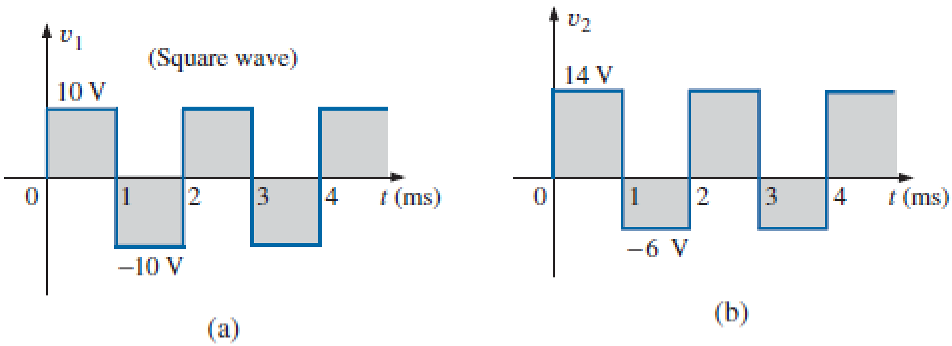


Since we know the area under the positive (or negative) pulse, we can easily determine the average value of the positive (or negative) region of a sine wave pulse:

$$\boxed{G = \frac{2\,A_m}{\pi} = 0.637\,A_m}$$



Example 11: Determine the average value of the waveforms in Figure below.



Solution:

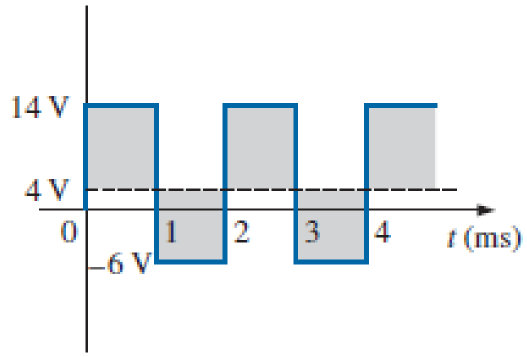
A) By inspection, the area above the axis equals the area below over one cycle, resulting in an average value of zero volts.

$$G = \frac{algebraic\ sum\ of\ areas}{length\ of\ curve} = \frac{(10\,V)(1\,ms) - (10\,V)(1\,ms)}{2\,ms} = \frac{0}{2\,ms} = \mathbf{0\,V}$$

B)

$$G = \frac{(14\,V)(1\,ms) - (6\,V)(1\,ms)}{2\,ms} = \frac{14\,V - 6V}{2} = \frac{8\,V}{2} = \mathbf{4\,V}$$

as shown in Figure below.

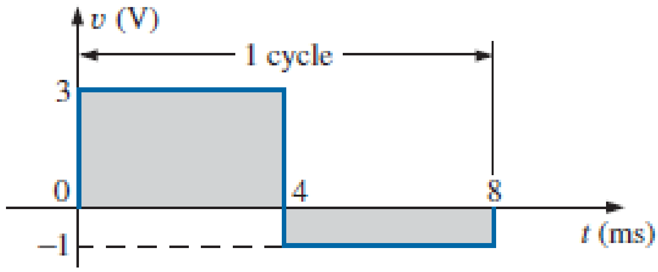


In reality, the waveform in the original Figure (b) is simply the square wave in the original Figure (b) with a dc shift of 4 V; that is,

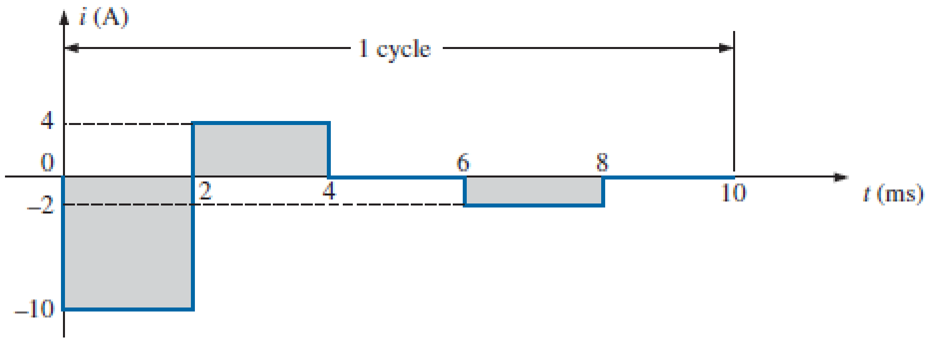
$$v_2 = v_1 + 4\,V$$

Example 12: Find the average values of the following waveforms over one full cycle:

A) Figure below.



B) Figure below.



Solution:

A)

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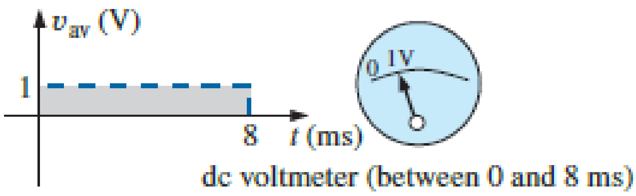
College: ***Petroleum Processes Engineering***
Department: ***Petroleum Systems Control Engineering***
Subject: ***Electrical Engineering Fundamentals***
Assistant Lecturer: ***Waladdin Mezher Shaher***
2023-2024

$$G = \frac{+(3\,V)(4\,ms) - (1\,V)(4\,ms)}{8\,ms} = \frac{12\,V - 4\,V}{8} = \frac{8\,V}{8} = \mathbf{1\,V}$$

Note Figure below.

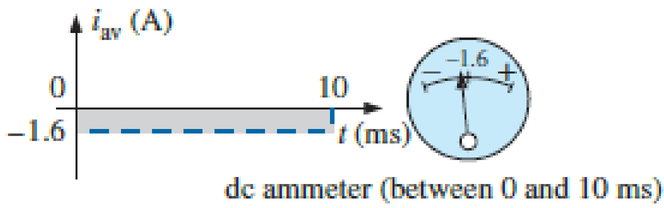


B)

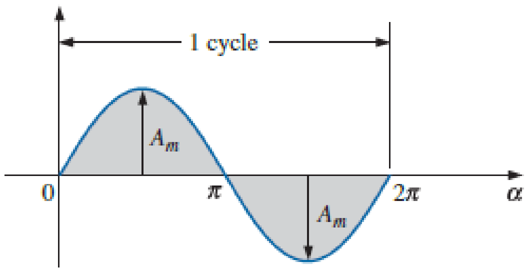
$$G = \frac{-(10\,V)(2\,ms) + (4\,V)(2\,ms) - (2\,V)(2\,ms)}{10\,ms}$$

$$G = \frac{-20\,V + 8\,V - 4\,V}{10} = \frac{-16\,V}{10} = \mathbf{-1.6\,V}$$

Note Figure below.



Example 13: Determine the average value of the sinusoidal waveform in Figure below.

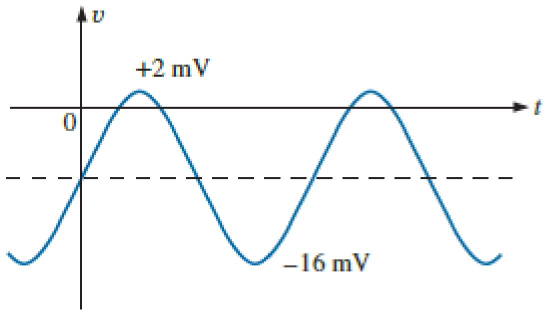


Solution: By inspection it is fairly obvious that

the average value of a pure sinusoidal waveform over one full cycle is zero.

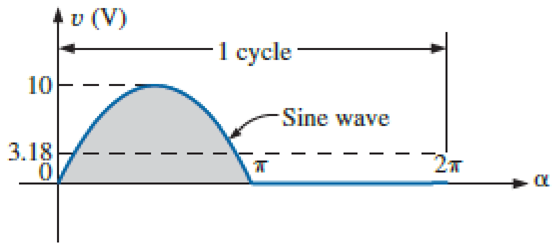
$$G = \frac{+2A_m - 2A_m}{2\pi} = \mathbf{0\,V}$$

Example 14: Determine the average value of the waveform in Figure below.



Solution: The peak-to-peak value of the sinusoidal function is $16\,mV + 2\,mV = 18\,mV$. The peak amplitude of the sinusoidal waveform is, therefore, $18\,mV/2 = 9\,mV$. Counting down 9 mV from 2 mV (or 9 mV up from -16 mV) results in an average or dc level of **-7 mV,** as noted by the dashed line in Figure above.

Example 15: Determine the average value of the waveform in Figure below.



Solution:

$$G = \frac{2A_m + 0}{2\pi} = \frac{2(10\,V)}{2\pi} \cong \mathbf{3.18\,V}$$

6.8 Effective (rms) Values:

A fixed relationship between ac and dc voltages and currents can be derived from the experimental setup shown in Figure (6.14). A resistor in a water bath is connected by switches to a dc and an ac supply. If switch 1 is closed, a dc current I, determined by the resistance R and battery voltage E, is established through the resistor R. The temperature reached by the water is determined by the dc power dissipated in the form of heat by the resistor.

If switch 2 is closed and switch 1 left open, the ac current through the resistor has a peak value of I_m. The temperature reached by the water is now determined by the ac power dissipated in the form of heat by the resistor. The ac input is varied until the temperature is the same as that reached with the dc input. When this is accomplished, the average electrical power delivered to the resistor R by the ac source is the same as that delivered by the dc source.

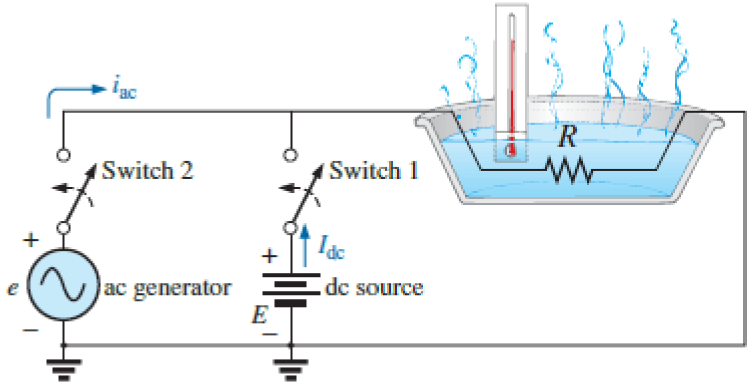


Figure (6.14):

The power delivered by the ac supply at any instant of time is:

$$P_{ac} = (i_{ac})^2 R = (I_m \sin \omega t)^2 R = (I_m^2 sin^2 \omega t) R$$

However,

$$sin^2 \omega t = \frac{1}{2}(1 - \cos 2\omega t) \qquad \text{(trigonometric identity)}$$

Therefore,

$$P_{ac} = I_m^2 \left[\frac{1}{2}(1 - \cos 2\omega t)\right] R$$

and

$$P_{ac} = \frac{I_m^2 R}{2} - \frac{I_m^2 R}{2} \cos 2\omega t$$

The average power delivered by the ac source is just the first term, since the average value of a cosine wave is zero even though the wave may have twice the frequency of the original input current waveform. Equating the average power delivered by the ac generator to that delivered by the dc source:

$$P_{av(ac)} = P_{dc}$$

$$\frac{I_m^2 R}{2} = I_{dc}^2 R$$

and

$$I_{dc} = \frac{I_m}{\sqrt{2}} = 0.707\, I_m$$

which, in words, states that: ***The equivalent dc value of a sinusoidal current or voltage is $1/\sqrt{2}$ or 0.707 of its peak value.***

The equivalent dc value is called the rms or effective value of the sinusoidal quantity.

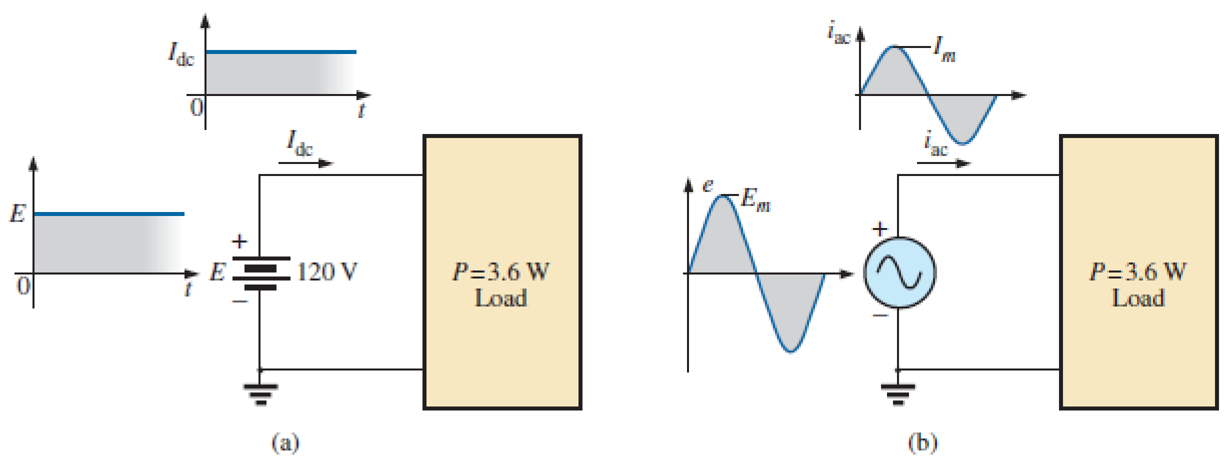
The effective value of any quantity plotted as a function of time can be found by using the following equation derived from the experiment just described:

Calculus format: $$I_{rms} = \sqrt{\frac{\int_0^T i^2(t)dt}{T}}$$

which means $$I_{rms} = \sqrt{\frac{Area(i^2(t))}{T}}$$

To find the rms value, the function $i(t)$ must first be squared. After $i(t)$ is squared, the area under the curve is found by integration. It is then divided by T, the length of the cycle or the period of the waveform. The final step is to take the square root of the mean value.

Example 16: The 120 V dc source in Figure below (a) delivers 3.6 W to the load. Determine the peak value of the applied voltage (E_m) and the current (I_m) if the ac source [Figure below (b)] is to deliver the same power to the load.



Solution:

$$P_{dc} = V_{dc}\, I_{dc}$$

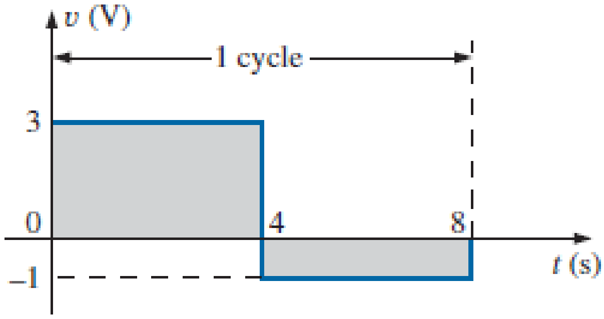
and

$$I_{dc} = \frac{P_{dc}}{V_{dc}} = \frac{3.6\ W}{120\ V} = 30\ mA$$

$$I_m = \sqrt{2}\, I_{dc} = \sqrt{2} \times 30\ mA = (1.414)(30\ mA) = \mathbf{42.42\ mA}$$

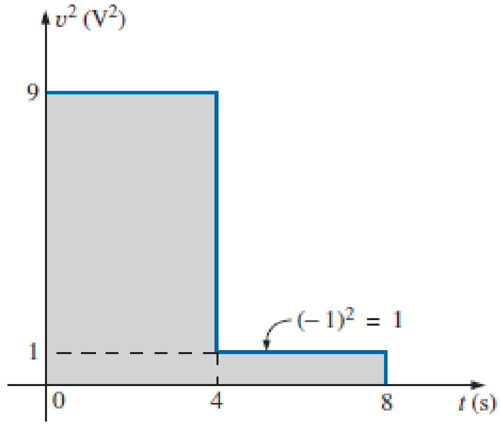
$$E_m = \sqrt{2}\, E_{dc} = \sqrt{2} \times 120\ V = (1.414)(120\ V) = \mathbf{169.68\ V}$$

Example 17: Find the rms value of the waveform in Figure below.

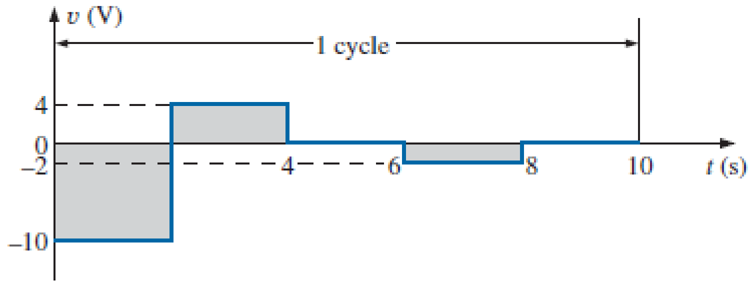


Solution: v^2 (Figure below):

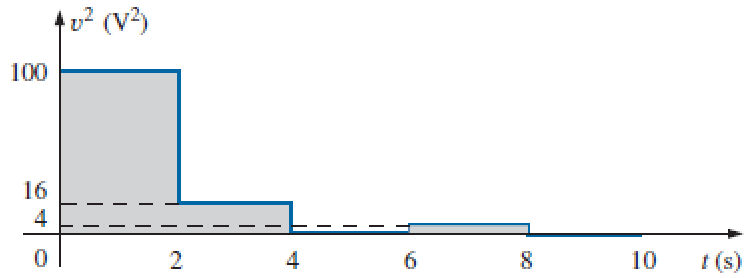
$$V_{rms} = \sqrt{\frac{(9)(4) + (1)(4)}{8}} = \sqrt{\frac{40}{8}} = \mathbf{2.24\ V}$$



Example 18: Calculate the rms value of the voltage in Figure below.

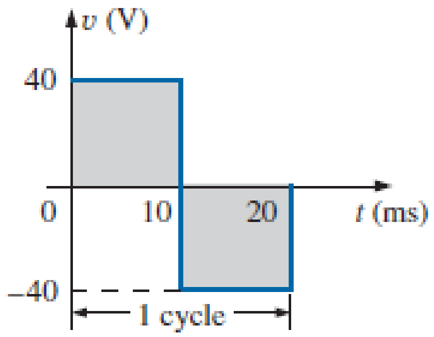


Solution: v^2 (Figure below):

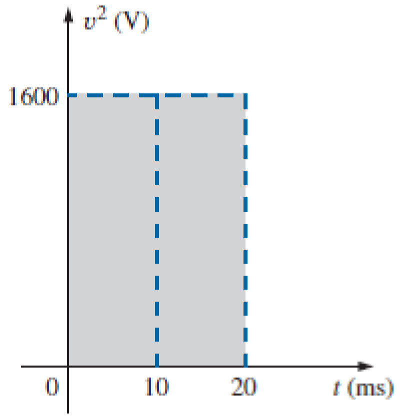


$$V_{rms} = \sqrt{\frac{(100\ V^2)(2s) + (16\ V^2)(s) + (4\ V^2)(2s)}{10\ s}} = \sqrt{\frac{240\ V^2}{10\ s}} = \mathbf{4.9\ V}$$

Example 19: Determine the average and rms values of the square wave in Figure below.



Solution: By inspection, the average value is zero. v^2 (Figure below):



$$V_{rms} = \sqrt{\frac{(1600)(10 \times 10^{-3}) + (1600)(10 \times 10^{-3})}{20 \times 10^{-3}}} = \sqrt{\frac{32000 \times 10^{-3}}{20 \times 10^{-3}}} = \mathbf{40\ V}$$

(the maximum value of the waveform in the original Figure).

6.9 DC and AC Components:

A unique situation arises if a waveform has both a dc and an ac component that may be due to a source, such as the one in Figure (6.15). The combination appears frequently in the analysis of electronic networks where both dc and ac levels are present in the same system.

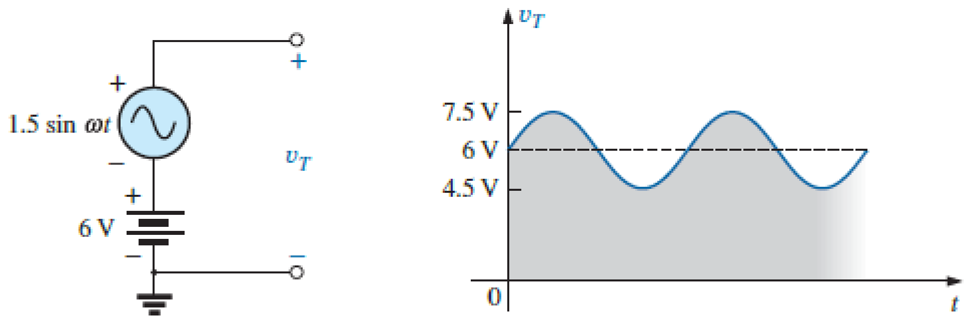


Figure (6.15): Generation and display of a waveform having a dc and an ac component.

The rms value is actually determined by:

$$V_{rms} = \sqrt{V_{dc}^2 + V_{ac(rms)}^2}$$

which for the waveform in Figure (6.15) is:

$$V_{rms} = \sqrt{(6\ V)^2 + (1.06\ V)^2} = \sqrt{37.124\ V^2} \cong \mathbf{6.1\ V}$$

Lecture 7: Series and Parallel AC Circuits

7.1 Response of Basic R, L, and C Elements to a Sinusoidal Voltage or Current:

7.1.1 Resistor:

For power-line frequencies and frequencies up to a few hundred kilohertz, resistance is, for all practical purposes, unaffected by the frequency of the applied sinusoidal voltage or current. For this frequency region, the resistor R in Figure (7.1) can be treated as a constant, and Ohm's law can be applied, as follows.

For $v = V_m \sin \omega t$,

$$i = \frac{v}{R} = \frac{V_m \sin \omega t}{R} = \frac{V_m}{R} \sin \omega t = I_m \sin \omega t$$

where

$$\boxed{I_m = \frac{V_m}{R}}$$

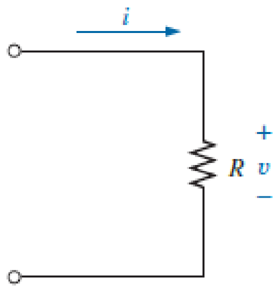


Figure (7.1): Determining the sinusoidal response for a resistive element.

In addition, for a given i,

$$v = iR = (I_m \sin \omega t)R = I_m R \ \sin \omega t = V_m \sin \omega t$$

where

$$\boxed{V_m = I_m R}$$

A plot of v and i in Figure (7.2) reveals that: ***For a purely resistive element, the voltage across and the current through the element are in phase, with their peak values related by Ohm's law.***

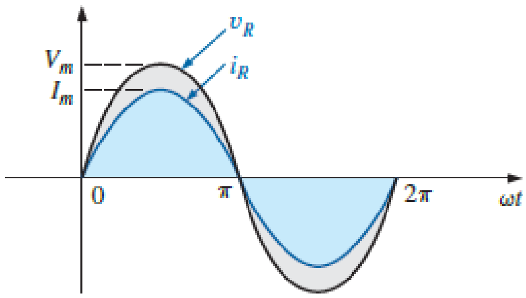


Figure (7.2): The voltage and current of a resistive element are in phase.

If we now write both the voltage and current in phasor form, we find that the phase angle associated with the voltage and current is zero degrees. That is:

$$v_R = V_m \sin \omega t \Rightarrow V_R = V\angle 0°$$

$$i_R = I_m \sin \omega t \Rightarrow I_R = I\angle 0°$$

If we apply phasor algebra as follows:

$$\mathbf{I}_R = \frac{\mathbf{V}_R}{\mathbf{R}} = \frac{V\angle 0°}{R\angle \theta_R} = \frac{V}{R}\underline{/0° - \theta_R}$$

Now since we know the angle associated with the current must also be zero degrees, the angle θ_R must be zero degrees.

$$I_R = \frac{V_R}{R} = \frac{V\angle 0°}{R\angle 0°} = \frac{V}{R}\angle 0°$$

so that in the time domain:

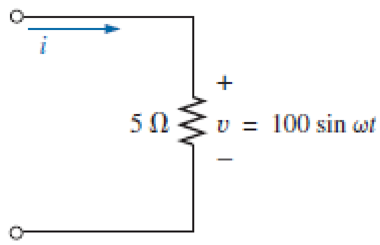
$$i_R = \sqrt{2}\left(\frac{V}{R}\right)\sin \omega t$$

For the future, therefore, whenever we encounter a resistor in the ac domain, we will assign an angle of zero degrees to form a complex number notation. The standard format will therefore be: $Z_R = R\angle 0°$

The quantity has both magnitude and angle, called the impedance of the resistive element and measured in ohms, it is a measure of how much the element will “impede” the flow of charge through the circuit.

Example 1: Using complex algebra,

A) Find the current i_R for the circuit in Figure below.



B) Sketch the waveforms of i_R and V_R.

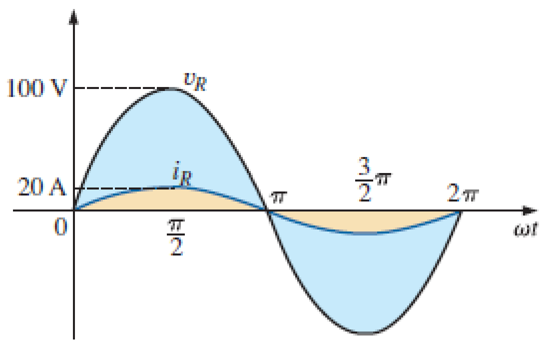
Solution:

A) $v = 100\ sin\ \omega t \Rightarrow$ phasor form $\boldsymbol{V} = 70.71\ V\ \angle 0°$

$$I_R = \frac{V_R}{Z_R} = \frac{V\angle 0°}{R\angle 0°} = \frac{70.71\ V\ \angle 0°}{5\ \Omega\ \angle 0°} = 14.14\ A\ \angle 0°$$

And $i_R = \sqrt{2}(14.14)\ sin\ \omega t = \boldsymbol{20\ sin\ \omega t}$

B) Note Figure below.



7.1.2 Inductor:

The voltage across the inductor of Figure (7.3) is directly related to the inductance of the coil and the rate of change of current through the coil. by the following equation:

$$v_L = L\frac{di_L}{dt}$$

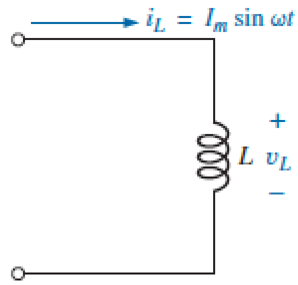


Figure (7.3): Investigating the sinusoidal response of an inductive element.

Consequently, the higher the frequency, the greater is the rate of change of current through the coil, and the greater is the magnitude of the voltage. In addition, the higher the inductance, the greater is the rate of change of the flux linkages, and the greater is the resulting voltage across the coil.

For a sinusoidal current defined by:

$$i_L = I_m \sin \omega t$$

we can calculate the voltage across the coil by differentiating the current through the coil and substituting into the basic equation above. That is:

$$v_L = L\frac{di_L}{dt} = L\frac{d}{dt}(I_m \sin \omega t) = LI_m \frac{d}{dt}(\sin \omega t) = LI_m(\omega \cos \omega t)$$

with the final solution of:

$$v_L = \omega L I_m \sin(\omega t + 90°)$$

The peak value of the resulting voltage is therefore directly related to the applied frequency ($\omega = 2\pi f$), the inductance of the coil L, and the peak value of the applied current I_m. A plot of v_L and i_L in Figure (7.4) reveals that for an inductor, v_L leads i_L by 90°, or i_L lags v_L by 90°.

The opposition to an applied voltage can be determined by simply substituting the peak values for V_m and I_m, as follows:

$$Opposition = \frac{cause}{effect} = \frac{V_m}{I_m} = \frac{\omega L I_m}{I_m} = \omega L$$

revealing that the opposition established by an inductor in an ac sinusoidal network is directly related to the product of the angular velocity ($\omega = 2\pi f$) and the inductance.

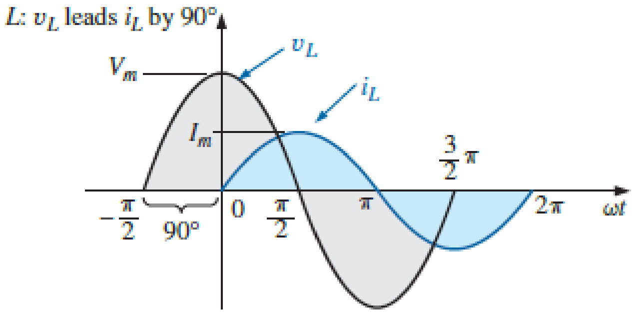


Figure (7.4): For a pure inductor, the voltage across the coil leads the current through the coil by 90°.

The quantity ωL, called the **reactance** (from the word reaction) of an inductor, is symbolically represented by X_L and is measured in ohms; that is:

$$X_L = \omega L \quad \text{(ohms, } \Omega\text{)}$$

In an Ohm's law format, its magnitude can be determined from:

$$X_L = \frac{V_m}{I_m} \quad \text{(ohms, } \Omega\text{)}$$

Inductive reactance is the opposition to the flow of current, which results in the continual interchange of energy between the source and the magnetic field of the inductor.

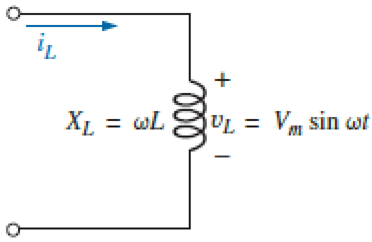
Once the reactance is known, the peak value of the voltage or current can be found from the other by simply applying Ohm's law, as follows:

$$I_m = \frac{V_m}{X_L}$$

and

$$V_m = I_m X_L$$

$$v_L = V_m \sin \omega t \Rightarrow \text{phase form } V = V\angle 0°$$



Applying Ohm's law, we find that:

$$\mathbf{I}_L = \frac{\mathbf{V}_L}{\mathbf{X}_L} = \frac{V\angle 0^\circ}{X_L\angle\theta_L} = \frac{V}{X_L}\underline{/0^\circ - \theta_L}$$

Since v_L leads i_L by 90°, i_L must have an angle of $-90°$ associated with it. To satisfy this condition, θ_L must equal $+90°$. Substituting $\theta_L = +90°$, we obtain:

$$\mathbf{I}_L = \frac{V\angle 0^\circ}{X_L\angle 90^\circ} = \frac{V}{X_L}\underline{/0^\circ - 90^\circ} = \frac{V}{X_L}\angle -90^\circ$$

so that in the time domain:

$$i_L = \sqrt{2}\left(\frac{V}{X_L}\right)\sin(\omega t - 90^\circ)$$

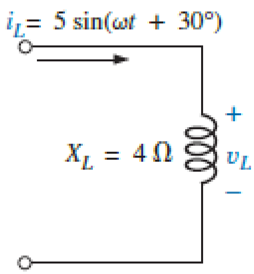
We use the fact that $\theta_L = 90°$ in the following polar format for inductive reactance to ensure the proper phase relationship between the voltage and current of an inductor:

$$Z_L = X_L\angle 90^\circ$$

Z_L, having both magnitude and an associated angle, is referred to as the impedance of an inductive element. It is measured in ohms and is a measure of how much the inductive element "controls or impedes" the level of current through the network.

Example 2: Using complex algebra,

A) Find the current i_L for the circuit in Figure below.



B) Sketch the waveforms of v_L and i_L.

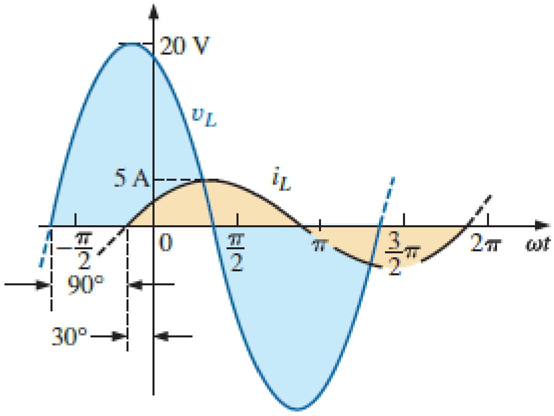
Solution:

A) $v_L = 24\,sin\,\omega t \Rightarrow$ phasor form $\boldsymbol{V_L} = 16.968\,V\angle 0°$

$$I = \frac{V_L}{Z_L} = \frac{V\angle\theta^\circ}{X_L\angle 90^\circ} = \frac{16.968\,V\angle 0^\circ}{3\,\Omega\angle 90^\circ} = 5.656\,A\angle -90^\circ$$

and $i = \sqrt{2}(5.656)sin\,(\omega t - 90°) = \mathbf{8.0}\ \boldsymbol{sin}\ (\boldsymbol{\omega t} - \mathbf{90}°)$

B) Note Figure below.



7.1.3 Capacitor:

For capacitive networks, the voltage across the capacitor is limited by the rate at which charge can be deposited on, or released by, the plates of the capacitor during the charging and discharging phases, respectively. In other words, an instantaneous change in voltage across a capacitor is opposed by the fact that there is an element of time required to deposit charge on (or release charge from) the plates of a capacitor, and $V = Q/C$.

Since capacitance is a measure of the rate at which a capacitor will store charge on its plates, ***for a particular change in voltage across the capacitor, the greater the value of capacitance, the greater is the resulting capacitive current.***

For the capacitor of Figure (7.5),

$$i_C = C\frac{dv_C}{dt}$$

Substituting:

$$v_C = V_m \sin \omega t$$

and, applying differentiation, we obtain:

$$i_C = C\frac{dv_C}{dt} = C\frac{d}{dt}(V_m \sin \omega t) = \omega C V_m \cos \omega t$$

so that:

$$i_C = \omega C V_m \sin(\omega t + 90°)$$

Note that the peak value of i_C is directly related to $\omega(= 2\pi f)$, C, and the peak value of the applied voltage.

