Electrical Engineering Fundamentals

First class

AC & DC

University: ***Tikrit***
College: ***Petroleum Processes Engineering***
Department: ***Petroleum Systems Control Engineering***
Assistant Lecturer: ***Waladdin Mezher Shaher***
2024-2025

Electrical Engineering Fundamentals

EE1

1 Basic Concepts & Basic Laws

1.1 Basic Concepts

1.1.1 System of Units

The basic SI units

Quantity	Basic unit	Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Thermodynamic temperature	kelvin	K
Luminous intensity	candela	cd

The SI prefixes

Multiplier	Prefix	Symbol
10^{18}	exa	E
10^{15}	peta	P
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10	deca	da
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f
10^{-18}	atto	a

Examples

$10\ MHz \Rightarrow 10 \times 10^{6}\ Hz$

$2\ mA = 2 \times 10^{-3} = 0.002\ A$

$5\ \mu s = 5 \times 10^{-6}\ s$

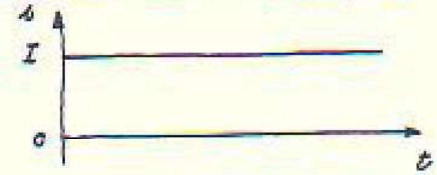
1.1.2 Charge and Current

The electric charge is an electrical property of the atomic particles of which matter consists, measured in coulombs (C). The charge of an electron is $(-1.602 \times 10^{-19} C)$.

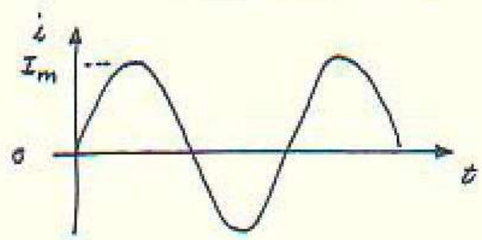
Electric Current : is the rate of change of charge, measured in amperes (A). The current (I) is defined mathematically as:

$$\boxed{i = \frac{dq}{dt}}$$

$$\therefore \; q = \int_{t_1}^{t_2} i \, dt$$



d.c current



a.c current

* A direct current (dc) is current that remains constant with time. The symbol (I) is usually used to represent such a constant current.

* An alternating current (ac) is a current that is varying sinusoidally with time. A time varying current is represented by the symbol (i).

Example : Determine the total charge entering a terminal between $t = 1s$ and $t = 2s$, if the current passing the terminal is $i = (3t^2 - t)$ A.

Solution :

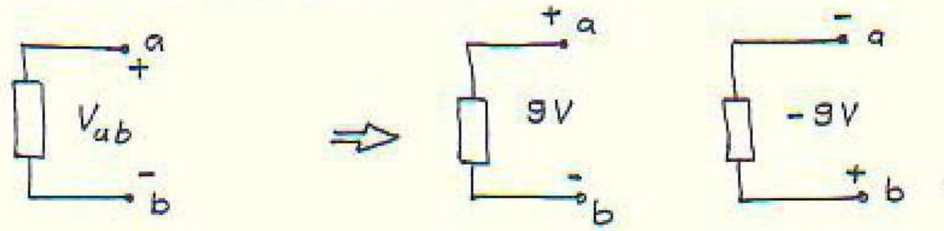
$$q = \int_{t_1}^{t_2} i \, dt$$

$$= \int_1^2 (3t^2 - t)\, dt = \left(t^3 - \frac{t^2}{2}\right)\Bigg|_1^2$$

$$= (8 - 2) - \left(1 - \frac{1}{2}\right) = \underline{\underline{5.5\ C}}$$

1.1.3 Voltage

: The voltage (or potential difference) is the energy required to move a unit charge through an element, measured in volts (V).



Polarity of Voltage V_{ab}

* For the voltage V_{ab} ⇒ This means that the potential of point $\underline{\underline{a}}$ is higher than that of point $\underline{\underline{b}}$

$$\boxed{V_{ab} = V_a - V_b}$$

1.1.4 Power and Energy

:

* Power : is the time rate of expending or absorbing energy, measured in watts (W)

$$\Rightarrow \quad p = \frac{dw}{dt}$$

where p is the power in watts (W), w is the energy in joules (J), and t is the time in seconds (s)

We have; $p = \frac{dw}{dt} \Rightarrow p = \frac{dw}{dt} \cdot \frac{dq}{dq}$

$$\Rightarrow p = \frac{dw}{dq} \cdot \frac{dq}{dt} = v.i$$

$$\boxed{\therefore \; p = vi}$$

* The energy absorbed or supplied by an element

from time $\underline{t_o}$ to time $\underline{t}$ is :

$$\boxed{w = \int_{t_o}^{t} p\,dt = \int_{t_o}^{t} vi\,dt}$$

Energy is the capacity to do work, measured in joules (J)

* The electric power utility companies measure energy in watt-hour (Wh), where

$$\boxed{1\ Wh = 3{,}600\ J}$$

Example: How much energy does a 100 W electric bulb consume in 2 hours ?

Solution:

$$w = pt = 100 \times 2 = 200\ \underline{\underline{Wh}}$$

or

$$\begin{aligned} w = pt &= (100\,W)(2 \times 60 \times 60) \\ &= 720000\ J \\ &= 720\ kJ \end{aligned}$$

which is the same result (if you convert from joules to watts or vise-versa).

1.1.5 Circuit Elements

: An electric circuit is an interconnection of electrical elementic.

* Circuit analysis is the process of determining voltages across (or the currents through) the elements of the circuit.

Circuit Elements
- Passive Elements (can not generate energy) ⇒ Resistors, capacitors, and inductors
- Active Elements (can generate energy) ⇒ Generators, batteries, operational amplifiers

* The most important active elements are voltage or current sources that generally deliver power to the circuit connected to them. EE1

Voltage (or current) sources — Independent sources / Dependent sources

* An ideal independent source is an active element that provides a specified voltage or current that is completely independent of other circuit variables.

* Dependent sources (or controlled sources) are active elements in which the source quantity is controlled by another voltage or current. (It will be discussed later).

1.2 Basic Laws :

1.2.1 Ohm's Law (1826) :

Ohm's Law states that the voltage $\underline{\underline{V}}$ across a resistor is directly proportional to the current $\underline{\underline{I}}$ flowing through the resistor.

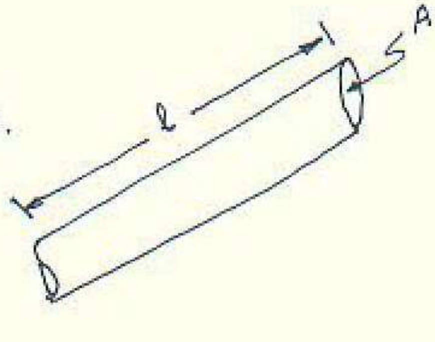
$$V \propto I$$

$$\Rightarrow \boxed{V = IR}$$ قانون أوم

where $\underline{\underline{R}}$ is the resistance. The resistance R denotes the ability of an element to resist the flow of electric current, it is measured in ohms (Ω).

For any material, the resistance R depends on its physical dimensions as follows:

$$R = \rho \frac{\ell}{A}$$



where ρ is the resistivity of the material.

⇒ Good conductors have low resistivities (such copper, aluminum, etc..)

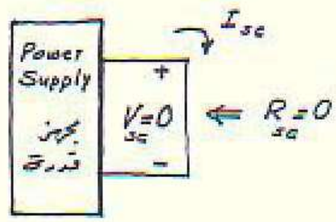
⇒ Insulators have high resistivities (such as mica, paper, etc...)

للمعلومات Resistivities of Common Materials ES1

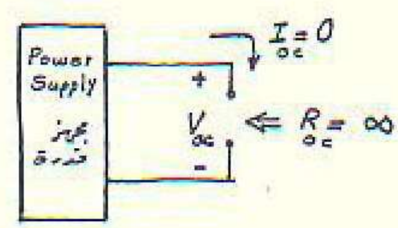
Material	Resistivity ($\Omega . m$)	Usage
Silver	1.64×10^{-8}	Conductor
Copper	1.72×10^{-8}	"
Aluminum	2.80×10^{-8}	"
Gold	2.45×10^{-8}	"
Carbon	4.00×10^{-5}	Semiconductor
Germanium	47.0×10^{-2}	"
Silicon	6.40×10^{-2}	"
Paper	10^{10}	Insulator
Mica	5×10^{11}	"
Glass	10^{12}	"
Teflon	3×10^{12}	"

* The resistance of a short circuit element is approaching zero

* The resistance of an open circuit is approaching infinity.



Short circuit with $R_{sc}=0$
$V_{sc}=0$



Open circuit with $R_{oc}=\infty$
$I_{oc}=0$

* Conductance (G)

A useful quantity in circuit analysis is the reciprocal of resistance (R), is called the conductance (G);

$$G = \frac{1}{R} = \frac{i}{v}$$

The conductance can be explained as the ability of an element to conduct electric current, it is measured in mhos (℧) or in siemens (S).

$$\therefore \quad i = Gv$$

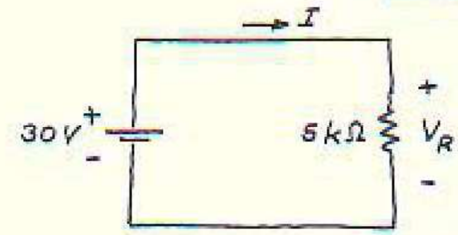
and:

$$p = vi = i^2 R = \frac{v^2}{R} \quad \text{watts (W)}$$

OR

$$p = vi = v^2 G = \frac{i^2}{G}$$

Example: In the circuit shown, calculate the current I, the conductance G, and the power P



Solution:

$V_R = V$

- the current $I = \frac{V_R}{R} = \frac{30}{5 \times 10^3} = 6 \times 10^{-3} = \underline{6\ mA}$

- the conductance $G = \frac{1}{R} = \frac{1}{5 \times 10^3} = 0.2 \times 10^{-3} = 0.2\ mS$

- the power $P = V_R I = 30(6 \times 10^{-3}) = \underline{180\ mW}$

Or

$P = I^2 R = (6 \times 10^{-3})^2 (5 \times 10^3) = \underline{180\ mW}$

or

$P = V_R^2 G = (30)^2 (0.2 \times 10^{-3}) = \underline{180\ mW}$

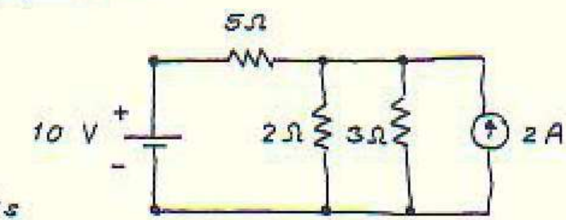
1.2.2 Nodes, branches and loops

* A branch represents a single element in the electric circuit, such as a voltage source or a resistor etc...

* A node represents the point of connection between two or more branches.

* A loop is any closed path in a circuit.

Example: For the circuit shown, determine the number of branches, nodes and the independent loops.

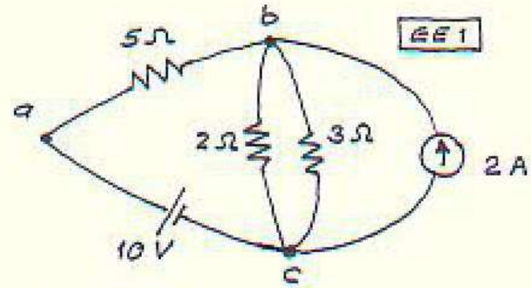


Solution Since there are 5 elements

$\Rightarrow$ Number of branches = $\underline{5}$ 10V, 5Ω, 2Ω, 3Ω, and 2A

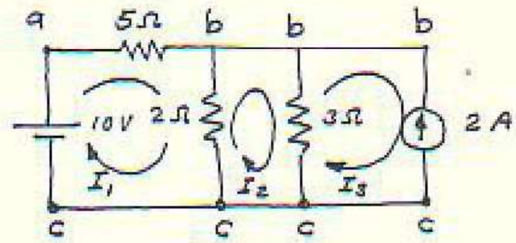
Number of nodes = $\underline{3}$ (as shown in the figure).

3 عقد



⇒ There are 3 nodes :
a , b and c

* The number of the independent loops = 3



⇒ loop 1 or loop abc :
contains (10V, 5Ω, 2Ω)

⇒ loop 2 or loop bcb :
contains (2Ω, 3Ω)

& ⇒ loop 3 or loop bcb :
contains (3Ω, 2A)

Notes : There are more than 3 (dependent) loops in this example, we had only calculated the INDEPENDENT loops which are only 3.

IN GENERAL; Any circuit with b branches, n nodes and ℓ independent loops, the following fundamental theorem of network topology:

$$\boxed{b = \ell + n - 1}$$

* Two or more elements are in **SERIES** if they are cascaded sequentially and consequently carry the **SAME** current.

* Two or more elements are in **PARALLEL** if they are connected to the same two nodes and have consequently the same **VOLTAGE** across them.

1.2.3 Kirchoff's Laws (1847) :

قانون كيرشوف للتيار * Kirchoff's current law (KCL) ; states that the algebraic sum of all current entering a node is zero or : The sum of currents entering a node is equal to the sum of curents leaving that node.

$$\sum_{n=1}^{N} I_n = 0$$

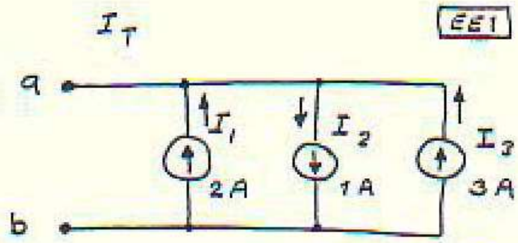
or

$$\boxed{\sum_{m=1}^{M} I_{mi} = \sum_{n=1}^{N} I_{no}}$$

where I_{mi} are the currents entering the node and I_{no} are the currents leaving the node.

EE1

Example: For the network shown, calculate the total current I_T



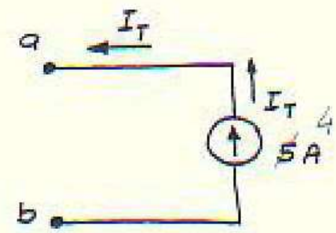
Current Sources in Parallel

Solution

According to KCL;

$$I_T = I_1 - I_2 + I_3 \quad 4$$
$$= 2 - 1 + 3 = \underline{5\ A}$$

∴ The equivalent circuit for the network can be as shown ⇒



* Kirchoff's Voltage Law (KVL); states that the algebraic sum of all voltages around a closed path (or loop) is zero.

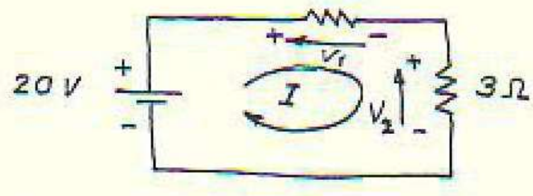
قانون كيرشوف للفولتية

∴ Mathematically KVL states that:

$$\boxed{\sum_{m=1}^{M} V_m = 0}$$

where M is the number of voltages in the loop (or the number of branches in the loop), and V_m is the $\underline{mth}$ voltage.

Example: For the circuit shown, find the voltages V_1 and V_2



Solution:

$$V_1 = 2I$$
$$V_2 = 3I$$

From KVL:

$$\sum V = 0 \Rightarrow 20 - V_1 - V_2 = 0 \Rightarrow 20 = 3I + 2I$$

$$\Rightarrow 5I = 20 \Rightarrow I = 4\ A$$

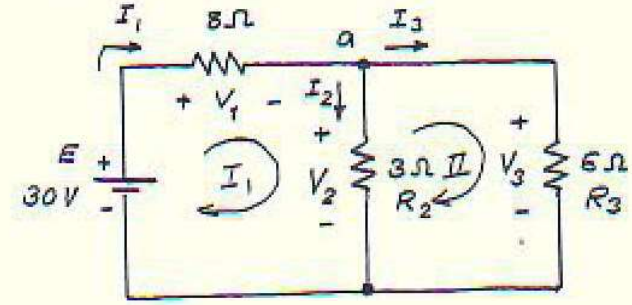
$$\therefore V_1 = 2I = \underline{8V} \quad \text{and} \quad V_2 = 3I = \underline{12\ V}$$

Tutorial Sheet № 1

Basic Concepts & Basic Laws

TS1

Example Using Kirchoff's laws, find the currents and voltages in the circuit shown.



Solution :

ملاحظة : المطلوب في المثال ايجاد كل من :
I_1 , I_2 , I_3 , V_1 , V_2 & V_3
باستخدام قانوني كيرشوف

* Using Ohm's law:

$$V_1 = I_1 R_1 = 8 I_1$$
$$V_2 = I_2 R_2 = 3 I_2$$
$$V_3 = I_3 R_3 = 6 I_3$$

* Applying KCL at node $\underline{\underline{a}}$:

$$I_1 = I_2 + I_3 \Rightarrow \boxed{I_1 - I_2 - I_3 = 0}$$

Eq(1)

* Applying KVL to loop 1 ;

$$E - V_1 - V_2 = 0 \Rightarrow 30 - V_1 - V_2 = 0$$

$$\Rightarrow \quad 30 - 8I_1 - 3I_2 = 0$$

$$\therefore \boxed{I_1 = \frac{30 - 3I_2}{8}} \quad Eq(2)$$

* Applying KVL to loop 2 ;

$$V_2 - V_3 = 0 \Rightarrow V_2 = V_3$$

وهذا واضح حيث R_2 & R_3 متوازيتان

$$\therefore 6I_3 = 3I_2$$

$$\boxed{\therefore I_3 = \frac{I_2}{2}} \quad Eq(3)$$

$\Rightarrow$ From Eq(1), Eq(2) & Eq(3)

$$\frac{30 - 3I_2}{8} - I_2 - \frac{I_2}{2} = 0 \quad \Rightarrow I_2 = \underline{\underline{2\ A}}$$

$$\text{and} \quad I_1 = \frac{30 - 3I_2}{8} = \frac{30 - 3(2)}{8} = \underline{\underline{3\ A}}$$

$$I_3 = \frac{I_2}{2} = \frac{2}{2} = 1\ A$$

TS1

$\Rightarrow \quad \because V_1 = 8 I_1 \quad \Rightarrow \quad \therefore V_1 = 8(3) = 24\ V$

Similarly $\quad V_2 = 3 I_2 = 3(2) = 6\ V$

and $\quad V_3 = 6 I_3 = 6(1) = 6\ V$

للتأكد من صحة الحل :

$I_1 = I_2 + I_3$

$3 = 2 + 1 \quad \Rightarrow \quad 3 = \underline{\underline{3}}$

كذلك فإن تطبيق قانون كيرشوف للفولتية على المسار المغلق رقم 1 ينتج ما يأتي :

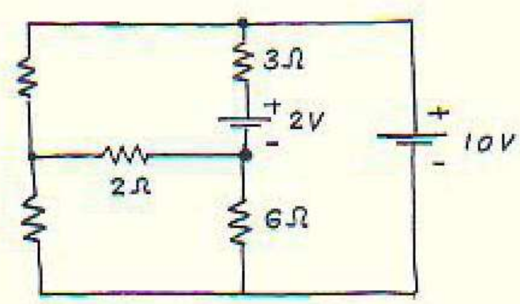
$E = V_1 + V_2$

$\therefore 30 = 24 + 6$

$\therefore 30 = 30$

∴ الحل صحيح

<u>Example</u>: Determine the number of branches, nodes and independent loops in the circuit shown.



<u>Solution</u>:

* There are $\underline{\underline{7}}$ element $\Rightarrow$ no. of branches $= \underline{\underline{7}}$

$\Rightarrow \underline{b = 7}$

a, b, c, d, e $\Leftarrow$ * There are 5 nodes as shown in the figure :

$\Rightarrow n = 5$

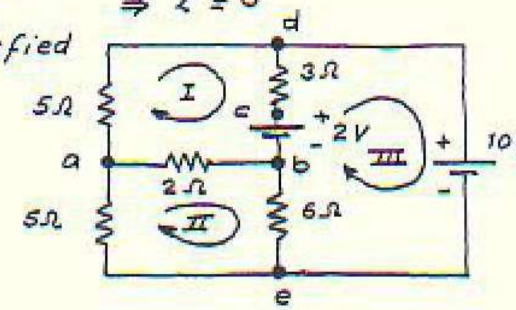
I, II, III $\Leftarrow$ * There are 3 independent loops :

$\Rightarrow \ell = 3$

$\therefore b = \ell + n - 1$ is satisfied

since $7 = 3 + 5 - 1$

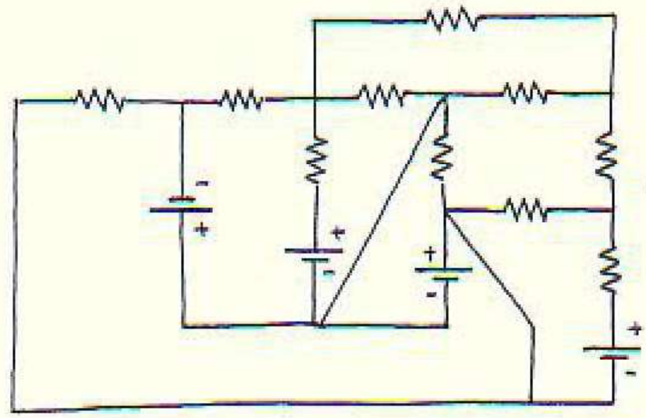
$\Rightarrow 7 = 7$



TS1

Practice Problem

: Identify all nodes, branches and independent loops in the circuit shown in the figure.



Answer

: no. of nodes = 8 $\qquad n = 8$

no. of branches = 14 $\qquad b = 14$

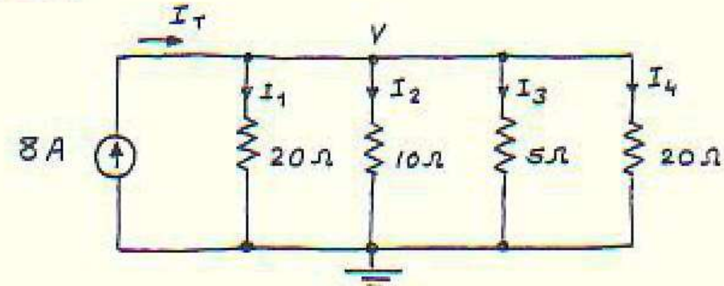
no. of independent loops = 7 $\qquad \ell = 7$

Check. Does this satisfy the fundamental theorem of network topology?

$$b = \ell + n - 1 = 7 + 8 - 1 = 14 \quad \underline{\text{YES}}$$

Example

: Determine all currents and voltages in the circuit of the figure shown.



Solution

: KCL ⇒

$$I_T = I_1 + I_2 + I_3 + I_4 \quad \Rightarrow \quad 8 = I_1 + I_2 + I_3 + I_4$$

Ohm's law

$$\begin{aligned} V &= 20\,I_1 & \Rightarrow \quad I_1 &= V/20 \\ &= 10\,I_2 & I_2 &= V/10 \\ &= 5\,I_3 & I_3 &= V/5 \\ &= 20\,I_4 & I_4 &= V/20 \end{aligned}$$

TS1

Substituting in the current equation;

$$\Rightarrow \quad 8 = \frac{V}{20} + \frac{V}{10} + \frac{V}{5} + \frac{V}{20}$$

$$\therefore \quad 160 = V + 2V + 4V + V$$

$$160 = 8V$$

$$\therefore \quad \underline{V = 20 \text{ Volts}}$$

$$\therefore \quad I_1 = \frac{V}{20} = \frac{20}{20} = 1\ A$$

$$I_2 = \frac{V}{10} = \frac{20}{10} = 2\ A$$

$$I_3 = \frac{V}{5} = \frac{20}{5} = 4\ A$$

$$I_4 = \frac{V}{20} = \frac{20}{20} = 1\ A$$

<u>Check</u> $\quad I_T = I_1 + I_2 + I_3 + I_4$

$$8 = 1 + 2 + 4 + 1$$

$$8 = 8 \quad \checkmark$$

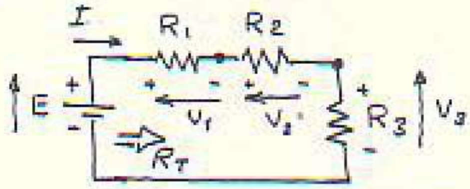
EE2

2. Circuit Transformations

2.1 Series Circuits

Series Circuits: Two elements are in series if they have only one point in common that is not connected to other current carrying elements of the network.

For the series cct. shown, using KVL we have:



$$E = V_1 + V_2 + V_3$$
$$= IR_1 + IR_2 + IR_3$$
$$= I(R_1 + R_2 + R_3)$$
$$= IR_T$$

$$\Rightarrow \boxed{R_T = R_1 + R_2 + R_3}$$

$$\therefore \boxed{I = \frac{E}{R_T}}$$

In general, for a series cct. consisting N resistors, then the total resistance of such a cct. R_T is given as

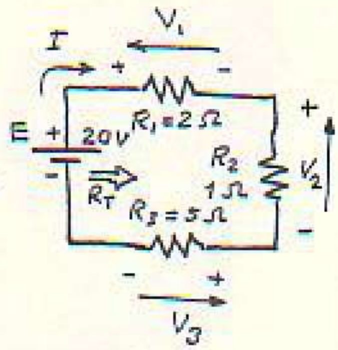
$$R_T = R_1 + R_2 + R_3 + \cdots R_N$$

Example: For the cct. shown;

a. Find the total resistance

b. Calculate the current I

c. Determine the voltages V_1, V_2 and V_3



Solution:

a. $R_T = R_1 + R_2 + R_3$
$= 2 + 1 + 5 = \underline{8\,\Omega}$

b. $I = \frac{E}{R_T} = \frac{20}{8} = \underline{2.5\text{ A}}$

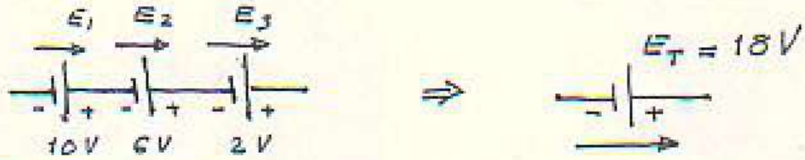
c. $V_1 = IR_1 = (2.5)(2) = 5\text{ V}$
$V_2 = IR_2 = (2.5)(1) = 2.5\text{ V}$
$V_3 = IR_3 = (2.5)(5) = 12.5\text{ V}$

ملاحظة
$V_1 + V_2 + V_3 = E$
$5 + 2.5 + 12.5 = 20\text{ V}$
$\therefore 20\text{ V} = 20\text{ V}$

2.1.1 Voltage Sources in Series

The net voltage will the algebraic sum of all sources that are connected in series.

Example:



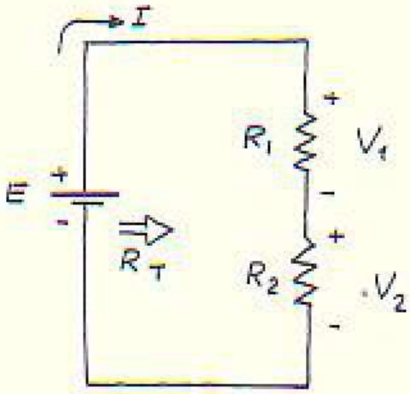
$$E_T = E_1 + E_2 + E_3$$
$$= 10 + 6 + 2 = 18\text{V}$$

* ملاحظة: لا يمكن ربط مصادر التيار بالتوالي لأن ذلك مخالف لقانون كيرشوف للتيار.

EE2

2.1.2 Voltage Divider Rule

: Consider the series cct. shown.



we have ;

$$R_T = R_1 + R_2$$

and

$$I = \frac{E}{R_T}$$

$$V_1 = IR_1 = \left(\frac{E}{R_T}\right) . R_1$$

$$= \frac{E . R_1}{R_T}$$

Similarly ;

$$V_2 = IR_2 = \left(\frac{E}{R_T}\right) . R_2$$

$$= \frac{E . R_2}{R_T}$$

Hence, we can write:

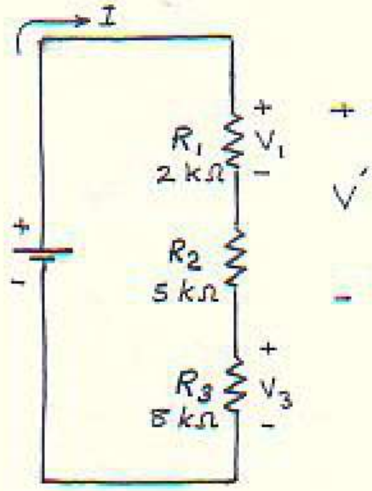
$$\boxed{V_x = \frac{E . R_x}{R_T}}$$

⇐ Voltage divider rule

This means that "the voltage divider rule" can be understood to state that :

> The voltage across a resistor in a series circuit is equal to the value of that resistor times the the total applied voltage across the series elements divided by the total resistance of the series elements.

Example : Determine the voltages V_1, V_3, and V' for the cct. shown.



Solution : Using the voltage divider rule:

$$V_1 = \frac{E . R_1}{R_T}$$

$$= \frac{(45).(2 \times 10^3)}{(2+5+8) \times 10^3}$$

$$= \underline{6\ V}$$

$$V_3 = \frac{E . R_3}{R_T} = \frac{(45)(8 \times 10^3)}{(2+5+8) \times 10^3}$$

$$= \underline{24\ V}$$

EE2

$$V' = \frac{E \cdot R'}{R_T} \Leftarrow R' = (2+5)\times 10^3\ \Omega = 7\times 10^3\ \Omega$$

$$= \frac{(45)\cdot(7\times 10^3)}{(2+5+8)\times 10^3}$$

$$= \underline{21\ V}$$

نلاحظ ان :

$$E = V_3 + V'$$
$$45\ V = 24\ V + 21\ V$$
$$45\ V = 45\ V$$

نلاحظ ان فرق الجهد بين طرفي اي عنصر من عناصر دائرة التوالي يتناسب مع مقاومة ذلك العنصر ، أي ان المقاومة الكبيرة يقابلها فرق جهد كبير والمقاومة الصغيرة يقابلها فرق جهد صغير ، وفي جميع الاحوال يجب ان يكون مجموع فروق الجهد مساوياً الى فولتية المصدر .

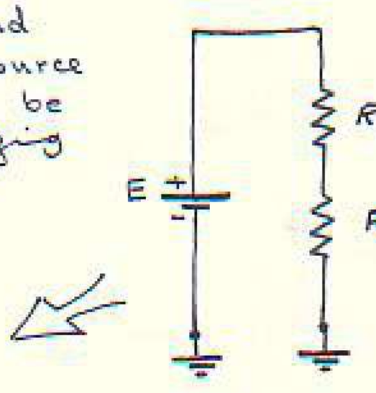
NOTATION

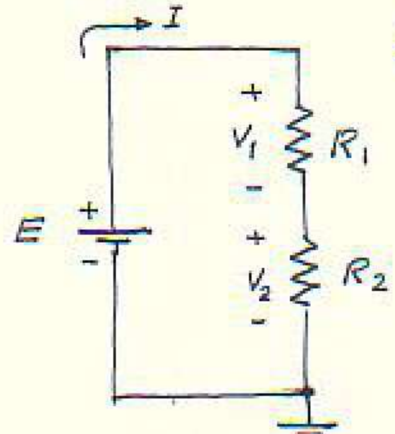
ground potential

It is common, for safety purposes and as a reference to ground electrical and electronic systems. The symbol for the ground connection is :

with its defined potential level (zero volts). As a consequence, the cct. might need to be redrawn in the ordinary form so as to be analyzed.