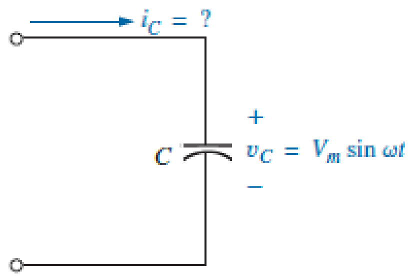


Figure (7.5): Investigating the sinusoidal response of a capacitive element.

A plot of v_C and i_C in Figure (7.6) reveals that:

For a capacitor, i_C leads v_C by 90°, or v_C lags i_C by 90°.

Applying:

$$Opposition = \frac{cause}{effect}$$

and substituting values, we obtain:

$$Opposition = \frac{V_m}{I_m} = \frac{V_m}{\omega C\, V_m} = \frac{1}{\omega C}$$

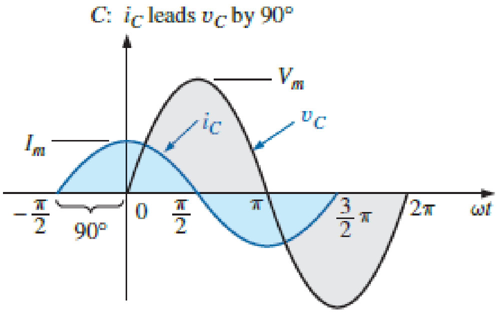


Figure (7.6): The current of a purely capacitive element leads the voltage across the element by 90°.

The quantity $1/\omega C$, called the **reactance** of a capacitor, is symbolically represented by X_C and is measured in ohms; that is:

$$X_C = \frac{1}{\omega C} \quad \text{(ohms, } \Omega)$$

In an Ohm's law format, its magnitude can be determined from:

$$X_C = \frac{V_m}{I_m} \quad \text{(ohms, } \Omega)$$

Capacitive reactance is the opposition to the flow of charge, which results in the continual interchange of energy between the source and the electric field of the capacitor. Like the inductor, the capacitor does not dissipate energy in any form (ignoring the effects of the leakage resistance). The peak value of the voltage or current can be found from the other by simply applying Ohm's law as follows:

$$I_m = \frac{V_m}{X_C}$$

and

$$V_m = I_m X_C$$

In the inductive circuit,

$$v_L = L\frac{di_L}{dt}$$

and through integration:

$$i_L = \frac{1}{L}\int v_L\, dt$$

In the capacitive circuit,

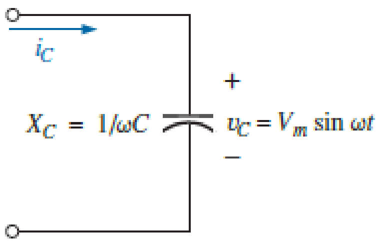
$$i_C = C\frac{dv_C}{dt}$$

and through integration:

$$v_C = \frac{1}{C}\int i_C\, dt$$

If the source current leads the applied voltage, the network is predominantly capacitive, and if the applied voltage leads the source current, it is predominantly inductive.

$$v_C = V_m \sin\omega t \Rightarrow \text{phase form } V = V\angle 0°$$



Applying Ohm's law, we find:

$$\mathbf{I}_C = \frac{\mathbf{V}_C}{\mathbf{X}_C} = \frac{V\angle 0^\circ}{X_C\angle\theta_C} = \frac{V}{X_C}\underline{/0^\circ - \theta_C}$$

Since i_C leads v_C by 90°, i_C must have an angle of +90° associated with it. To satisfy this condition, θ_C must equal -90°. Substituting $\theta_C = -90°$ yields:

$$\mathbf{I}_C = \frac{\mathbf{V}_C}{\mathbf{X}_C} = \frac{V\angle 0^\circ}{X_C\angle -90^\circ} = \frac{V}{X_C}\underline{/0^\circ - (-90^\circ)} = \frac{V}{X_C}\angle 90^\circ$$

so, in the time domain:

$$i_C = \sqrt{2}\left(\frac{V}{X_C}\right)\sin(\omega t + 90°)$$

We use the fact that $\theta_C = -90°$ in the following polar format for capacitive reactance to ensure the proper phase relationship between the voltage and current of a capacitor:

$$Z_C = X_C \angle -90°$$

Z_C, having both magnitude and an associated angle, is referred to as the impedance of a capacitive element. It is measured in ohms and is a measure of how much the capacitive element "controls or impedes" the level of current through the network.

Example 3: The voltage across a resistor is provided below. Find the sinusoidal expression for the current if the resistor is 10 **Ω**. Sketch the curves for v and i.

A) $v = 100 \sin 377t$

B) $v = 25\, sin(377t + 60°)$

Solution:

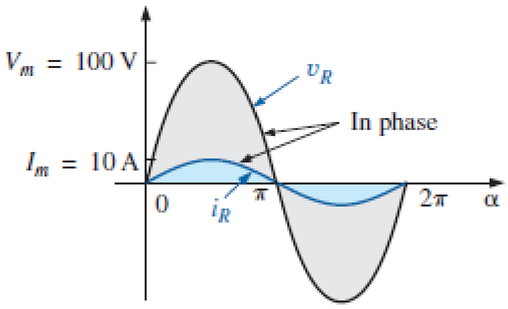
A)

$$I_m = \frac{V_m}{R} = \frac{100\ V}{10\ \Omega} = 10\ A$$

(v and i are in phase), resulting in:

$$i = \mathbf{10\, sin\, 377t}$$

The curves are sketched in Figure below.



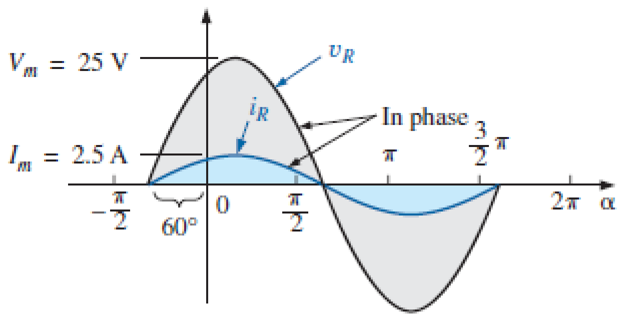
B)

$$I_m = \frac{V_m}{R} = \frac{25\ V}{10\ \Omega} = 2.5\ A$$

(v and i are in phase), resulting in:

$$i = \mathbf{2.5\ sin(377t + 60°)}$$

The curves are sketched in Figure below.



Example 4: The current through a 0.1 H coil is provided. Find the sinusoidal expression for the voltage across the coil. Sketch the v and i curves.

A) $i = 10\ sin\ 377t$

B) $i = 7\ sin(377t\ -\ 70°)$

Solution:

A)

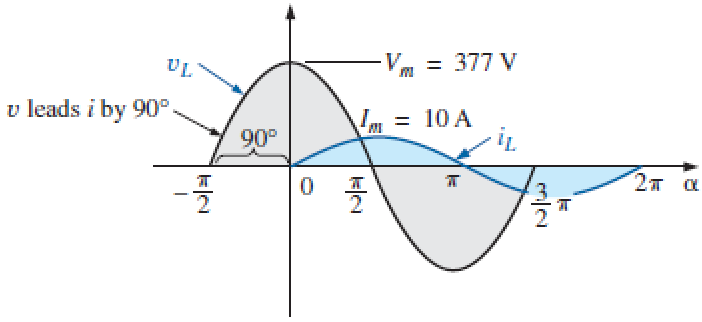
$$X_L = \omega L = (377\ rad/s)(0.1\ H) = 37.7\ \Omega$$

$$V_m = I_m X_L = (10\ A)(37.7\ \Omega) = 377\ V$$

and we know that for a coil v leads i by 90°. Therefore,

$$v = \mathbf{377\ sin(377t + 90°)}$$

The curves are sketched in Figure below.



B) X_L remains at 37.7 Ω.

$$V_m = I_m X_L = (7\,A)(37.7\,\Omega) = 263.9\,V$$

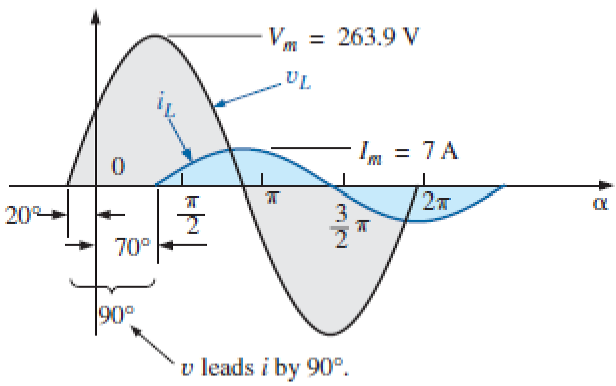
and we know that for a coil v leads i by 90°. Therefore,

$$v = 263.9\,sin(377t - 70° + 90°)$$

and

$$v = \mathbf{263.9\,sin(377t + 20°)}$$

The curves are sketched in Figure below.



Example 5: The voltage across a 0.5 H coil is provided below. What is the sinusoidal expression for the current?

$$v = 100\,sin\,20t$$

Solution:

$$X_L = \omega L = (20\,rad/s)(0.5\,H) = 10\,\Omega$$

$$I_m = \frac{V_m}{X_L} = \frac{100\,V}{10\,\Omega} = 10\,A$$

and we know the i lags v by 90°. Therefore,

$$i = \mathbf{10}\ \boldsymbol{sin}(\mathbf{20}\boldsymbol{t} - \mathbf{90}°)$$

Example 6: The voltage across a 1 μF capacitor is provided below. What is the sinusoidal expression for the current? Sketch the v and i curves.

$$v = 30\ sin\ 400t$$

Solution:

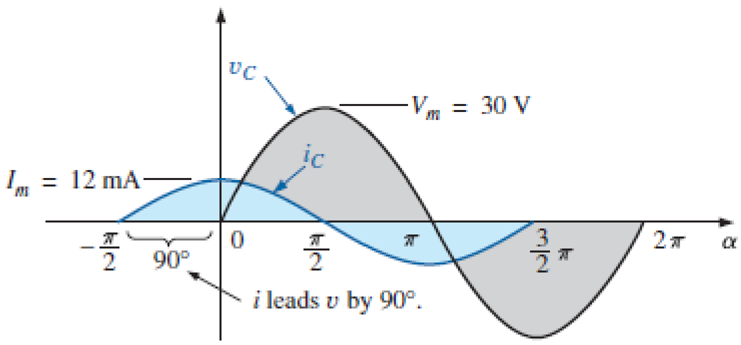
$$X_C = \frac{1}{\omega C} = \frac{1}{(400\ rad/s)(1 \times 10^{-6}\ F)} = \frac{10^6\ \Omega}{400} = 2500\ \Omega$$

$$I_m = \frac{V_m}{X_C} = \frac{30\ V}{2500\ \Omega} = 0.0120\ A = 12\ mA$$

and we know that for a capacitor i leads v by 90°. Therefore,

$$i = \mathbf{12}\ \times \mathbf{10}^{-3}\ \boldsymbol{sin}(\mathbf{400}\boldsymbol{t}\ + \mathbf{90}°)$$

The curves are sketched in Figure below.



Example 7: For the following pairs of voltages and currents, determine whether the element involved is a capacitor, an inductor, or a resistor. Determine the value of C, L, or R if sufficient data are provided (Figure below):

A) $v = 100\ sin(\omega t + 40°)$

$i = 20\ sin(\omega t + 40°)$

B) $v = 1000\ sin(377t + 10°)$

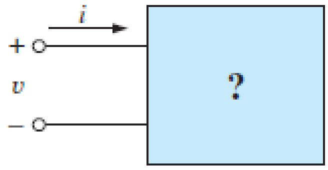
$i = 5\ sin(377t - 80°)$

C) $v = 500\ sin(157t + 30°)$

$i = 1\ sin(157t + 120°)$

D) $v = 50\ cos(\omega t + 20°)$

$i = 5\,sin(\omega t + 110°)$



Solution:

A) Since v and i are *in phase*, the element is a *resistor*, and:

$$R = \frac{V_m}{I_m} = \frac{100\ V}{20\ A} = \mathbf{5}\ \Omega$$

B) Since v *leads* i by 90°, the element is an *inductor*, and:

$$X_L = \frac{V_m}{I_m} = \frac{1000\ V}{5\ A} = \mathbf{200}\ \Omega$$

so that:

$$X_L = \omega L = 200\ \Omega\ \ or$$

$$L = \frac{X_L}{\omega} = \frac{200\ \Omega}{377\ rad/s} = \mathbf{0.53}\ \boldsymbol{H}$$

C) Since i *leads* v by 90°, the element is a *capacitor*, and:

$$X_C = \frac{V_m}{I_m} = \frac{500\ V}{1\ A} = \mathbf{500}\ \Omega$$

so that:

$$X_C = \frac{1}{\omega C} = 500\ \Omega\ \ or$$

$$C = \frac{1}{\omega\, X_C} = \frac{1}{157\ rad/s \times 500\ \Omega} = \mathbf{12.74}\ \boldsymbol{F}$$

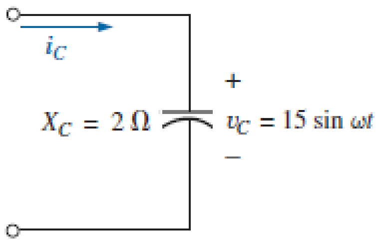
D) $v = 50\,cos(\omega t + 20°) = 50\,sin(\omega t\ 20° + 90°) = \ 50\,sin(\omega t + 110°)$

Since v and i are in phase, the element is a resistor, and:

$$R = \frac{V_m}{I_m} = \frac{50\ V}{5\ A} = \mathbf{10}\ \Omega$$

Example 8: Using complex algebra,

A) Find the current i_C for the circuit in Figure below.



B) Sketch the waveforms of v_C and i_C.

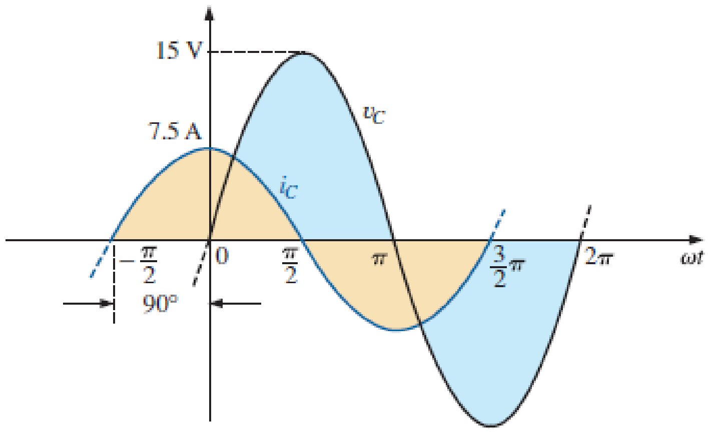
Solution:

A) $v_C = 15\, sin\, \omega t \Rightarrow$ phasor notation $\boldsymbol{V_L} = 10.605\, V \angle 0°$

$$I_C = \frac{V_C}{Z_C} = \frac{V \angle \theta°}{X_C \angle -90°} = \frac{10.605\, V \angle 0°}{2\, \Omega \angle -90°} = 5.303\, A \angle 90°$$

and $i_C = \sqrt{2}(5.303) sin\, (\omega t + 90°) = \boldsymbol{7.5\, sin\, (\omega t + 90°)}$

B) Note Figure below.



7.2 Impedance Diagram:

Now that an angle is associated with resistance, inductive reactance, and capacitive reactance, each can be placed on a complex plane diagram, as shown in Figure (7.7). For any network, the resistance will always appear on the positive real axis, the inductive reactance on the positive imaginary axis, and the capacitive reactance on the negative imaginary axis. The result is an impedance diagram that can reflect the individual and total impedance levels of an ac network.

networks combining different types of elements will have total impedances that extend from -90° to +90°. If the total impedance has an angle of 0°, it is said to be resistive in nature. If it is closer to 90°, it is inductive in nature. If it is closer to -90°, it is capacitive in nature.

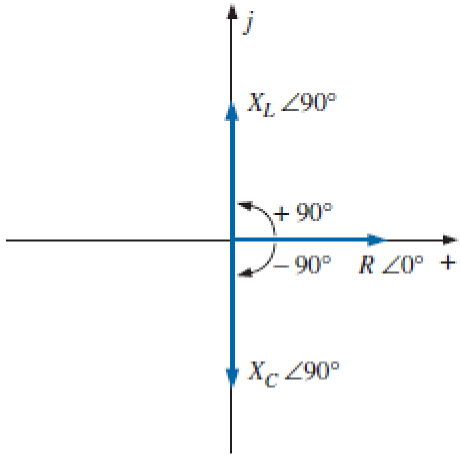
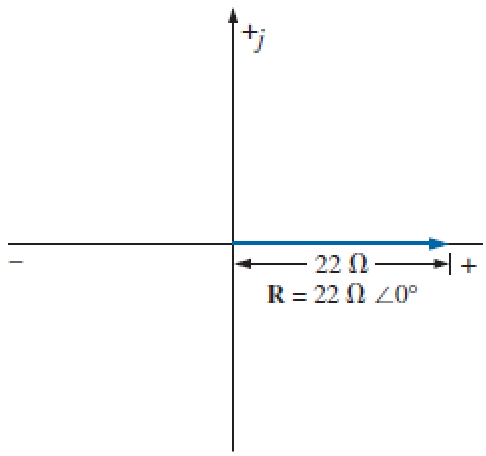


Figure (7.7): Impedance diagram.

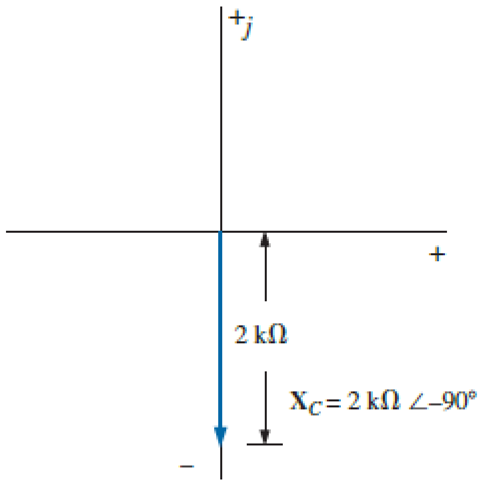
Example 9: Sketch the impedance diagram for a 22 ohm resistor.

Solution: Note Figure below.



Example 10: Sketch the impedance diagram of a 2 KΩ capacitive

Solution: Note Figure below.



7.3 Series Configuration:

The overall properties of series ac circuits (Figure (7.8)) are the same as those for dc circuits. For instance, the total impedance of a system is the sum of the individual impedances and the current I is the same through each impedance.

$$Z_T = Z_1 + Z_2 + Z_3 + \cdots + Z_N$$

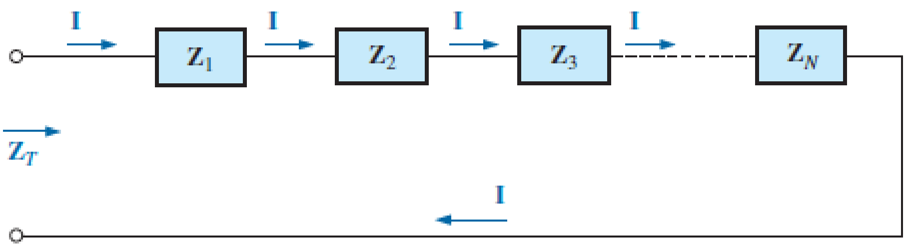


Figure (7.8): Series impedances.

For the representative **series ac configuration** in Figure (7.9) having two impedances, *the current is the same through each element* and is determined by Ohm’s law:

$$Z_T = Z_1 + Z_2$$

and

$$I = \frac{E}{Z_T}$$

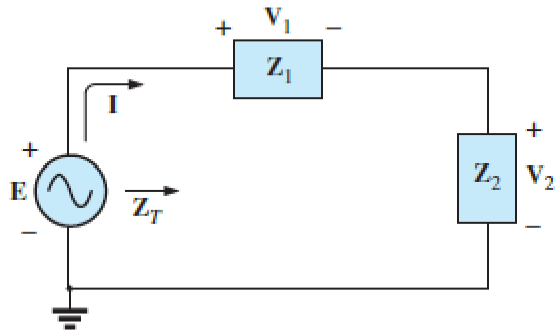


Figure (7.9): Series ac circuit.

The voltage across each element can then be found by another application of Ohm's law:

$$V_1 = I\,Z_1$$
$$V_2 = I\,Z_2$$

7.3.1 Kirchhoff's Voltage Law (KVL):

Kirchhoff's voltage law can then be applied in the same manner as it is employed for dc circuits. We have:

$$E - V_1 - V_2 = 0$$

or

$$E = V_1 + V_2$$

The power to the circuit can be determined by:

$$P = EI\cos\theta_T$$

where θ_T is the phase angle between E and I.

7.3.2 Voltage Divider Rule (VDR):

The basic format for the **voltage divider rule** in ac circuits is exactly the same as that for dc circuits:

$$V_x = \frac{Z_x\,E}{Z_T}$$

where V_x is the voltage across one or more elements in a series that have total impedance Z_x, E is the total voltage appearing across the series circuit, and Z_T is the total impedance of the series circuit.

Example 11: For the circuit in Figure below,

A) Calculate I, V_R, V_L, and V_C in phasor form.

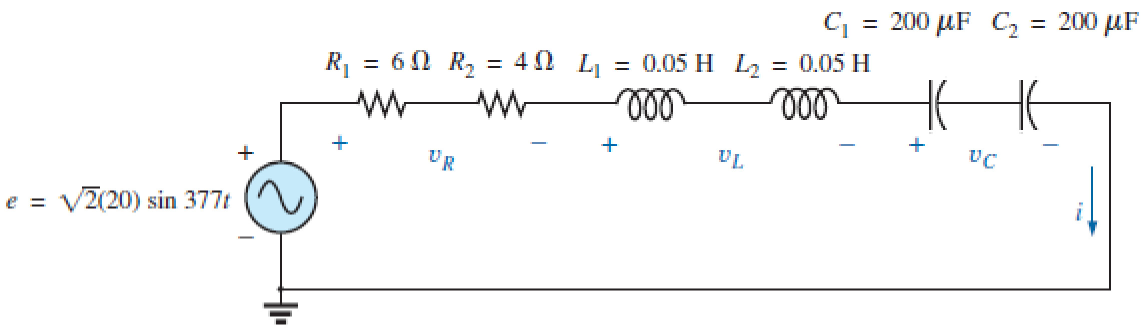
B) Calculate the total power factor.

C) Calculate the average power delivered to the circuit.

D) Draw the phasor diagram.

E) Obtain the phasor sum of V_R, V_L, and V_C, and show that it equals the input voltage E.

f. Find V_R and V_C using the voltage divider rule.



Solution:

A) Combining common elements and finding the reactance of the inductor and capacitor, we obtain:

$$R_T = 6\ \Omega + 4\ \Omega = 10\ \Omega$$

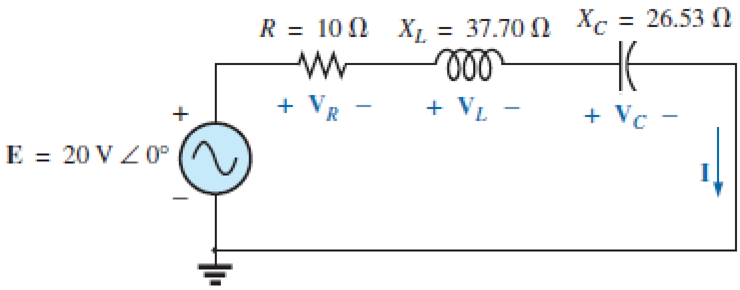
$$L_T = 0.05\ H + 0.05\ H = 0.1\ H$$

$$C_T = \frac{200\ \mu F}{2} = 100\ \mu F$$

$$X_L = \omega L = (377\ rad/s)(0.1\ H) = 37.70\ \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{(377\ rad/s)(100\ \times 10^{-6}\ F)} = \frac{10^6\ \Omega}{37{,}700} = 26.53\ \Omega$$

Redrawing the circuit using phasor notation results in Figure below.



For the circuit in Figure above:

$$Z_T = R\,\angle 0° + X_L\,\angle 90° + X_C\,\angle -90°$$

$$Z_T = 10\,\Omega + j\,37.70\,\Omega - j\,26.53\,\Omega = 10\,\Omega + j\,11.17\,\Omega = \mathbf{15\,\Omega\,\angle 48.16°}$$

The current ***I*** is:

$$I = \frac{E}{Z_T} = \frac{20\,V\,\angle 0°}{15\,\Omega\,\angle 48.16°} = \mathbf{1.33\,A\,\angle -48.16°}$$

The voltage across the resistor, inductor, and capacitor can be found using Ohm's law:

$$V_R = IZ_R = (I\,\angle\theta)(R\,\angle 0°) = (1.33\,A\,\angle -48.16°)(10\,\Omega\,\angle 0°)$$

$$V_R = \mathbf{13.30\,V\,\angle -48.16°}$$

$$V_L = IZ_L = (I\,\angle\theta)(X_L\,\angle 90°) = (1.33\,A\,\angle -48.16°)(37.70\,\Omega\,\angle 90°)$$

$$V_L = \mathbf{50.14\,V\,\angle 41.84°}$$

$$V_C = IZ_C = (I\,\angle\theta)(X_C\,\angle -90°) = (1.33\,A\,\angle -48.16°)(26.53\,\Omega\,\angle -90°)$$

$$V_C = \mathbf{35.28\,V\,V\,\angle -138.16°}$$

B) The total power factor, determined by the angle between the applied voltage **E** and the resulting current **I,** is 48.16°:

$$F_P = \cos\theta = cos\,48.16° = \mathbf{0.667\ lagging}$$

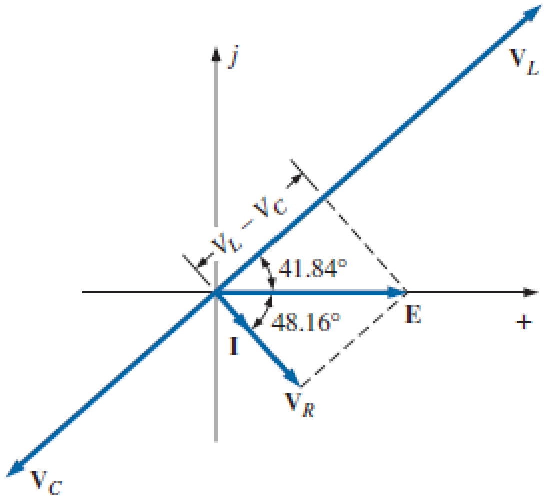
or

$$F_P = \cos\theta = \frac{R}{Z_T} = \frac{10\,\Omega}{15\,\Omega} = \mathbf{0.667\ lagging}$$

C) The total power in watts delivered to the circuit is:

$$P = EI\cos\theta_T = (20\,V)(1.33\,A)(0.667) = \mathbf{17.74\,W}$$

D) The phasor diagram appears in Figure below.



E) The phasor sum of V_R, V_L, and V_C is:

$$E = V_R + V_L + V_C = 13.30\ V\angle -48.16° + 50.14\ V\angle 41.84° + 35.28\ V\ \angle -138.16°$$

$$E = 13.30\ V\ \angle -48.16° + 14.86\ V\ \angle 41.84°$$

Therefore,

$$E = \sqrt{(13.30\ V)^2 + (14.86\ V)^2} = \mathbf{20\ V}$$

and

$$\theta_E = \mathbf{0}°\ \text{(from phasor diagram)}$$

and

$$E = \mathbf{20\ V\ \angle 0°}$$

F)

$$V_R = \frac{Z_R\ E}{Z_T} = \frac{(10\ \Omega\ \angle 0°)(20\ V\ \angle 0°)}{15\ \Omega\ \angle 48.16°} = \frac{200\ V\ \angle 0°}{15\ \angle 48.16°} = \mathbf{13.3\ V\ \angle -48.16°}$$

$$V_C = \frac{Z_C\ E}{Z_T} = \frac{(26.5\ \Omega\ \angle -90°)(20\ V\ \angle 0°)}{15\ \Omega\ \angle 48.16°} = \frac{530.6\ V\ \angle -90°}{15\ \angle 48.16°}$$

$$V_C = \mathbf{35.37\ V\ \angle -138.16°}$$

7.4 Parallel AC Elements:

For the network of Figure (7.10) with any number of parallel elements the total impedance has the same format as encountered for dc networks:

$$\frac{1}{Z_T} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} + \cdots + \frac{1}{Z_N}$$

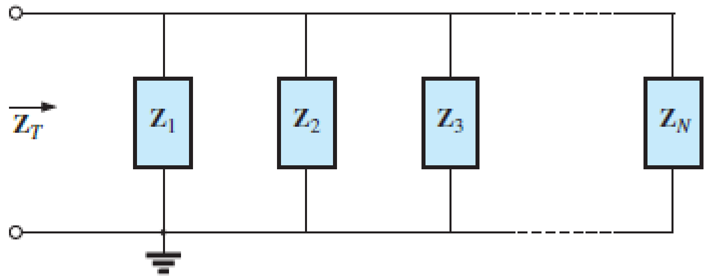


Figure (7.10): Parallel impedances.

which can be written in the following form:

$$Z_T = \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} + \cdots + \frac{1}{Z_N}}$$

For two impedances in parallel:

$$\frac{1}{Z_T} = \frac{1}{Z_1} + \frac{1}{Z_2}$$

which will become the following after a few mathematical manipulations:

$$Z_T = \frac{Z_1\, Z_2}{Z_1 + Z_2}$$

For three impedances in parallel the resulting equation is the following:

$$Z_T = \frac{Z_1\, Z_2\, Z_3}{Z_1\, Z_2 + Z_2\, Z_3 + Z_1\, Z_3}$$

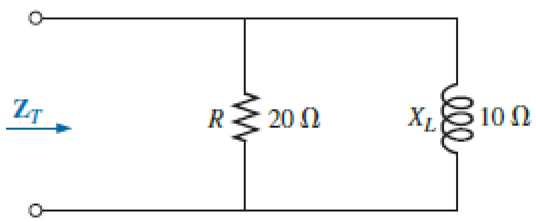
And for any number of impedances in parallel of the same content the following equation can be applied:

$$Z_T = \frac{Z_1}{N}$$

Example 12: For the network in Figure below:

A) Determine the input impedance.

B) Draw the impedance diagram.



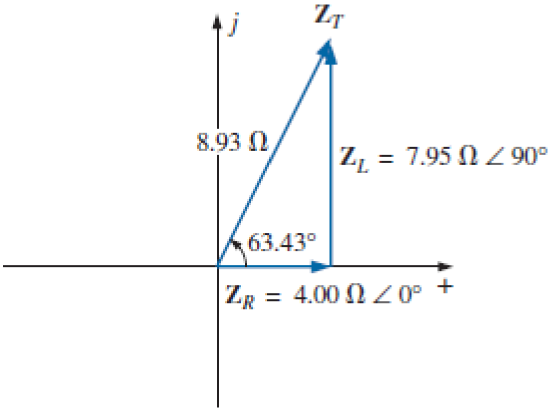
Solution:

A)

$$Z_T = \frac{Z_R\, Z_L}{Z_R + Z_L} = \frac{(20\ \Omega\ \angle 0°)(10\ \Omega\ \angle 90°)}{20\ \Omega + j10\ \Omega} = \frac{200\ \Omega\ \angle 90°}{22.361\ \angle 26.57°} = \mathbf{8.93\ \Omega\ \angle 63.43°}$$

$$= \mathbf{4.00}\ \Omega + \boldsymbol{j7.95}\ ° = R_T + jX_L = \mathbf{8.93\ \Omega\ \angle 63.43°}$$

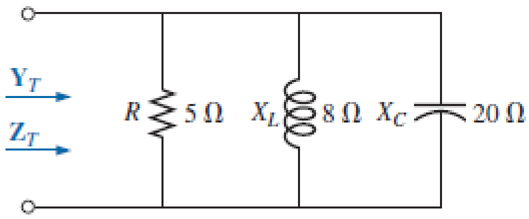
B) The impedance diagram appears in Figure below.



Example 13: For the network in Figure below:

A) Determine the total impedance.

B) Sketch the impedance diagram.



Solution:

A)

$$Z_T = \frac{1}{\frac{1}{Z_R} + \frac{1}{Z_L} + \frac{1}{Z_C}} = \frac{1}{\frac{1}{5\ \Omega\ \angle 0°} + \frac{1}{8\ \Omega\ \angle 90°} + \frac{1}{20\ \Omega\ \angle -90°}}$$

$$Z_T = \frac{1}{0.2\ S\ \angle 0° + 0.125\ S\ \angle -90° + 0.05\ S\ \angle 90°} = \frac{1}{0.2\ S - j\ 0.075\ S}$$

$$Z_T = \frac{1}{0.2136\ S\ \angle -20.56°} = \mathbf{4.68\ \Omega\ \angle 20.56°}$$

or

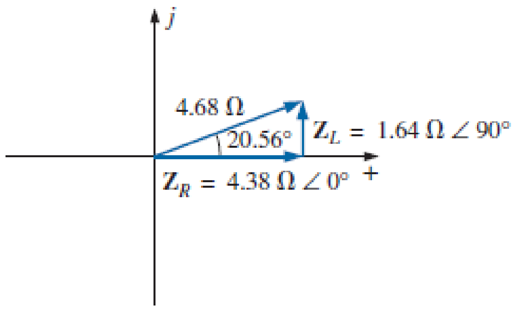
$$Z_T = \frac{Z_R\, Z_L\, Z_C}{Z_R\, Z_L + Z_L\, Z_C + Z_R\, Z_C}$$

$$= \frac{(5\ \Omega\ \angle 0°)(8\ \Omega\ \angle 90°)(20\ \Omega\ \angle -90°)}{(5\ \Omega\ \angle 0°)(8\ \Omega\ \angle 90°) + (8\ \Omega\ \angle 90°)(20\ \Omega\ \angle -90°) + (5\ \Omega\ \angle 0°)(20\ \Omega\ \angle -90°)}$$

$$= \frac{800\ \Omega\ \angle 0°}{40\ \angle 90° \ +\ 160\ \angle 0° \ +\ 100\ \angle -90°} = \frac{800\ \Omega}{160\ +\ j\,40\ -\ j\,100} = \frac{800\ \Omega}{160\ -\ j\,60}$$

$$Z_T = \frac{800\ \Omega}{170.88\ \angle -20.56°} = \mathbf{4.68}\ \Omega\ \angle \mathbf{20.56}° = \mathbf{4.38}\ \Omega + \boldsymbol{j}\mathbf{1.64}\ \Omega$$

B) The impedance diagram appears in Figure below.



7.5 Total Admittance:

In ac circuit, admittance (Y) is a measure of how well an ac circuit will admit, or allow, current to follow in the circuit.

For ac parallel circuits the terminology applied is **admittance**, which has the symbol **Y** and is measured in **siemens (S)**.

- **Resistive Elements:** For resistors the admittance is defined by:

$$Y_R = \frac{1}{Z_R} = \frac{1}{R\angle 0°} = G\angle 0° \qquad \text{(siemens, S)}$$

- **Inductive Elements:** For inductive elements the admittance is defined by:

$$Y_L = \frac{1}{Z_L} = \frac{1}{X_L\angle 90°} = \frac{1}{X_L}\angle -90° \qquad \text{(siemens, S)}$$

The ratio $1/X_L$ is called the **susceptance** of the inductive element, is given the symbol $\boldsymbol{B_L}$, and is measured in **siemens (S)**. Therefore,

$$B_L = \frac{1}{X_L} \qquad \text{(siemens, S)}$$

and

$$Y_L = B_L \angle -90^\circ \quad \text{(siemens, S)}$$

❖ **Capacitive Elements:** For capacitive elements the admittance is defined by:

$$Y_C = \frac{1}{Z_C} = \frac{1}{X_C \angle -90^\circ} = \frac{1}{X_C} \angle 90^\circ \quad \text{(siemens, S)}$$

The ratio $1/X_C$ is also called the **susceptance** of the capacitive element, is given the symbol $\boldsymbol{B_C}$, and is measured in **siemens (S)**. Therefore,

$$B_C = \frac{1}{X_C} \quad \text{(siemens, S)}$$

and

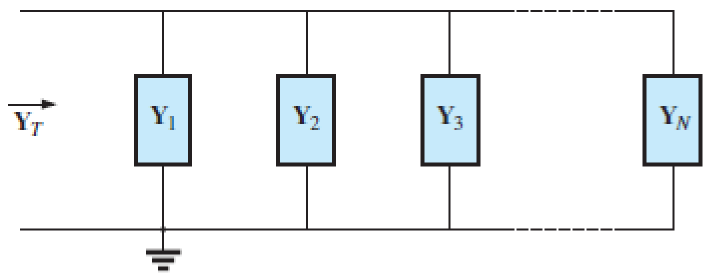
$$Y_C = B_C \angle 90^\circ \quad \text{(siemens, S)}$$

For dc circuits with simply resistive elements we found that the total conductance of parallel resistive elements was simply the sum of the conductance values as shown below.

$$G_T = G_1 + G_2 + G_3 + \cdots + G_N \quad \text{(siemens, S)}$$

For ac parallel networks, the total admittance is simply the sum of the admittance levels of all the parallel branches of Figure below. That is:

$$Y_T = Y_1 + Y_2 + Y_3 + \cdots + Y_N \quad \text{(siemens, S)}$$



In any case, whether the total impedance or admittance is first found, the other can be found using the simple equation:

$$Y_T = \frac{1}{Z_T} \quad \text{(siemens, S)}$$

For any configuration (series, parallel, series-parallel, and so on), the angle associated with the total admittance is the angle by which the source current leads the applied voltage. For inductive networks, θ_T is negative, whereas for capacitive networks, θ_T is positive.

7.6 Parallel AC Networks:

For the representative parallel ac network in Figure (7.11), the source current is determined by Ohm's law as follows:

$$I = \frac{E}{Z_T} = E\,Y_T$$

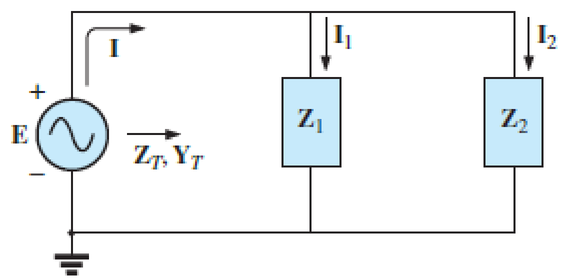


Figure (7.11): Parallel ac network.

Since the voltage is the same across parallel elements, the current through each branch can then be found through another application of Ohm's law:

$$I_1 = \frac{E}{Z_1} = E\,Y_1$$

$$I_2 = \frac{E}{Z_2} = E\,Y_2$$

7.6.1 Kirchhoff's Current Law (KCL):

Kirchhoff's current law can then be applied in the same manner as used for dc networks. We have:

$$I - I_1 - I_2 = 0$$

or

$$I = I_1 + I_2$$

Although the product of the voltage and current is not always the power delivered, it is a power rating of significant usefulness in the description and analysis of sinusoidal ac networks and in the maximum rating of a number of electrical components and systems. It is called the **apparent power** and is represented symbolically by *S*. Since it is simply the product of voltage and current, its units are *volt-amperes* (VA). Its magnitude is determined by: $S = V\,I^*$ (volt-amperes, VA)

Therefore, the average power to the network can be determined by:

$$P = S\cos\theta_T = EI\cos\theta_T$$

and the power factor of the circuit is: $F_P = \dfrac{P}{S} = \cos\theta_T$

where θ_T is the phase angle between $\boldsymbol{E}$ and $\boldsymbol{I}$.

Example 14: For the circuit of figure below:

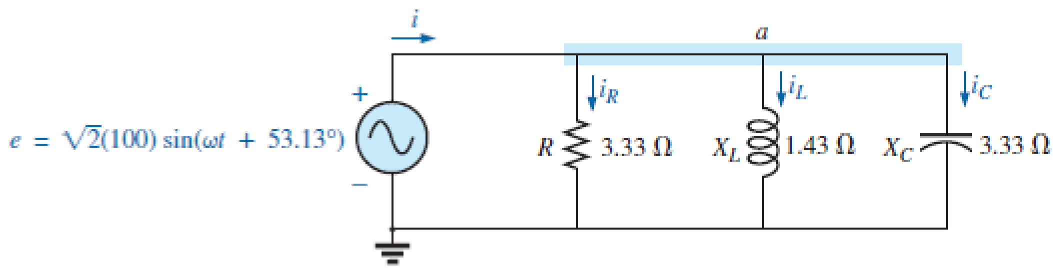
A) Calculate the total impedance and draw the admittance diagram.

B) Calculate I, I_R, I_L, and $I_C$in phasor form and draw the phaser diagram.

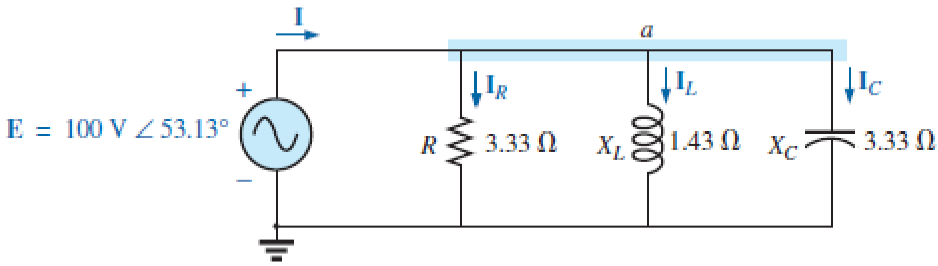
C) Sketch the waveforms for the parallel R-L-C network.

D) Calculate the average power delivered to the circuit and the total power factor.

E) Find the input current I using Ohm's law.



Solution: Phasor notation: As shown in Figure below.



A)

$$Y_T = Y_R + Y_L + Y_C = G\angle 0° + B_L \angle -90° + B_C \angle 90°$$

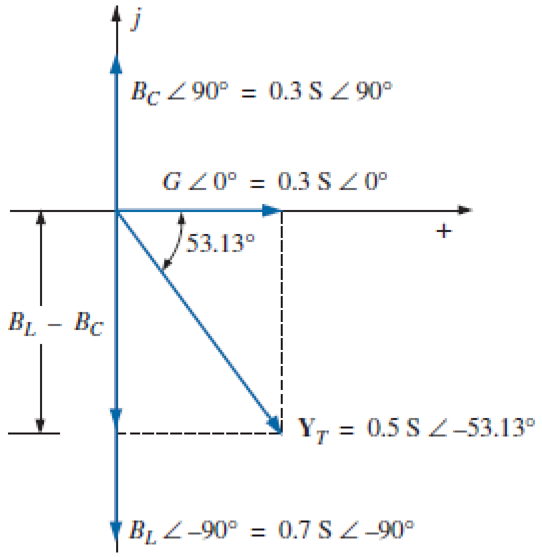
$$Y_T = \frac{1}{3.33\ \Omega}\angle 0° + \frac{1}{1.43\ \Omega}\angle -90° + \frac{1}{3.33\ \Omega}\angle 90°$$

$$Y_T = 0.3\ S \angle 0° + 0.7\ S \angle -90° + 0.3\ S \angle 90° = 0.3\ S - j\,0.7\ S + j\,0.3\ S$$

$$Y_T = 0.3\ S - j\,0.4\ S = \mathbf{0.5\ S \angle -53.13°}$$

$$Z_T = \frac{1}{Y_T} = \frac{1}{0.5\ S \angle -53.13°} = \mathbf{2\ \Omega \angle 53.13°}$$

Admittance diagram: As shown in Figure below.



B)

$$I = \frac{E}{Z_T} = E\,Y_T = (100\ V \angle 53.13°)(0.5\ S \angle -53.13°) = \mathbf{50\ A \angle 0°}$$

$$I_R = (E \angle \theta)(G \angle 0°) = (100\ V \angle 53.13°)(0.3\ S \angle 0°) = \mathbf{30\ A \angle 53.13°}$$

$$I_L = (E \angle \theta)(B_L \angle -90°)$$

$$I_L = (100\ V \angle 53.13°)(0.7\ S \angle -90°) = \mathbf{70\ A \angle -36.87°}$$

$$I_C = (E \angle \theta)(B_C \angle 90°) = (100\ V \angle 53.13°)(0.3\ S \angle +90°) = \mathbf{30\ A \angle 143.13°}$$

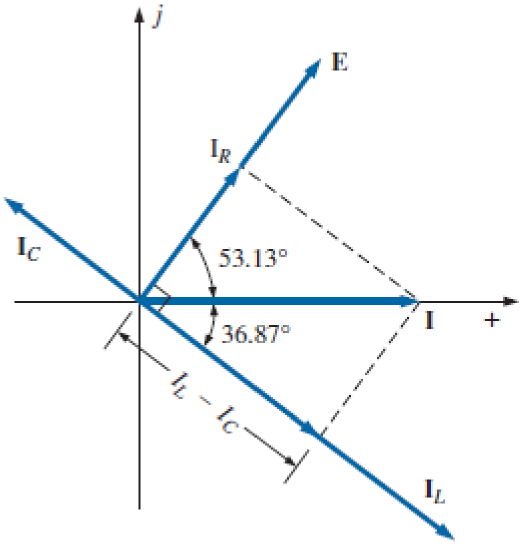
Kirchhoff's current law: At node *a*,

$$I - I_R - I_L - I_C = 0$$

or

$$I = I_R + I_L + I_C$$

Phasor diagram: The phasor diagram in Figure below indicates that the impressed voltage $\boldsymbol{E}$ is in phase with the current $\boldsymbol{I_R}$ through the resistor, leads the current $\boldsymbol{I_L}$ through the inductor by 90°, and lags the current $\boldsymbol{I_C}$ of the capacitor by 90°.



C) Time domain:

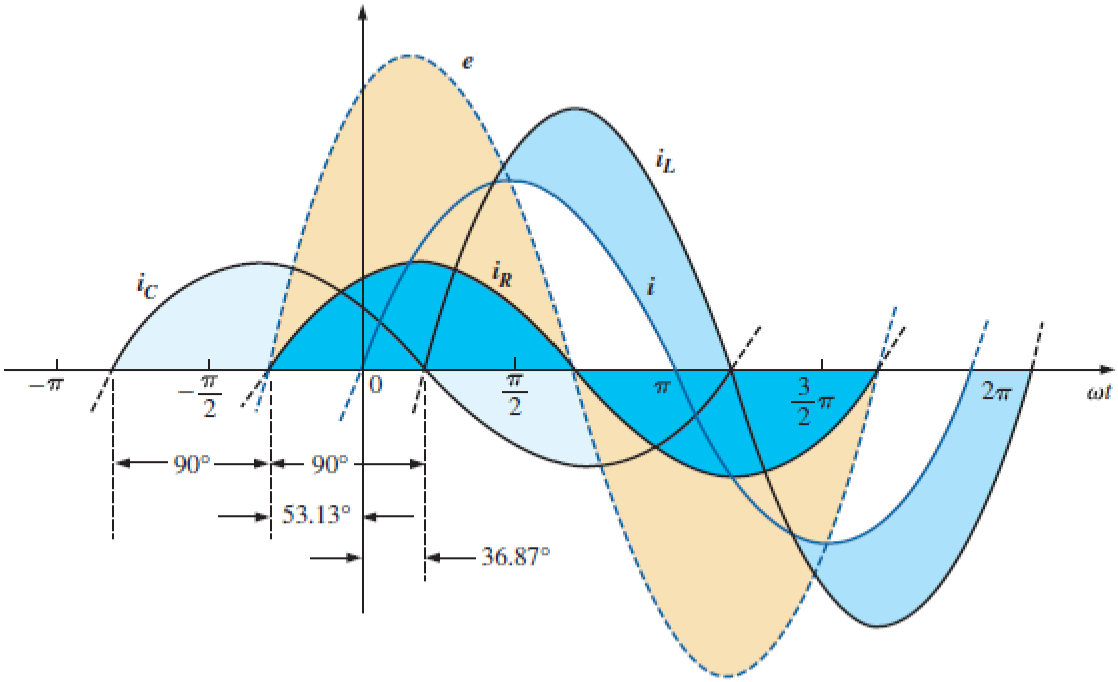
$$i = \sqrt{2}(50)\sin\omega t = \boldsymbol{70.70\ sin\ \omega t}$$

$$i_R = \sqrt{2}(30)\sin(\omega t + 53.13°) = \boldsymbol{42.42\ sin(\omega t + 53.13}°)$$

$$i_L = \sqrt{2}(70)\sin(\omega t - 36.87°) = \boldsymbol{98.98\ sin(\omega t - 36.87}°)$$

$$i_C = \sqrt{2}(30)\sin(\omega t + 143.13°) = \boldsymbol{42.42\ sin(\omega t + 143.13}°)$$

A plot of all of the currents and the impressed voltage appears in Figure below.



D) ***Power:*** The total power in watts delivered to the circuit is

$$P_T = EI\cos\theta_T = (100\ V)(50\ A)\cos 53.13° = (5000)(0.6) = \mathbf{3000\ W}$$

or

$$P_T = E^2 G = (100\ V)^2(0.3\ S) = \mathbf{3000\ W}$$

or, finally,

$$P_T = P_R + P_L + P_C = EI_R\cos\theta_R + EI_L\cos\theta_L + EI_C\cos\theta_C$$

$$= (100\ V)(30\ A)\cos 0° + (100\ V)(70\ A)\cos 90° + (100\ V)(30\ A)\cos 90°$$

$$P_T = 3000\ W + 0 + 0 = \mathbf{3000\ W}$$

Power factor: The power factor of the circuit is:

$$F_P = \cos\theta_T = \cos 53.13° = \mathbf{0.6\ lagging}$$

or

$$F_P = \cos\theta_T = \frac{G}{Y_T} = \frac{0.3\ S}{0.5\ S} = \mathbf{0.6\ lagging}$$

E) ***Impedance approach:*** The input current **I** can also be determined by first finding the total impedance in the following manner:

$$Z_T = \frac{Z_R\ Z_L\ Z_C}{Z_R\ Z_L + Z_L\ Z_C + Z_R\ Z_C} = \mathbf{2\ \Omega\ \angle 53.13°}$$

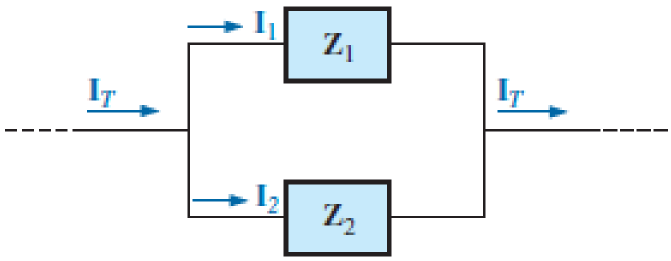
and, applying Ohm's law, we obtain:

$$I = \frac{E}{Z_T} = \frac{100\ V\angle 53.13°}{2\ \Omega\ \angle 53.13°} = \mathbf{50\ A\ \angle 0°}$$

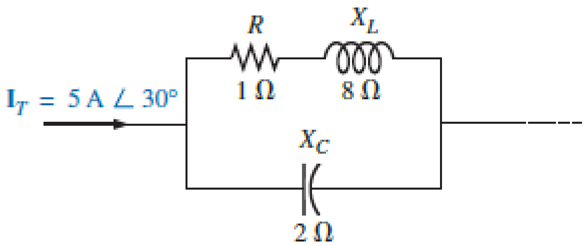
7.6.2 Current Divider Rule (CDR):

The basic format for the **current divider rule** in ac circuits is exactly the same as that for dc circuits; that is, for two parallel branches with impedances $\boldsymbol{Z_1}$ and $\boldsymbol{Z_2}$ as shown in Figure below.

$$I_1 = \frac{Z_2\ I_T}{Z_1 + Z_2}\quad or\quad I_1 = \frac{Z_1\ I_T}{Z_1 + Z_2}$$



Example 15: Using the current divider rule, find the current through each parallel branch in Figure below.



Solution:

$$I_{R-L} = \frac{Z_C\ I_T}{Z_C + Z_{R-L}} = \frac{(2\ \Omega\ \angle -90°)(5\ A\ \angle 30°)}{-j\ 2\ \Omega + 1\ \Omega + j\ 8\ \Omega} = \frac{10\ A\ \angle -60°}{1 + j\ 6}$$

$$I_{R-L} = \frac{10\ A\ \angle -60°}{6.083\ \angle 80.54°} \cong \mathbf{1.64\ A\ \angle -140.54°}$$

$$I_C = \frac{Z_{R-L}\ I_T}{Z_C + Z_{R-L}} = \frac{(1\ \Omega + j\ 8\ \Omega)(5\ A\ \angle 30°)}{6.083\ \angle 80.54°} = \frac{(8.06\ \angle 82.87°)(5\ A\ \angle 30°)}{6.083\ \angle 80.54°}$$

$$I_C = \frac{40.30\ A\ \angle 112.87°}{6.083\ \angle 80.54°} = \mathbf{6.63\ A\ \angle 32.33°}$$

7.7 Series-Parallel Networks:

In general, when working with series-parallel ac networks, consider the following approach:

1) Redraw the network, using block impedances to combine obvious series and parallel elements, which will reduce the network to one that clearly reveals the fundamental structure of the system.
2) Study the problem and make a brief mental sketch of the overall approach you plan to use. Doing this may result in time- and energy-saving shortcuts. In some cases, a lengthy, drawn-out analysis may not be necessary. A single application of a fundamental law of circuit analysis may result in the desired solution.
3) After the overall approach has been determined, it is usually best to consider each branch involved in your method independently before tying them together in series parallel combinations. In most cases, work back from the obvious series and parallel combinations to the source to determine the total impedance of the network. The source current can then be determined, and the path back to specific unknowns can be defined. As you progress back to the source, continually define those unknowns that have not been lost in the reduction process. It will save time when you have to work back through the network to find specific quantities.

Example 16: For the network in Figure below:

A) Calculate $\boldsymbol{Z_T}$.

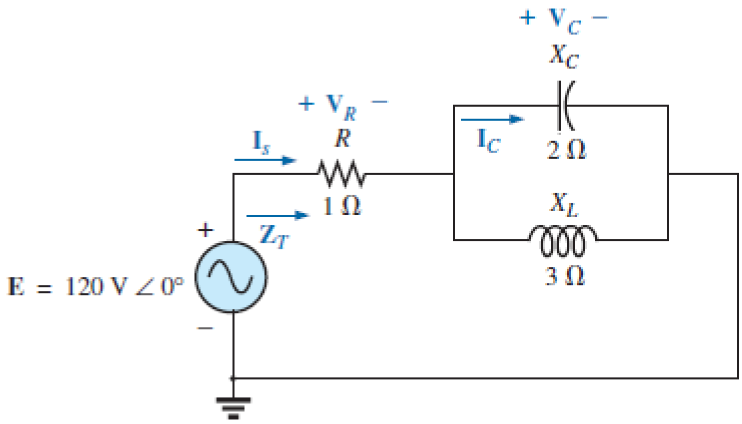
B) Determine $\boldsymbol{I_S}$.

C) Calculate $\boldsymbol{V_R}$ and $\boldsymbol{V_C}$.

D) Find $\boldsymbol{I_C}$..

E) Compute the power delivered.

A) Find $\boldsymbol{F_P}$. of the network.



Solution:

As suggested in the introduction, the network has been redrawn with block impedances, as shown in Figure above. The impedance $\boldsymbol{Z_1}$ is simply the resistor R of 1 Ω, and $\boldsymbol{Z_2}$ is the parallel combination of X_C and X_L. The network now clearly reveals that it is fundamentally a series circuit, suggesting a direct path toward the total impedance and the source current. For many such problems, you must work back to the source to find first the total impedance and then the source current. When the unknown quantities are found in terms of these subscripted impedances, the numerical values can then be substituted to find the magnitude and phase angle of the unknown. In other words, try to find the desired solution solely in terms of the subscripted impedances before substituting numbers. This approach will usually enhance the clarity of the chosen path toward a solution while saving time and preventing careless calculation errors. Note also in Figure below that all the unknown quantities except $\boldsymbol{I_C}$ have been preserved, meaning that we can use Figure below to determine these quantities rather than having to return to the more complex network in Figure above.

The total impedance is defined by:

$$Z_T = Z_1 + Z_2$$

with

$$Z_1 = R\angle 0° = 1\,\Omega\angle 0°$$

$$Z_2 = Z_C \parallel Z_L = \frac{(X_C\angle -90°)(X_L\angle 90°)}{-jX_C + jX_L} = \frac{(2\,\Omega\angle -90°)(3\,\Omega\angle 90°)}{-j\,2\,\Omega + j\,3\,\Omega}$$

$$Z_2 = \frac{6\Omega \angle 0°}{j1} = \frac{6\Omega \angle 0°}{1 \angle 90°} = 6\Omega \angle -90°$$

and

$$Z_T = Z_1 + Z_2 = 1\,\Omega - j\,6\,\Omega = \mathbf{6.08}\,\Omega \angle -\mathbf{80.54}°$$

B)

$$I_S = \frac{E}{Z_T} = \frac{120\ V \angle 0°}{6.08\ \Omega \angle -80.54°} = \mathbf{19.74\ A \angle 80.54}°$$

C) Referring to Network in Figure above after assigning the block impedances, we find that $\boldsymbol{V_R}$ and $\boldsymbol{V_C}$ can be found by a direct application of Ohm's law:

$$V_R = I_S Z_1 = (19.74\ A \angle 80.54°)(1\ \Omega \angle 0°) = \mathbf{19.74\ V \angle 80.54}°$$

$$V_C = I_S Z_2 = (19.74\ A \angle 80.54°)(6\ \Omega \angle -90°) = \mathbf{118.44\ V \angle -9.46}°$$

D) Now that $\boldsymbol{V_C}$ is known, the current $\boldsymbol{I_C}$ can also be found using Ohm's law:

$$I_C = \frac{V_C}{Z_C} = \frac{118.44\ V \angle -9.46°}{2\ \Omega \angle -90°} = \mathbf{59.22\ A \angle 80.54}\_$$

E)

$$P_{del} = I_S^2 R = (19.74\ A)^2 (1\ \Omega) = \mathbf{389.67\ W}$$

F)

$$F_P = \cos\theta = \cos 80.54° = \mathbf{0.164\ leading}$$

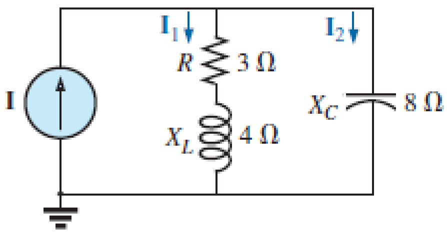
The fact that the total impedance has a negative phase angle (revealing that $\boldsymbol{I_S}$ leads $\boldsymbol{E}$) is a clear indication that the network is capacitive in nature and therefore has a leading power factor. The fact that the network is capacitive can be determined from the original network by first realizing that, for the parallel *L*-*C* elements, the smaller impedance predominates and results in an *R*-*C* network.

Example 17: For the network in Figure below:

A) If **I** is 50 A ∠30°, calculate $\boldsymbol{I_1}$ using the current divider rule.

B) Repeat part (A) for $\boldsymbol{I_2}$.

C) Verify Kirchhoff's current law at one node.

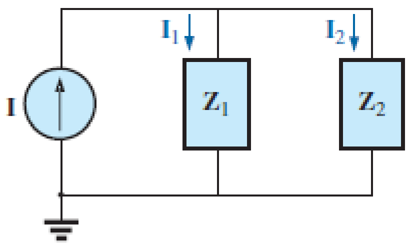


Solution:

A) Redrawing the circuit as in Figure below, we have:

$$Z_1 = R + jX_L = 3\,\Omega + j\,4\,\Omega = 5\,\Omega\,\angle 53.13°$$

$$Z_2 = -jX_L = -j\,8\,\Omega = 8\,\Omega\,\angle -90°$$



Using the current divider rule yields:

$$I_1 = \frac{Z_2\,I}{Z_1 + Z_2} = \frac{(8\,\Omega\,\angle -90°)(50\,A\,\angle 30°)}{3\,\Omega + j\,4\,\Omega - j\,8\,\Omega} = \frac{400\,A\,\angle -60°}{3 - j4}$$

$$I_1 = \frac{400\,A\,\angle -60°}{5\,\angle -53.13°} = \mathbf{80\,A\,\angle -6.87°}$$

B)

$$I_1 = \frac{Z_1\,I}{Z_1 + Z_2} = \frac{(5\,\Omega\,\angle 53.13°)(50\,A\,\angle 30°)}{5\,\angle -53.13°} = \frac{250\,A\,\angle 83.13°}{5\,\angle -53.13°} = \mathbf{50\,A\,\angle 136.26°}$$

C)

$$I = I_1 + I_2$$

$$50\ A\ \angle 30° = 80\ A\ \angle -6.87° + 50\ A\ \angle 136.26°$$

$$= (79.43\ -\ j\ 9.57) + (-36.12\ +\ j\ 34.57) = 43.31\ +\ j\ 25.0$$

$$50\ A\ \angle 30° = 50\ A\ \angle 30° \qquad \text{(checks)}$$

Lecture 8: Methods of AC and DC Analysis

For networks with two or more sources that are not in series or parallel, the methods mentioned before cannot be applied. Rather, methods such as mesh analysis or nodal analysis must be used.

8.1 DC Source Conversions:

In reality, all sources—whether they are voltage sources or current sources—have some internal resistance in the relative positions shown in Figure (8.1). For the voltage source, if $R_S = 0\ \Omega$, or if it is so small compared to any series resistors that it can be ignored, then we have an "ideal" voltage source for all practical purposes.

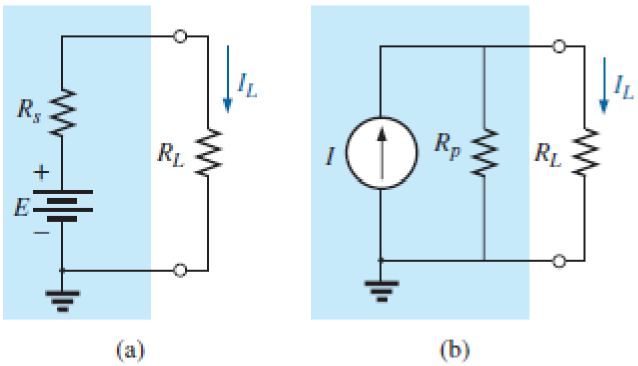


Figure (8.1): Practical sources: (a) voltage; (b) current.

For the current source, since the resistor R_P is in parallel, if $R_P = \infty\ \Omega$, or if it is large enough compared to any parallel resistive elements that it can be ignored, then we have an "ideal" current source.

Unfortunately, however, ideal sources *cannot be converted* from one type to another. That is, a voltage source cannot be converted to a current source, and vice versa—*the internal resistance must be present*. If the voltage source in Figure (8.1(a)) is to be equivalent to the source in Figure (8.1(b)), any load connected to the sources such as R_L should receive the same current, voltage, and power from each configuration.

For the voltage source equivalent, the voltage is determined by a simple application of Ohm's law to the current source: $E = I\ R_P$. For the current source equivalent, the current is again determined by applying Ohm's law to the voltage source: $I = E/R_S$.

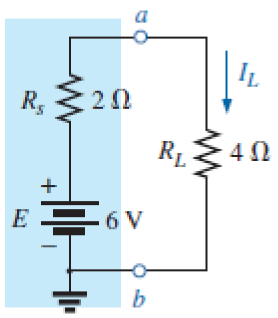
It is important to realize, that ***the equivalence between a current source and a voltage source exists only at their external terminals***

Example 1: For the circuit in Figure below:

A) Determine the current I_L.

B) Convert the voltage source to a current source.

C) Using the resulting current source of part (B), calculate the current through the load resistor, and compare your answer to the result of part (A).



Solution:

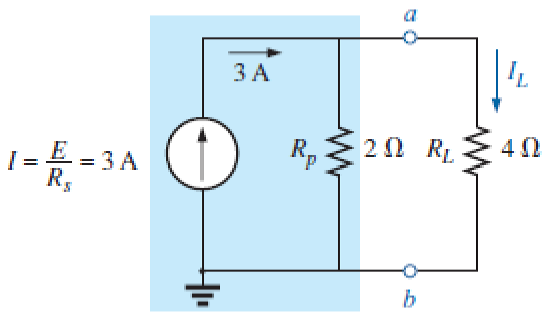
A) Applying Ohm's law gives:

$$I_L = \frac{E}{R_S + R_L} = \frac{6\ V}{2\ \Omega + 4\ \Omega} = \frac{6\ V}{6\ \Omega} = \mathbf{1\ A}$$

B) Using Ohm's law again gives:

$$I = \frac{E}{R_S} = \frac{6\ V}{2\ \Omega} = \mathbf{3\ A}$$

and the equivalent source appears in Figure below with the load reapplied.



C) Using the current divider rule gives:

$$I_L = \frac{R_P\, I}{R_P + R_L} = \frac{(2\,\Omega)(3\,A)}{2\,\Omega + 4\,\Omega} = \frac{1}{3}(3\,A) = \mathbf{1\,A}$$

We find that the current I_L is the same for the voltage source as it was for the equivalent current source—the sources are therefore equivalent.

8.2 AC Source Conversions:

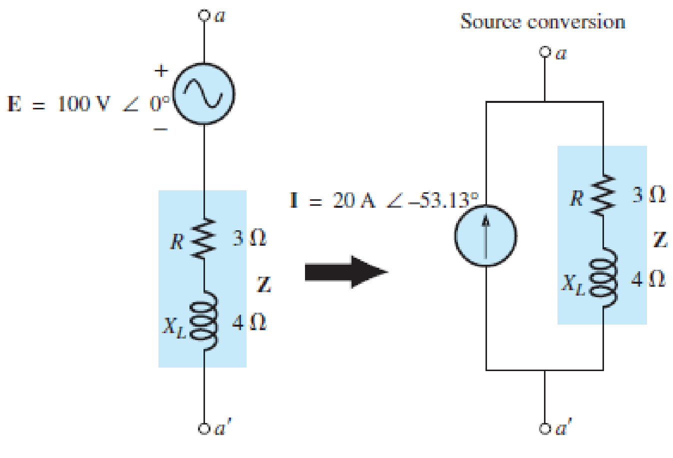
When applying the methods to be discussed, it may be necessary to convert a current source to a voltage source or a voltage source to a current source. This **source conversion** can be accomplished in much the same manner as for dc circuits, except that now we shall be dealing with phasors and impedances instead of just real numbers and resistors.

Example 2: Convert the voltage source in Figure below (a) to a current source.