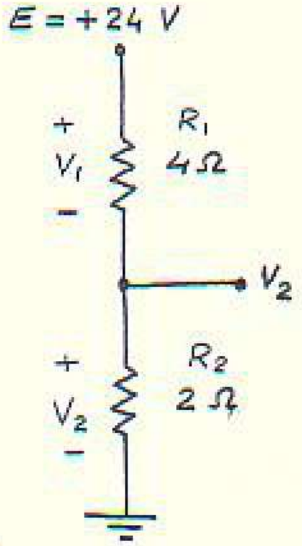
Example : For the cct. shown ; with ground potentials connected for the source and to elements; this cct.can be redrawn to make it easy analyzing it.





EE2

Example: Using the voltage divider rule, determine the voltages V_1 and V_2 for the cct shown:

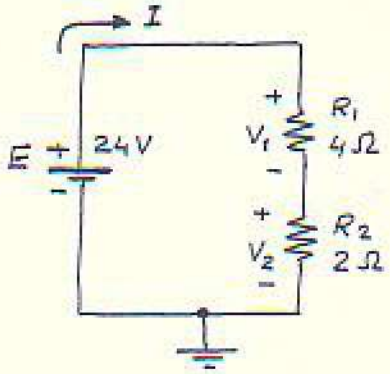


Solution: Redrawing the cct. with the standard battery symbol, then the cct. will be as shown below:

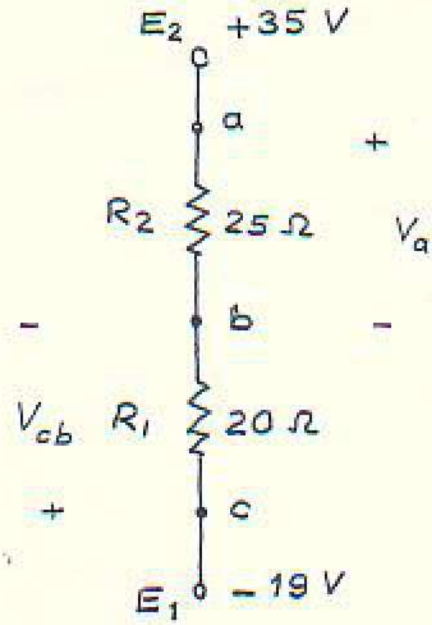
$$\therefore \; V_1 = \frac{R_1 E}{R_T} = \frac{(4)\cdot(24)}{4+2} = \underline{16\ V}$$

and

$$V_2 = \frac{R_2 E}{R_T} = \frac{(4).(24)}{4+2} = \underline{8\ V}$$

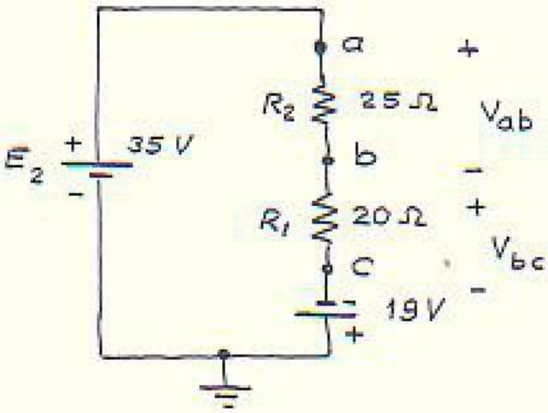


Example: For the cct. shown, determine V_{ab}, V_{cb} and V_b.



EE2

Solution: The cct. is redrawn as shown;



$$\therefore I = \frac{E_1 + E_2}{R_T} = \frac{19 + 35}{20 + 25}$$
$$= 1.2 \text{ A}$$

$$V_{ab} = IR_2 = (1.2)(25)$$
$$= 30 \text{ V}$$

and $V_{cb} = -V_{bc} = -IR_1$
$$= -(1.2)(20)$$
$$= -24 \text{ V}$$

$$V_c = -E_1 = -19 \text{ V}$$

ملاحظة ١/ : يمكن الحل باستخدام voltage divider rule مع ملاحظة ان $V_{cb} = -V_{bc}$.

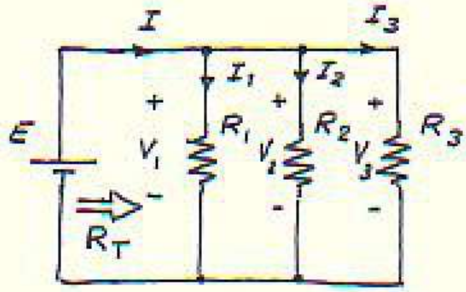
ملاحظة ٢/ : ان V_{ab} تعني فرق جهد النقطة a عن النقطة b ، فاذا كانت V_{ab} موجبة فان هذا يعني ان V_a اكبر من V_b

ملاحظة ٣/ : ان V_a تعني جهد النقطة a بالنسبة الى الارضي .

2.2 Parallel Circuits

: Two branches or elements or networks are in parallel if they have two points in common.

For the parallel cct. shown ; using KCL , we have :



$$I = I_1 + I_2 + I_3$$

$$= \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}$$

since $V_1 = V_2 = V_3 = E = V$

$$\therefore \; I = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$E = V_1 = V_2 = V_3$$

$$\therefore \; \frac{V}{R_T} = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) \qquad I = \frac{E}{R_T} = \frac{V}{R_T}$$

$$\therefore \; \frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

In general , for N resistors connected in parallel , then :

$$\boxed{\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots \frac{1}{R_N}}$$

Notation Conductance G

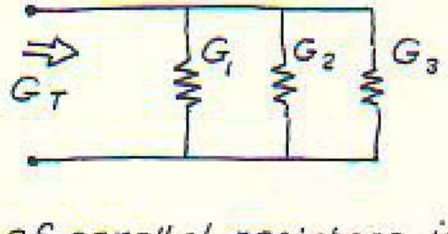
For parallel networks , it is common to use the idea of conductane in the cct. analysis . The conductance (G) is defined as :

$$G = \frac{1}{R} \qquad \text{Siemens} \quad (S)$$

So, we can write the total conductance G_T for the parallel cct shown, as :

$$G_T = G_1 + G_2 + G_3$$

$$\Rightarrow \; R_T = \frac{1}{G_T}$$



NOTE that , the total resistance R_T of parallel resistors is always less than the value of the smallest resistor .

EE2

Special Cases : The general relation for the total resistance of parallel resistors is :

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_N}$$

* **Case 1** For equal resistors in parallel , ie, when

$$R_1 = R_2 = R_3 \cdots R_N = R$$

then ; $$\frac{1}{R_T} = \underbrace{\frac{1}{R} + \frac{1}{R} + \frac{1}{R} + \cdots \frac{1}{R}}_{N}$$

$$\therefore \frac{1}{R_T} = N\left(\frac{1}{R}\right) \Rightarrow \boxed{R_T = \frac{R}{N}}$$

* **Case 2** : For two resistors in parallel , then R_T is given as :

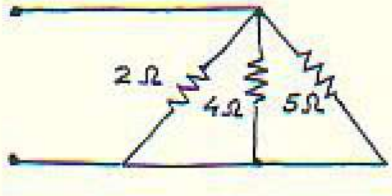
$$\boxed{R_T = \frac{R_1 R_2}{R_1 + R_2}}$$

This means that, the total resistance of the two parallel resistors is the product of the two divided by their sum.

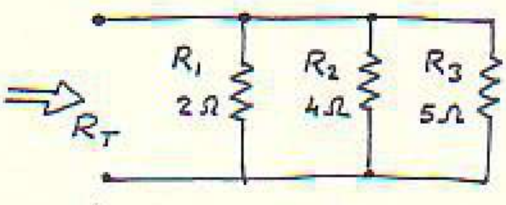
* **Case 3** : For three resistors in parallel, then R_T is given as :

$$\boxed{R_T = \frac{R_1 R_2 R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3}}$$

Example : Determine the total resistance for the network shown :



Solution Redraw the cct. to be as shown ;



$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$
$$= \frac{1}{2} + \frac{1}{4} + \frac{1}{5}$$
$$= 0.5 + 0.25 + 0.2$$
$$= 0.95 \Rightarrow \therefore R_T = \frac{1}{0.95} = \underline{1.053\ \Omega}$$

Example: For the parallel network shown;

a. Calculate R_T

b. Determine I_T

c. Calculate I_1 and I_2

d. Determine the power to each resistive load.

e. Determine the power delivered by the source and compare it with the total power dissipated by the resistive elements.

Solution:

a. $$R_T = \frac{R_1 R_2}{R_1 + R_2} = \frac{(9)(18)}{9+18} = \frac{162}{27} = \underline{6\ \Omega}$$

b. $$I_T = \frac{E}{R_T} = \frac{27}{6} = \underline{4.5\ A}$$

c. $$I_1 = \frac{V_1}{R_1} = \frac{E}{R_1} = \frac{27}{9} = \underline{3\ A}$$

and

$$I_2 = \frac{V_2}{R_2} = \frac{E}{R_2} = \frac{27}{18} = \underline{1.5\ A}$$

نلاحظ ان : $I_T = I_1 + I_2$

d. $$P_1 = I_1 V_1 = E I_1 = (27)(3) = \underline{81\ W}$$

$$P_2 = I_2 V_2 = E I_2 = (27)(1.5) = \underline{40.5\ W}$$

e. $$P_s = E I_T = (27)(4.5) = 121.5\ W$$

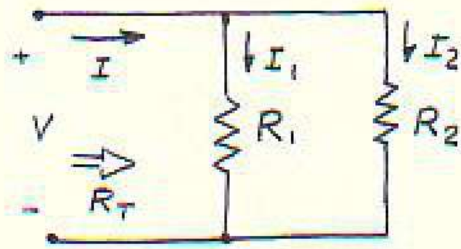
$$P_s = P_1 + P_2 = 81 + 40.5 = 121.5\ W$$

* نلاحظ ان مجموع القدرة المستهلكة في المقاومتين R_1 و R_2 يساوي القدرة التي يجهزها المصدر E

2.2.1 Current Divider Rule

Consider the parallel cct. shown;

We have; $I = \frac{V}{R_T} \Rightarrow V = I R_T$



and $$R_T = \frac{R_1 R_2}{R_1 + R_2}$$

but $$I_1 = \frac{V}{R_1} = \frac{I R_T}{R_1} = \frac{I \cdot \frac{R_1 R_2}{R_1 + R_2}}{R_1}$$

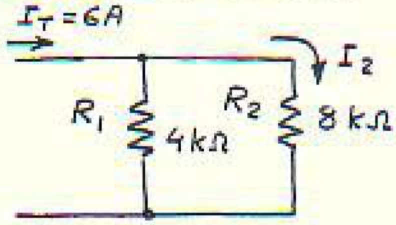
$$\therefore \boxed{I_1 = I \cdot \frac{R_2}{R_1 + R_2}}$$

Similarly, for I_2, we have:

$$I_2 = \frac{V}{R_2} = \frac{I R_T}{R_2} = I . \frac{\frac{R_1 R_2}{R_1 + R_2}}{R_2}$$

$$\therefore \boxed{I_2 = I . \frac{R_1}{R_1 + R_2}}$$

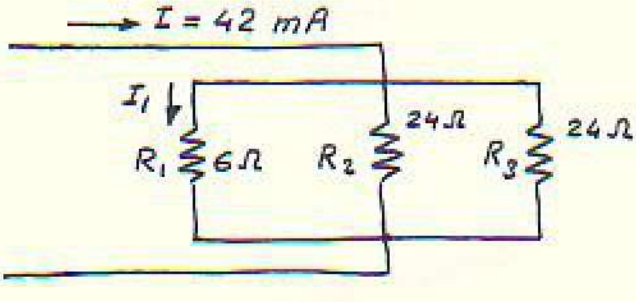
<u>Example</u>: Determine the current I_2 for the network shown;



<u>solution</u>: $I_2 = I_T . \frac{R_1}{R_1 + R_2}$

$$= 6 \frac{4 \times 10^3}{(4+8) \times 10^3} = 6 . \frac{4}{12} = \underline{2\ A}$$

<u>Example</u>: Find the current I_1 for the network shown;



<u>Solution</u>:

$$I_1 = I . \frac{R_T}{R_1}$$

$$R_T = \frac{R_1 R_2 R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

or

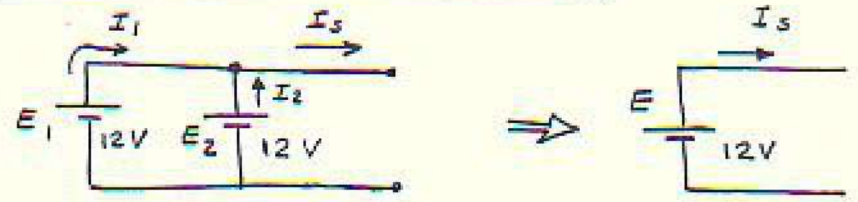
$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{6} + \frac{1}{24} + \frac{1}{24}$$

$$\Rightarrow R_T = 4\,\Omega$$

$$\therefore I_1 = 42 \times 10^{-3} \frac{4}{6}$$

$$= 28 \times 10^{-3} = \underline{28\ mA}$$

2.2.2 Voltage Sources in Parallel :



$E_1 = E_2 = E$ and $I_s = I_1 + I_2$

To increase the current rating of the source, two or more batteries in parallel of the same terminal voltage would be used.

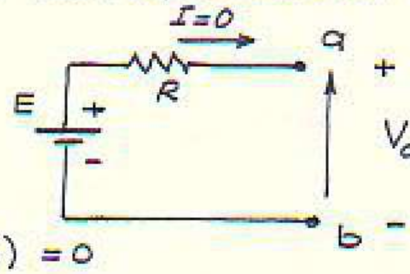
2.3 Open and Short Circuits

: We often need to apply the open and short circuits in the analysis fof electric networks.

* **Open Circuit** : An open circuit is simply two isolated terminals not connected by an element of any kind.

Consider the circuit shown ; with open circuit terminals a and b.

$$V_{\text{open circuit}} = V_{oc} = V_{ab} = E$$

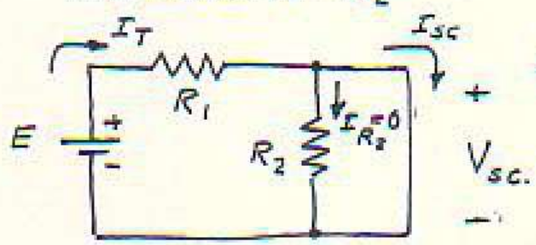


since I (in the open circuit) = 0

In general : An open circuit CAN HAVE a potential difference (voltage) across its terminals but the current is always ZERO.

* **Short Circuit** : A short circuit is a direct connection of zero ohms across an element or combination of elements.