Solution:

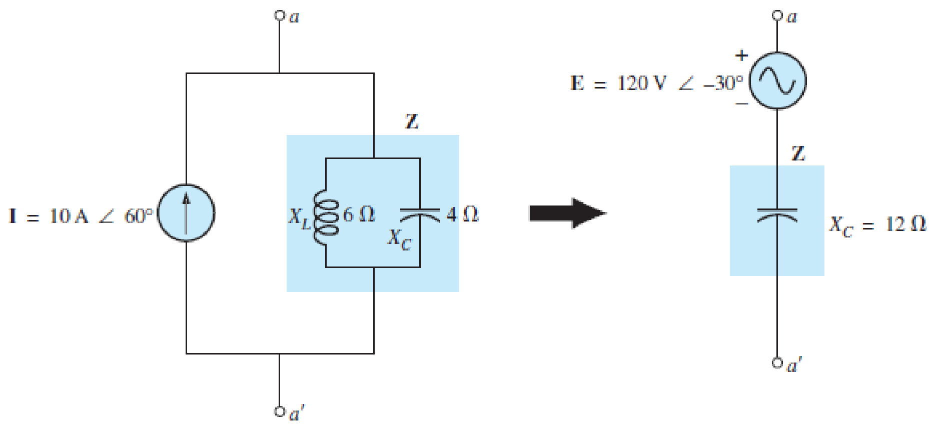
$$I = \frac{E}{Z} = \frac{100\ \angle 0^o}{5\ \Omega\ \angle 53.13^o}$$

$I = 20\ A\ \angle{-53.13^o}$ **[Figure above (b)]**



(a) (b)

Example 3: Convert the current source in Figure below (a) to a voltage source.



(a) (b)

Solution:

$$Z = \frac{Z_C \, Z_L}{Z_C + Z_L} = \frac{(X_C \angle -90^o)(X_L \angle 90^o)}{-jX_C + jX_L}$$

$$Z = \frac{(4\,\Omega \angle -90^o)(6\,\Omega \angle 90^o)}{-j4\,\Omega + j6\,\Omega} = \frac{24\,\Omega \angle 0^o}{2 \angle 90^o} = \mathbf{12\,\Omega \angle -90^o}$$

$E = I\,Z = (10\,A \angle 60^o)(12\,\Omega \angle -90^o) = \mathbf{120\,V \angle -30^o}$ **[Figure above (b)]**

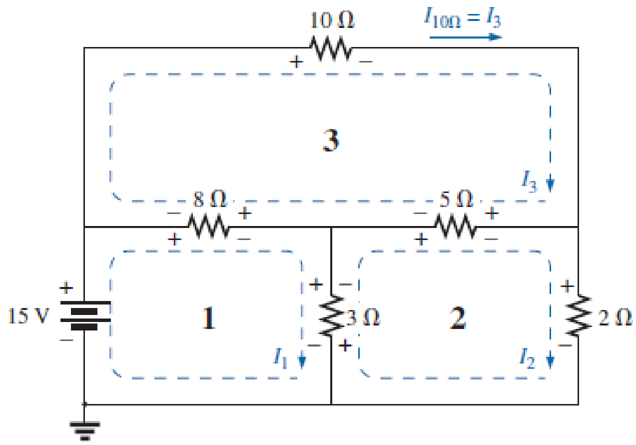
8.3 Mesh Analysis (Format Approach):

In AC network, the only change from the dc coverage is to substitute impedance for resistance and admittance for conductance in the procedure.

Mesh Analysis Procedure:

1) Assign a loop current to each independent closed loop in a clockwise direction.
2) The number of required equations is equal to the number of chosen independent closed loops. Column 1 of each equation is formed by summing the **impedance/resistance** values of those **impedances/resistances** through which the loop current of interest passes and multiplying the result by that loop current.
3) We must now consider the mutual terms that are always subtracted from the terms in the first column. It is possible to have more than one mutual term if the loop current of interest has an element in common with more than one other loop current. Each mutual term is the product of the mutual impedance/resistance and the other loop current passing through the same element.
4) The column to the right of the equality sign is the algebraic sum of the voltage sources through which the loop current of interest passes. Positive signs are assigned to those sources of voltage having a polarity such that the loop current passes from the negative to the positive terminal. Negative signs are assigned to those potentials for which the reverse is true.
5) Solve the resulting simultaneous equations for the desired loop currents.

Example 4: Find the current through the 10 Ω resistor of the network in Figure below.



Solution:

I_1: $\quad (8\,\Omega + 3\,\Omega)I_1 - (8\,\Omega)I_3 - (3\,\Omega)I_2 = 15\,V$

I_2: $\quad (3\,\Omega + 5\,\Omega + 2\,\Omega)I_2 - (3\,\Omega)I_1 - (5\,\Omega)I_3 = 0$

I_1: $\quad (8\,\Omega + 10\,\Omega + 5\,\Omega)I_3 - (8\,\Omega)I_1 - (5\,\Omega)I_2 = 0$

$$11\,I_1 - 3\,I_2 - 8\,I_3 = 15\,V$$

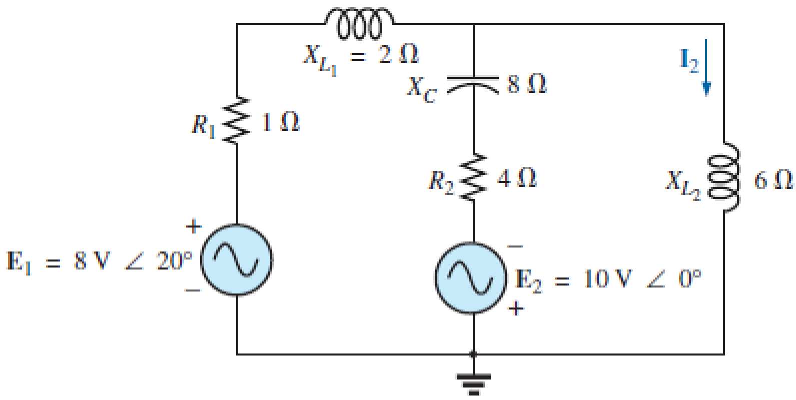
$$-3\,I_1 + 10\,I_2 - 5\,I_3 = 0$$

$$-8\,I_1 - 5\,I_2 + 23\,I_3 = 0$$

and

$$I_3 = I_{10\,\Omega} = \frac{\begin{vmatrix} 11 & -3 & 15 \\ -3 & 10 & 0 \\ -8 & -5 & 0 \end{vmatrix}}{\begin{vmatrix} 11 & -3 & -8 \\ -3 & 10 & -5 \\ -8 & -5 & 23 \end{vmatrix}} = \mathbf{1.22\ A}$$

Example 5: Using the format approach to mesh analysis, find the current I_2 in Figure below.



Solution: The network is redrawn in Figure below.

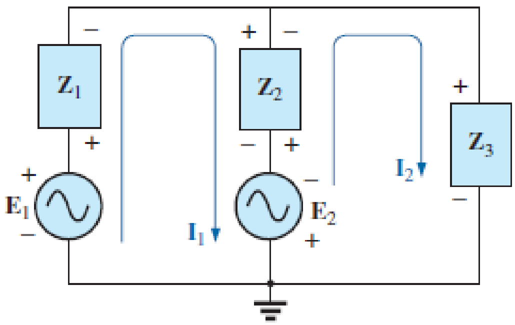
$Z_1 = R_1 + jX_{L_1} = 1\,\Omega + j\,2\,\Omega \qquad \boldsymbol{E_1} = 8\ V\angle 20^o$

$Z_2 = R_2 - jX_C = 4\,\Omega - j\,8\,\Omega \qquad \boldsymbol{E_2} = 10\ V\angle 0^o$

$Z_3 = +jX_{L_2} = +j\,6\,\Omega$

Note the reduction in complexity of the problem with the substitution of the subscripted impedances.

Step 1: is as indicated in Figure below.



Steps 2 to 4:

$$I_1(Z_1 + Z_2) - I_2 Z_2 = E_1 + E_2$$
$$I_2(Z_2 + Z_3) - I_1 Z_2 = -E_2$$

which are rewritten as:

$$I_1(Z_1 + Z_2) - I_2 Z_2 \qquad = E_1 + E_2$$
$$-I_1 Z_2 \qquad + I_2(Z_2 + Z_3) = -E_2$$

Step 5: Using determinants, we have:

$$I_2 = \frac{\begin{vmatrix} Z_1 + Z_2 & E_1 + E_2 \\ -Z_2 & -E_2 \end{vmatrix}}{\begin{vmatrix} Z_1 + Z_2 & -Z_2 \\ -Z_2 & Z_2 + Z_3 \end{vmatrix}} = \frac{-(Z_1 + Z_2)E_2 + Z_2(E_1 + E_2)}{(Z_1 + Z_2)(Z_2 + Z_3) - Z_2^2}$$

$$I_2 = \frac{Z_2\, E_1 - Z_1\, E_2}{Z_1\, Z_2 + Z_1\, Z_3 + Z_2\, Z_3}$$

Substituting numerical values yields:

$$I_2 = \frac{(4\,\Omega - j\,8\,\Omega)(8\,V\angle 20^o) - (1\,\Omega + j\,2\,\Omega\,)(10\,V\angle 0^o)}{(1\,\Omega + j\,2\,\Omega)(4\,\Omega - j\,8\,\Omega) + (1\,\Omega + j\,2\,\Omega)(+j\,6\,\Omega) + (4\,\Omega - j\,8\,\Omega)(+j\,6\,\Omega)}$$

$$I_2 = \frac{(4\,\Omega - j\,8\,\Omega)(7.52 + j\,2.74) - (10\; + \; j\,20\,)}{20 + (j\,6 - 12) + (j\,24 + 48)}$$

$$I_2 = \frac{(52.0 - j\,49.20) - (10 + j\,20)}{56 + j\,30} = \frac{42.0 - j\,69.20}{56 + j\,30} = \frac{80.95\,A\angle - 58.74°}{63.53\,\angle 28.18°}$$

$$I_2 = \mathbf{1.27\,A\,\angle 286.92^o}$$

8.4 Nodal Analysis (Format Approach):

Nodal analysis—a method that provides the nodal voltages of a network, that is, the voltage from the various **nodes** (junction points) of the network to ground. The method is developed through the use of Kirchhoff's current law in much the same manner as Kirchhoff's voltage law was used to develop the mesh analysis approach.

The number of nodes for which the voltage must be determined using nodal analysis is 1 less than the total number of nodes.

The number of equations required to solve for all the nodal voltages of a network is 1 less than the total number of independent nodes.

A major requirement, however, is that *all voltage sources must first be converted to current sources before the procedure is applied*.

Nodal Analysis Procedure:

1) Choose a reference node, and assign a subscripted voltage label to the (N-1) remaining nodes of the network.

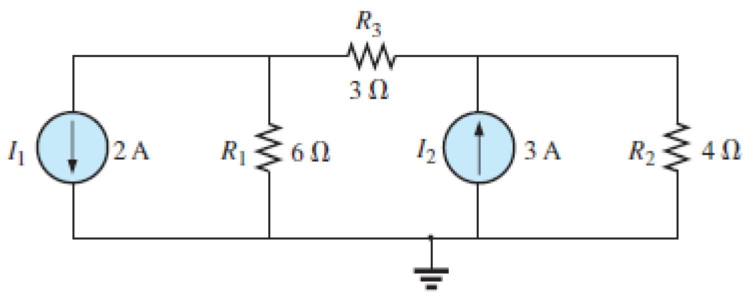
2) The number of equations required for a complete solution is equal to the number of subscripted voltages (N-1). Column 1 of each equation is formed by summing the **conductances/admittances** tied to the node of interest and multiplying the result by that subscripted nodal voltage.

3) The mutual terms are always subtracted from the terms of the first column. It is possible to have more than one mutual term if the nodal voltage of interest has an element in common with more than one other nodal voltage. Each mutual term is the product of the mutual **admittance/conductance** and the other nodal voltage tied to that **admittance/conductance**.

4) The column to the right of the equality sign is the algebraic sum of the current sources tied to the node of interest. A current source is assigned a positive sign if it supplies current to a node and a negative sign if it draws current from the node.

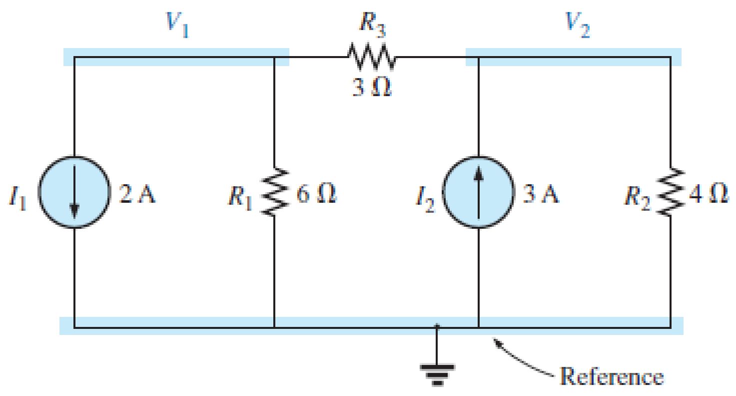
5) Solve the resulting simultaneous equations for the desired voltages.

Example 6: Write the nodal equations for the network in Figure below.



Solution:

Step 1: Redraw the figure with assigned subscripted voltages in Figure below.



Steps 2 to 4:

$$V_1:\ \underbrace{\left(\frac{1}{6\,\Omega}+\frac{1}{3\,\Omega}\right)}_{\substack{\text{Sum of}\\ \text{conductances}\\ \text{connected}\\ \text{to node 1}}}V_1-\underbrace{\left(\frac{1}{3\,\Omega}\right)}_{\substack{\text{Mutual}\\ \text{conductance}}}V_2=\overset{\substack{\text{Drawing current}\\ \text{from node 1}\\ \downarrow}}{-2\text{ A}}$$

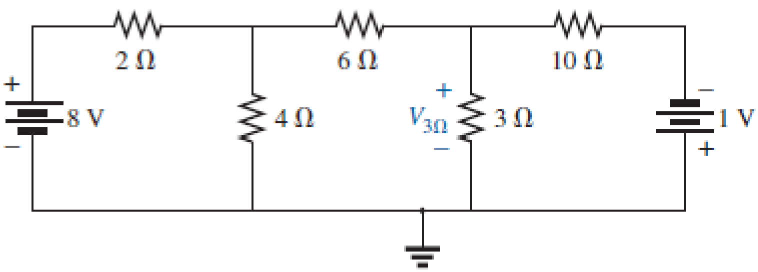
$$V_2:\ \underbrace{\left(\frac{1}{4\,\Omega}+\frac{1}{3\,\Omega}\right)}_{\substack{\text{Sum of}\\ \text{conductances}\\ \text{connected}\\ \text{to node 2}}}V_2-\underbrace{\left(\frac{1}{3\,\Omega}\right)}_{\substack{\text{Mutual}\\ \text{conductance}}}V_1=\overset{\substack{\text{Supplying current}\\ \text{to node 2}\\ \downarrow}}{+3\text{ A}}$$

and

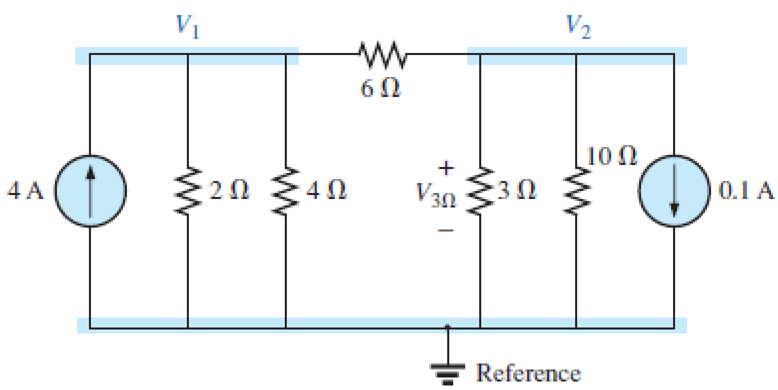
$$\frac{1}{2}V_1-\frac{1}{3}V_2=-2$$

$$-\frac{1}{3}V_1+\frac{7}{12}V_2=3$$

Example 7: Find the voltage across the 3 Ω resistor in Figure below by nodal analysis.



Solution: Converting sources and choosing nodes (Figure below), we have:



$$\left(\frac{1}{2\,\Omega}+\frac{1}{4\,\Omega}\right)V_1-\left(\frac{1}{6\,\Omega}\right)V_2=+4\,A$$

$$\left(\frac{1}{10\ \Omega}+\frac{1}{3\ \Omega}+\frac{1}{6\ \Omega}\right)V_2-\left(\frac{1}{6\ \Omega}\right)V_1=-0.1\ A$$

$$\frac{11}{12}V_1-\frac{1}{6}V_2=4$$

$$\frac{-1}{6}V_1+\frac{3}{5}V_2=-0.1$$

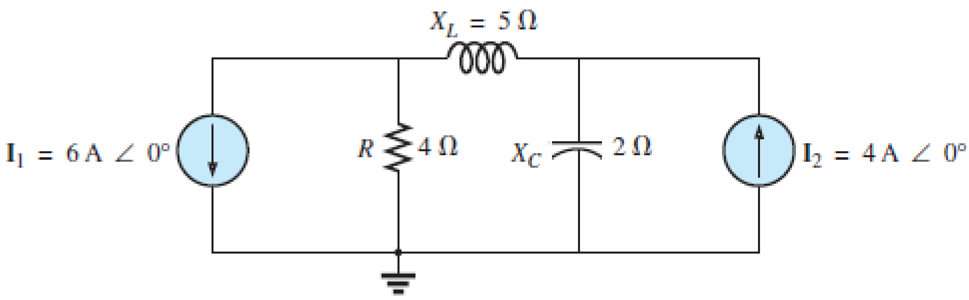
resulting in:

$$11\,V_1-2V_2=48$$

$$-5\,V_1+18\,V_2=-3$$

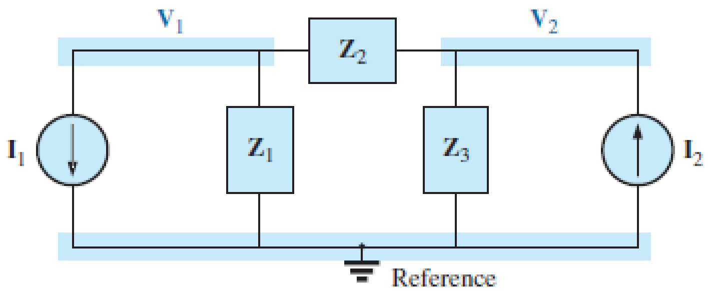
$$V_2=V_{3\,\Omega}=\frac{\begin{vmatrix}11 & 48\\ -5 & -3\end{vmatrix}}{\begin{vmatrix}11 & -2\\ -5 & 18\end{vmatrix}}=\frac{-33+240}{198-10}=\frac{207}{188}=\mathbf{1.10\ V}$$

Example 8: Using the format approach to nodal analysis, find the voltage across the 4 Ω resistor in Figure below.



Solution: Choosing nodes (Figure below) and writing the nodal equations, we have:

$$Z_1=R=4\ \Omega\qquad Z_2=jX_L=j5\ \Omega\qquad Z_3=-jX_C=-j2\ \Omega$$



$$V_1\left(\frac{1}{Z_1}+\frac{1}{Z_2}\right)-V_2\left(\frac{1}{Z_2}\right)=-I_1$$

$$V_2\left(\frac{1}{Z_3}+\frac{1}{Z_2}\right)-V_1\left(\frac{1}{Z_2}\right)=+I_2$$

$$V_1\left(\frac{1}{Z_1}+\frac{1}{Z_2}\right)-V_2\left(\frac{1}{Z_2}\right)\qquad=-I_1$$

$$-V_1\left(\frac{1}{Z_2}\right)\qquad+V_2\left(\frac{1}{Z_3}+\frac{1}{Z_2}\right)=+I_2$$

Using determinants yields:

$$V_1=\frac{\begin{vmatrix}-I_1 & -\frac{1}{Z_2}\\ +I_2 & \frac{1}{Z_3}+\frac{1}{Z_2}\end{vmatrix}}{\begin{vmatrix}\frac{1}{Z_1}+\frac{1}{Z_2} & -\frac{1}{Z_2}\\ -\frac{1}{Z_2} & \frac{1}{Z_3}+\frac{1}{Z_2}\end{vmatrix}}=\frac{-\left(\frac{1}{Z_3}+\frac{1}{Z_2}\right)I_1+\frac{1}{Z_2}\,I_2}{\left(\frac{1}{Z_1}+\frac{1}{Z_2}\right)\left(\frac{1}{Z_3}+\frac{1}{Z_2}\right)-\left(\frac{1}{Z_2}\right)^2}$$

$$V_1=\frac{-\left(\frac{1}{Z_3}+\frac{1}{Z_2}\right)I_1+\frac{1}{Z_2}\,I_2}{\frac{1}{Z_1}\frac{1}{Z_3}+\frac{1}{Z_2}\frac{1}{Z_3}+\frac{1}{Z_1}\frac{1}{Z_2}}$$

Substituting numerical values, we have:

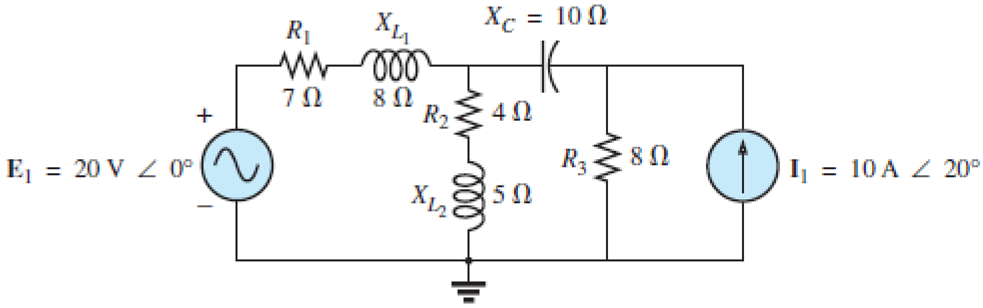
$$V_1=\frac{-\left(\frac{1}{-j2\ \Omega}+\frac{1}{j5\ \Omega}\right)6\ A\ \angle 0^\circ+4\ A\ \angle 0^\circ\left(\frac{1}{j5\ \Omega}\right)}{\left(\frac{1}{4\ \Omega}\right)\left(\frac{1}{-j2\ \Omega}\right)+\left(\frac{1}{j5\ \Omega}\right)\left(\frac{1}{-j2\ \Omega}\right)+\left(\frac{1}{4\ \Omega}\right)\left(\frac{1}{j5\ \Omega}\right)}$$

$$V_1=\frac{-(+j\,0.5\ -\ j\,0.2)6\ \angle 0^\circ\ +\ 4\ \angle 0^\circ(-j\,0.2)}{(1/-j\,8)\ +\ (1/10)\ +\ (1/j\,20)}$$

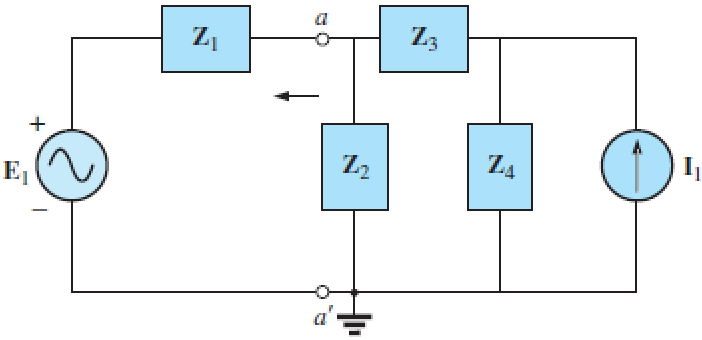
$$V_1=\frac{(-0.3\ \angle 90^\circ)(6\ \angle 0^\circ)\ +\ (4\ \angle 0^\circ)(0.2\ \angle-90^\circ)}{j\,0.125+0.1-j\,0.05}=\frac{-1.8\ \angle 90^\circ\ +\ 0.8\ \angle-90^\circ}{0.1\ +\ j\,0.075}$$

$$V_1=\frac{2.6\ V\ \angle-90^\circ}{0.125\ \angle 36.87^\circ}=\mathbf{20.80\ V}\ j\angle\mathbf{-126.87^o}$$

Example 9: Using the format approach, write the nodal equations for the network in Figure below.



Solution: The circuit is redrawn in Figure below, where:



$Z_1 = R_1 + jX_{L_1} = 7\ \Omega + j\ 8\ \Omega \qquad \boldsymbol{E_1} = 20\ V\angle 0^o$

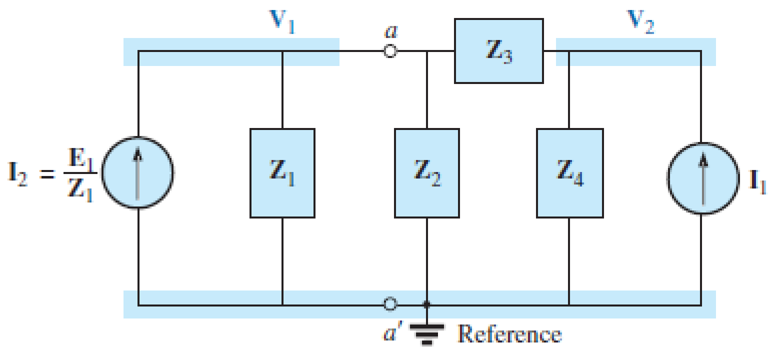
$Z_2 = R_2 + jX_{L_2} = 4\ \Omega + j\ 5\ \Omega \qquad \boldsymbol{E_2} = 10\ V\angle 20^o$

$Z_3 = -jX_C = -j\ 10\ \Omega$

$Z_4 = R_3 = 8\ \Omega$

Converting the voltage source to a current source and choosing nodes, we obtain Figure below. Note the "neat" appearance of the network using the subscripted impedances. Working directly with the original Figure would be more difficult and could produce errors.

Write the nodal equations:



$$V_1\left(\frac{1}{Z_1}+\frac{1}{Z_2}+\frac{1}{Z_3}\right)-V_2\left(\frac{1}{Z_3}\right)=+I_2$$

$$V_2\left(\frac{1}{Z_3}+\frac{1}{Z_4}\right)-V_1\left(\frac{1}{Z_3}\right)=+I_1$$

which are rewritten as:

$$\begin{aligned} V_1\left(\frac{1}{Z_1}+\frac{1}{Z_2}+\frac{1}{Z_3}\right)-V_2\left(\frac{1}{Z_3}\right) &= +I_2 \\ -V_1\left(\frac{1}{Z_3}\right)+V_2\left(\frac{1}{Z_3}+\frac{1}{Z_4}\right) &= +I_1 \end{aligned}$$

Lecture 9: AC and DC Network Theorems

Network theorems can be used to solve networks and provide an opportunity to determine the impact of a particular source or element on the response of the entire system.

9.1 Superposition Theorem:

9.1.1 DC Networks:

The superposition theorem states the following: ***The current through, or voltage across, any element of a network is equal to the algebraic sum of the currents or voltages produced independently by each source.***

In other words, this theorem allows us to find a solution for a current or voltage using only one source at a time.

If we are to consider the effects of each source, the other sources obviously must be removed.

When removing a voltage source from a network schematic, replace it with a direct connection (short circuit) of zero ohms. Any internal resistance associated with the source must remain in the network. While, when removing a current source from a network schematic, replace it by an open circuit of infinite ohms. Any internal resistance associated with the source must remain in the network.

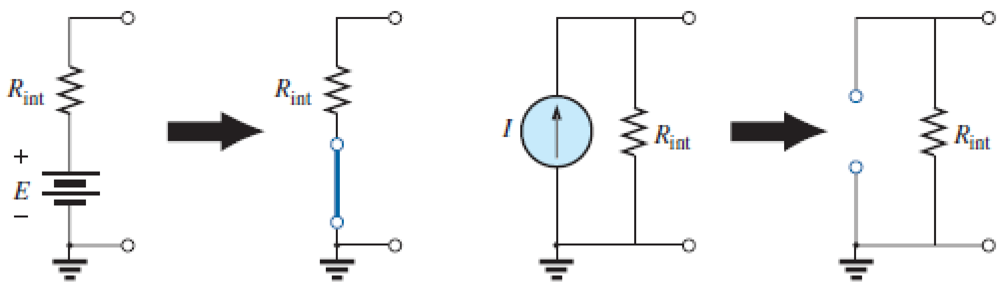


Figure (9.1): Removing a voltage source and a current source to permit the application of the superposition theorem.

Since the effect of each source will be determined independently, the number of networks to be analyzed will equal the number of sources.

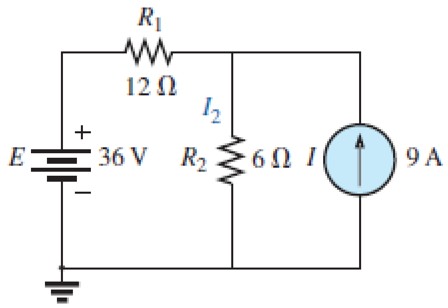
Superposition cannot be applied to power effects because the power is related to the square of the voltage across a resistor or the current through a resistor. The squared term results in a nonlinear (a curve, not a straight line) relationship between the power and the determining current or voltage. For example, doubling the current through a resistor does not double the power to the resistor (as defined by a linear relationship) but, in fact, increases it by a factor of 4 (due to the squared term). Tripling the current increases the power level by a factor of 9.

The total power delivered to a resistive element must be determined using the total current through or the total voltage across the element and cannot be determined by a simple sum of the power levels established by each source.

Example 1:

A) Using the superposition theorem, determine the current through resistor R_2 for the network in Figure below.

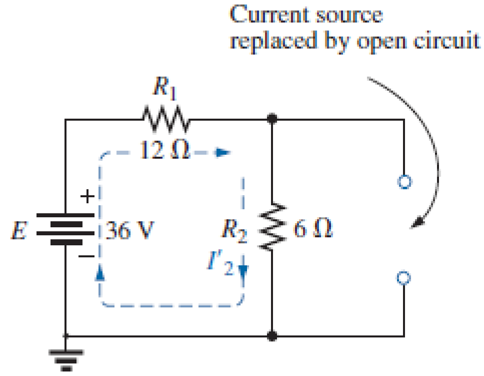
B) Demonstrate that the superposition theorem is not applicable to power levels.



Solution:

A) In order to determine the effect of the 36 V voltage source, the current source must be replaced by an open-circuit equivalent as shown in Figure below. The result is a simple series circuit with a current equal to:

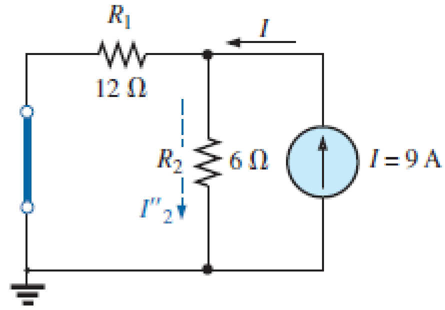
$$I_2' = \frac{E}{R_T} = \frac{E}{R_1 + R_2} = \frac{36\ V}{12\ \Omega + 6\ \Omega} = \frac{36\ V}{18\ \Omega} = 2\ A$$



Examining the effect of the 9 A current source requires replacing the 36 V voltage source by a short-circuit equivalent as shown in Figure below. The result is a parallel combination of resistors R_1 and R_2.

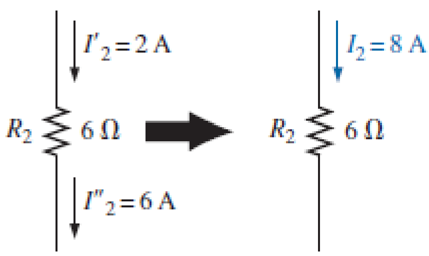
Applying the current divider rule results in:

$$I_2'' = \frac{R_1(I)}{R_1 + R_2} = \frac{(12\ \Omega)(9\ A)}{12\ \Omega + 6\ \Omega} = 6\ A$$



Since the contribution to current I_2 has the same direction for each source, as shown in Figure below, the total solution for current I_2 is the sum of the currents established by the two sources. That is:

$$I_2 = I_2' + I_2'' = 2\ A + 6\ A = \mathbf{8\ A}$$



B) Using the open-circuit equivalent of Figure above and the results obtained, we find the power delivered to the 6 Ω resistor:

$$P_1 = (I_2')^2(R_2) = (2\ A)^2(6\ \Omega) = \mathbf{24\ W}$$

Using the short-circuit equivalent of Figure above and the results obtained, we find the power delivered to the 6 Ω resistor:

$$P_2 = (I_2'')^2(R_2) = (6\ A)^2(6\ \Omega) = \mathbf{216\ W}$$

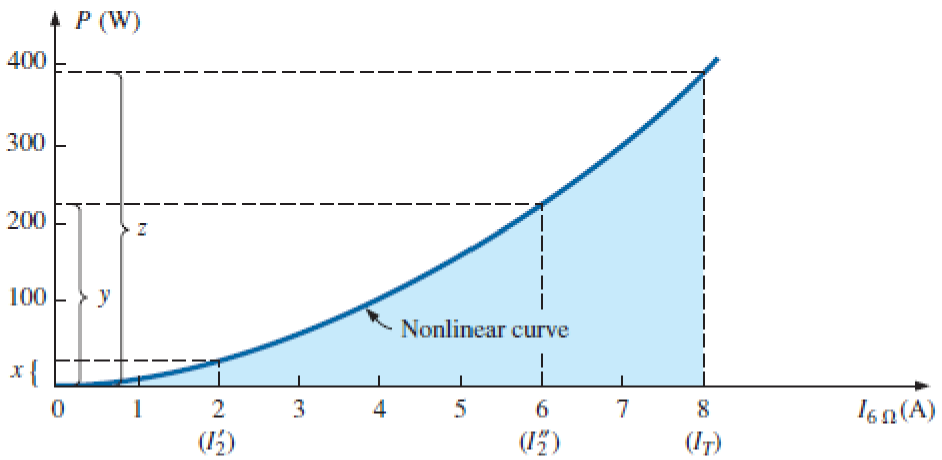
Using the total results of Figure above, we obtain the power delivered to the 6 Ω resistor:

$$P_T = I_T^2\, R_2 = (8\ A)^2(6\ \Omega) = \mathbf{384\ W}$$

It is now quite clear that the power delivered to the 6 Ω resistor using the total current of 8 A is not equal to the sum of the power levels due to each source independently. That is:

$$P_1 + P_2 = 24\ W + 216\ W = 240\ W \neq P_T = 384\ W$$

To expand on the above conclusion and further demonstrate what is meant by a *nonlinear relationship*, the power to the 6 Ω resistor versus current through the 6 Ω resistor is plotted in Figure below. Note that the curve is not a straight line but one whose rise gets steeper with increase in current level.