Consider the circuit shown , with a short circuit across the resistor R_2

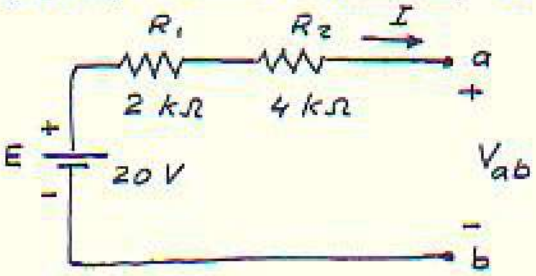


$$I_{sc} = I_T = \frac{E}{R_1} \quad \text{since } I_{R_2} = 0$$

$$V_{\text{short cct.}} = V_{sc} = \underline{\underline{0}}$$

In general: A short circuit CAN CARRY a current of any level but the potential difference (voltage) across its terminals is always ZERO.

Examples: (a). For the network shown, determine V_{ab}.

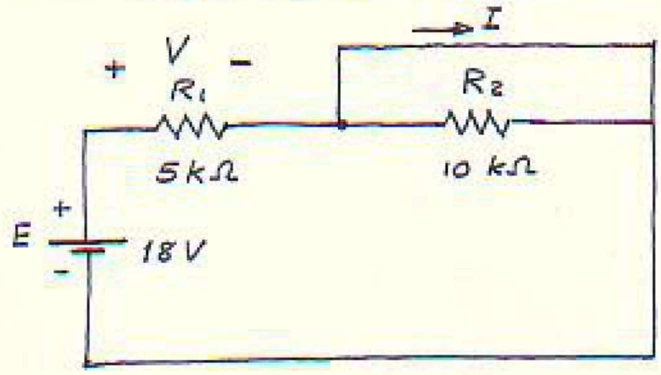


Solution: - We have an open cct across the terminal a and b, the

$I = 0 \Rightarrow V_1 = 0$ and $V_2 = 0$

- Applying KVL $\Rightarrow V_{ab} = E = 20\,V$

(b). Calculate I and V for the network shown;



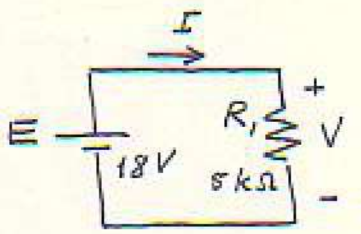
Solution: - We have a short cct. across R_2

$\Rightarrow$ No current through R_2

Note: the cct can be redrawn to be as shown

$$\Rightarrow I = \frac{E}{R_T} = \frac{E}{R_1 + 0} = \frac{18}{5\,k\Omega}$$

$$= \underline{3.6\ mA}$$



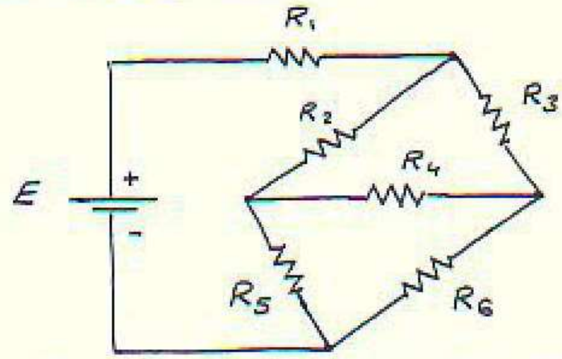
$\Leftrightarrow$

$$\therefore V = I R_1 = E = \underline{18\,V}$$

Wye - Delta Transformation

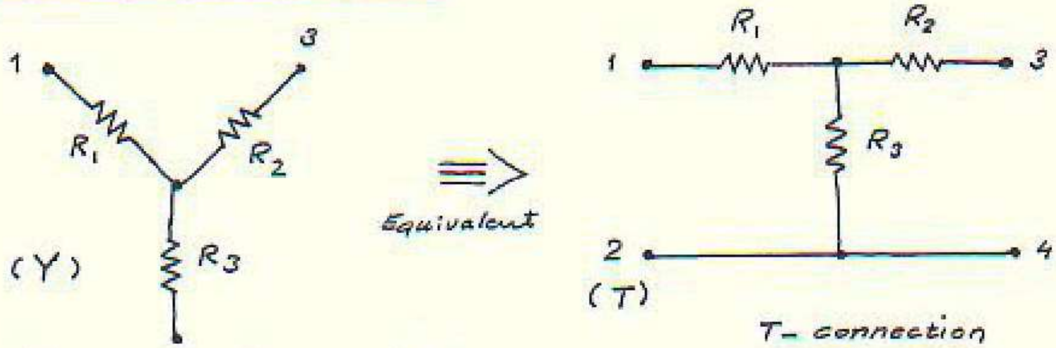
EE2

: There are some cases often arises in circuit analysis, when the resistors are neither in parallel nor in series. For example, consider the circuit shown :



In this circuit, R_1, R_2, R_3 ... R_6 are neither in series nor in parallel

* Wye (Y or star) connection

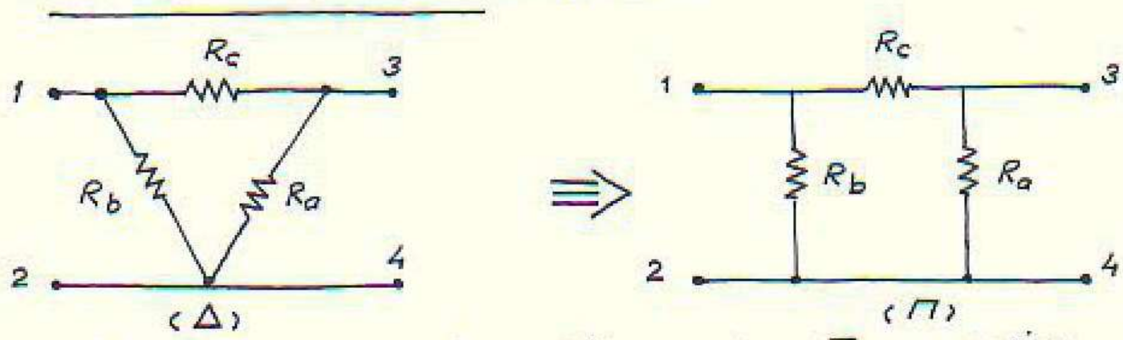


Y or star circuit connection

T- connection

Y and T connections

* Delta (Δ or π) circuit connection



Delta circuit connection & its equivalen Π- connection

Δ $\qquad$ Y

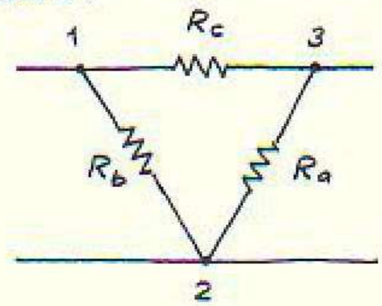
* Delta to Wye Transformation :

- We have Δ and want to get its equivalent star circuit
- Consider the Δ circuit shown; to be transformed into its equivalent star shown below:

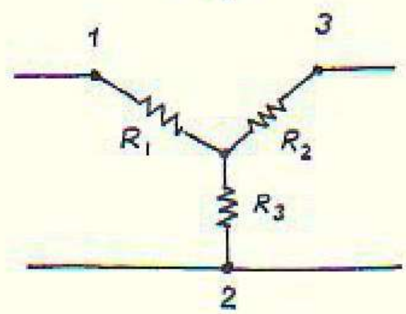
$$R_{12} = \frac{R_b(R_a + R_c)}{R_a + R_b + R_c}$$

$$R_{13} = \frac{R_c(R_a + R_b)}{R_a + R_b + R_c}$$

$$R_{23} = \frac{R_a(R_b + R_c)}{R_a + R_b + R_c}$$



Delta



$$R_{12} = R_1 + R_3$$

$$R_{13} = R_1 + R_2$$

$$R_{23} = R_2 + R_3$$

$$\therefore \quad R_1 + R_3 = \frac{R_b(R_a + R_c)}{R_a + R_b + R_c} \qquad \cdots (1)$$

and

$$R_1 + R_2 = \frac{R_c(R_b + R_c)}{R_a + R_b + R_c} \qquad \cdots (2)$$

$$R_2 + R_3 = \frac{R_a(R_b + R_c)}{R_a + R_b + R_c} \qquad \cdots (3)$$

Subtraction Eq.(3) from Eq.(1) and adding the resulting equation to Eq.(1) results in:

$$\boxed{R_1 = \frac{R_b R_c}{R_a + R_b + R_c}}$$

Similarly;

$$\boxed{R_2 = \frac{R_c R_a}{R_a + R_b + R_c}}$$

and;

$$\boxed{R_3 = \frac{R_a R_b}{R_a + R_b + R_c}}$$

IN GENERAL

> Each resistor in the Y network is the product of the resistors in the two adjacent Δ branches, divided by the sum of the three resistors

Wye to Delta Transformation

- We have Y connected circuit and want to get its equivalent Δ.
- Consider the Y circuit shown, its equivalent Δ is shown below;

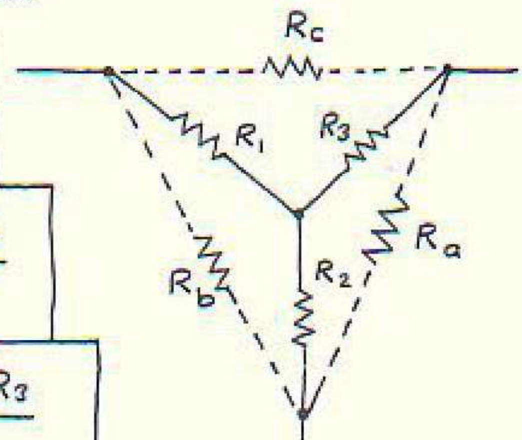
Using the previous sets of equations, then we have:

$$\boxed{R_a = \frac{R_1R_2 + R_2R_3 + R_1R_3}{R_1}}$$

$$\boxed{R_b = \frac{R_1R_2 + R_2R_3 + R_1R_3}{R_3}}$$

and

$$\boxed{R_c = \frac{R_1R_2 + R_2R_3 + R_1R_3}{R_2}}$$



IN GENERAL

> Each resistor in the $\underline{\Delta}$ network is the sum of all possible products of $\underline{Y}$ resistors taken two at a time, divided by the opposite $\underline{Y}$ resistor.

Notes:

* The Y and Δ networks are said to be balanced when:

$$R_1 = R_2 = R_3 = R_Y$$

and

$$R_a = R_b = R_c = R_\Delta$$

* Under balance condition, the conversion equations become:

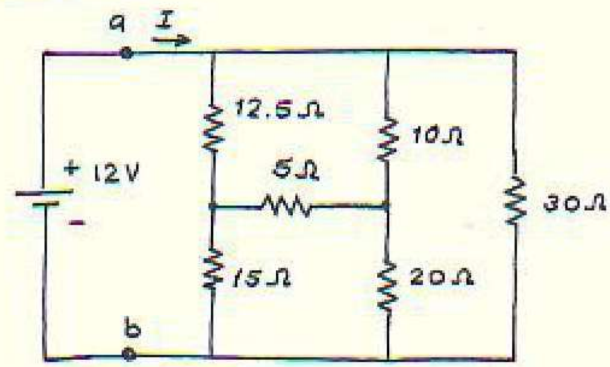
$$\boxed{R_Y = \frac{R_\Delta}{3}}$$

or

$$\boxed{R_\Delta = 3R_Y}$$

Example

Obtain the quivalent resistance R_{ab} for the circuit shown and use it to find the current I



Solution

* We can't use the relations of series connected or parallel connected resistors to obtain R_{ab}.
* We try to use Δ-Y transformations or Y-Δ to get R_{ab}.

EE2

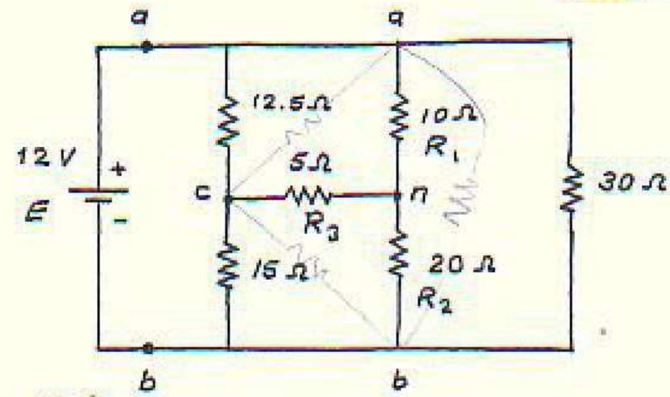
* If we transform the Y consisting of :

$R_1 = 10\,\Omega$

$R_2 = 20\,\Omega$

and $R_3 = 5\,\Omega$

∴ the equivalent Δ circuit contains :



$$R_a = \frac{R_1R_2 + R_2R_3 + R_1R_3}{R_1}$$

$$= \frac{10\times20 + 20\times5 + 5\times10}{10} = \frac{350}{10}$$

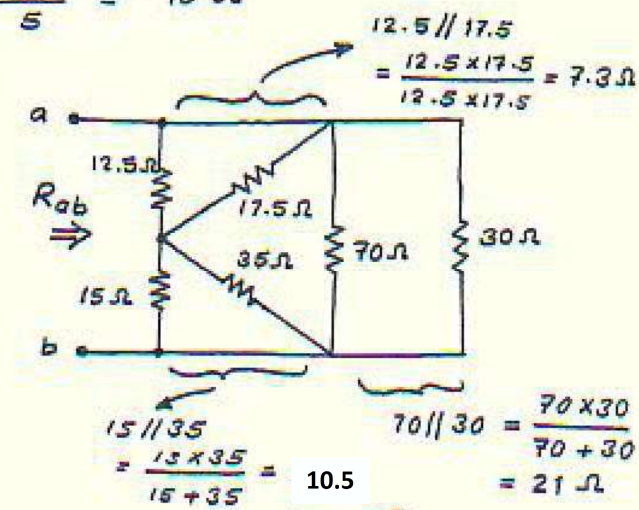
$$= 35\,\Omega$$

Similarly;

$$R_b = \frac{R_1R_2 + R_2R_3 + R_1R_3}{20} = \frac{350}{20} = 17.5\,\Omega$$

and:

$$R_c = \frac{R_1R_2 + R_2R_3 + R_1R_3}{5} = \frac{350}{5} = 70\,\Omega$$



$$\therefore R_{ab} = (7.3 + 10.5) \,//\, 30$$

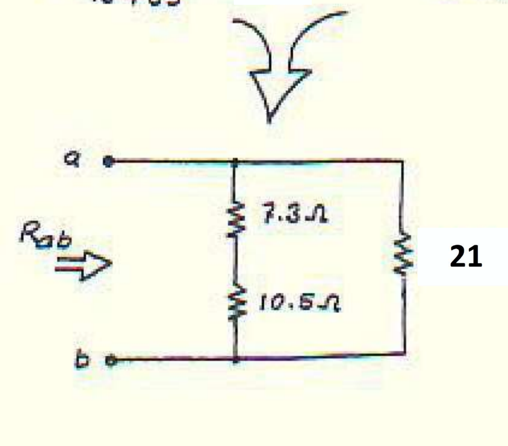
$$= \frac{17.8 \times 21}{17.8 + 21}$$

$$\Rightarrow R_{ab} = \underline{9.632\,\Omega}$$

$$\therefore I = \frac{E}{R_{eq}} = \frac{E}{R_{ab}}$$

$$= \frac{12}{9.632}$$

$$\Rightarrow I = \underline{1.246\ A}$$



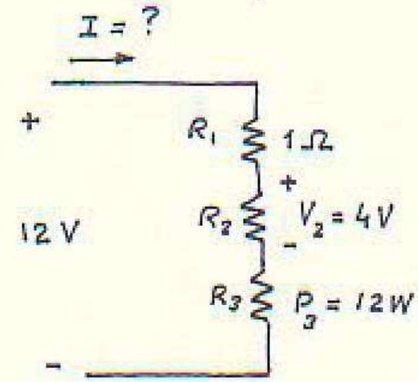
Tutorial Sheet № 2

ملاحظة : جميع اسئلة الكتاب والامثلة المحلولة فيه مطلوبة

Example 1: Three resistors are connected in series across a 12 V battery. The first resistor has a value of 1Ω, the second has a voltage drop of 4 V, and the third has a power dissipation of 12 W.

Calculate the value of the circuit current.

Solution:



$R_2 = ?$
$R_3 = ?$

We have

$$P_3 = I^2 R_3 = 12\ W \quad \text{——} (1)$$

and

$$V_2 = I R_2 = 4\ V \quad \text{——} (2)$$

From (2)

$$\therefore\ I = \frac{4}{R_2}$$

From (1) & (2)

$$\left(\frac{4}{R_2}\right)^2 R_3 = 12$$

$$\therefore\ R_3 = \frac{3}{4} R_2^2$$

From the circuit shown, we have:

$$12 = I\,(R_1 + R_2 + R_3) = I\,(1 + R_2 + R_3)$$

Substituting for I and R_2, we have:

$$12 = \frac{4}{R_2}\left(1 + R_2 + \frac{3}{4} R_2^2\right)$$

$$\therefore\ 3R_2^2 - 8R_2 + 4 = 0$$

$$\therefore\ R_2 = \frac{8 \pm \sqrt{64 - 48}}{6} = 2\Omega \quad \text{or} \quad \frac{2}{3}\Omega$$

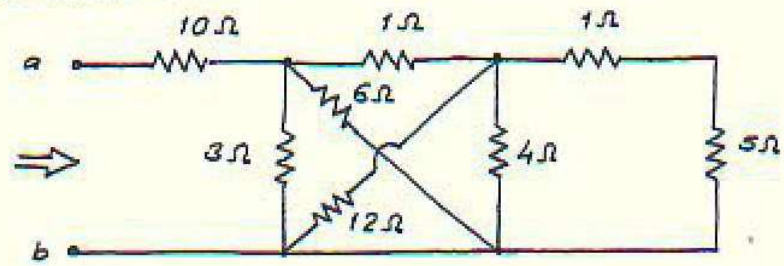
$$\therefore\ R_3 = \frac{3}{4} R_2^2 \implies R_3 = 3\Omega \quad \text{or} \quad \frac{1}{3}\Omega$$

$$\therefore\ I = \frac{12}{R_1 + R_2 + R_3} = \frac{12}{1 + 2 + 3} = \underline{2\ A}$$

or

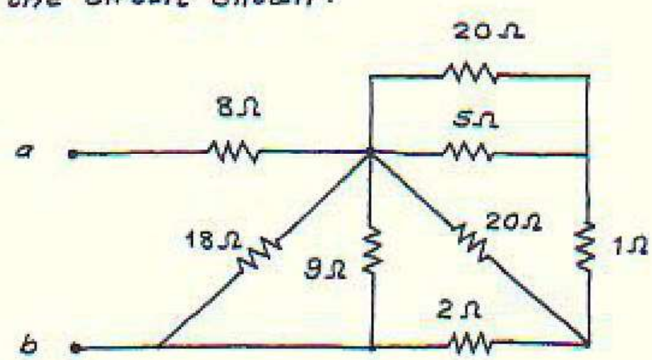
$$I = \frac{12}{1 + \left(\frac{2}{3}\right) + \left(\frac{1}{3}\right)} = \underline{6\ A}$$

Practice Problem : Calculate the equivalent resistance R_{ab} in the circuit shown.



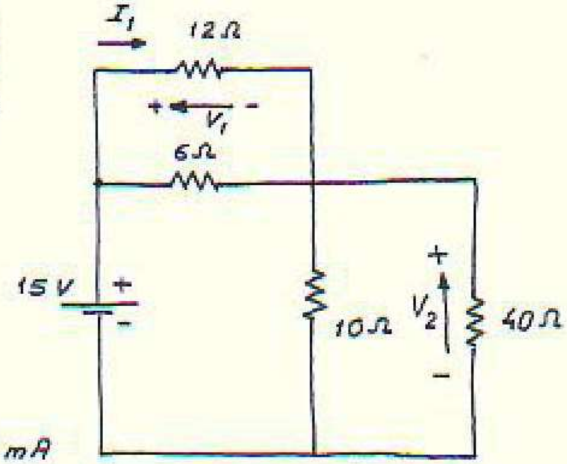
Answer : $R_{ab} = 11.2\ \Omega$

Practice Problem : Find R_{ab} for the circuit shown:



Answer : $R_{ab} = 11\ \Omega$

Practice Problem : Find V_1 and V_2 in the circuit shown. Also calculate I_1 and I_2 and the power dissipated in the 12 Ω and 40 Ω resistors.



Answer :

$V_1 = 5V$, $V_2 = 10V$

$I_1 = 416.7\ mA$, $I_2 = 250\ mA$

$P_1 = 2.083\ W$, $P_2 = 2.5\ W$

3. Techniques of Circuit Analysis

EE3

3.1 Determinants

: Consider the two simultaneous equations

$$a_1 x + b_1 y = C_1$$

and

$$a_2 x + b_2 y = C_2$$

where x and y are the unknown variables, and a_1, a_2, b_1, b_2, C_1 and C_2 are constants.

Using the determinants, the following formats are obtained for each of the variables; x and y:

$$x_1 = \frac{\Delta_1}{\Delta} = \frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

where; $\Delta_1 = c_1 b_2 - c_2 b_1$

$\Delta = a_1 b_2 - a_2 b_1$

$$y = \frac{\Delta_2}{\Delta} = \frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

where; $\Delta_2 = a_1 c_2 - a_2 c$

Δ, Δ_1 and Δ_2 are called second order determinants, since it contains two rows and two columns.

Third order determinants are used to solve three simultaneous linear equations. Consider, the following three simultaneous equations:

$$a_1 x + b_1 y + c_1 z = d_1$$

$$a_2 x + b_2 y + c_2 z = d_2$$

$$a_3 x + b_3 y + c_3 z = d_3$$

The unkown variables, x, y, and z are determined as follows:

$$x_1 = \frac{\Delta_1}{\Delta} = \frac{\begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}}$$

$$y = \frac{\Delta_2}{\Delta} = \frac{\begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}}{\Delta}$$

$$z = \frac{\Delta_3}{\Delta} = \frac{\begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}}{\Delta}$$

The third order determinant can be evaluated as:

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \Longrightarrow \begin{array}{c} \qquad\qquad\qquad 4(-)\ \ 5(-)\ \ 6(-) \\ \left|\begin{array}{ccc|cc} a_1 & b_1 & c_1 & a_1 & b_1 \\ a_2 & b_2 & c_2 & a_2 & b_2 \\ a_3 & b_3 & c_3 & a_3 & b_3 \end{array}\right. \\ \qquad\qquad\qquad 1(+)\ \ 2(+)\ \ 3(+) \end{array}$$

$$\Longrightarrow \Delta = \overset{1(+)}{a_1 b_2 c_3} + \overset{2(+)}{b_1 c_2 a_3} + \overset{3(+)}{c_1 a_2 b_3} - \overset{4(-)}{a_3 b_2 c_1} - \overset{5(-)}{b_3 c_2 a_1} \underset{- c_3 a_2 b}{\overset{}{6(-)}}$$

$$\therefore\ \Delta = a_1 b_2 c_3 + b_1 c_2 a_3 + c_1 a_2 b_3 - a_3 b_2 c_1 - b_3 c_2 a_1 - c_3 a_2 b_1$$

<u>Example</u> : Find x and y :

$$\begin{aligned} -x + 2y &= 3 \\ 3x - 2y &= -2 \end{aligned} \Longrightarrow x = \frac{\begin{vmatrix} 3 & 2 \\ -2 & -2 \end{vmatrix}}{\begin{vmatrix} -1 & 2 \\ 3 & -2 \end{vmatrix}} = \frac{(3)(-2) - (-2)(2)}{(-1)(-2) - (3)(2)} = \frac{-2}{-4} = \frac{1}{2}$$

$$\text{and } y = \frac{\begin{vmatrix} -1 & 3 \\ 3 & -2 \end{vmatrix}}{\begin{vmatrix} -1 & 2 \\ 3 & -2 \end{vmatrix}} = \frac{(-1)(-2) - (3)(3)}{-4} = \frac{-7}{-4} = \frac{7}{4}$$

EE3

Example: Find x, y and z, for the following simultaneous equation.

$$x - 2z = -1$$
$$3y + z = 2$$
$$x + 2y + 3z = 0$$

Solution: Arrange the equations to be as:

$$1x + 0y - 2z = -1$$
$$0x + 3y + 1z = 2$$
$$1x + 2y + 3z = 0$$

then

$$x = \frac{\Delta_1}{\Delta} = \frac{\left|\begin{array}{ccc|cc} -1 & 0 & -2 & -1 & 0 \\ 2 & 3 & 1 & 2 & 3 \\ 0 & 2 & 3 & 0 & 2 \end{array}\right.}{\left|\begin{array}{ccc|cc} 1 & 0 & -2 & 1 & 0 \\ 0 & 3 & 1 & 0 & 3 \\ 1 & 2 & 3 & 1 & 2 \end{array}\right.}$$

$$\therefore x = \frac{(-1)(3)(3) + (0)(1)(0) + (-2)(2)(2) - [(0)(3)(-2) + (2)(1)(-1) + (3)(2)(0)]}{(1)(3)(3) + (0)(1)(1) + (-2)(0)(2) - [(1)(3)(-2) + (2)(1)(1) + (3)(0)(0)]}$$

$$= \frac{-15}{13} = -\frac{15}{13}$$

and

$$y = \frac{\begin{vmatrix} 1 & -1 & -2 \\ 0 & 2 & 1 \\ 1 & 0 & 3 \end{vmatrix}}{\Delta} = \frac{\left|\begin{array}{ccc|cc} 1 & -1 & -2 & 1 & -1 \\ 0 & 2 & 1 & 0 & 2 \\ 1 & 0 & 3 & 1 & 0 \end{array}\right.}{13}$$

$$= \frac{(1)(2)(3) + (-1)(1)(1) + (-2)(0)(0) - [(1)(2)(-2) + (0)(1)(1) + (3)(0)(-1)}{13}$$

$$\therefore y = \frac{5+4}{13} = \frac{9}{13}$$

$$z = \frac{\Delta_3}{\Delta} = \frac{\left|\begin{array}{ccc|cc} 1 & 0 & -1 & 1 & 0 \\ 0 & 3 & 2 & 0 & 3 \\ 1 & 2 & 0 & 1 & 2 \end{array}\right.}{13}$$

$$\therefore z = \frac{(1)(3)(0) + (0)(2)(1) + (-1)(0)(2) - [(1)(3)(-1) + (2)(2)(1) + (0)(0)(0)]}{13}$$

$$\Rightarrow z = \frac{0-1}{13} = -\frac{1}{13}$$

2.4 Source Transformation :

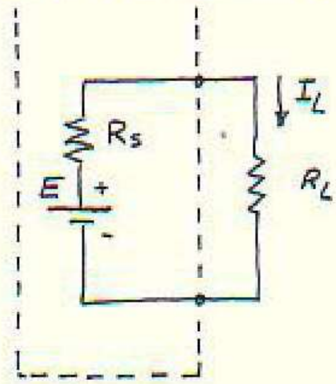
EE2

It is often necessary or convenient to have a voltage source rather than a current source or a current source rather than a voltage source.
In the cct. shown, we have a voltage source connected to a load resistance R_L

We have :

$$I_L = \frac{E}{R_T} = \frac{E}{R_S + R_L}$$

Multiplying the numerator by $(R_S/R_S = 1)$, we have :



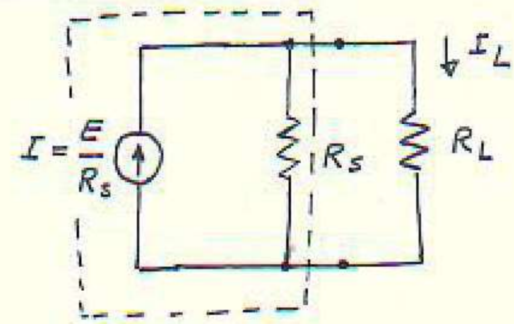
Voltage Source

$$I_L = \frac{(R_S/R_S)E}{R_S + R_L} = \frac{R_S(E/R_S)}{R_S + R_L}$$

$I = \frac{E}{R_S}$

$$\therefore \; I_L = \frac{R_S \cdot I}{R_S + R_L}$$

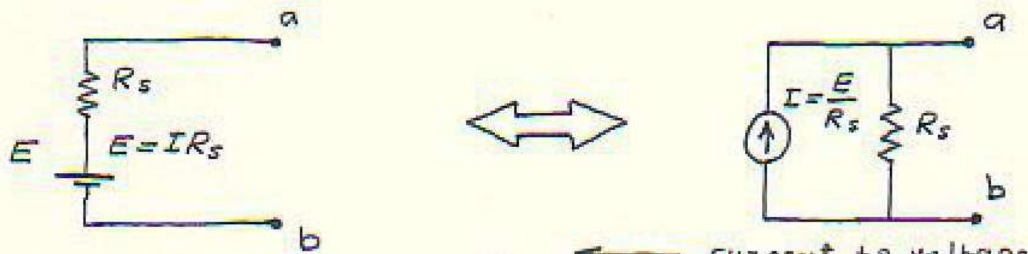
This is a current divider equation, which can be represented by the circuit below; which is the equivalent cct. of the voltage source.



Current source

In general

A voltage source with voltage E and series resistor R_S can be replaced by a current source with a current I and parallel resistor R_S as shown:



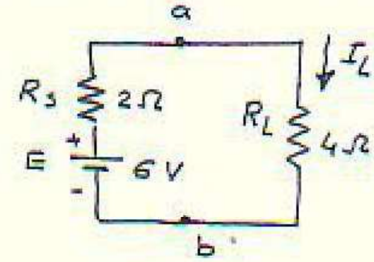
⟵ current to voltage source

voltage source to current source ⟶

EE2

Example

: Convert the voltage source, in the cct below, to a current source, then calculate the current through the load for each source.



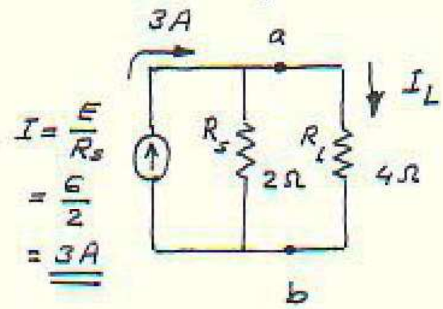
Solution

:

* For the voltage source cct;

$$I_L = \frac{E}{R_s + R_L} = \frac{6}{2+4}$$

$$= \underline{\underline{1\ A}}$$



current source

* For the current source cct;

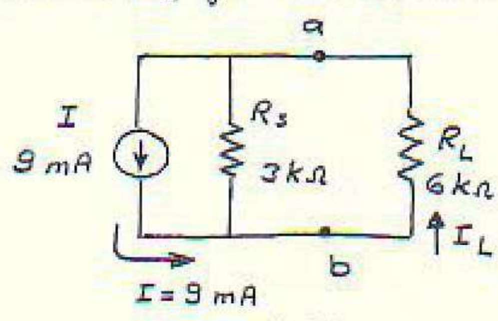
$$I_L = \frac{I R_s}{R_s + R_L} = \frac{(3)(2)}{2+4}$$

$$= \underline{1\ A}$$

نلاحظ ان I_L متساوي في الحالتين وهذا صحيح.