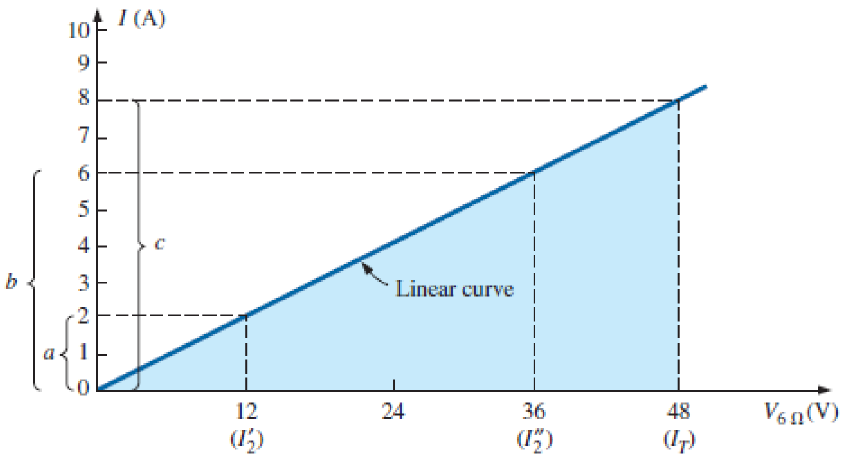


Recall from the original Figure above that the power level was 24 W for a current of 2 A developed by the 36 V voltage source, shown in Figure above. From the short-circuit equivalent of Figure above, we found that the current level was 6 A for a power level of 216 W, shown in Figure above. Using the total current of 8 A, we find that the power level in 384 W, shown in Figure above. Quite clearly, the sum of power levels due to the 2 A and 6 A current levels does not equal that due to the 8 A level. That is:

$$x + y \neq z$$

Now, the relationship between the voltage across a resistor and the current through a resistor is a linear (straight line) one, as shown in Figure below, with:

$$c = a + b$$



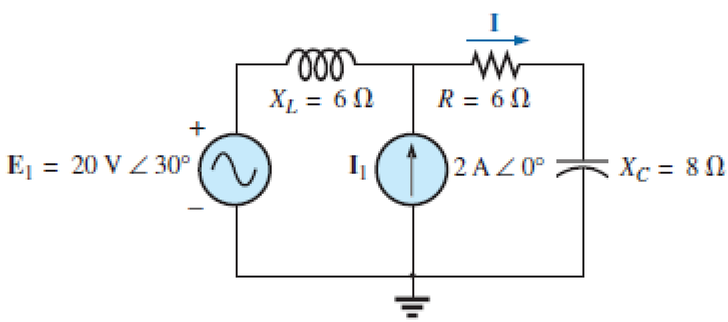
9.1.2 AC Networks:

The only variation in applying this method to ac networks with independent sources is that we are now working with impedances and phasors instead of just resistors and real numbers.

For a network with a dc source and ac source, the total power can be determined by the sum of the powers delivered by each source.

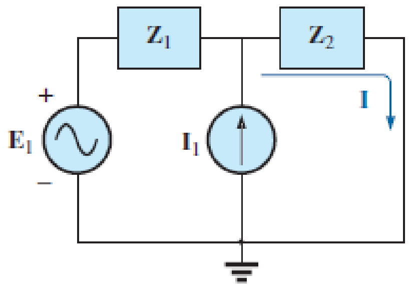
If the sources had different frequencies, the impedances of the elements would change with each applied frequency.

Example 2: Using superposition, find the current **I** through the 6 Ω resistor in Figure below.



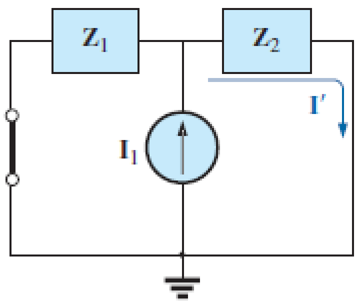
Solution: For the redrawn circuit (Figure below),

$$Z_1 = 6\ \Omega \qquad Z_2 = 6\ \Omega - j8\ \Omega$$



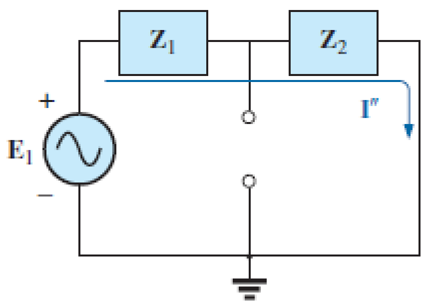
Consider the effects of the current source (Figure below). Applying the current divider rule, we have:

$$I' = \frac{Z_1\, I_1}{Z_1 + Z_2} = \frac{(6\ \Omega)(2\ A)}{6\ \Omega + 6\ \Omega - j8\ \Omega} = \frac{j\ 12\ A}{6\ -\ j\ 2} = \frac{12\ A\ \angle 90°}{6.32\ \angle -18.43°} = 1.9\ A\ \angle 108.43°$$



Consider the effects of the voltage source (Figure below). Applying Ohm's law gives us:

$$I'' = \frac{E_1}{Z_T} = \frac{E_1}{Z_1 + Z_2} == \frac{20\ V\ \angle 30°}{6.32\ \angle -18.43°} = 3.16\ A\ \angle 48.43°$$

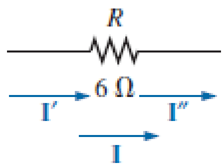


The total current through the 6 Ω resistor (Figure below) is:

$$I = I' + I'' = 1.9\ A\ \angle 108.43°\ +\ 3.16\ A\ \angle 48.43°$$

$$I = (-0.60\ A\ +\ j\ 1.80\ A) + (2.10\ A\ +\ j\ 2.36\ A) = 1.50\ A\ +\ j\ 4.16A$$

$$I = \mathbf{4.42\ A\ \angle 70.2^o}$$



9.2 Independent DC and AC Sources:

The analysis with each source will be performed independently and the total result for the voltage or current will be the sum of the two sources. The total power can be determined by the sum of the power delivered by each source, as demonstrated below:

The effective value of the resulting voltage is determined by the following equation, as introduced in equation $V_{rms} = \sqrt{V_{dc}^2 + V_{ac(rms)}^2}$:

$$V_{eff} = \sqrt{V_{dc}^2 + V_{rms}^2}$$

The power to the load is then:

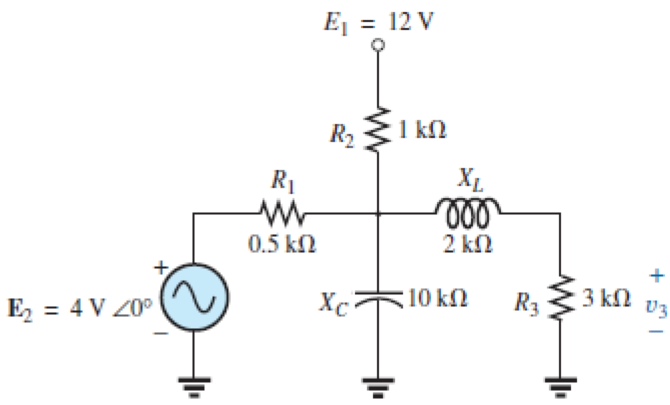
$$P = \frac{V_{eff}^2}{R} = \frac{\left(\sqrt{V_{dc}^2 + V_{rms}^2}\right)^2}{R} = \frac{V_{dc}^2}{R} + \frac{V_{rms}^2}{R} = P_{dc} + P_{ac}$$

which breaks down to the sum of the dc and ac power distributions.

Example 3: For the network of Figure below:

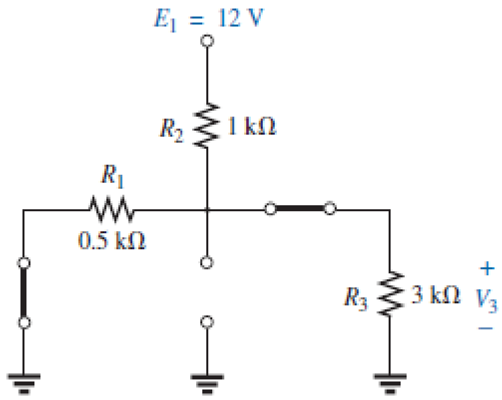
A) Determine the sinusoidal expression for the voltage v_3.

B) Calculate the power delivered to R_3.



Solution:

A) For the dc analysis, the capacitor can be replaced by an open-circuit equivalent and the inductor by a short-circuit equivalent. The result is the network in Figure below.



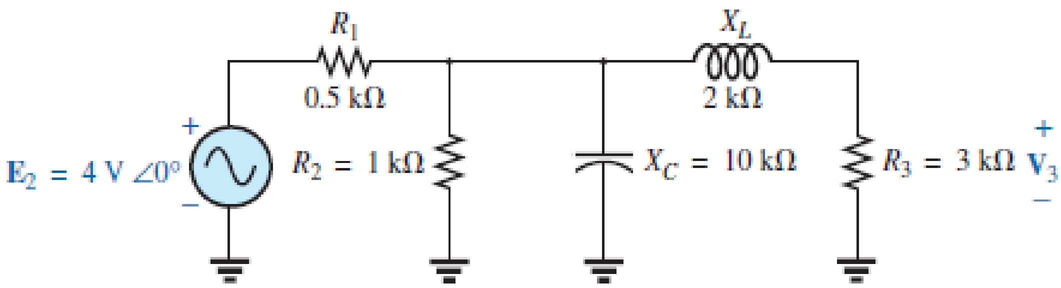
The resistors R_1 and R_3 are then in parallel, and the voltage V_3 can be determined using the voltage divider rule:

$$R' = R_1 \parallel R_3 = 0.5\ K\Omega \parallel 3\ K\Omega = 0.429\ K\Omega$$

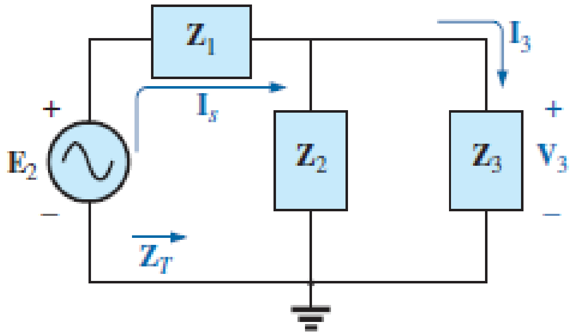
and

$$V_3 = \frac{R'\ E_1}{R' + R_2} = \frac{(0.429\ K\Omega)(12\ V)}{0.429\ K\Omega + \ 1\ K\Omega} = \frac{5.148\ V}{1.429} \cong \mathbf{3.6\ V}$$

For the ac analysis, the dc source is set to zero and the network is redrawn, as shown in Figure below.



The block impedances are then defined as in Figure below, and series-parallel techniques are applied as follows:



$$Z_1 = 0.5\ K\Omega\ \angle 0°$$

$$Z_2 = (R_2\ \angle 0° \parallel (X_C \angle -90°) = \frac{(1\ K\Omega\ \angle 0°)(10\ K\Omega \angle -90°)}{1\ K\Omega - j\ 10\ K\Omega} = \frac{10\ K\Omega \angle -90°}{10.05 \angle -84.29°}$$

$$Z_2 = 0.995\ K\Omega \angle -5.71°$$

$$Z_3 = R_3 + jX_L = 3\ k\ +\ j\ 2\ K\Omega = \ 3.61\ K\Omega\ \angle 33.69°$$

and

$$Z_T = Z_1 + Z_2 \parallel Z_3 = 0.5\ K\Omega\ + (0.995\ K\Omega \angle -5.71°) \parallel (3.61\ K\Omega\ \angle 33.69°)$$

$$Z_T = \ 1.312\ K\Omega\ \angle 1.57°$$

9.3 Thévenin's Theorem:

9.3.1 DC Networks:

Thévenin's theorem states the following:

Any two-terminal dc network can be replaced by an equivalent circuit consisting solely of a voltage source and a series resistor as shown in Figure (9.2).

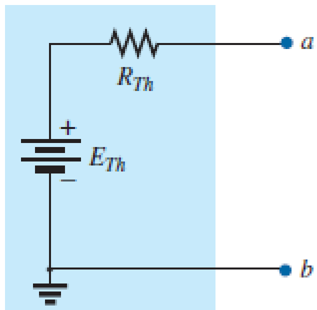


Figure (9.2): Thévenin equivalent circuit.

9.3.2 AC Networks:

Thévenin's theorem, as stated for sinusoidal ac circuits, is changed only to include the term impedance instead of resistance; that is, any two-terminal linear ac network can be replaced with an equivalent circuit consisting of a voltage source and an impedance in series, as shown in Figure (9.3).

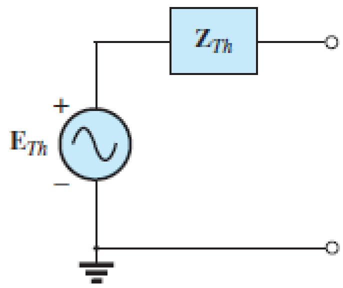


Figure (9.3): Thévenin equivalent circuit for ac networks.

9.3.3 Thévenin's Theorem Procedure:

Preliminary:

1) Remove that portion of the network where the Thévenin equivalent circuit is found.
2) Mark (○, •, and so on) the terminals of the remaining two-terminal network.

R_{Th}/Z_{Th}:

3) For DC network, calculate R_{Th} by first setting all sources to zero (voltage sources are replaced by short circuits and current sources by open circuits) and then finding the resultant resistance between the two marked terminals. For AC network, calculate Z_{Th} by first setting all voltage and current sources to zero (short circuit and open circuit, respectively) and then finding the resulting impedance between the two marked terminals.

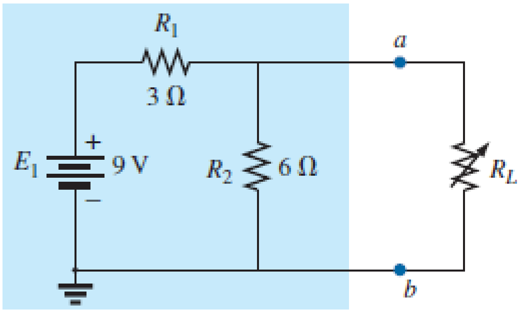
E_{Th}:

4) Calculate E_{Th} by first returning all sources to their original position and finding the open-circuit voltage between the marked terminals.

Conclusion:

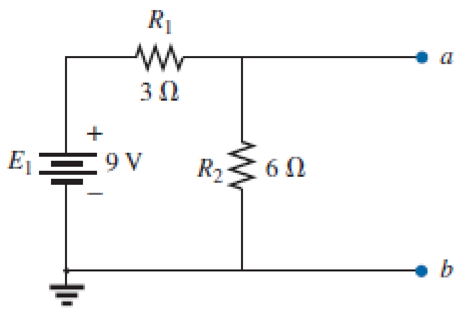
5) Draw the Thévenin equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the equivalent circuit.

Example 4: Find the Thévenin equivalent circuit for the network in the shaded area of the network in Figure below. Then find the current through R_L for values of 2 Ω, 10 Ω, and 100 Ω.



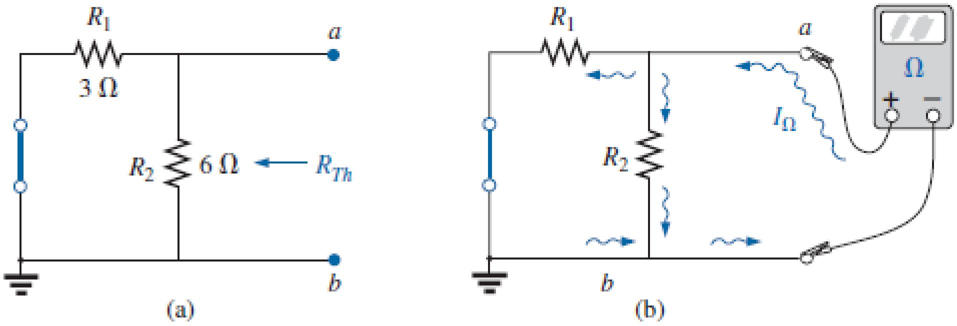
Solution:

Steps 1 and 2: These produce the network in Figure below. Note that the load resistor R_L has been removed and the two "holding" terminals have been defined as *a* and *b*.



Step 3: Replacing the voltage source E_1 with a short-circuit equivalent yields the network in Figure below (a), where:

$$R_{Th} = R_1 \parallel R_2 = \frac{(3\ \Omega)(6\ \Omega)}{3\ \Omega + 6\ \Omega} = \mathbf{2}\ \Omega$$



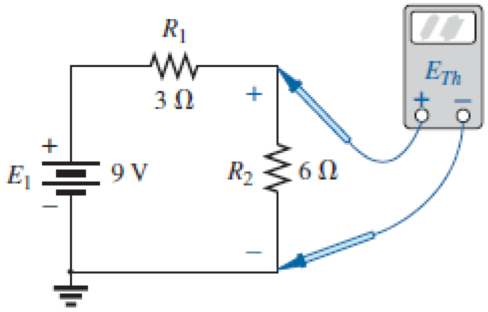
The importance of the two marked terminals now begins to surface. They are the two terminals across which the Thévenin resistance is measured. It is no longer the total resistance as seen by the source . If some difficulty develops when determining R_{Th} with regard to whether the resistive elements are in series or parallel, consider recalling that the ohmmeter sends out a trickle current into a resistive combination and senses the level of the resulting voltage to establish the measured resistance level. In Figure above (b), the trickle current of the ohmmeter approaches the network through terminal *a*, and when it reaches the junction of R_1 and R_2, it splits as shown. The fact that the trickle current splits and then recombines at the lower node reveals that the resistors are in parallel as far as the ohmmeter reading is concerned. In essence, the path of the sensing current of the ohmmeter has revealed how the resistors are connected to the two terminals of interest and how the Thévenin resistance should be determined. Remember this as you work through the various examples in this section.

Step 4: Replace the voltage source (Figure below). For this case, the open-circuit voltage E_{Th} is the same as the voltage drop across the 6 Ω resistor.

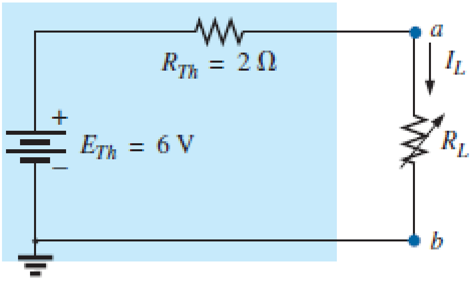
Applying the voltage divider rule gives:

$$E_{Th} = \frac{R_2\,E_1}{R_2 + R_1} = \frac{(6\,\Omega)(9\text{ V})}{6\,\Omega + 3\,\Omega} = \frac{54\,V}{9} = \boldsymbol{6\,V}$$

It is particularly important to recognize that E_{Th} is the open-circuit potential between points *a* and *b*. Remember that an open circuit can have any voltage across it, but the current must be zero. In fact, the current through any element in series with the open circuit must be zero also. The use of a voltmeter to measure E_{Th} appears in Figure below. Note that it is placed directly across the resistor R_2 since E_{Th} and V_{R_2} are in parallel.



Step 5: (Figure below):



$$I_L = \frac{E_{Th}}{R_{Th} + R_L}$$

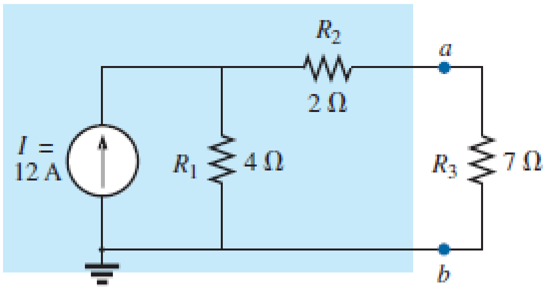
$$\boldsymbol{R_L = 2\ \Omega:}\quad I_L = \frac{6\ V}{2\ \Omega + 2\ \Omega} = \mathbf{1.5\ A}$$

$$\boldsymbol{R_L = 10\ \Omega:}\quad I_L = \frac{6\ V}{2\ \Omega + 10\ \Omega} = \mathbf{0.5\ A}$$

$$\boldsymbol{R_L = 100\ \Omega:}\quad I_L = \frac{6\ V}{2\ \Omega + 100\ \Omega} = \mathbf{0.06\ A}$$

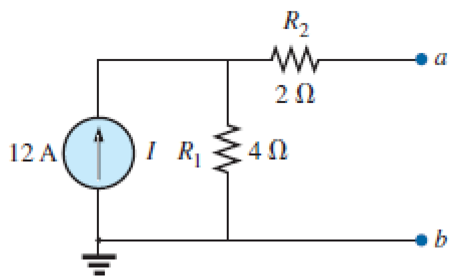
If Thévenin's theorem were unavailable, each change in R_L would require that the entire network in the original Figure be reexamined to find the new value of R_L.

Example 5: Find the Thévenin equivalent circuit for the network in the shaded area of the network in Figure below.

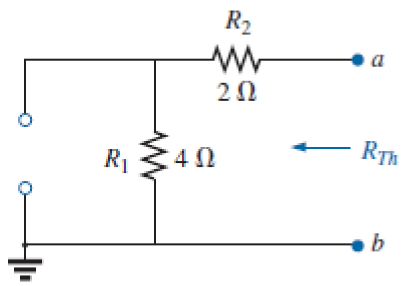


Solution:

Steps 1 and 2: See Figure below.



Step 3: See Figure below. The current source has been replaced with an open-circuit equivalent and the resistance determined between terminals *a* and *b*.



In this case, an ohmmeter connected between terminals *a* and *b* sends out a sensing current that flows directly through R_1 and R_2 (at the same level). The result is that R_1 and R_2 are in series and the Thévenin resistance is the sum of the two:

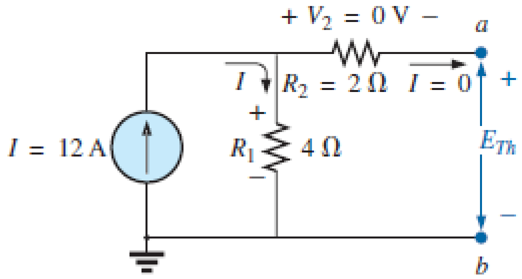
$$R_{Th} = R_1 + R_2 = 4\ \Omega + 2\ \Omega = \mathbf{6}\ \Omega$$

Step 4: See Figure below. In this case, since an open circuit exists between the two marked terminals, the current is zero between these terminals and through the 2 Ω resistor. The voltage drop across R_2 is, therefore:

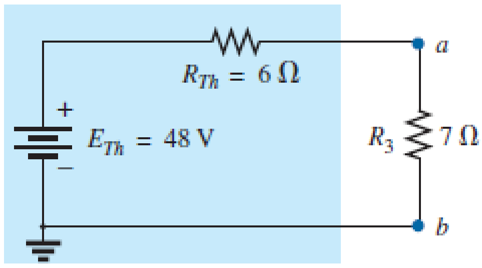
$$V_2 = I_2\,R_2 = (0)R_2 = 0V$$

and

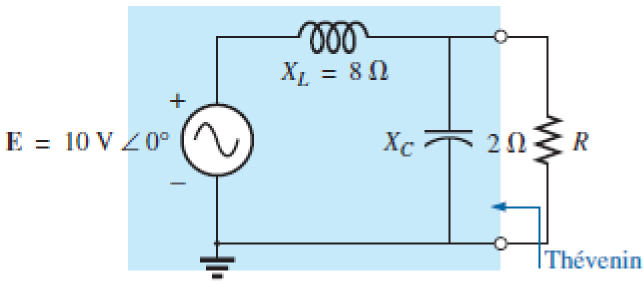
$$E_{Th} = V_1 = I_1\,R_1 = (12\ A)(4\ \Omega) = \mathbf{48\ V}$$



Step 5: See Figure below.



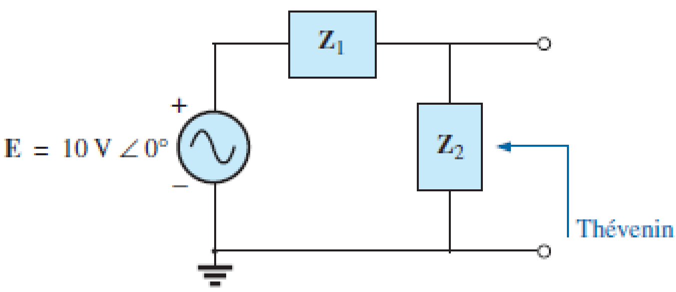
Example 6: Find the Thévenin equivalent circuit for the network external to resistor *R* in Figure below.



Solution:

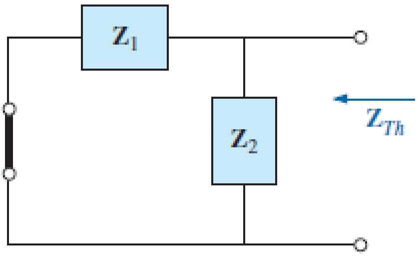
Steps 1 and 2 (Figure below):

$$Z_1 = jX_L = j8\ \Omega \qquad Z_2 = -jX_C = -j2\ \Omega$$



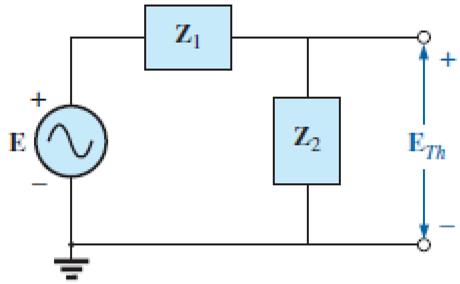
Step 3 (Figure below):

$$Z_{Th} = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{(j8\ \Omega)(-j2\ \Omega)}{j8\ \Omega - j2\ \Omega} = \frac{-j^2 16\ \Omega}{j6} = \frac{16\ \Omega}{6\angle 90^o} = \mathbf{2.67\ \Omega\ \angle{-90^o}}$$

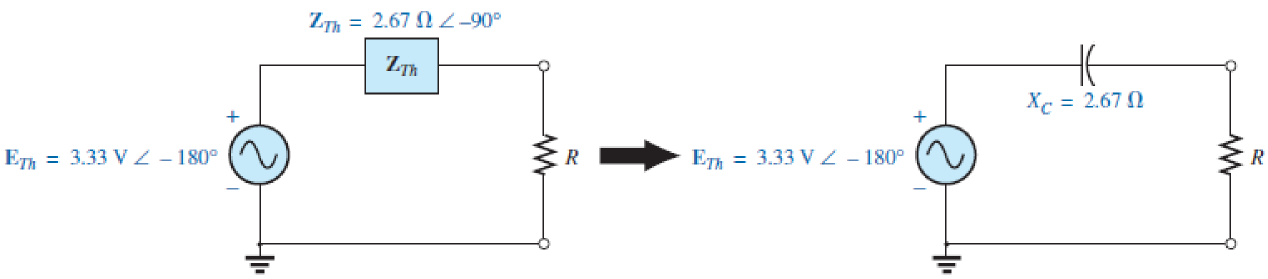


Step 4 (Figure below):

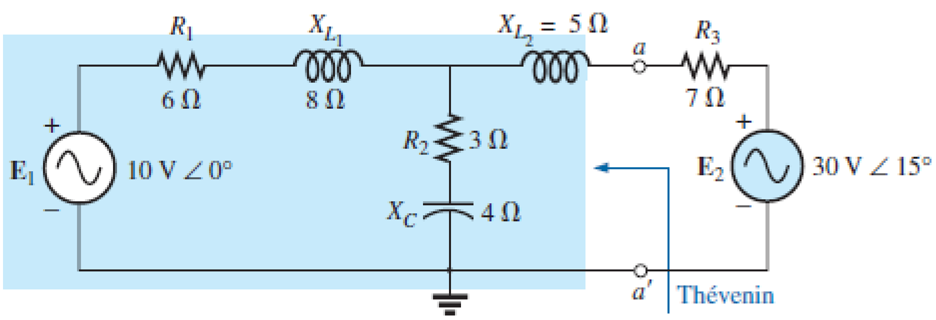
$$E_{Th} = \frac{Z_2\, E}{Z_1 + Z_2} = \frac{(-j2\,\Omega)(10\,V)}{j8\,\Omega - j2\,\Omega} = \frac{-j20\,V}{j6} = \mathbf{3.33\,V\,\angle{-180^o}}$$



Step 5: The Thévenin equivalent circuit is shown in Figure below.



Example 7: Find the Thévenin equivalent circuit for the network external to branch $a - a'$ in Figure below.



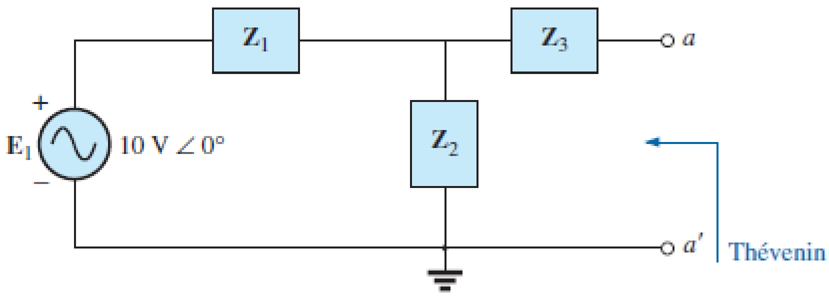
Solution:

Steps 1 and 2 (Figure below): Note the reduced complexity with subscripted impedances:

$$Z_1 = R_1 + jX_{L_1} = 6\,\Omega + j8\,\Omega$$

$$Z_2 = R_2 - jX_C = 3\,\Omega - j4\,\Omega$$

$$Z_3 = jX_{L_2} = j5\,\Omega$$

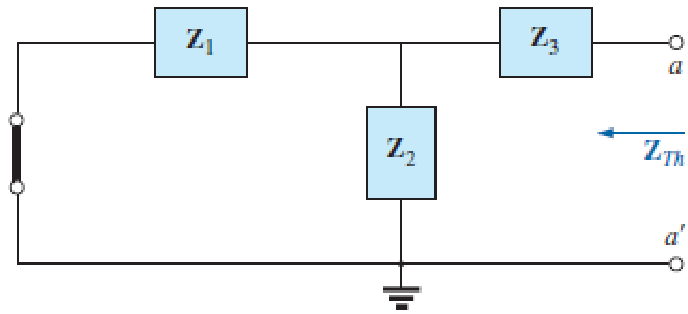


Step 3 (Figure below):

$$Z_{Th} = Z_3 + \frac{Z_1 Z_2}{Z_1 + Z_2} = j5\,\Omega + \frac{(6\,\Omega + j8\,\Omega)(3\,\Omega - j4\,\Omega)}{6\,\Omega + j8\,\Omega + 3\,\Omega - j4\,\Omega}$$

$$Z_{Th} = \frac{(10\,\Omega\,\angle 53.13°)(5\,\Omega\,\angle -53.13°)}{9\ +\ j\,4} = \frac{50\ \ \angle 0°}{9.85\,\angle 23.96°} = j\,5 + 5.08\,\angle -23.96°$$

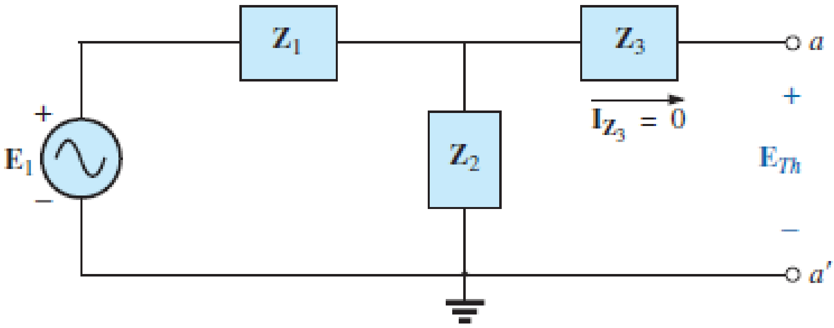
$$Z_{Th} = \ j\,5 + 4.64 - j\,2.06 = \mathbf{4.64}\,\Omega + \boldsymbol{j\,2.94}\,\Omega = \ \mathbf{5.49}\,\Omega\,\angle\mathbf{32.36^{o}}$$



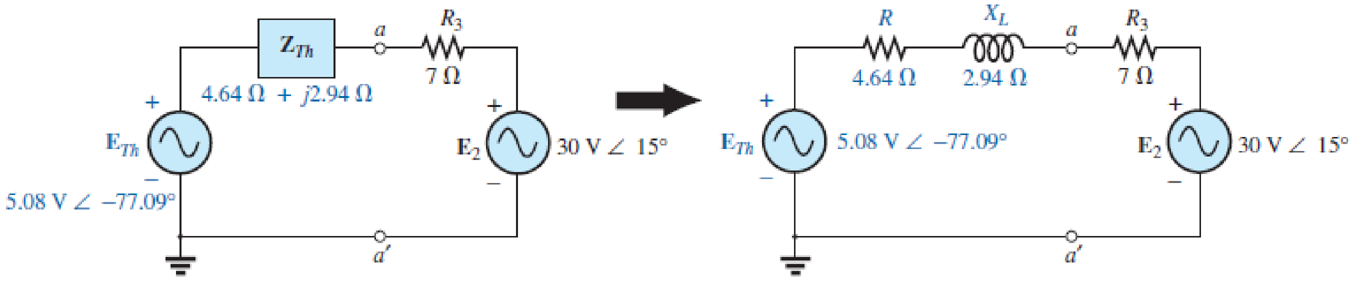
Step 4 (Figure below): Since $a - a'$ is an open circuit, $\boldsymbol{I_{Z_3}} = 0$. Then E_{Th} is the voltage drop across $\boldsymbol{Z_2}$:

$$E_{Th} = \frac{Z_2\,E}{Z_1 + Z_2} = \frac{(5\,\Omega\,\angle -53.13°)(10\,V\,\angle 0°)}{9.85\,\Omega\,\angle 23.96°}$$

$$E_{Th} = \frac{50\,V\,\angle -53.13°}{9.85\,\Omega\,\angle 23.96°} = \mathbf{5.08}\,\boldsymbol{V}\,\angle\mathbf{-77.09^{o}}$$



Step 5: The Thévenin equivalent circuit is shown in Figure below.



9.4 Maximum Power Transfer Theorem:

9.4.1 DC Networks:

When applied to DC circuits, the **maximum power transfer theorem** states that: A load will receive maximum power from a network when its resistance is exactly equal to the Thévenin resistance of the network applied to the load. That is:

$$R_L = R_{Th}$$

In other words, for the Thévenin equivalent circuit in Figure (9.4), when the load is set equal to the Thévenin resistance, the load will receive maximum power from the network.

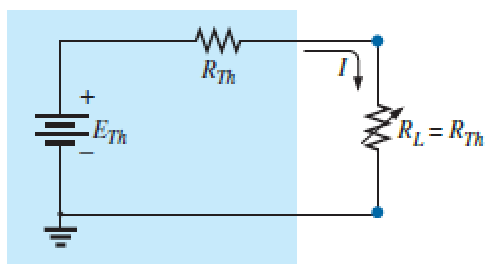


Figure (9.4): Defining the conditions for maximum power to a load using the Thévenin equivalent circuit.

Using Figure (9.5), we can determine the maximum power delivered to the load by first finding the current:

$$I_L = \frac{E_{Th}}{R_{Th} + R_L} = \frac{E_{Th}}{R_{Th} + R_{Th}} = \frac{E_{Th}}{2\,R_{Th}}$$

Then we substitute into the power equation:

$$P_L = I_L^2\,R_L = \left(\frac{E_{Th}}{2\,R_{Th}}\right)^2 (R_{Th}) = \frac{E_{Th}^2\,R_{Th}}{4\,R_{Th}^2}$$

and

$$P_{L_{max}} = \frac{E_{Th}^2}{4\,R_{Th}}$$

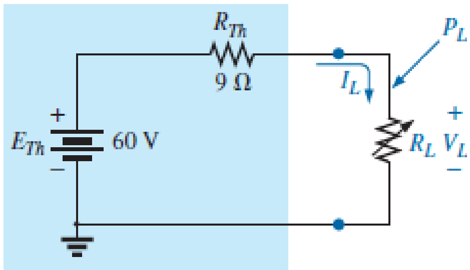


Figure (9.5): Thévenin equivalent network to be used to validate the maximum power transfer theorem.

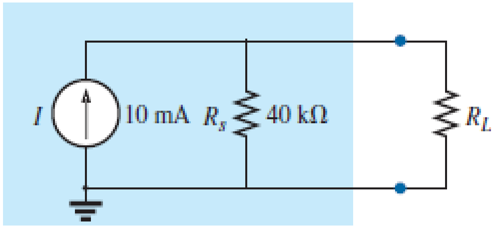
The total power delivered by a supply such as E_{Th} is absorbed by both the Thévenin equivalent resistance and the load resistance. Any power delivered by the source that does not get to the load is lost to the Thévenin resistance.

Example 8: The analysis of a transistor network resulted in the reduced equivalent in Figure below.

A) Find the load resistance that will result in maximum power transfer to the load, and find the maximum power delivered.

B) If the load were changed to 68 KΩ, would you expect a fairly high level of power transfer to the load based on the results of part (A)? What would the new power level be? Is your initial assumption verified?

C) If the load were changed to 8.2 KΩ, would you expect a fairly high level of power transfer to the load based on the results of part (A)? What would the new power level be? Is your initial assumption verified?



Solution:

A) Replacing the current source by an open-circuit equivalent results in:

$$R_{Th} = R_S = 40\ K\Omega$$

Restoring the current source and finding the open-circuit voltage at the output terminals results in:

$$E_{Th} = V_{oc} = I\ R_S = (10\ mA)(40\ K\Omega)\ =\ 400\ V$$

For maximum power transfer to the load,

$$R_L = R_{Th} = \mathbf{40\ K\Omega}$$

with a maximum power level of:

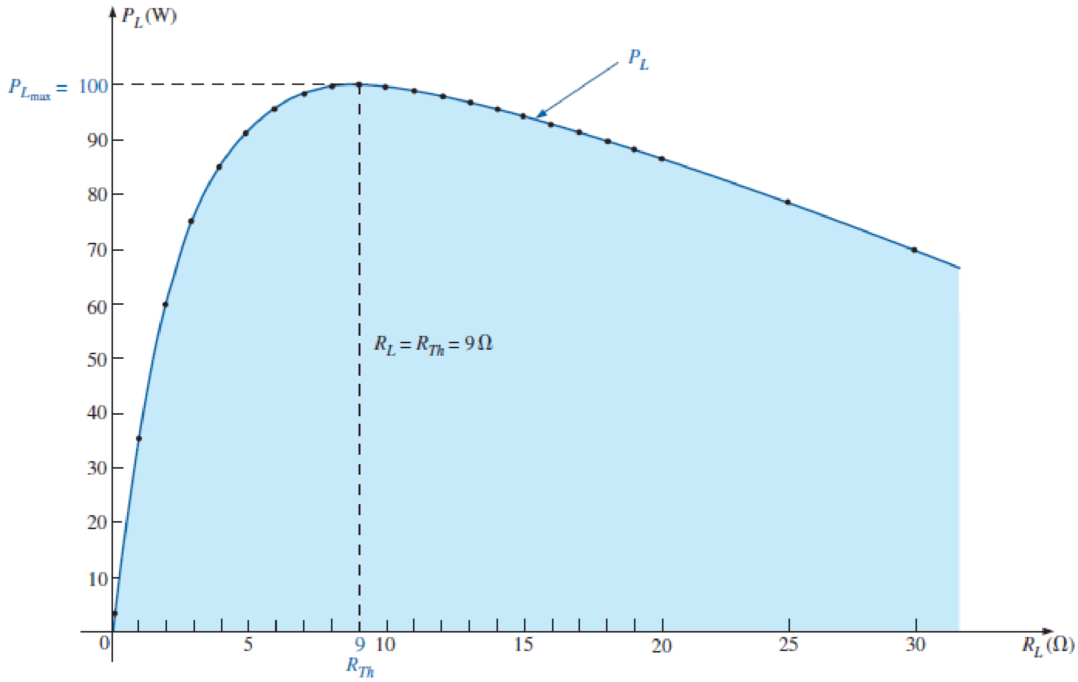
$$P_{L_{max}} = \frac{E_{Th}^2}{4\ R_{Th}} = \frac{(400\ V)^2}{4(40\ K\Omega)} = \mathbf{1\ W}$$

B) Yes, because the 68 $K\Omega$ load is greater (note Figure below) than the 40 $K\Omega$ load, but relatively close in magnitude.

$$I_L = \frac{E_{Th}}{R_{Th} + R_L} = \frac{400\ V}{40\ K\Omega + 68\ K\Omega} = \frac{400\ V}{108\ K\Omega} \cong 3.7\ mA$$

$$P_L = I_L^2\ R_L = (3.7\ mA)^2(68\ K\Omega) \cong \mathbf{0.93\ W}$$

Yes, the power level of 0.93 W compared to the 1 W level of part (A) verifies the assumption.



C) No, 8.2 KΩ is quite a bit less (note Figure above) than the 40 KΩ value.

$$I_L = \frac{E_{Th}}{R_{Th} + R_L} = \frac{400\ V}{40\ K\Omega + 8.2\ K\Omega} = \frac{400\ V}{48.2\ K\Omega} \cong 8.3\ mA$$

$$P_L = I_L^2\ R_L = (8.3\ mA)^2 (8.2\ K\Omega) \cong \mathbf{0.57\ W}$$

Yes, the power level of 0.57 W compared to the 1 W level of part (A) verifies the assumption.

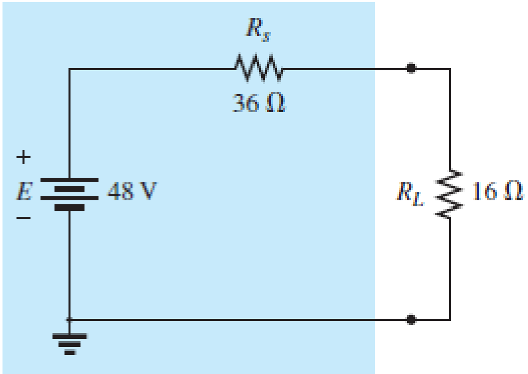
Example 9: In Figure below, a fixed load of 16 Ω is applied to a 48 V supply with an internal resistance of 36 Ω.

A) For the conditions in Figure above, what is the power delivered to the load and lost to the internal resistance of the supply?

B) If the designer has some control over the internal resistance level of the supply, what value should he or she make it for maximum power to the load? What is the maximum power to the load? How does it compare to the level obtained in part (A)?

C) Without making a single calculation, find the value that would result in more power to the load if the designer could change the internal resistance to 22 Ω or

8.2 Ω. Verify your conclusion by calculating the power to the load for each value.



dc supply

Solution:

A)

$$I_L = \frac{E_{Th}}{R_S + R_L} = \frac{48\ V}{36\ \Omega + 16\ \Omega} = \frac{48\ V}{52\ \Omega} = 923.1\ mA$$

$$P_{R_S} = I_L^2\ R_S = (923.1\ mA)^2(36\ \Omega) = \mathbf{30.68\ W}$$

$$P_L = I_L^2\ R_L = (923.1\ mA)^2(16\ \Omega) = \mathbf{13.63\ W}$$

B) Be careful here. The quick response is to make the source resistance R_S equal to the load resistance to satisfy the criteria of the maximum power transfer theorem. However, this is a totally different type of problem from what was examined earlier in this section. If the load is fixed, the smaller the source resistance R_S, the more applied voltage will reach the load and the less will be lost in the internal series resistor. In fact, the source resistance should be made as small as possible. If zero ohms were possible for R_S, the voltage across the load would be the full supply voltage, and the power delivered to the load would equal:

$$P_L = \frac{V_L^2}{R_L} = \frac{(48\ V)^2}{16\ \Omega} = \mathbf{144\ W}$$

which is more than 10 times the value with a source resistance of 36 Ω.

C) Again, forget the impact in Figure below: The smaller the source resistance, the greater is the power to the fixed 16 **Ω** load. Therefore, the 8.2**Ω** resistance level results in a higher power transfer to the load than the 22 **Ω** resistor.

For *Rs* = 8.2 Ω:

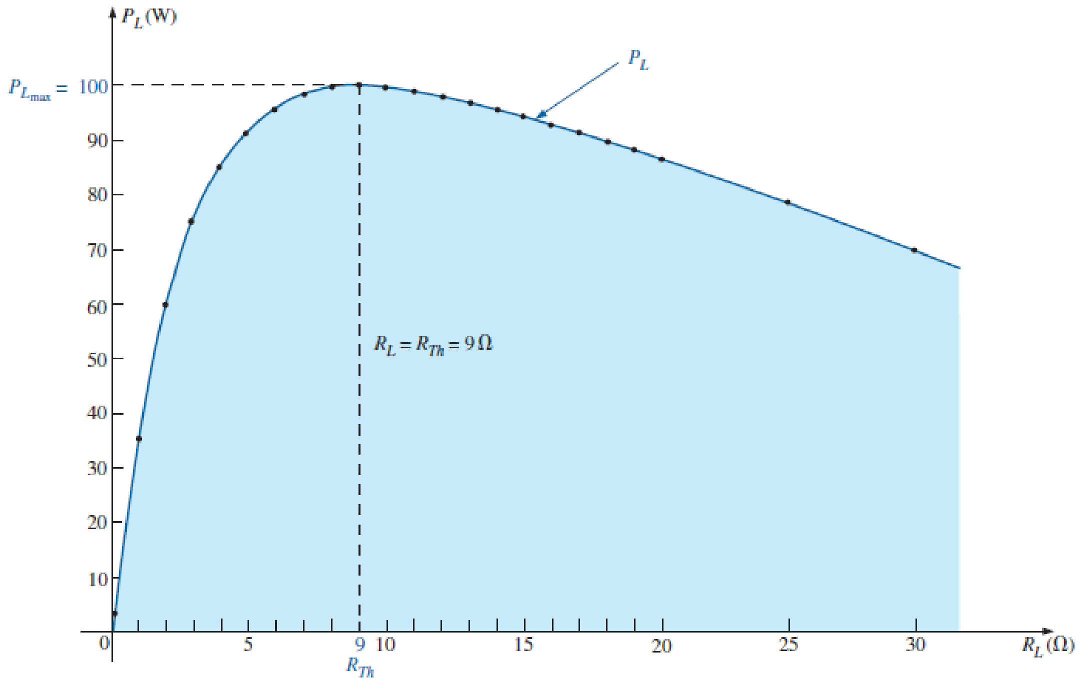
$$I_L = \frac{E_{Th}}{R_S + R_L} = \frac{48\ V}{8.2\ \Omega + 16\ \ \Omega} = \frac{48\ V}{24.2\ \Omega} = 1.983\ A$$

$$P_L = I_L^2\ R_L = (1.983\ A)^2(16\ \Omega) \cong \mathbf{62.92\ W}$$

For *Rs* = 22 Ω:

$$I_L = \frac{E_{Th}}{R_S + R_L} = \frac{48\ V}{22\ \Omega + 16\ \ \Omega} = \frac{48\ V}{28\ \Omega} = 1.263\ A$$

$$P_L = I_L^2\ R_L = (1.263\ A)^2(22\ \Omega) \cong \mathbf{25.52\ W}$$



9.4.1 AC Networks:

When applied to ac circuits, the **maximum power transfer theorem** states that: ***Maximum power will be delivered to a load when the load impedance is the conjugate of the Thévenin impedance across its terminals.***

For maximum power transfer to the load:

$$Z_L = Z_{Th} \qquad \theta_L = -\theta_{Z_{Th}}$$

or, in rectangular form:

$$R_L = R_{Th} \qquad \pm jX_{load} = \mp jX_{Th}$$

The total impedance of the circuit appear purely resistive, as indicated in Figure (9.6):

$$Z_T = (R_{Th} \pm jX_{Th}) + (R_{Th} \mp jX_{Th})$$

and

$$Z_T = 2\,R_{Th}$$

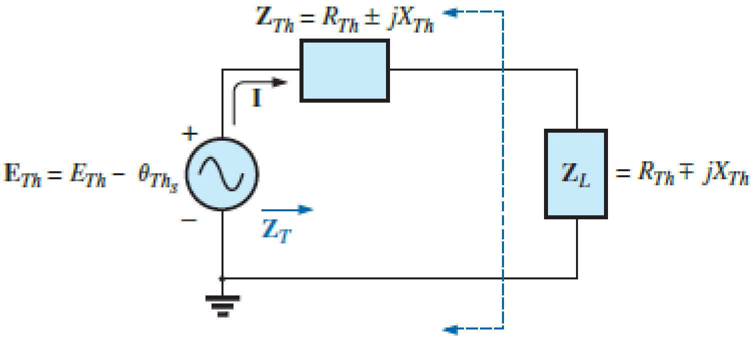


Figure (9.6): Conditions for maximum power transfer to Z_L.

Since the circuit is purely resistive, the power factor of the circuit under maximum power conditions is 1; that is:

$$F_P = 1 \qquad \text{(maximum power transfer)}$$

The magnitude of the current I in Figure (9.6) is:

$$I = \frac{E_{Th}}{Z_T} = \frac{E_{Th}}{2\,R_{Th}}$$

The maximum power to the load is:

$$P_{max} = I^2\,R_{Th} = \left(\frac{E_{Th}}{2\,R_{Th}}\right)^2 R_{Th}$$

and

$$P_{max} = \frac{E_{Th}^2}{4\,R_{Th}}$$

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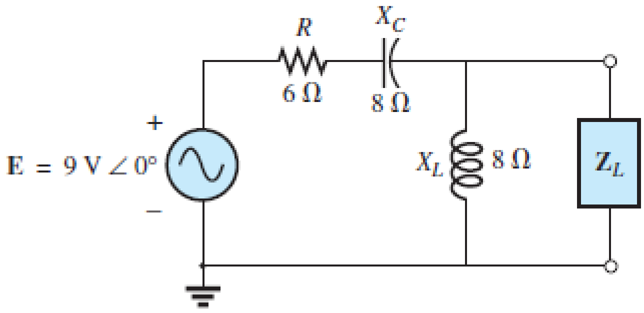
Example 10: Find the load impedance in Figure below for maximum power to the load, and find the maximum power.



Solution: Determine Z_{Th} [Figure below]:

$Z_1 = R - jX_C = 6\,\Omega - j8\,\Omega$

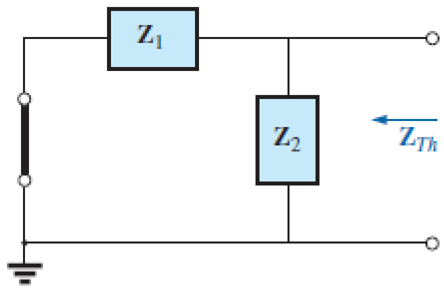
$Z_2 = +jX_L = j8\,\Omega$

$$Z_{Th} = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{(6\,\Omega - j8\,\Omega)(j8\,\Omega)}{6\,\Omega - j8\,\Omega + j8\,\Omega} = \frac{(10\,\Omega \angle -53.13°)(8\,\Omega\ \angle 90°)}{6\,\Omega \angle 0°}$$

$$Z_{Th} = \frac{80\,\Omega \angle 36.87°}{6\angle 0^o} = \mathbf{13.33}\,\Omega \angle \mathbf{36.87}° = \mathbf{10.66}\,\Omega + \boldsymbol{j}\,\mathbf{8}\,\Omega$$

and

$Z_L = \mathbf{13.33}\,\Omega \angle -\mathbf{36.87}° = \mathbf{10.66}\,\Omega - \boldsymbol{j}\,\mathbf{8}\,\Omega$



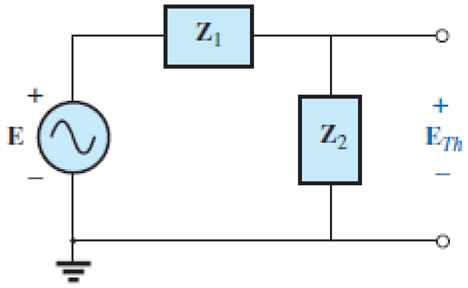
To find the maximum power, we must first find E_{Th} [Figure below], as follows:

$$E_{Th} = \frac{Z_2\,E}{Z_2 + Z_1} \quad \text{(voltage divider rule)}$$

$$E_{Th} = \frac{(8\,\Omega \angle 90°)(9\,V\angle 0°)}{j\,8\,\Omega +\ 6\,\Omega -\ j\,8\,\Omega} = \frac{72\,V \angle 90°}{6\angle 0°} = 12\,V \angle 90°$$

Then:

$$P_{max} = \frac{E_{Th}^2}{4\ R_{Th}} = \frac{(12\ V)^2}{4(0.66\ \Omega)} = \frac{144}{42.64} = \mathbf{3.38\ W}$$



Lecture 10: Transformers

Mutual inductance is a phenomenon basic to the operation of the transformer. **A transformer** is an electrical device used today in almost every field of electrical engineering. This device plays an integral part in power distribution systems and can be found in many electronic circuits and measuring instruments.

10.1 Mutual Inductance:

A transformer is constructed of two coils placed so that the changing flux developed by one links the other, as shown in Figure (10.1). This results in an induced voltage across each coil. To distinguish between the coils, we will apply the transformer convention that ***the coil to which the source is applied is called the primary, and the coil to which the load is applied is called the secondary***

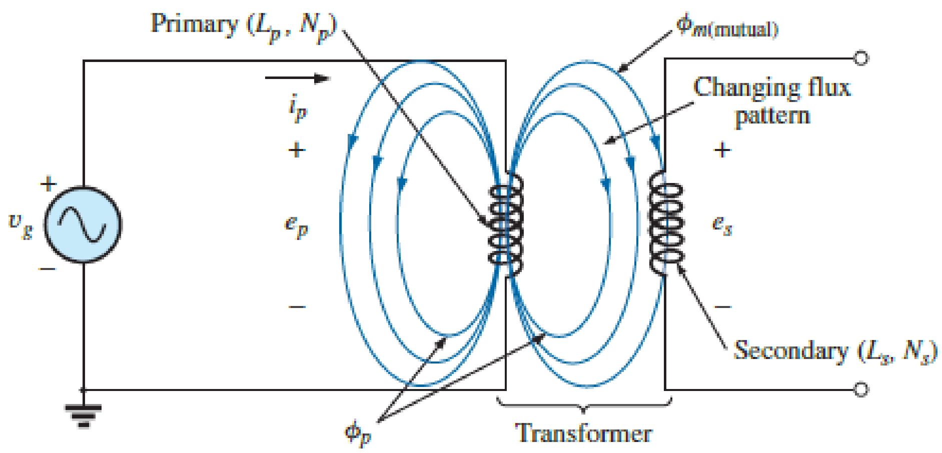


Figure (10.1): Defining the components of a transformer.

Primary is the coil or winding to which the source of electrical energy is normally applied. For the primary of the transformer in Figure (10.1), an application of Faraday's law results in:

$$e_P = N_P \frac{d\phi_P}{dt} \quad \text{(volts, V)}$$

revealing that the voltage induced across the primary is directly related to the number of turns in the primary and the rate of change of magnetic flux linking the primary coil.

The magnitude of e_S, the voltage induced across the secondary, is determined by:

$$e_S = N_S \frac{d\phi_m}{dt} \quad \text{(volts, V)}$$

Where N_S is the number of turns in the secondary winding and ϕ_m is the portion of the primary flux ϕ_P that links the secondary winding.

Secondary is the coil or winding to which the load is normally applied. If all of the flux linking the primary links the secondary, then:

$$\phi_m = \phi_P$$

and

$$e_S = N_S \frac{d\phi_P}{dt} \quad \text{(volts, V)}$$

Coefficient of coupling (k) is a measure of the magnetic coupling of two coils that ranges from a minimum of 0 to a maximum of 1. The coefficient of coupling (k) between two coils is determined by:

$$\text{coefficient of coupling } (k) = \frac{\phi_m}{\phi_P}$$

Since the maximum level of ϕ_m is ϕ_P, the coefficient of coupling between two coils can never be greater than 1.

For the secondary, we have:

$$e_S = N_S \frac{d\phi_P}{dt} = N_S \frac{dk\phi_P}{dt}$$

and

$$e_S = kN_S \frac{d\phi_P}{dt} \quad \text{(volts, V)}$$

The mutual inductance is determined by:

$$M = k\sqrt{L_P L_S} \quad \text{(henries, H)}$$

The greater the coefficient of coupling (greater flux linkages), or the greater the inductance of either coil, the higher is the mutual inductance between the coils.

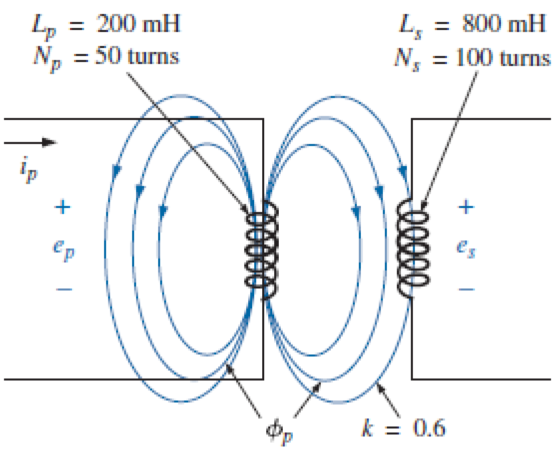
Example 1: For the transformer in Figure below:

A) Find the mutual inductance M.

B) Find the induced voltage e_P if the flux ϕ_P changes at the rate of 450 mWb/s.

C) Find the induced voltage e_S for the same rate of change indicated in part (B).

D) Find the induced voltages e_P and es if the current i_P changes at the rate of 0.2 A/ms.



Solution:

A)

$$M = k\sqrt{L_P L_S} = 0.6\sqrt{(200\ mH)(800\ mH)} = 0.6\sqrt{16 \times 10^{-2}} = (0.6)(400 \times 10^{-3})$$

$$M = \mathbf{240\ mH}$$

B)

$$e_P = N_P \frac{d\phi_P}{dt} = (50)(450\ mWb/s) = \mathbf{22.5\ V}$$

C)

$$e_S = kN_S \frac{d\phi_P}{dt} = (0.6)(100)(450\ mWb/s) = \mathbf{27\ V}$$

D)

$$e_P = L_P \frac{di_P}{dt} = (200\ mH)(0.2\ A/ms) = (200\ mH)(200\ A/s) = \mathbf{40\ V}$$

E)

$$e_S = M \frac{di_P}{dt} = (240\ mH)(200\ A/s) = \mathbf{48\ V}$$

10.2 The Iron-Core Transformer:

An iron-core transformer under loaded conditions is shown in Figure (10.2). The iron core will serve to increase the coefficient of coupling between the coils by increasing the mutual flux ϕ_m. Since the magnetic flux lines always take the path of least reluctance, which in this case is the iron core.

In the analyses in this lecture, we assume that all of the flux linking coil 1 will link coil 2. In other words, the coefficient of coupling is its maximum value, 1, and $\phi_m = \phi_P = \phi_S$. We neglect losses such as the geometric or dc resistance of the coils, the leakage reactance due to the flux linking either coil that forms no part of ϕ_m, and the hysteresis and eddy current losses. Most transformers manufactured today can be considered almost ideal.

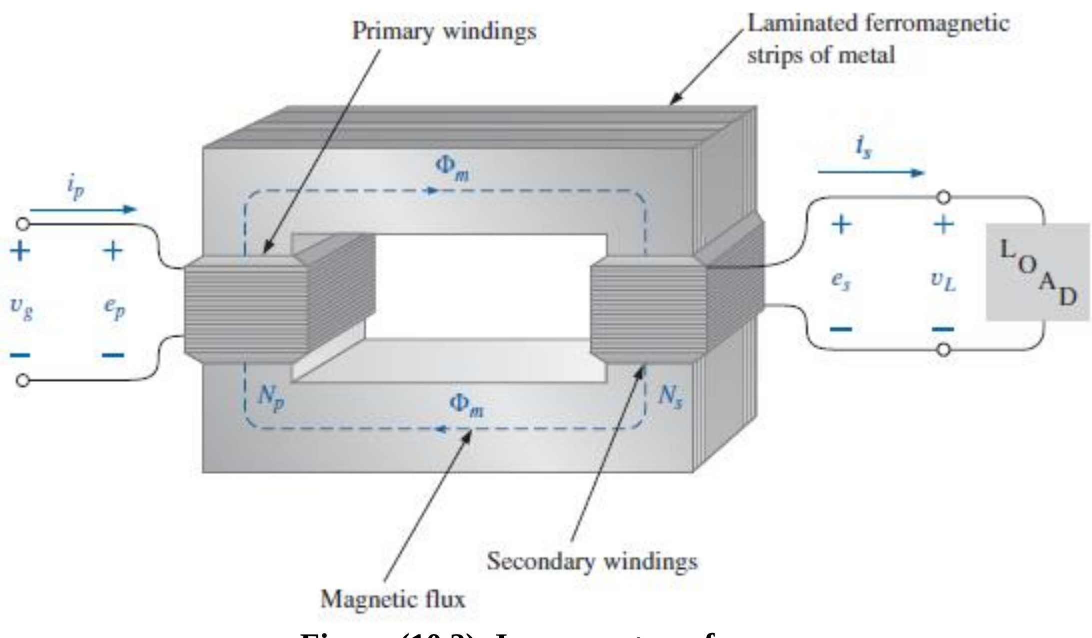


Figure (10.2): Iron-core transformer.

When the current i_P through the primary circuit of the iron-core transformer is a maximum, the flux ϕ_m linking both coils is also a maximum. In fact, the magnitude of the flux is directly proportional to the current through the primary windings. Therefore, the two are in phase, and for sinusoidal inputs, the magnitude of the flux varies as a sinusoid also. That is, if:

$$i_P = \sqrt{2}\, I_P \sin \omega t$$

then:

$$\phi_m = \Phi_m \sin \omega t$$

The induced voltage across the primary due to a sinusoidal input can be determined by Faraday's law:

$$e_P = N_P \frac{d\phi_P}{dt} = N_P \frac{d\phi_m}{dt}$$

Substituting for ϕ_m gives us:

$$e_P = N_P \frac{d}{dt}(\Phi_m \sin \omega t)$$

and differentiating, we obtain:

$$e_P = \omega N_P \Phi_m \cos \omega t$$

or

$$e_P = \omega N_P \Phi_m \sin(\omega t + 90°)$$

indicating that the induced voltage e_P leads the current through the primary coil by 90°.

The effective value of e_P is:

$$e_P = \frac{\omega N_P \Phi_m}{\sqrt{2}} = \frac{2\pi f N_P \Phi_m}{\sqrt{2}}$$

and

$$E_P = 4.442\pi f N_P \Phi_m$$

which is an equation for the rms value of the voltage across the primary coil in terms of the frequency of the input current or voltage, the number of turns of the primary, and the maximum value of the magnetic flux linking the primary.

If we repeat the procedure just described for the induced voltage across the secondary, we get:

$$E_S = 4.442\pi f N_S \Phi_m$$

Dividing equation E_P by E_S as:

$$\frac{E_P}{E_S} = \frac{4.442\pi f N_P \Phi_m}{4.442\pi f N_S \Phi_m}$$

we obtain:

$$\frac{E_P}{E_S} = \frac{N_P}{N_S}$$

The ratio of the magnitudes of the induced voltages is the same as the ratio of the corresponding turns.

If we consider that:

$$e_P = N_P \frac{d\phi_m}{dt} = and\ e_S = N_S \frac{d\phi_m}{dt}$$

and divide one by the other, that is:

$$\frac{e_P}{e_S} = \frac{N_P \frac{d\phi_m}{dt}}{N_S \frac{d\phi_m}{dt}}$$

then:

$$\frac{e_P}{e_S} = \frac{N_P}{N_S}$$

The *instantaneous* values of e_1 and e_2 are therefore related by a constant determined by the turns ratio. Since their instantaneous magnitudes are related by a constant, the induced voltages are in phase.

Since $V_g = E_1$ and $V_L = E_2$ for the ideal situation:

$$\boxed{\frac{V_g}{V_L} = \frac{N_P}{N_S}}$$

The ratio $\frac{N_P}{N_S}$, usually represented by the lowercase letter a, is referred to as the **transformation ratio**:

$$\boxed{a = \frac{N_P}{N_S}}$$

Transformation ratio ($\boldsymbol{a}$) The ratio of primary to secondary turns of a transformer.

If $a < 1$, the transformer is called a **step-up transformer** since the voltage $E_S > E_P$, that is:

$$\frac{E_P}{E_S} = \frac{N_P}{N_S} = a \quad or\ E_S = \frac{E_P}{a}$$

and, if $a < 1$, $\quad E_S > E_P$

Step-up transformer is a transformer whose secondary voltage is greater than its primary voltage. The magnitude of the transformation ratio a is less than 1.

If $a > 1$, the transformer is called a **step-down transformer** since $E_S < E_P$; that is:

$$E_P = a\,E_S$$

and, if $a > 1$, then:

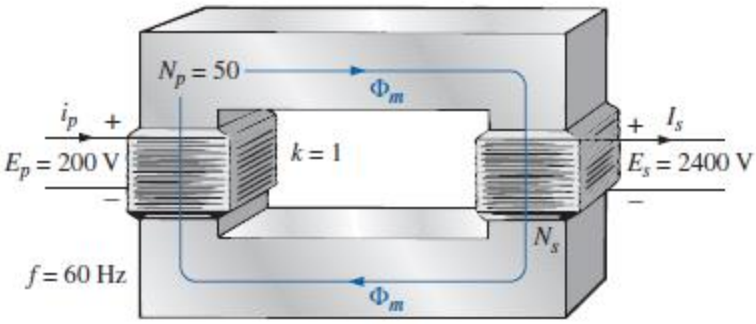
$$E_P > E_S$$

Step-down transformer is a transformer whose secondary voltage is less than its primary voltage. The transformation ratio a is greater than 1.

Example 2: For the iron-core transformer in Figure below:

A) Find the maximum flux Φ_m.

B) Find the secondary turns Ns.