Example

: Convert the current source, in the cct. shown, to a voltage source and determine I_L for each source.



Solution

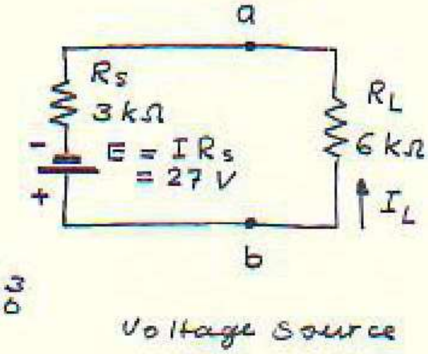
:

* For the current source cct;

current divider rule $\Rightarrow$

$$I_L = \frac{I \cdot R_s}{R_s + R_L} = \frac{(9\times10^{-3})(3\times10^{3})}{(3+\ )\times10^{3}}$$

$$= 3\times10^{-3} = \underline{3\ mA}$$



Voltage Source

* For the voltage source cct;

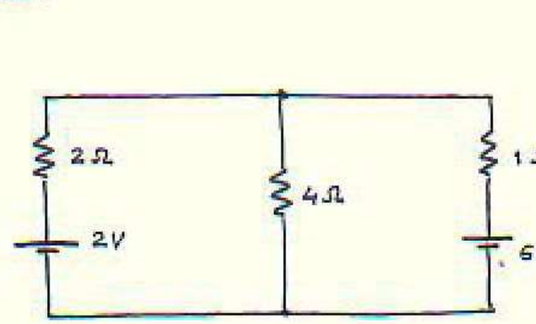
$$I_L = \frac{E}{R_T} = \frac{E}{R_s + R_L} = \frac{27}{(3+6)\times10^{3}}$$

$$= 3\times10^{-3} = \underline{3\ mA}$$

3.2 Loop (Mesh) Current Method

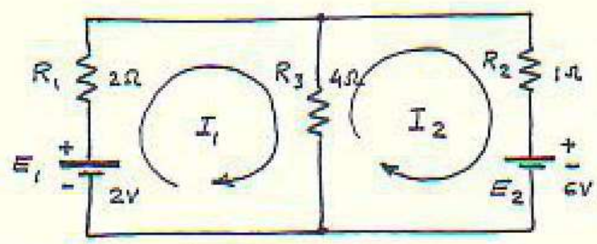
EE3

Consider the circuit shown below:



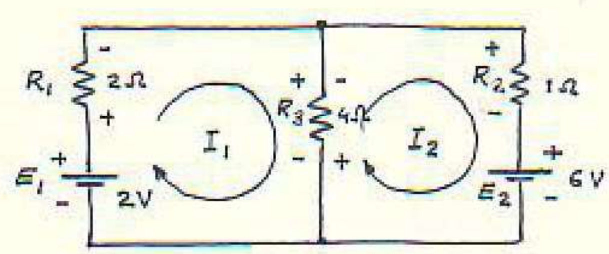
To analyze this circuit using the loop (mesh) method, the following steps must be followed.

STEP 1: Assign a distinct current in the clockwise direction to each independent loop of the network



Note: there are only two independent loops.

STEP 2: Indicate the polarities <u>within</u> each loop for <u>each resistor</u> as determined by the assumed direction of loop current for that loop.



STEP 3: Apply (KVL) around each closed loop in the clockwise direction.

<u>for loop 1</u> $\Rightarrow E_1 - V_1 - V_3 = 0 \quad \Rightarrow E_1 - I_1R_1 - (I_1 - I_2)R_3 = 0$

$$2 - 2I_1 - 4(I_1 - I_2) = 0$$

<u>for loop 2</u>

$\Rightarrow -E_2 - V_3 - V_2 = 0 \quad \Rightarrow -E_2 - R_3(I_2 - I_1) - I_2R_2 = 0$

$$-6 - 4(I_2 - I_1) - 1I_2 = 0$$

Notes : * If a resistor has two or assumed currents through it, the total current must be taken into account.

EE3

* The polarity of the voltage source is unaffected by the loop currents passing through it.

STEP 4

: Solve the resulting simultaneous equations for the assumed loop currents.

⇒ The equations for loop 1 and loop 2 are rewritten to be as:

Loop 1

$$2 = 6I_1 - 4I_2$$

Loop 2

$$-6 = -4I_1 + 5I_2$$

Solving by determinants, then :

$$I_1 = \frac{\begin{vmatrix} 2 & -4 \\ -6 & 5 \end{vmatrix}}{\begin{vmatrix} 6 & -4 \\ -4 & 5 \end{vmatrix}} = \frac{(2)(5)-(6)(4)}{(6)(5)-(4)(4)} = \frac{-14}{+14} = \underline{\underline{-1\ A}}$$

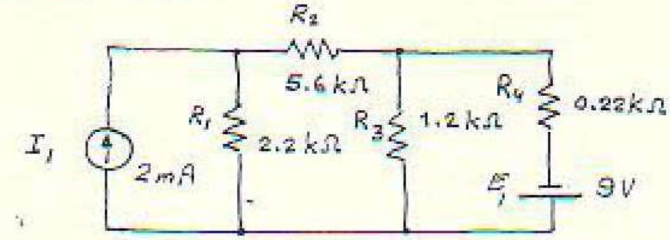
and

$$I_2 = \frac{\begin{vmatrix} 6 & 2 \\ -4 & -6 \end{vmatrix}}{\begin{vmatrix} 6 & -4 \\ -4 & 5 \end{vmatrix}} = \frac{(-6)(6)-(4)(2)}{+14} = \frac{-28}{+14} = \underline{\underline{-2\ A}}$$

$$\therefore I_{4\Omega} = I_1 - I_2$$
$$= -1 - (-2)$$
$$= 1\ A \quad \text{in the direction of } I_1$$

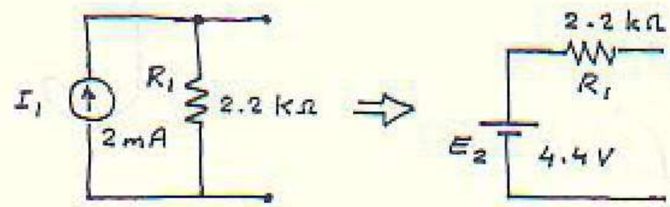
Example

: Using the mesh analysis, determine the current through the 9V battery for the network shown.



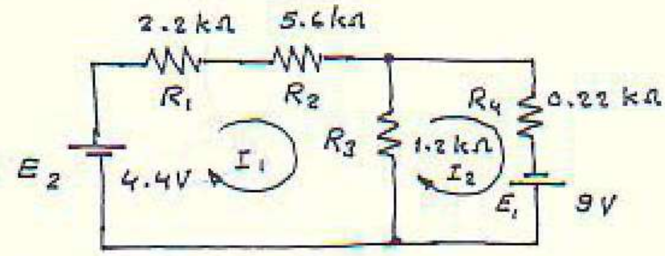
Solution: EE3

* First, the current source has to be converted to a voltage source as shown:



$E_2 = I_1 R$
$= 2\times10^{-3} \times 2.2\times10^{3}$
$= 4.4\ V$

* The original circuit will be as shown;



∴ For loop 1, we have:

$$E_2 - I_1(R_1 + R_2 + R_3) + I_2 R_3 = 0$$

$$4.4 - I_1(2.2\times10^3 + 5.6\times10^3 + 1.2\times10^3) - 1.2\times10^3 I_2 = 0$$

$$\Rightarrow 9\times10^3 I_1 - 1.2\times10^3 I_2 = 4.4$$

for loop 2, we have:

$$E_1 - I_2(R_3 + R_4) + I_1 R_3 = 0$$

$$\Rightarrow -1.2\times10^3 I_1 + 1.42\times10^3 I_2 = 9$$

Solving for I_2

$$\therefore I_2 = \frac{\begin{vmatrix} 9\times10^3 & 4.4 \\ -1.2\times10^3 & 9 \end{vmatrix}}{\begin{vmatrix} 9\times10^3 & -1.2\times10^3 \\ -1.2\times10^3 & 1.42\times10^3 \end{vmatrix}}$$

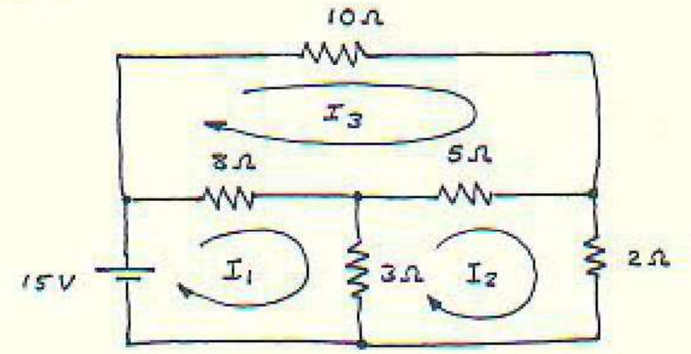
$$= \frac{86.28}{11.34\times10^3} = 7.608\times10^{-3}$$

$$\therefore I_2 = 7.608\ mA$$

Example

$\boxed{EE3}$

: Find the current through the $10\,\Omega$ resistor of the network shown.



Solution

: The loop equations are:

Loop 1:

$$(8+3)I_1 - 3I_2 - 8I_3 = 15$$

Loop 2:

$$(3+5+2)I_2 - 3I_1 - 5I_3 = 0$$

Loop 3:

$$(5+8+10)I_3 - 8I_2 - 5I_2 = 0$$

Rearrange, then:

$$11I_1 - 3I_2 - 8I_3 = 15$$
$$-3I_1 + 10I_2 - 5I_3 = 0$$
$$-8I_1 - 5I_2 + 23I_3 = 0$$

$$\therefore\ I_3 = \frac{\Delta_3}{\Delta} = \frac{\begin{vmatrix} 11 & -3 & 15 \\ -3 & 10 & 0 \\ -8 & -5 & 0 \end{vmatrix}}{\begin{vmatrix} 11 & -3 & -8 \\ -3 & 10 & -5 \\ -8 & -5 & 23 \end{vmatrix}} = \underline{1.22\ A}$$

$$\therefore\ I_3 = I_{10\Omega} = 1.22\ A$$

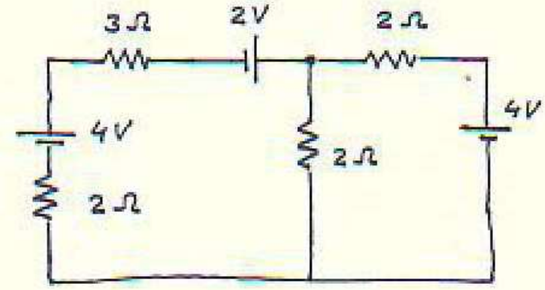
Tutorial Sheet № 3

TS3

ملاحظة : جميع امثلة الكتاب المنهجي والاسئلة المحلوله فيه مطلوبة

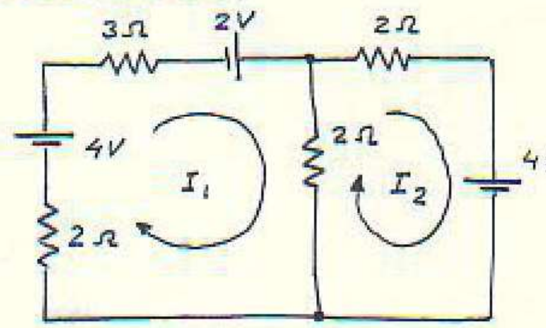
Example (1)

: For the circuit shown, find the current in the $3\,\Omega$ resistor using : (a) loop current method, (b) nodal voltage method.



Solution

: (a) Using loop current method;



Loop 1

$$4+2 = I_1(2+3+2) - I_2(2)$$

$$\Rightarrow \quad 7I_1 - 2I_2 = 6$$

Loop 2

$$-4 = I_2(2+2) - I_1(2)$$

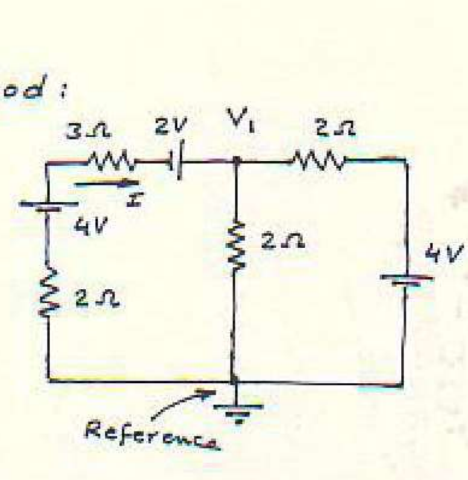
$$\Rightarrow \quad -2I_1 + 4I_2 = -4$$

$$\therefore \quad I_1 = \frac{\begin{vmatrix} 6 & -2 \\ -4 & 4 \end{vmatrix}}{\begin{vmatrix} 7 & -2 \\ -2 & 4 \end{vmatrix}} = \frac{(6)(4)-(-2)(-4)}{(7)(4)-(-2)(-2)} = \frac{24-8}{28-4} = \frac{16}{24}$$

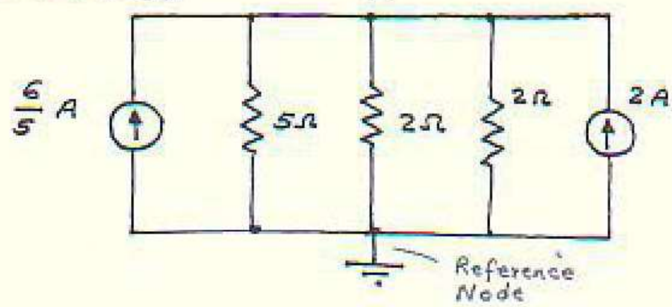
$$\therefore \quad I_1 = \frac{2}{3}\ A$$

(b) using nodal voltage method :

* There is one independent node and a reference node as shown;



* Converting the voltage sources to current sources as shown; TS3
* We have only ONE independent node (V_1), so we have one equation to find V_1



$$\therefore \quad V_1\left(\frac{1}{5}+\frac{1}{2}+\frac{1}{2}\right)=\frac{6}{5}+2$$

Simplifying, we get;

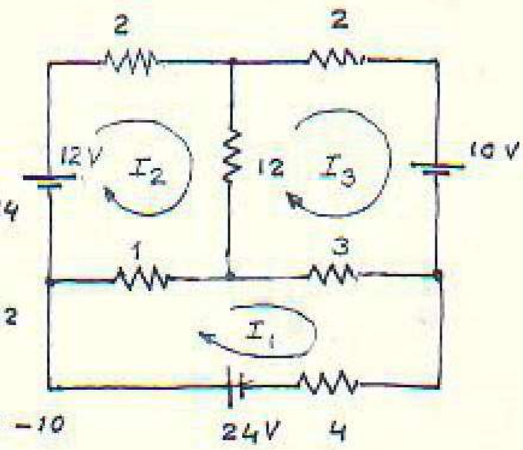
$$\Rightarrow \quad V_1=\frac{8}{3}\ V$$

Returning to the original circuit, the current through the 3Ω resistor is:

$$I_1=\frac{4+2-V_1}{3+2}=\frac{6-(8/3)}{5}$$

$$\therefore \quad I=\frac{2}{3}\ A$$

Example: Determine the current in the 4Ω resistor for the circuit shown, using loop current method. All resistor values are in Ohms.