Solution:

A)

$$E_P = 4.442\pi f N_P \Phi_m$$

Therefore,

$$\Phi_m = \frac{E_P}{4.442\pi f N_P} = \frac{200\ V}{(4.44)(50\ t)(60\ Hz)} = \mathbf{15.02\ mWb}$$

B)

$$\frac{E_P}{E_S} = \frac{N_P}{N_S}$$

Therefore,

$$N_S = \frac{N_P E_S}{E_P} = \frac{(50\ t)(2400\ V)}{200\ V} = \mathbf{600\ turns}$$

10.3 Reflected Impedance and Power:

$$\frac{V_g}{V_L} = \frac{N_P}{N_S} = a \quad and \quad \frac{I_P}{I_S} = \frac{N_S}{N_P} = \frac{1}{a}$$

Dividing the first by the second, we have:

$$\frac{V_g/V_L}{I_P/I_S} = \frac{a}{1/a}$$

or

$$\frac{V_g/V_L}{I_P/I_S} = a^2 \quad and \quad \frac{V_g}{I_P} = a^2\frac{V_L}{I_S}$$

However, since:

$$Z_P = \frac{V_g}{I_P} \quad and \quad Z_L = \frac{V_L}{I_S}$$

then: $$Z_P = a^2 Z_L$$

That is, the impedance of the primary circuit of an ideal transformer is the transformation ratio squared times the impedance of the load. If a transformer is used, therefore, an impedance can be made to appear larger or smaller at the primary by placing it in the secondary of a step-down (a>1) or step-up (a<1) transformer, respectively. For the ideal iron-core transformer:

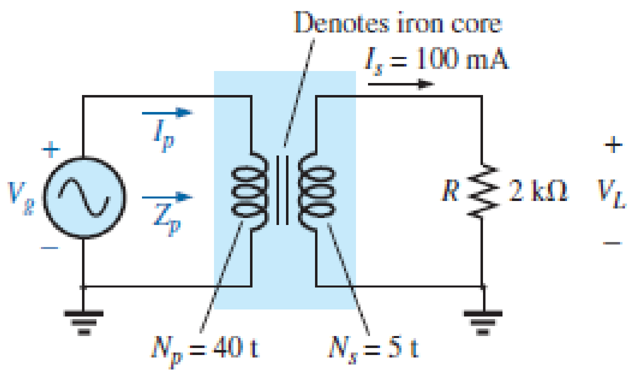
$$\frac{E_P}{E_S} = a = \frac{I_S}{I_P}$$

or

$$E_P I_P = E_S I_S$$

and $$P_{in} = P_{out}$$ (Ideal conditions)

Example 3: For the iron-core transformer in Figure below:



A) Find the magnitude of the current in the primary and the impressed voltage across the primary.

B) Find the input resistance of the transformer.

Solution:

A)

$$\frac{I_P}{I_S} = \frac{N_S}{N_P}$$

$$I_P = \frac{N_S}{N_P} I_S = \left(\frac{5\ t}{40\ t}\right)(0.1\ A) = \mathbf{12.5\ mA}$$

$$V_L = I_S Z_L = (0.1\ A)(2\ K\Omega) = 200\ V$$

Also,

$$\frac{V_g}{V_L} = \frac{N_P}{N_S}$$

$$V_g = \frac{N_P}{N_S} V_L = \left(\frac{40\ t}{5\ t}\right)(200\ V) = \mathbf{1600\ V}$$

B)

$$Z_P = a^2 Z_L$$

$$a = \frac{N_P}{N_S} = \left(\frac{40\ t}{5\ t}\right) = 8$$

$$Z_P = a^2 Z_L = (8)^2 (2\ K\Omega) = R_P = \mathbf{128\ K\Omega}$$

10.4 Equivalent Circuit of Iron-Core Transformer:

For the nonideal or practical iron-core transformer, the equivalent circuit appears as in Figure (10.3). As indicated, part of this equivalent circuit includes an ideal transformer. The remaining elements of Figure (10.3) are those elements that contribute to the nonideal characteristics of the device. The resistances R_P and R_S are simply the dc or geometric resistance of the primary and secondary windings, respectively.

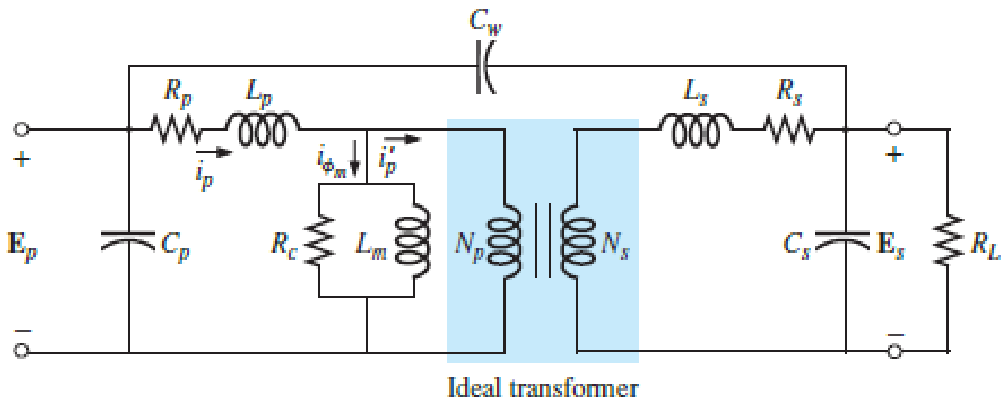


Figure (10.3): Equivalent circuit for the practical iron-core transformer.

For the primary and secondary coils of a transformer, there is a small amount of flux that links each coil but does not pass through the core, as shown in Figure (10.4) for the primary winding. This **leakage flux**, representing a definite loss in the system, is represented by an inductance L_P in the primary circuit and an inductance L_S in the secondary.

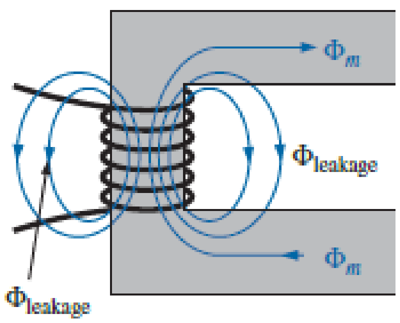


Figure (10.4): Identifying the leakage flux of the primary.

The resistance R_C represents the hysteresis and eddy current losses (core losses) within the core due to an ac flux through the core. The inductance L_m (magnetizing inductance) is the inductance associated with the magnetization of the core, that is, the establishing of the flux in the core. The capacitances C_P and C_S are the lumped capacitances of the primary and secondary circuits, respectively, and C_w represents the equivalent lumped capacitances between the windings of the transformer.

Since i'_P is normally considerably larger than i_{ϕ_m} (the magnetizing current), we will ignore i_{ϕ_m} for the moment (set it equal to zero), resulting in the absence of R_C and L_m in the reduced equivalent circuit in Figure (10.5). The capacitances C_P, C_w, and C_S do not appear in the equivalent circuit in Figure (10.5) since their reactance at typical operating frequencies do not appreciably affect the transfer characteristics of the transformer.

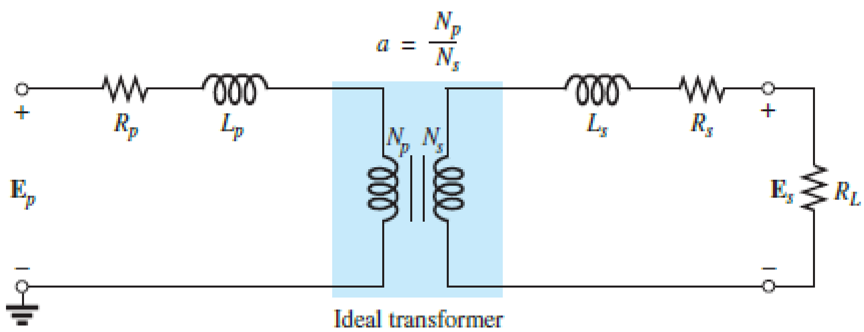


Figure (10.5): Reduced equivalent circuit for the nonideal iron-core transformer.

If we now reflect the secondary circuit through the ideal transformer, we will have the load and generator voltage in the same continuous circuit. The total resistance and inductive reactance of the primary circuit are determined by:

$$R_{equivalent} = R_e = R_P + a^2 R_S$$

and

$$X_{equivalent} = X_e = X_P + a^2 X_S$$

The load voltage can be obtained directly from the circuit in Figure (10.6) through the voltage divider rule:

$$aV_L = \frac{R_i\, V_g}{(R_e + R_i) + jX_e}$$

and

$$V_L = \frac{a\, R_L\, V_g}{(R_e + a^2 R_L) + jX_e}$$

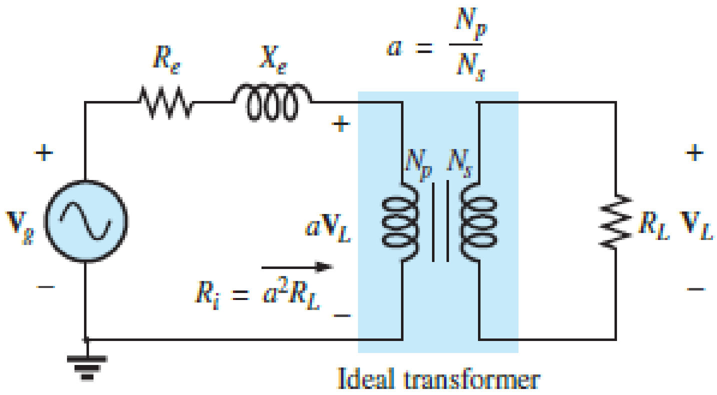


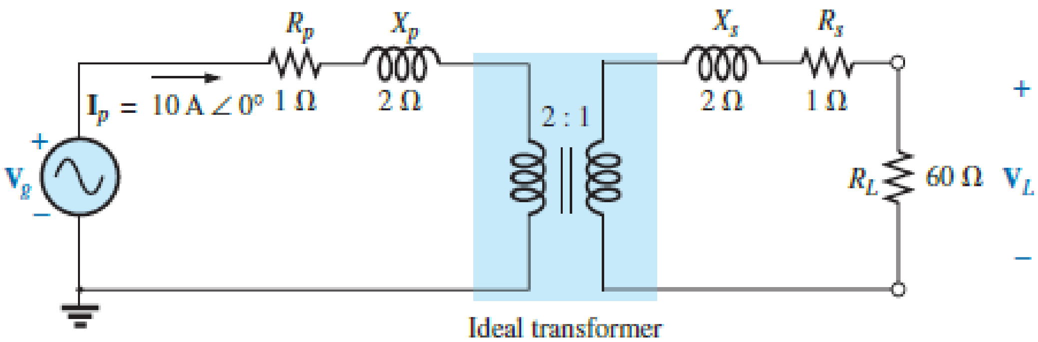
Figure (10.6): Reflecting the secondary circuit into the primary side of the iron-core transformer.

Example 4: For a transformer having the equivalent circuit in Figure below:

A) Determine R_e and X_e.

B) Determine the magnitude of the voltages V_L and V_g.

C) Determine the magnitude of the voltage V_g to establish the same load voltage in part (B) if R_e and $X_e = 0\ \Omega$. Compare with the result of part (B).



Solution:

A)

$$R_e = R_P + a^2 R_S = 1\,\Omega + (2)^2(1\,\Omega) = \mathbf{5}\,\Omega$$

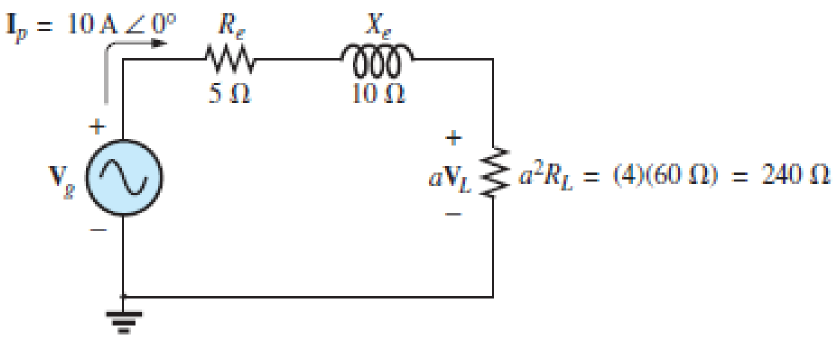
$$X_e = X_P + a^2 X_S = 2\,\Omega + (2)^2(2\,\Omega) = \mathbf{10}\,\Omega$$

B) The transformed equivalent circuit appears in Figure below.

$$V_g = I_P(R_e + a^2 R_L + jX_e) = 10\,A(5\,\Omega + 240\,\Omega + j\,10\,\Omega)$$

$$V_g = 10\,A(245\,\Omega + j\,10\,\Omega)2450\,V + j\,100\,V = 2452.04\,V\,\angle 2.34°$$

$$V_g = \mathbf{2452.04}\,\boldsymbol{V}\,\angle\mathbf{2.34}°$$



C) For R_e and $X_e = 0$, $V_g = aV_L = (2)(1200\,V) = 2400\,V$.

Therefore, it is necessary to increase the generator voltage by 52.04 V (due to R_e and X_e) to obtain the same load voltage.

10.5 Types of Transformers:

Transformers are available in many different shapes and sizes. Some of the more common types include the power transformer, audio transformer, IF (intermediate frequency) transformer, and RF (radio frequency) transformer. Each is designed to fulfill a particular requirement in a specific area of application. The symbols for some of the basic types of transformers are shown in Figure (10.7).

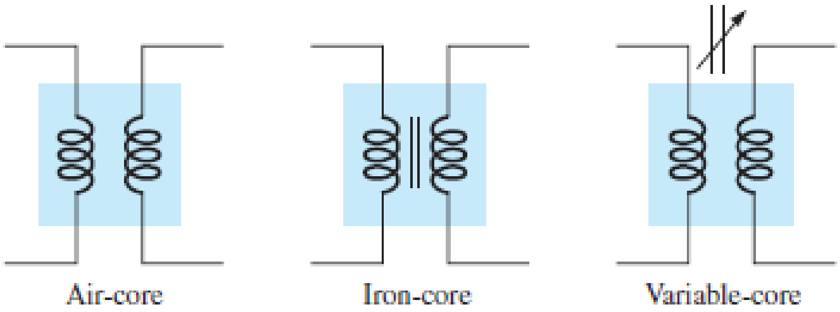


Figure (10.7): Transformer symbols.

The method of construction varies from one transformer to another. Two of the many different ways in which the primary and secondary coils can be wound around an iron core are shown in Figure (10.8). In either case, the core is made of laminated sheets of ferromagnetic material separated by an insulator to reduce the eddy current losses. The sheets themselves also contain a small percentage of silicon to increase the electrical resistivity of the material and further reduce the eddy current losses.

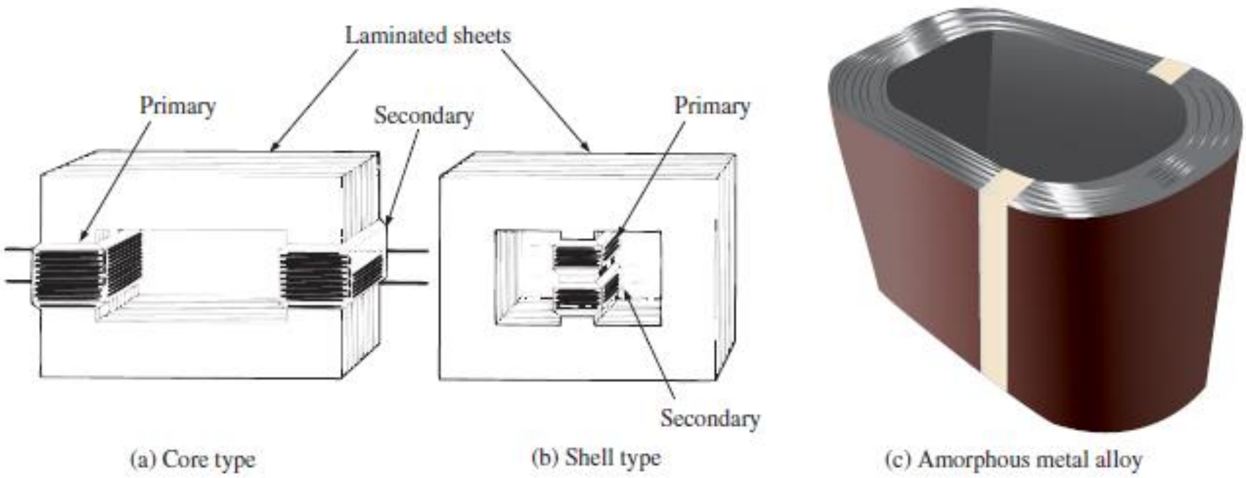


Figure (10.8): Types of ferromagnetic core construction.

In recent years, there has been a move toward using an amorphous metal alloy to form the core of the transformer. Using this alloy can result in a 70 to 80% drop in the no-load losses of a transformer. No-load losses are those that occur in the absence of a load on the transformer to draw current through the windings.

A variation of the core-type transformer appears in Figure (10.9). This transformer is designed for low-profile (the 1.1 VA size has a maximum height of only 1 in.) applications in power, control, and instrumentation applications. There are actually two transformers on the same core, with the primary and secondary of each wound side by side. The schematic is provided in Figure (10.9) for a single 115 V, 50/60 Hz input using a series connection with center-tap for the output. For this unit, the output voltage is 10 V line to center-tap with a current rating of 0.11 A, satisfying the condition that (10 V) (0.11 A) = 1.1 VA as indicated above. Note the dot convention and the commercial representation of the transformer coils.

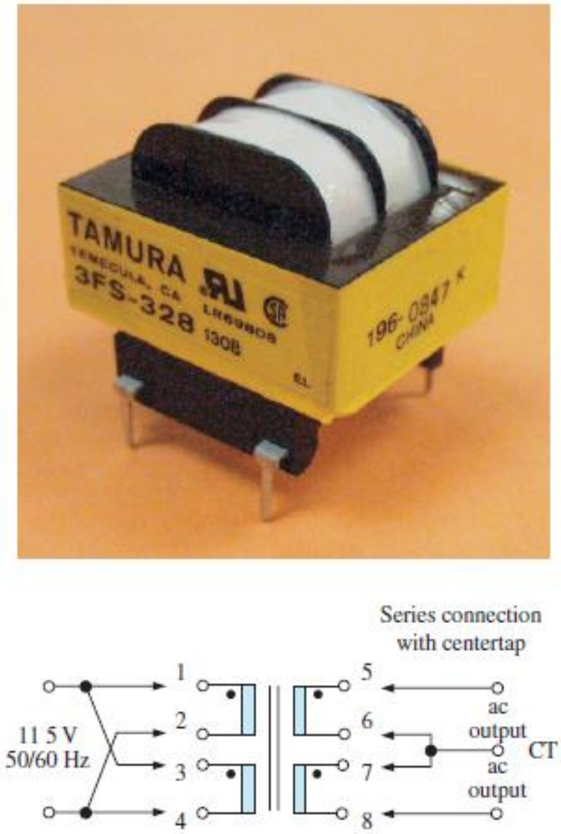


Figure (10.9): Laminated power transformer. (Tamura Corporation of America)

Autotransformer is a transformer with one winding common to both the primary and the secondary circuits. A loss in isolation is balanced by the increase in its kilovolt-ampere rating. The autotransformer [Figure (10.10(b))] is a type of power transformer that, instead of employing the two-circuit principle (complete isolation between coils), has one winding common to both the input and the output circuits. The induced voltages are related to the turns ratio in the same manner as that described for the two-circuit transformer. If the proper connection is used, a two-circuit power transformer can be used as an autotransformer. The advantage of using it as an autotransformer is that a larger apparent power can be transformed. This can be demonstrated by the two-circuit transformer of Figure (10.10(a)), shown in Figure (10.10(b)) as an autotransformer.

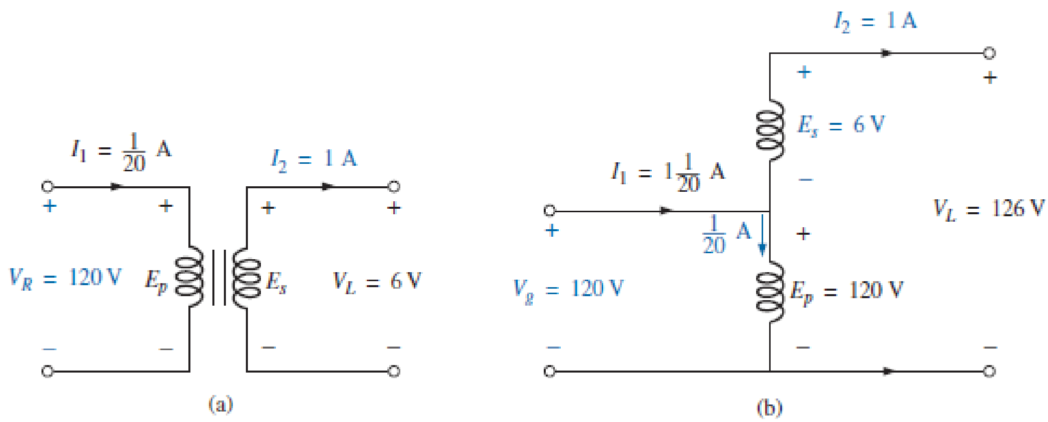


Figure (10.10): (a) Two-circuit transformer; (b) autotransformer.

For the two-circuit transformer, note that $S = \left(\frac{1}{20}\ A\right)(120\ V) = 6\ VA$, whereas for the autotransformer, $S = \left(1\frac{1}{20}\ A\right)(120\ V) = 126\ VA$, which is many times that of the two-circuit transformer. Note also that the current and voltage of each coil are the same as those for the two-circuit configuration. The disadvantage of the autotransformer is obvious: loss of the isolation between the primary and secondary circuits.

10.6 Tapped and Multiple-Load transformers:

Tapped transformer A transformer having an additional connection between the terminals of the primary or secondary windings.

For the **center-tapped** (primary) **transformer** in Figure (10.11), where the voltage from the center tap to either outside lead is defined as $E_P/2$, the relationship between E_P and E_S is:

$$\frac{E_P}{E_S} = \frac{N_P}{N_S}$$

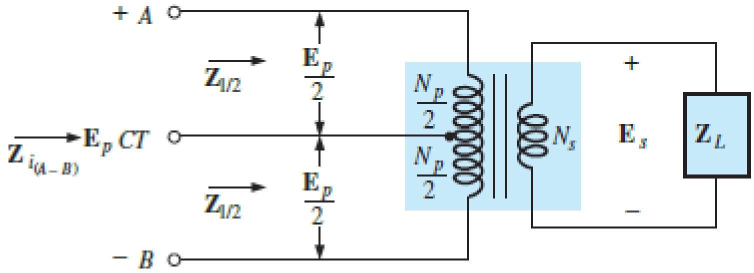


Figure (10.11): Ideal transformer with a center-tapped primary.

For each half-section of the primary,

$$Z_{1/2} = \left(\frac{N_P/2}{N_S}\right)^2 Z_L = \frac{1}{4}\left(\frac{N_P}{N_S}\right)^2 Z_L$$

with

$$Z_{i_{(A-B)}} = \left(\frac{N_P}{N_S}\right)^2 Z_L$$

Therefor,

$$Z_{1/2} = \frac{1}{4} Z_i$$

For the **multiple-load transformer** in Figure (10.12), the following equations apply:

$$\frac{E_i}{E_2} = \frac{N_1}{N_2} \qquad \frac{E_i}{E_3} = \frac{N_1}{N_3} \qquad \frac{E_2}{E_3} = \frac{N_2}{N_3}$$

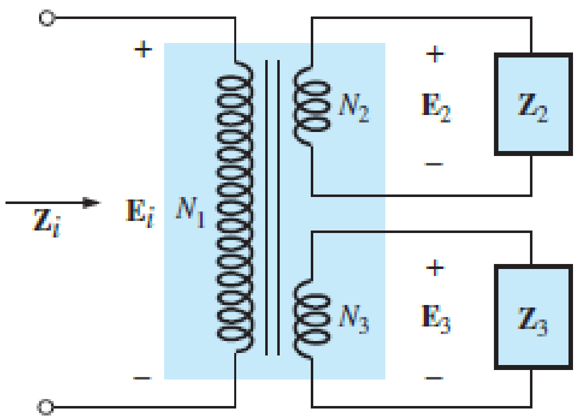


Figure (10.12): Ideal transformer with multiple loads.

The total input impedance can be determined by first noting that, for the ideal transformer, the power delivered to the primary is equal to the power dissipated by the load; that is:

$$P_1 = P_{L_2} + P_{L_3}$$

and for resistive loads ($Z_i = R_i$, $Z_2 = R_2$, and $Z_3 = R_3$),

$$\frac{E_i^2}{R_i} = \frac{E_2^2}{R_2} + \frac{E_3^2}{R_3}$$

then

$$\frac{E_i^2}{R_i} = \frac{[(N_2/N_1)E_i]^2}{R_2} + \frac{[(N_3/N_1)E_i]^2}{R_3}$$

and

$$\frac{E_i^2}{R_i} = \frac{E_i^2}{[N_2/N_1]^2 R_2} + \frac{E_i^2}{[N_3/N_1]^2 R_3}$$

Thus,

$$\frac{1}{R_i} = \frac{1}{[N_2/N_1]^2 R_2} + \frac{1}{[N_3/N_1]^2 R_3}$$

indicating that the load resistances are reflected in parallel.

Lecture 11: Three-Phase Circuits

An ac generator designed to develop a single sinusoidal voltage for each rotation of the shaft (rotor) is referred to as a **single-phase ac generator**. **Single-phase ac generator** is an electromechanical source of ac power that generates a single sinusoidal voltage having a frequency determined by the speed of rotation and the number of poles of the rotor. If the number of coils on the rotor is increased in a specified manner, the result is a **polyphase ac generator**, which develops more than one ac phase voltage per rotation of the rotor. **Polyphase ac generator** is an electromechanical source of ac power that generates more than one sinusoidal voltage per rotation of the rotor. The frequency generated is determined by the speed of rotation and the number of poles of the rotor.

11.1 Three-Phase Generator:

The three-phase generator in Figure (11.1(a)) has three induction coils placed 120° apart on the stator, as shown symbolically by Figure (11.1(b)). Since the three coils have an equal number of turns, and each coil rotates with the same angular velocity, the voltage induced across each coil has the same peak value, shape, and frequency. As the shaft of the generator is turned by some external means, the induced voltages e_{AN}, e_{BN}, and e_{CN} are generated simultaneously, as shown in Figure (11.2). Note the 120° phase shift between waveforms and the similarities in appearance of the three sinusoidal functions.

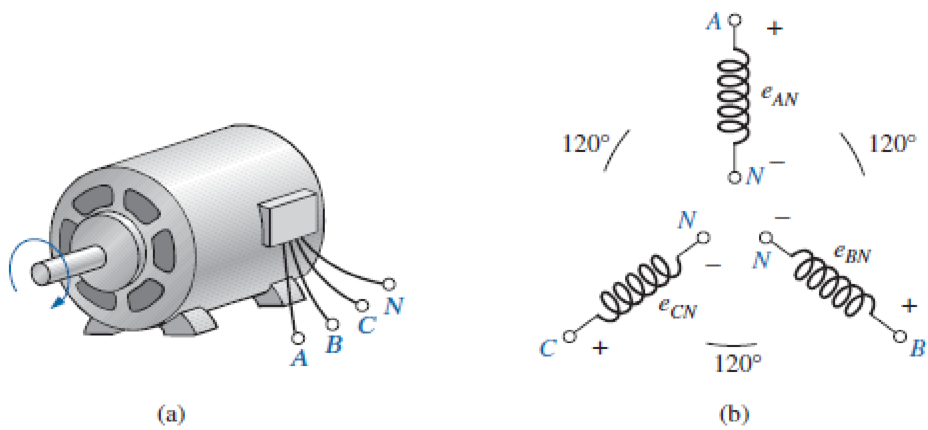


Figure (11.1): (a) Three-phase generator; (b) induced voltages of a three-phase generator.

In particular, note that ***at any instant of time, the algebraic sum of the three phase voltages of a three-phase generator is zero.***

Phase voltage is the voltage that appears between the line and neutral of a Y-connected generator and from line to line in a Δ-connected generator.

This is shown at $\omega t = 0$ in Figure (11.2), where it is also evident that *when one induced voltage is zero, the other two are 86.6% of their positive or negative maximums. In addition, when any two are equal in magnitude and sign (at* $0.5E_m$*), the remaining induced voltage has the opposite polarity and a peak value.*

The respective sinusoidal expressions for the induced voltages in Figure (11.2) are:

$$e_{AN} = E_{m(AN)} \sin \omega t$$

$$e_{BN} = E_{m(BN)} \sin(\omega t - 120°)$$

$$e_{CN} = E_{m(CN)} \sin(\omega t - 240°) = E_{m(CN)} \sin(\omega t + 120°)$$

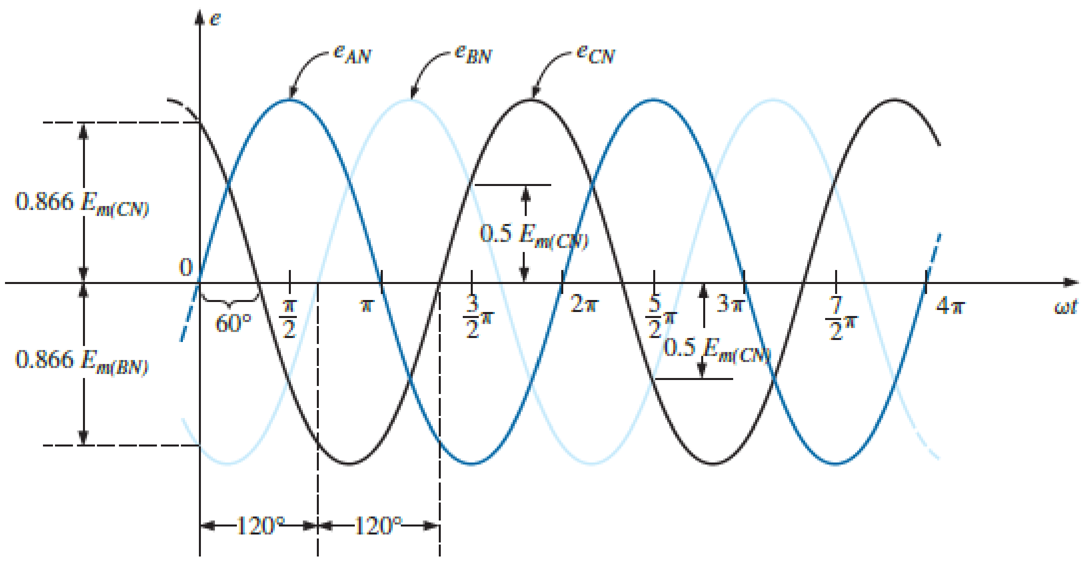


Figure (11.2): Phase voltages of a three-phase generator.

The phasor diagram of the induced voltages is shown in Figure (11.3), where the effective value of each is determined by:

$$E_{AN} = 0.707 E_{m(AN)}$$

$$E_{BN} = 0.707 E_{m(BN)}$$

$$E_{CN} = 0.707 E_{m(CN)}$$

and

$$E_{AN} = E_{AN} \angle 0°$$

$$E_{BN} = E_{BN} \angle -120°$$

$$E_{CN} = E_{CN} \angle +120°$$

The phasor sum of the phase voltages in a three-phase system is zero. That is:

$$E_{AN} + E_{BN} + E_{CN} = 0$$

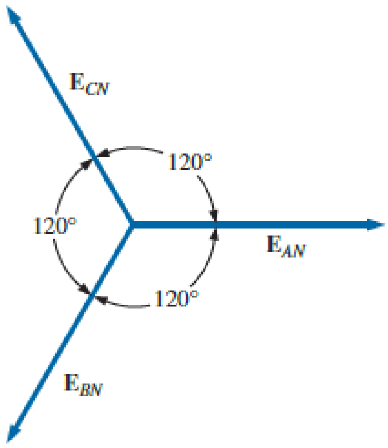


Figure (11.3): Phasor diagram for the phase voltages of a three-phase generator.

11.2 Y-Connected Generator:

Y-connected three-phase generator is a three-phase source of ac power in which the three phases are connected in the shape of the letter Y. If the three terminals denoted *N* in Figure (11.1(b)) are connected together, the generator is referred to as a **Y-connected three-phase generator** (Figure (11.4)).

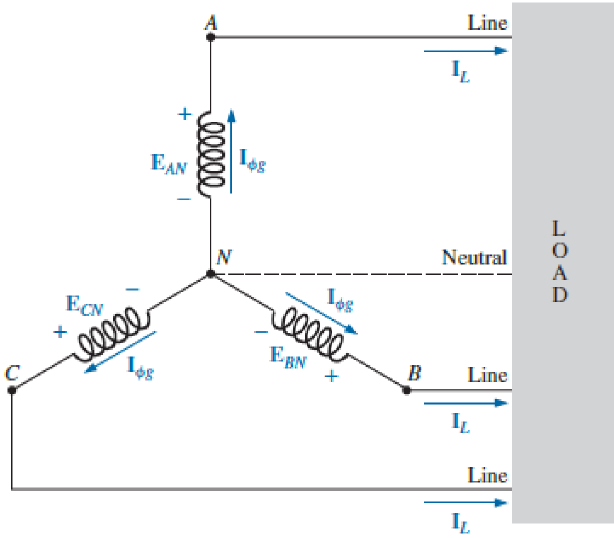


Figure (11.4): Y-connected generator.

The point at which all the terminals are connected is called the *neutral point.* If a conductor is not attached from this point to the load, the system is called a *Y-connected, three-phase, three-wire generator.* If the neutral is connected, the system is a *Y-connected, three-phase, four-wire generator.*

Neutral connection is the connection between the generator and the load that, under balanced conditions, will have zero current associated with it.

The three conductors connected from *A*, *B*, and *C* to the load are called *lines*. For the Y-connected system, the **line current** equals the **phase current** for each phase; that is:

$$I_L = I_{\phi g}$$

Line current is the current that flows from the generator to the load of a single-phase or polyphase system. **Phase current** is the current that flows through each phase of a single-phase (or polyphase) generator or load.

where ϕ is used to denote a phase quantity, and g is a generator parameter.