Solution:

Loop 1

$$I_1(4+1+3)-I_2(1)-I_3(3)=24$$

Loop 2

$$I_2(1+2+12)-I_1(1)-I_3(12)=12$$

Loop 3

$$I_3(3+12+2)-I_2(12)-I_1(3)=-10$$

Rearrange, we get

$$8I_1-I_2-3I_3=24$$

$$-I_1+15I_2-12I_3=12$$

$$-3I_1-12I_2+17I_3=-10$$

$$I_1=\frac{\Delta_1}{\Delta}=\frac{\begin{vmatrix}24 & -1 & -3\\ 12 & 15 & -12\\ -10 & -12 & 17\end{vmatrix}}{\begin{vmatrix}8 & -1 & -3\\ -1 & 15 & -12\\ -3 & -12 & 17\end{vmatrix}}$$

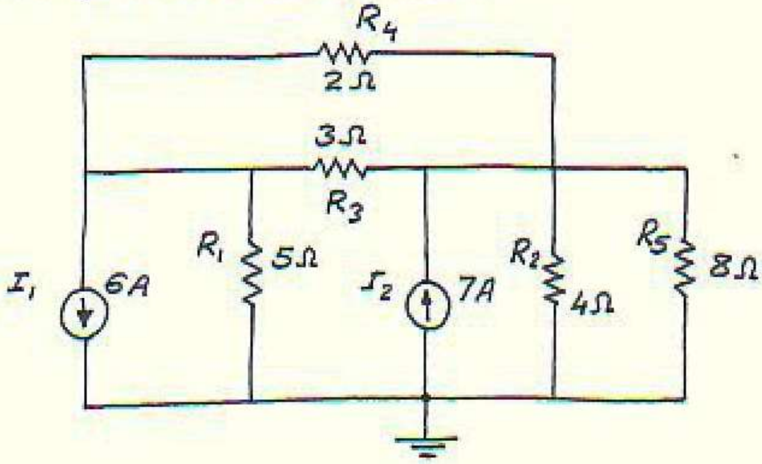
$$\Rightarrow \quad I_1=\frac{2730}{664}=\underline{4.1\ A}$$

TS3

Practice Problem

a. Write the nodal equations for the circuit shown; and solve for the nodal voltages.

b. Determine the magnitude and polarity of the voltage across each resistor.



Answer:

$V_1 = -2.556\ V$, $V_2 = 4.03\ V$

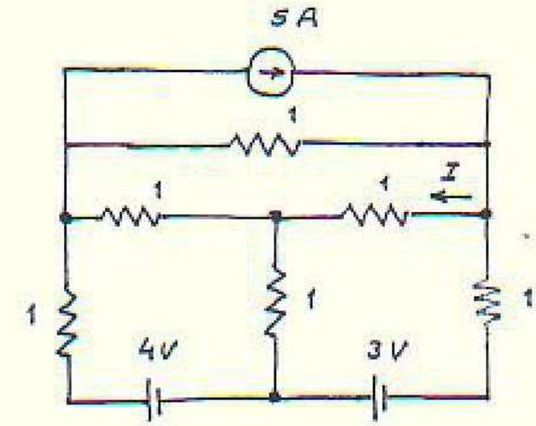
$V_{R_1} = V_1 = -2.556\ V$

$V_{R_2} = V_{R_5} = V_2 = 4.03\ V$

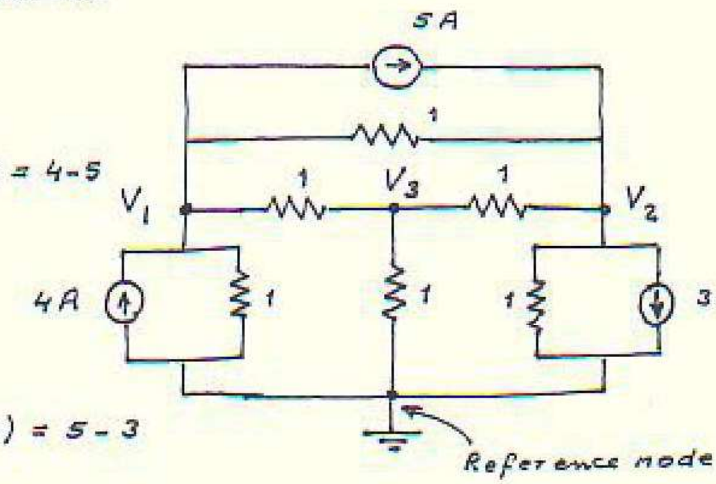
$V_{R_4} = V_{R_3} = V_2 - V_1 = 6.586\ V$

$\boxed{TS3}$

Example: Using the nodal voltage method, find, the current I in the circuit shown. All resistors are in Ohms.



Solution: * First Convert voltage sources to current sources.
There are three independent nodes and a reference node as shown.



For node 1

$$V_1(1+1+1) - V_2(1) - V_3(1) = 4 - 5$$

$$\rightarrow 3V_1 - V_2 - V_3 = -1$$

For node 2

$$V_2(1+1+1) - V_1(1) - V_3(1) = 5 - 3$$

$$\rightarrow -V_1 + 3V_2 - V_3 = 2$$

For node 3

$$V_3(1+1+1) - V_2(1) - V_1(1) = 0$$

$$\rightarrow -V_1 - V_2 + 3V_3 = 0$$

$$V_2 = \frac{\Delta_2}{\Delta} = \frac{\begin{vmatrix} 3 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & 0 & 3 \end{vmatrix}}{\begin{vmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{vmatrix}} = \frac{12}{16} = \frac{3}{4}\ V$$

$$\therefore V_2 = \frac{3}{4}\ V$$

$$V_3 = \frac{\Delta_3}{\Delta} = \frac{\begin{vmatrix} 3 & -1 & -1 \\ -1 & 3 & 2 \\ -1 & -1 & 0 \end{vmatrix}}{16} = \frac{4}{16} = \frac{1}{4}\ V$$

$$\therefore V_3 = \frac{1}{4}\ V$$

$$I = \frac{V_2 - V_3}{1\,\Omega} = \frac{\frac{3}{4} - \frac{1}{4}}{1} = \frac{1}{2}\ A$$

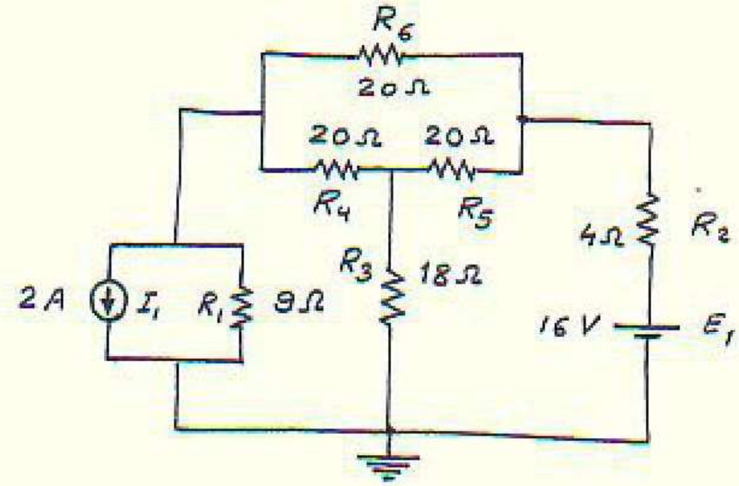
$$\therefore I = 0.5\ A$$

ملاحظة : أعد الحل باستخدام طريقة
Loop current method
وتأكد من الحصول على النتيجة نفسها

Practice Problem

TS3

: For the network shown in the Fig. below, write the nodal equations and solve for the nodal voltages.

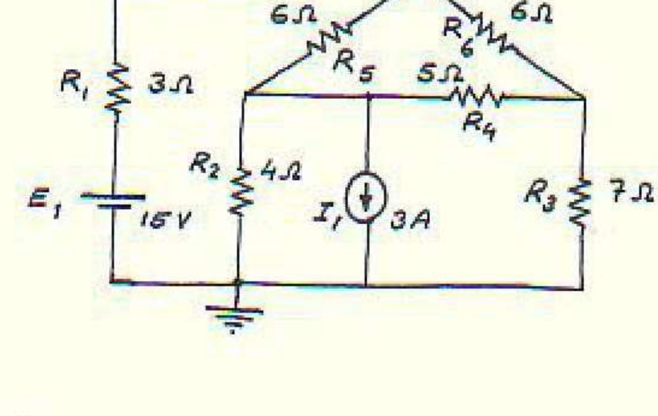


Answer

: $V_1 = -6.64\ V$, $V_2 = 1.288\ V$ and $V_3 = 10.676\ V$

Practice Problem

: For the circuit shown, write the nodal equations and solve for the nodal voltages.



Answer

: $V_1 = 7.238\ V$

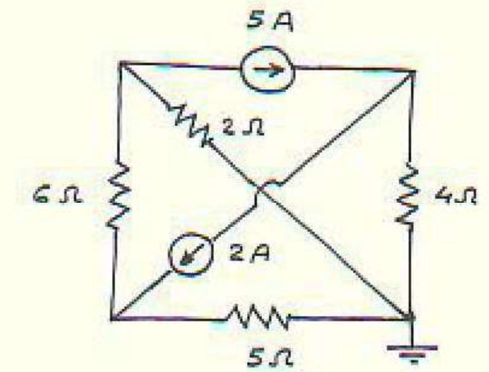
$V_2 = -2.453\ V$

$V_3 = 1.405\ V$

Example

TS3

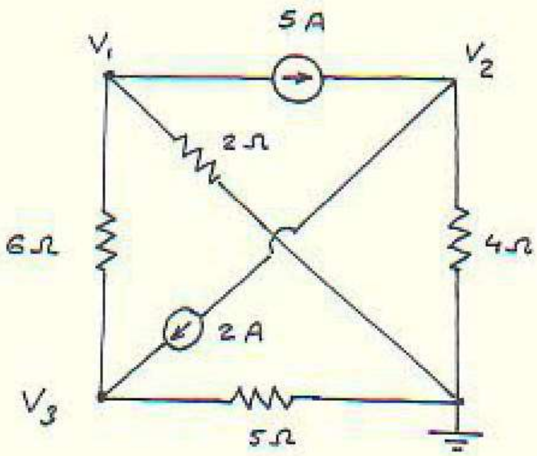
: For the network shown, write the nodal equations and solve for the nodal voltages.



Solution

: There are 3 independent nodes and a reference node as shown;

* The independent nodes are V_1, V_2, and V_3



for node 1:

$$V_1\left(\frac{1}{2}+\frac{1}{6}\right)-V_3\left(\frac{1}{6}\right)=-5$$

for node 2:

$$V_2\left(\frac{1}{4}\right) = 5-2 = 3$$

for node 3:

$$V_3\left(\frac{1}{5}+\frac{1}{6}\right)-V_1\left(\frac{1}{6}\right) = 2$$

Solving the three equations results in:

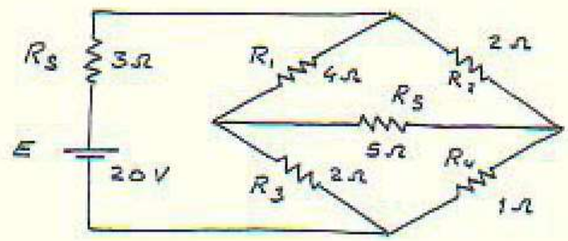
$$V_1 = -6.917\ V$$

$$V_2 = 12\ V$$

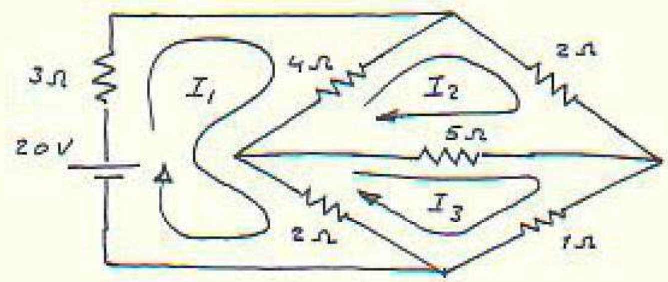
$$V_3 = 2.3\ V$$

TS3

Example: For the bridge network shown, using the loop current method find the current in R_5.



Solution:



Loop1:

$$I_1(3+4+2) - (4)I_2 - (2)I_3 = 20$$

Loop2:

$$I_2(4+2+5) - (4)I_1 - (5)I_3 = 0$$

Loop3:

$$I_3(2+5+1) - (2)I_1 - (5)I_2 = 0$$

Rearrange, we have:

$$9I_1 - 4I_2 - 2I_3 = 20$$
$$-4I_1 + 11I_2 - 5I_3 = 0$$
$$-2I_1 - 5I_2 + 8I_3 = 0$$

Solving using determinants, we have

$I_1 = 4\ A$

$I_2 = 2.67\ A \quad \Rightarrow \quad \therefore I_{R_5} = I_2 - I_3$

$I_3 = 2.67\ A \quad = 2.67 - 2.67$

$= \underline{\text{zero}}$

4. Circuit Theorems

EE4

3.1 Superposition Theorem

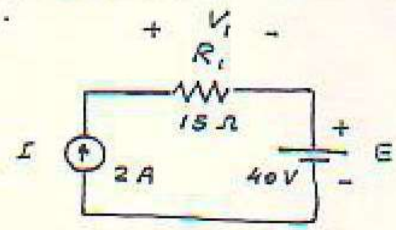
: The theorem states that: "the current through (or the voltage across) an element in a linear bilateral network is equal to the algebraic sum of the currents (or voltages) produced independently by each source.

* To apply this theorem to find the current (or voltage) in a certain part of a network, remove the sources of the network and find the current (or voltage) in the existance of only one source each time. The resultant current (or voltage) will be the algebraic sum of currents (or voltages) due to all sources when acting independently once a time.

* Removing the sources means: SHORT CIRCUITING the voltage source and OPEN CIRCUITING the current source.

Example

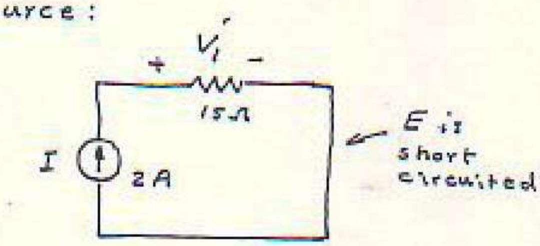
: Using the superposition theorem, determine V_1 for the network shown.



Solution

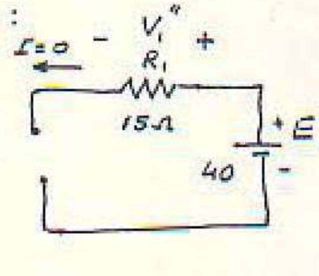
: * Due to the current source:

$$V_1' = I R_1 = (2)(15) = 30 \text{ V}$$



* Due to the voltage source:

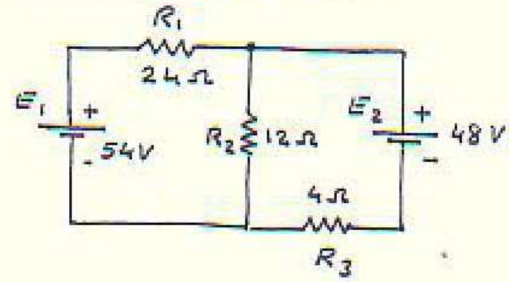
$$V_1'' = I_1 R_1 = (0)(15) = 0 \text{ V}$$



$$\therefore V_1 = V_1' + V_1'' = 30 - 0 = 30 \text{ V}$$

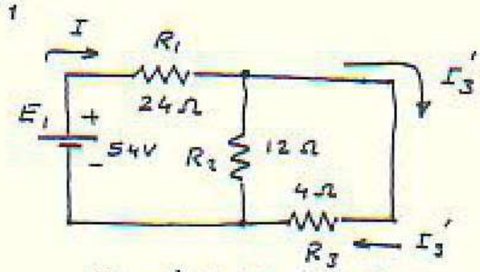
EE4

Example: Using the superposition theorem, determine the current through the 4-Ω resistor for the network shown.