The voltage from one line to another is called a **line voltage. Line voltage** is the potential difference that exists between the lines of a single-phase or polyphase system. On the phasor diagram (Figure (11.5)), it is the phasor drawn from the end of one phase to another in the counterclockwise direction.

Applying Kirchhoff's voltage law around the indicated loop in Figure (11.5), we obtain:

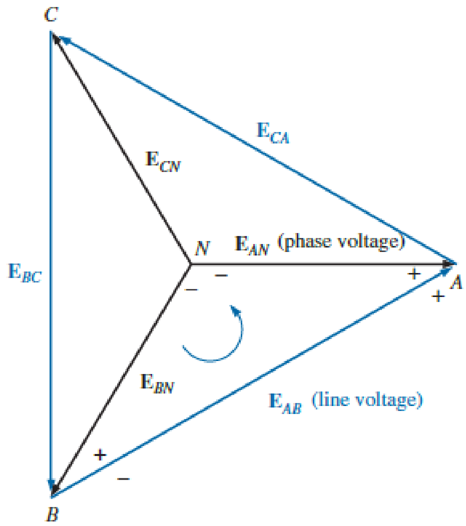


Figure (11.5): Line and phase voltages of the Y-connected three-phase generator.

$$E_{AB} - E_{AN} + E_{BN} = 0$$

or

$$E_{AB} = E_{AN} - E_{BN} = E_{AN} - E_{NB}$$

11.3 Y-Connected Generator with a Y-Connected Load:

Loads connected to three-phase supplies are of two types: the Y and the Δ. If a Y-connected load is connected to a Y-connected generator, the system is symbolically represented by Y-Y. The physical setup of such a system is shown in Figure (11.6).

If the load is balanced, the **neutral connection** can be removed without affecting the circuit in any manner; that is, if:

$$Z_1 = Z_2 = Z_3$$

then I_N will be zero.

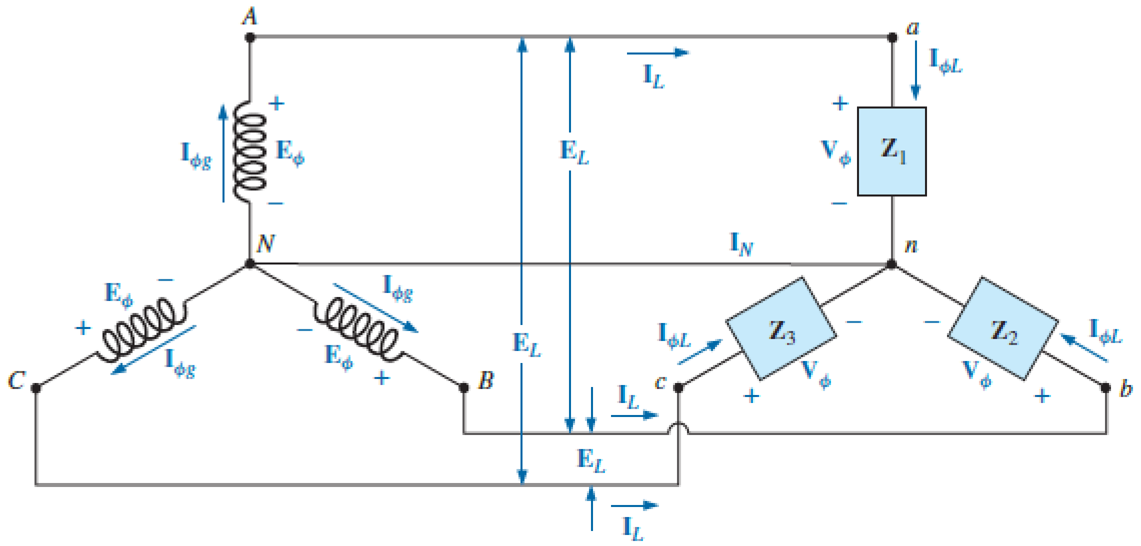


Figure (11.6): Y-connected generator with a Y-connected load.

The current passing through each phase of the generator is the same as its corresponding line current, which in turn for a Y-connected load is equal to the current in the phase of the load to which it is attached:

$$I_{\phi g} = I_L = I_{\phi L}$$

For a balanced or an unbalanced load, since the generator and load have a common neutral point, then:

$$V_\phi = E_\phi$$

In addition, since $I_{\phi L} = V_\phi / Z_\phi$, the magnitude of the current in each phase is equal for a balanced load and unequal for an unbalanced load. Recall that for the Y-connected generator, the magnitude of the line voltage is equal to $\sqrt{3}$ times the phase voltage. This same relationship can be applied to a balanced or an unbalanced four-wire Y-connected load:

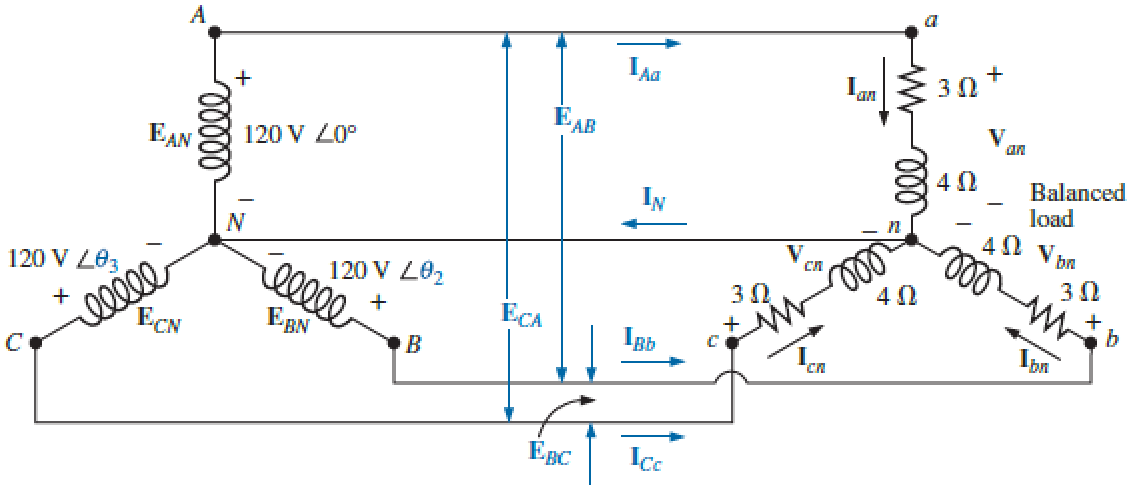
Example 1: The phase sequence of the Y-connected generator in Figure below is *ABC*.

A) Find the phase angles θ_2 and θ_3.

B) Find the magnitude of the line voltages.

C) Find the line currents.

D) Verify that, since the load is balanced, $I_N = 0$.



Solution:

A) For an *ABC* phase sequence,

$$\theta_2 = \mathbf{-120°} \qquad \text{and} \qquad \theta_3 = \mathbf{+120°}$$

B) $E_L = \sqrt{3}E_\phi = (1.73)(120\ V) = \mathbf{208\ V}$. Therefore,

$$E_{AB} = E_{BC} = E_{CA} = \mathbf{208\ V}$$

C) $V_\phi = E_\phi$. Therefore,

$$V_{an} = E_{AN} \qquad V_{bn} = E_{BN} \qquad V_{cn} = E_{CN}$$

$$I_{\phi L} = I_{an} = \frac{V_{an}}{Z_{an}} = \frac{120\ V\ \angle 0°}{3\ \Omega + j\ 4\ \Omega} = \frac{120\ V\ \angle 0°}{5\ \Omega\ \angle 53.13°} = 24\ A\ \angle -53.13°$$

$$I_{bn} = \frac{V_{bn}}{Z_{bn}} = \frac{120\ V\ \angle -120°}{5\ \Omega\ \angle 53.13°} = 24\ A\ \angle -173.13°$$

$$I_{cn} = \frac{V_{cn}}{Z_{cn}} = \frac{120\ V\ \angle +120°}{5\ \Omega\ \angle 53.13°} = 24\ A\ \angle 66.87°$$

and, since $I_L = I_{\phi L}$:

$$I_{Aa} = I_{an} = \mathbf{24\ A\ \angle -53.13°}$$

$$I_{Bb} = I_{bn} = \mathbf{24\ A\ \angle -173.13°}$$

$$I_{Cc} = I_{cn} = \mathbf{24\ A\ \angle 66.87°}$$

D) Applying Kirchhoff's current law, we have:

$$I_N = I_{Aa} + I_{Bb} + I_{Cc}$$

In rectangular form,

$$I_{Aa} = 24\ A\angle -53.13° = 14.40\ A\ -\ j\ 19.20\ A$$

$$I_{Bb} = 24\ A\angle -173.13° = -22.83\ A\ -\ j\ 2.87\ A$$

$$I_{Cc} = 24\ A\angle 66.87° = 9.43\ A\ +\ j\ 22.07\ A$$

$$\sum(I_{Aa} + I_{Bb} + I_{Cc}) = 0 + j0$$

and $\boldsymbol{I_N}$ is in fact equals to **zero,** as required for a balanced load.

11.4 Y-Δ System:

There is no neutral connection for the Y-Δ system in Figure (11.7). Any variation in the impedance of a phase that produces an unbalanced system simply varies the line and phase currents of the system.

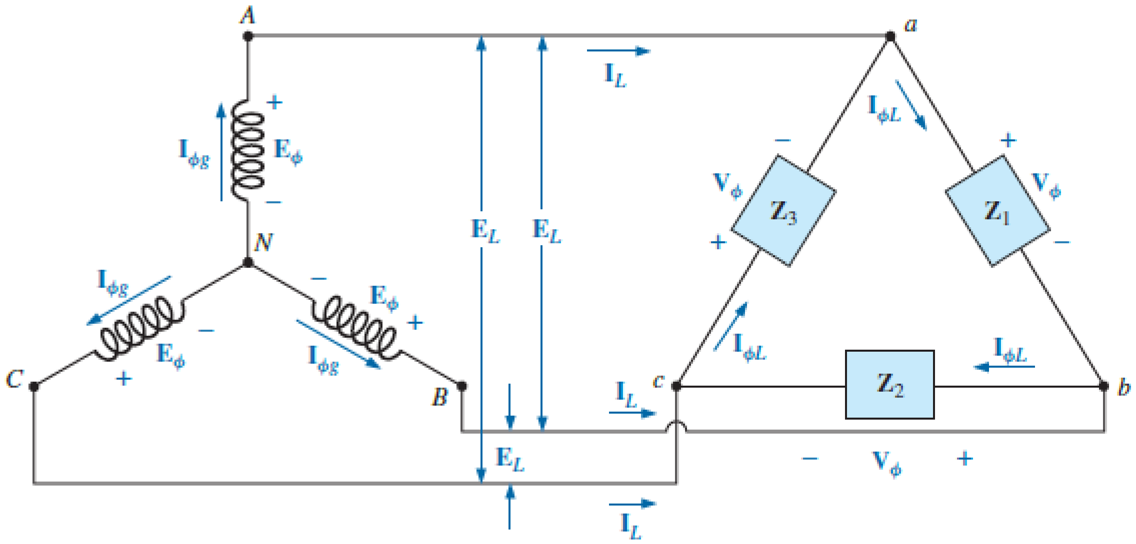


Figure (11.7): Y-connected generator with a Δ-connected load.

For a balanced load, $Z_1 = Z_2 = Z_3$

The voltage across each phase of the load is equal to the line voltage of the generator for a balanced or an unbalanced load:

$$V_\phi = E_L$$

$$I_L = \sqrt{3}\, I_\phi$$

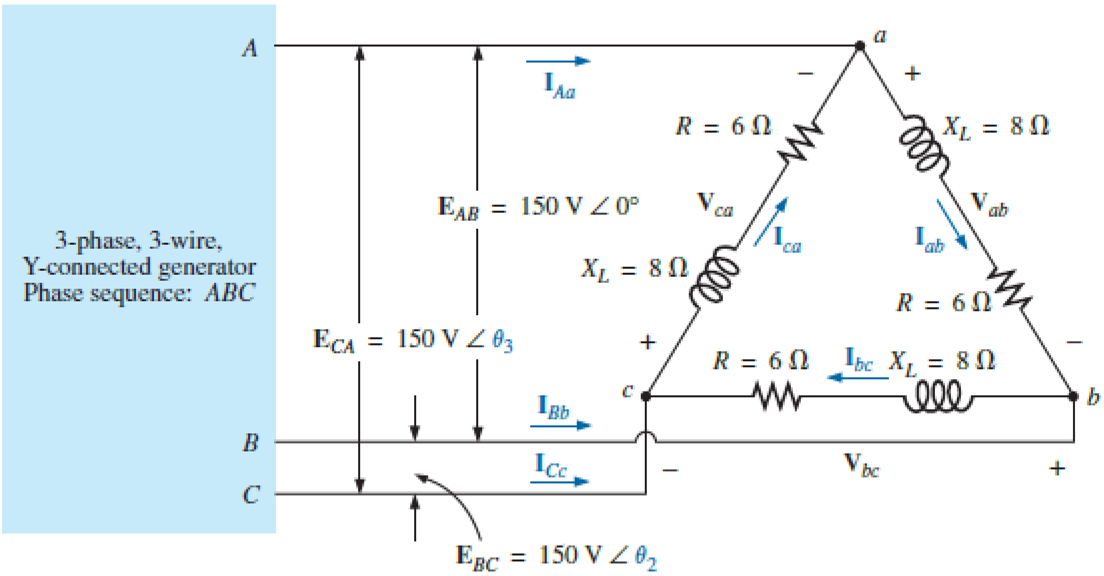
and the phase angle between a line current and the nearest phase current is 30°.

Example 2: For the three-phase system in Figure below:

A) Find the phase angles θ_2 and θ_3.

B) Find the current in each phase of the load.

C) Find the magnitude of the line currents.



Solution:

A) For an *ABC* phase sequence,

$$\theta_2 = -\mathbf{120°} \qquad \text{and} \qquad \theta_3 = +\mathbf{120°}$$

B) $V_\phi = E_L$. Therefore,

$$V_{ab} = E_{AB} \qquad V_{ca} = E_{CA} \qquad V_{bc} = E_{BC}$$

The phase currents are:

$$I_{ab} = \frac{V_{ab}}{Z_{ab}} = \frac{150\ V\ \angle 0°}{8\ \Omega + j\ 6\ \Omega} = \frac{150\ V\ \angle 0°}{10\ \Omega\ \angle 53.13°} = \mathbf{15\ A \angle -53.13°}$$

$$I_{bc} = \frac{V_{bc}}{Z_{bc}} = \frac{150\ V\ \angle -120°}{5\ \Omega\ \angle 53.13°} = \mathbf{15\ A \angle -173.13°}$$

$$I_{cn} = \frac{V_{cn}}{Z_{cn}} = \frac{150\ V\ \angle +120°}{5\ \Omega\ \angle 53.13°} = \mathbf{15\ A\ \angle 66.87°}$$

C) $I_L = \sqrt{3} I_\phi = (1.73)(15\ A) = \mathbf{25.95\ A}$. Therefore,

$$I_{Aa} = I_{Bb} = I_{Cc} = \mathbf{25.95\ A}$$

11.5 Power:

11.5.1 Y-Connected Balanced Load:

Please refer to Figure (11.8) for the following discussion.

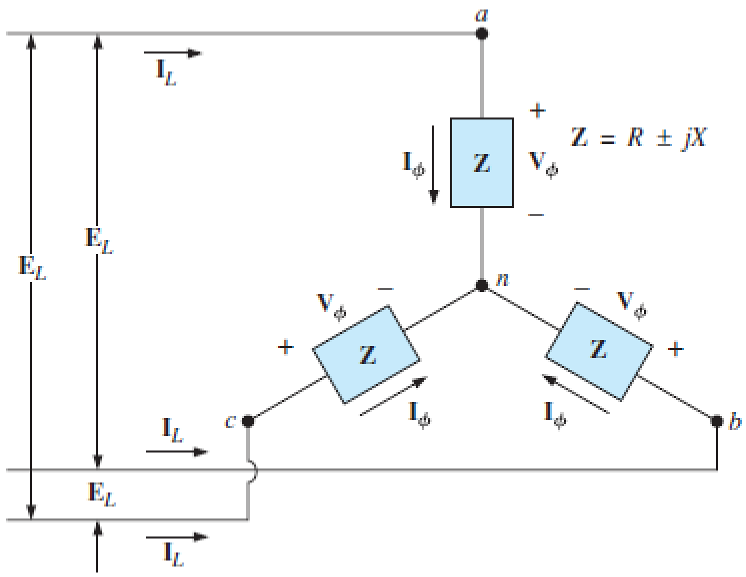


Figure (11.8): Y-connected balanced load.

Average Power The average power delivered to each phase can be determined by:

$$P_\phi = V_\phi I_\phi \cos \phi_{I_\phi}^{V_\phi} = I_\phi^2 R_\phi = \frac{V_\phi^2}{R_\phi} \quad \text{(watts, W)}$$

where $\phi_{I_\phi}^{V_\phi}$ indicates that u is the phase angle between V_ϕ and I_ϕ.

The total power delivered can be determined by:

$$P_T = 3P_\phi \quad \text{(W)}$$

or, since

$$V_\phi = \frac{E_L}{\sqrt{3}} \qquad \text{and} \qquad I_\phi = I_L$$

then

$$P_T = 3\frac{E_L}{\sqrt{3}}\, I_L \cos \phi_{I_\phi}^{V_\phi}$$

But

$$\left(\frac{3}{\sqrt{3}}\right)(1) = \left(\frac{3}{\sqrt{3}}\right)\left(\frac{\sqrt{3}}{\sqrt{3}}\right) = \frac{3\sqrt{3}}{3} = \sqrt{3}$$

Therefore,

$$P_T = \sqrt{3}E_L\, I_L \cos\phi_{I_\phi}^{V_\phi} = 3\, I_L^2\, R_\phi \quad \text{(W)}$$

Reactive Power The reactive power of each phase (in volt-amperes reactive) is:

$$Q_\phi = V_\phi I_\phi \sin\phi_{I_\phi}^{V_\phi} = I_\phi^2\, X_\phi = \frac{V_\phi^2}{X_\phi} \quad \text{(VAR)}$$

The total reactive power of the load is:

$$Q_T = 3Q_\phi \quad \text{(VAR)}$$

or, proceeding in the same manner as above, we have:

$$Q_T = \sqrt{3}E_L\, I_L \sin\phi_{I_\phi}^{V_\phi} = 3\, I_L^2\, X_\phi \quad \text{(VAR)}$$

Apparent Power The apparent power of each phase is:

$$S_\phi = V_\phi I_\phi \quad \text{(VA)}$$

The total apparent power of the load is:

$$S_T = 3S_\phi \quad \text{(VA)}$$

or, as before,

$$S_T = \sqrt{3}E_L\, I_L \quad \text{(VA)}$$

Power Factor The power factor of the system is given by:

$$F_P = \frac{P_T}{S_T} = \cos\phi_{I_\phi}^{V_\phi} \quad \text{(leading or lagging)}$$

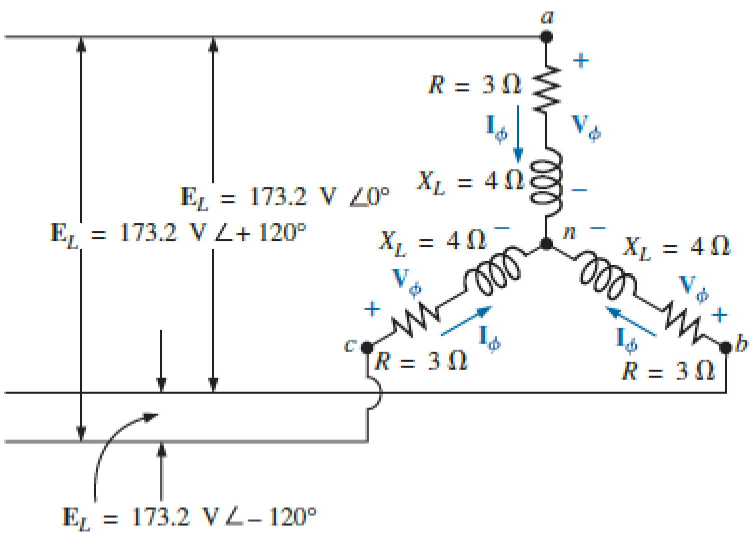
Example 3: For the Y-connected load in Figure below:

A) Find the average power to each phase and the total load.

B) Determine the reactive power to each phase and the total reactive power.

C) Find the apparent power to each phase and the total apparent power.

D) Find the power factor of the load.



Solution:

A) The *average power* is:

$$P_\phi = V_\phi I_\phi \cos\phi_{I_\phi}^{V_\phi} = (100\,V)(20\,A)\cos 53.13° = (2000)(0.6) = \mathbf{1200\ W}$$

$$P_\phi = I_\phi^2 R_\phi = (20\,A)^2(3\,\Omega) = (400)(3) = \mathbf{1200\ W}$$

$$P_\phi = \frac{V_\phi^2}{R_\phi} = \frac{(60\,V)^2}{3\,\Omega} = \frac{3600}{3} = \mathbf{1200\ W}$$

$$P_T = 3P_\phi = (3)(1200\,W) = \mathbf{3600\ W}$$

or

$$P_T = \sqrt{3}E_L\,I_L\cos\phi_{I_\phi}^{V_\phi} = (1.732)(173.2\,V)(20\,A)(0.6) = \mathbf{3600\ W}$$

B) The *reactive power* is:

$$Q_\phi = V_\phi I_\phi \sin\phi_{I_\phi}^{V_\phi} = (100\,V)(20\,A)\sin 53.13° = (2000)(0.8) = \mathbf{1600\ VAR}$$

or

$$Q_{\phi} = I_{\phi}^2 X_{\phi} = (20\ A)^2(4\ \Omega) = (400)(4) = \mathbf{1600\ VAR}$$

$$Q_T = 3Q_{\phi} = (3)(1600\ VAR) = \mathbf{4800\ VAR}$$

or

$$Q_T = \sqrt{3}E_L\ I_L \sin\phi_{I_\phi}^{V_\phi} = (1.732)(173.2\ V)(20\ A)(0.8) = \mathbf{4800\ VAR}$$

C) The *apparent power* is:

$$S_{\phi} = V_{\phi}I_{\phi} = (100\ V)(20\ A) = \mathbf{2000\ VA}$$

$$S_T = 3S_{\phi} = (3)(2000\ VA) = \mathbf{6000\ VA}$$

or

$$S_T = \sqrt{3}E_L\ I_L = (1.732)(173.2\ V)(20\ A) = \mathbf{6000\ VA}$$

D) The *power factor* is:

$$F_P = \frac{P_T}{S_T} = \frac{3600\ W}{6000\ VA} = \mathbf{0.6\ lagging}$$

11.5.2 Δ-Connected Balanced Load:

Δ-connected ac generator is a three-phase generator having the three phases connected in the shape of the capital Greek letter (delta (Δ). Please refer to Figure (11.9) for the following discussion.

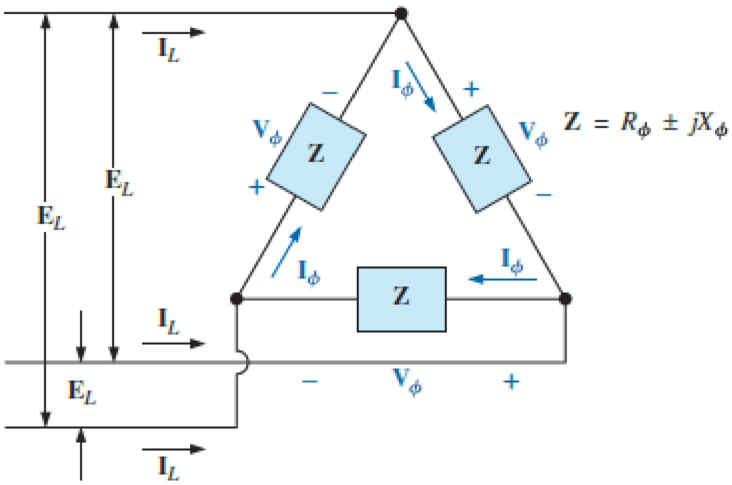


Figure (11.9): Δ-connected balanced load.

Average Power:

$$P_\phi = V_\phi I_\phi \cos \phi_{I_\phi}^{V_\phi} = I_\phi^2 R_\phi = \frac{V_\phi^2}{R_\phi} \quad \text{(W)}$$

$$P_T = 3P_\phi \quad \text{(W)}$$

Reactive Power:

$$Q_\phi = V_\phi I_\phi \sin \phi_{I_\phi}^{V_\phi} = I_\phi^2 X_\phi = \frac{V_\phi^2}{X_\phi} \quad \text{(VAR)}$$

$$Q_T = 3Q_\phi \quad \text{(VAR)}$$

Apparent Power:

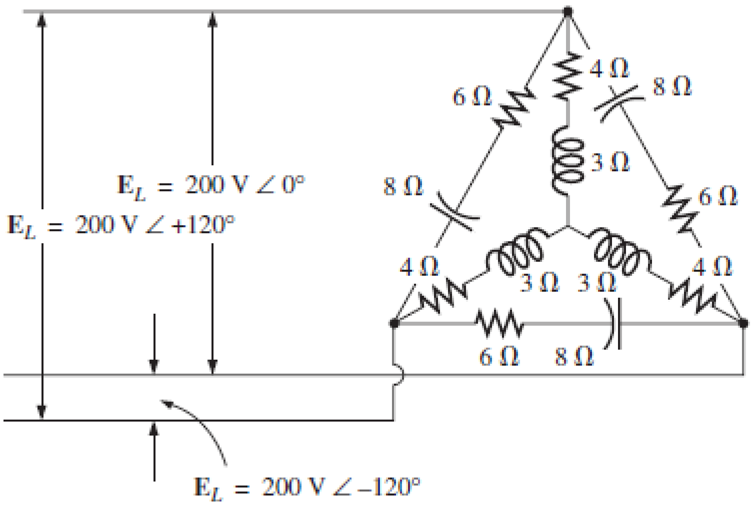
$$S_\phi = V_\phi I_\phi \quad \text{(VA)}$$

$$S_T = 3S_\phi = \sqrt{3} E_L I_L \quad \text{(VA)}$$

Power Factor:

$$F_P = \frac{P_T}{S_T} \quad \text{(leading or lagging)}$$

Example 4: For the Δ-Y connected load in Figure below, find the total average, reactive, and apparent power. In addition, find the power factor of the load.



Solution: Consider the Δ and Y separately.

For the Δ:

$$Z_{\Delta} = 6\Omega - j8\Omega = 10\ \Omega\angle -53.13°$$

$$I_{\phi} = \frac{E_L}{Z_{\Delta}} = \frac{200\ V}{10\ \Omega} = 20\ A$$

$$P_{T_{\Delta}} = 3\ I_{\phi}^2\ R_{\phi} = (3)(20\ A)^2(6\ \Omega) = \mathbf{7200\ W}$$

$$Q_{T_{\Delta}} = 3\ I_{\phi}^2\ X_{\phi} = (3)(20\ A)^2(8\ \Omega) = \mathbf{9600\ VAR\ (C)}$$

$$S_{T_{\Delta}} = 3V_{\phi}I_{\phi} = (3)(200\ V)(20\ A) = \mathbf{12,000\ VA}$$

For the Y:

$$Z_Y = 4\Omega + j3\Omega = 5\ \Omega\angle 36.87°$$

$$I_{\phi} = \frac{E_L/\sqrt{3}}{Z_Y} = \frac{200\ V/\sqrt{3}}{5\ \Omega} = \frac{116\ V}{5\ \Omega} = 23.12\ A$$

$$P_{T_Y} = 3\ I_{\phi}^2\ R_{\phi} = (3)(23.12\ A)^2(4\ \Omega) = \mathbf{6414.41\ W}$$

$$Q_{T_Y} = 3\ I_{\phi}^2\ X_{\phi} = (3)(23.12\ A)^2(3\ \Omega) = \mathbf{4810.81\ VAR\ (L)}$$

$$S_{T_Y} = 3V_{\phi}I_{\phi} = (3)(116\ V)(23.12\ A) = \mathbf{8045.76\ VA}$$

For the total load:

$$P_T = P_{T_{\Delta}} + P_{T_Y} = 7200\ W + 6414.41\ W = \mathbf{13,614.41\ W}$$

$$Q_T = Q_{T_\Delta} - Q_{T_Y} = 9600\ VAR\ (C) - 4810.81\ VAR\ (L) = \mathbf{4789.19\ VAR\ (C)}$$

$$S_T = \sqrt{P_T^2 + Q_T^2} = \sqrt{(13{,}614.41\ W)^2 + (4789.19\ VAR)^2} = \mathbf{14{,}432.2\ VA}$$

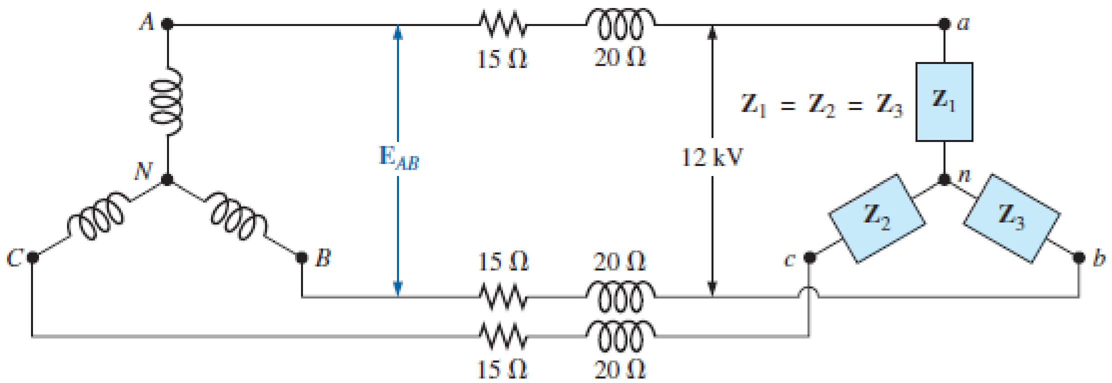
$$F_P = \frac{P_T}{S_T} = \frac{13{,}614.41\ W}{14{,}432.2\ VA} = \mathbf{0.943\ leading}$$

Example 5: Each transmission line of the three-wire, three-phase system in Figure below has an impedance of 15 Ω + *j* 20 Ω. The system delivers a total power of 160 KW at 12,000 V to a balanced three-phase load with a lagging power factor of 0.86.

A) Determine the magnitude of the line voltage E_{AB} of the generator.

B) Find the power factor of the total load applied to the generator.

C) What is the efficiency of the system?



Solution:

A)

$$V_\phi(load) = \frac{V_L}{\sqrt{3}} = \frac{12{,}000\ V}{1.73} = 6936.42\ V$$

$$P_T(load) = 3V_\phi I_\phi \cos\theta$$

and

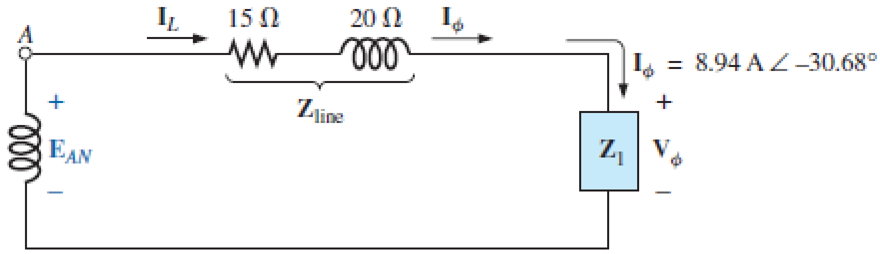
$$I_\phi = \frac{P_T(load)}{3V_\phi \cos\theta} = \frac{160{,}000\ W}{3(6936.42\ V)(0.86)} = \mathbf{8.94\ A}$$

Since $\theta = \cos^{-1} 0.86 = 30.68°$, assigning V_ϕ an angle of 0° or $V_\phi = V_\phi \angle 0°$, a lagging power factor results in:

$$I_\phi = \mathbf{8.94\ A \angle -30.68°}$$

For each phase, the system will appear as shown in Figure below, where:

$$E_{AN} = I_\phi Z_{line} - V_\phi = 0$$



or

$$E_{AN} = I_\phi Z_{line} + V_\phi = (8.94\ A\angle -30.68°)(25\ \Omega\angle 53.13°) + 6936.42\ V\angle 0°$$

$$E_{AN} = 223.5\ V\angle 22.45° + 6936.42\ V\angle 0°$$

$$E_{AN} = 206.56\ V + j\,85.35\ V + 6936.42\ V$$

$$E_{AN} = 7142.98\ V + j\,85.35\ V = 7143.5\ V\angle 0.68°$$

Then

$$E_{AB} = \sqrt{3}\,E_{\phi g} = (1.73)(7143.5\ V) = \mathbf{12,358.26\ V}$$

B)

$$P_T = P_{load} + P_{line} = 160\ kW + 3(I_L)^2 R_{line} = 160\ KW + 3(8.94\ A)^2 15\ \Omega$$

$$P_T = 160{,}000\ W + 3596.55\ W = 163{,}596.55\ W$$

and

$$P_T = \sqrt{3}\,V_L I_L \cos\theta_T$$

or

$$\cos\theta_T = \frac{P_T}{\sqrt{3}\,V_L I_L} = \frac{163{,}596.55\ W}{(1.73)(12{,}358.26\ V)(8.94\ A)}$$

and

$$F_P = \mathbf{0.856} < 0.86\ of\ load$$

C)

$$\eta = \frac{P_o}{P_i} = \frac{P_o}{P_o + P_{losses}} = \frac{160\ KW}{160\ KW + 3596.55\ W} = 0.978 = \mathbf{97.8\%}$$