Solution: Consider the effect of E_1

$$I = \frac{E_1}{R_T} = \frac{54}{27} = 2\ A$$



$R_T = (R_2 // R_3) + R_1$

$(12 // 4) + 24 = 3 + 24$

$= 27\ \Omega$

Using the current division rule ∴

$$\therefore I_3' = I \frac{R_2}{R_2 + R_3}$$

$$= 2 \frac{12}{12+4} = \underline{1.5\ A}$$

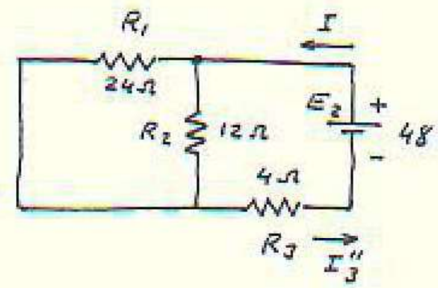
* Consider the effect of E_2:

$$I = I_3'' = \frac{E_2}{R_T}$$

$$R_T = (24 // 12) + 4$$

$$= 8 + 4$$

$$= 12\ \Omega$$

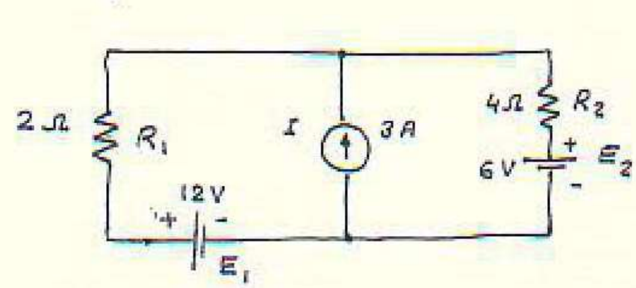


$$\therefore I_3'' = \frac{48}{12} = \underline{4\ A}$$

$$\therefore I_3 = I_3'' - I_3'$$

$$= 4 - 1.5 = \underline{2.5\ A}$$ (in the direction of I_3'').

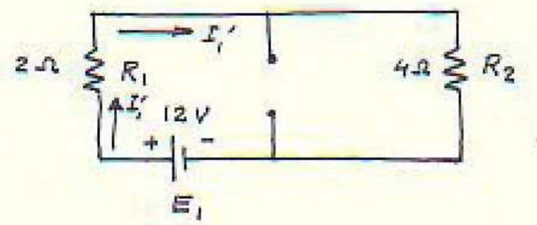
Example: Using the superposition theorem, find the current through the 2-Ω resistor of the network shown.



EE4

Solution: * The effect of E_1

Remove the voltage source E_2 (short circuited) and the current source I (open circuited); the network will be as shown:

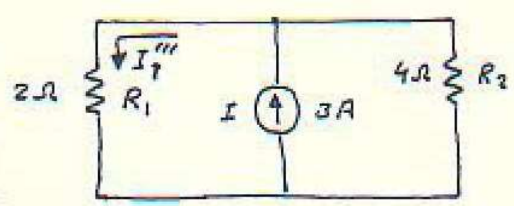


$$\therefore\ I_1' = \frac{E_1}{R_T} = \frac{12}{2+4} = 2\,A$$

* The effect of E_2: removing E_1 & I, the network will be as shown:

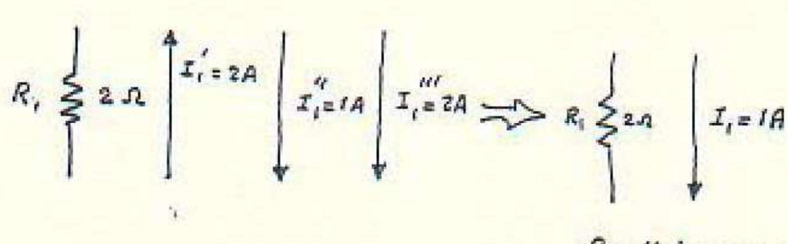
$$\therefore\ I_1'' = \frac{E_2}{R_T} = \frac{6}{2+4} = 1\ A$$

* The effect of I: removing E_1 and E_2, the network will be as shown:



$$\therefore\ I_1''' = I\,\frac{R_2}{R_1+R_2} = (3)\,\frac{4}{4+2} = 2\ A$$

$$\therefore\ I_1 = \underbrace{I_1'' + I_1'''}_{\text{same direction}} - \underbrace{I_1'}_{\text{opposite direction}} \Rightarrow I_1 = 1+2-1 = \underline{1\ A}$$



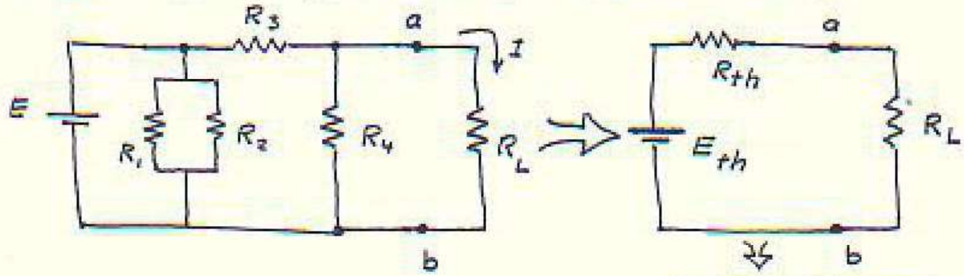
Resulting current in R_2

3.2 Thevenin's Theorem

EE4

: Thevenins theorem states that " Any two-terminal linear bilateral dc network can be replaced by an equivalent circuit consisting of a voltage source and a series resistor.

Consider the network shown, it can be replaced by the voltage source E_{th} and the series resistor R_{th} :



* To find I through the resistance R_L ⇒ $\boxed{I = \dfrac{E_{th}}{R_{th} + R_L}}$

* Steps to find E_{th} and R_{th} :

STEP 1 : Remove that portion of the network across which the Thevenins equivalent circuit is to be found.

STEP 2 : Mark the terminals of the remaining two-terminal network.

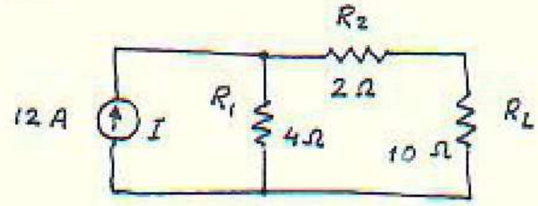
STEP 3 (R_{th}) : Calculate R_{th} by first setting all sources to zero (voltage sources are replaced by short circuits and current sources are replaced by open circuits), and finding the resultant resistance between the two marked terminals.

STEP 4 (E_{th}) : Calculate E_{th} by first returning all sources to their original positions and finding the open circuit voltage between the marked terminals

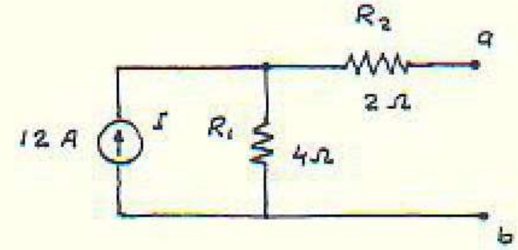
STEP 5 : Draw the Thevenins equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the equivalent circuit.

EE4

Example: Using Thevenins theorem, find the current in the $R_L = 10\,\Omega$ of the network shown.

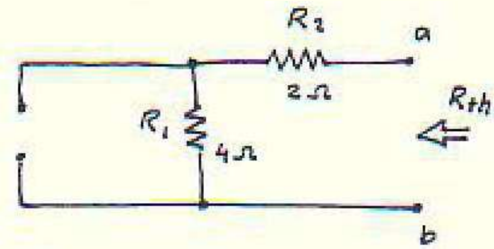


Solution: steps 1 and 2:



step 3: $R_{th} = ?$

Remove the current source I, then calculate R_{th} between the terminals a and b;



$$\therefore\ R_{th} = R_1 + R_2 = 4 + 2 = 6\,\Omega$$

step 4: $E_{th} = ?$

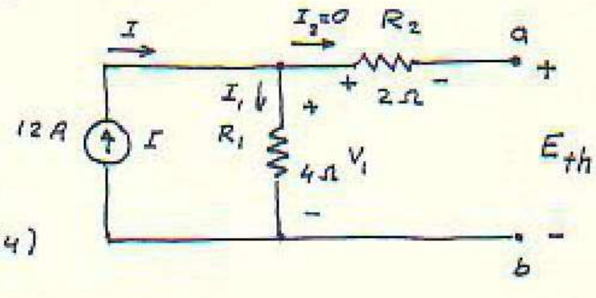
Return the current source to its original position then determine E_{th} across the open circuit terminals a and b.

$I_2 = 0$

$\Rightarrow I_2 R_2 = 0$

Then

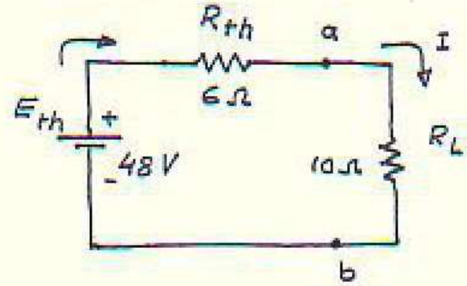
$$E_{th} = I_1 R_1 - I_2 R_2$$
$$= I_1 R_1 = 12(4)$$
$$= 48\ V$$



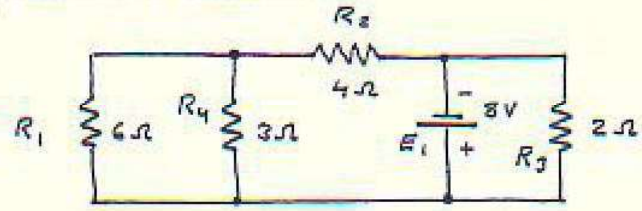
step 5: Draw the Thevenins equivalent circuit representing the network between points a and b with R_L added.

EE4

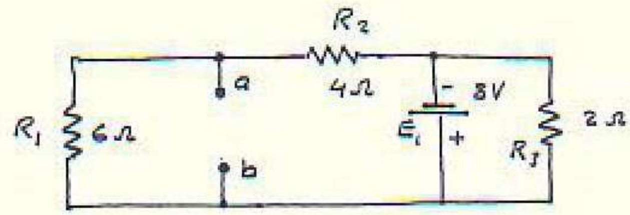
$$\therefore I = \frac{E_{th}}{R_{th} + R_L} = \frac{48}{6 + 10} = 3\ A$$



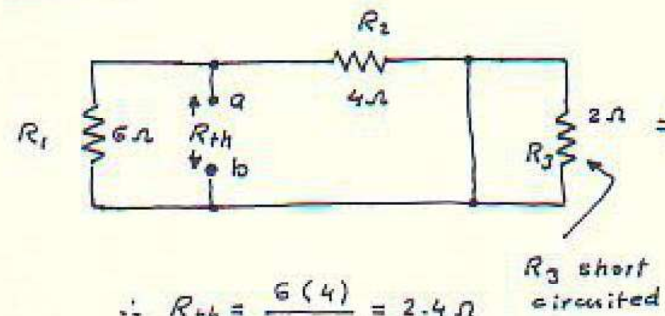
Example: For the circuit shown, find the current in the 3-Ω resistor using Thevenin's theorem.

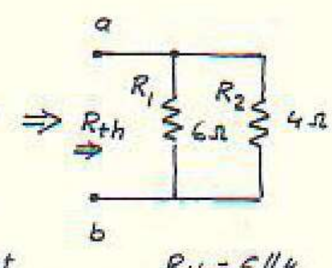


Solution: steps 1 and 2



step 3: $R_{th} = ?$

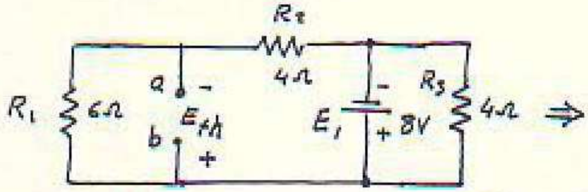




$$\therefore R_{th} = \frac{6(4)}{6+4} = 2.4\ \Omega$$

$R_{th} = 6 /\!/ 4 = 2.4\ \Omega$

step 4: $E_{th} = ?$

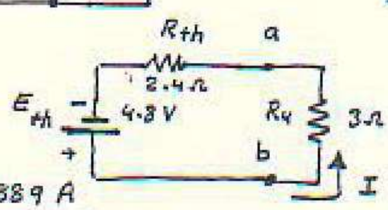


using the voltage divider rule,

$$E_{th} = E_1 \frac{R_1}{R_1 + R_2} = 8 \frac{6}{6+4} = 4.8\ V$$

step 5

$$\therefore I = \frac{E_{th}}{R_{th} + R_4} = \frac{4.8}{2.4 + 3} = \underline{0.889\ A}$$

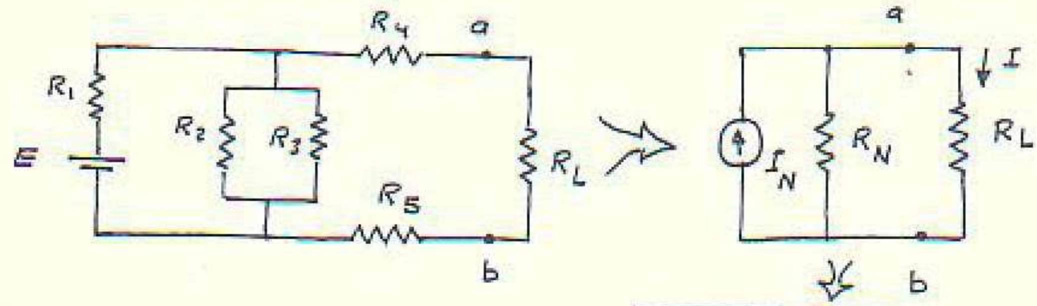


EE4

3.3. Norton's Theorem

: Norton's theorem states that " Any two terminal linear bilateral dc network can be replaced by an equivalent circuit consisting of a current source and a parallel resistor."

Consider the network shown, it can be replaced by the current source I_N and the parallel resistor R_N ;



To find the current through R_L $\Rightarrow$ $\boxed{I = \dfrac{I_N R_N}{R_N + R_L}}$

How to find I_N and R_N

STEP 1

: Remove that portion of the network across which the Norton equivalent circuit is found.

STEP 2

: Mark the terminals of the remaining two-terminal network.

STEP 3 (R_N)

: Calculate R_N by first removing all the sources (voltage sources replaced by short circuits and current sources replaced by open circuits) and then finding the resultant resistance between the two marked terminals.

STEP 4 (I_N)

: Calculate I_N by first returning all sources to their original position and then finding the short circuit current between the marked terminals

STEP 5

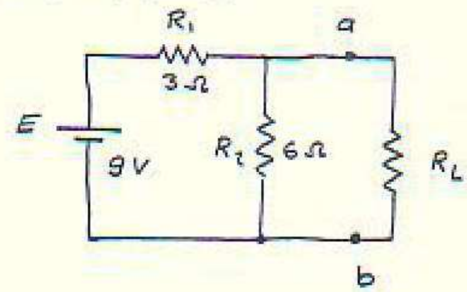
: Draw the Norton equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the equivalent circuit.

Relation between Norton equivalent circuit and Thevenin's equivalent circuit: The Norton and Thevenin equivalent circuits can also be found from each other by using the source transformation previously discussed, as shown;

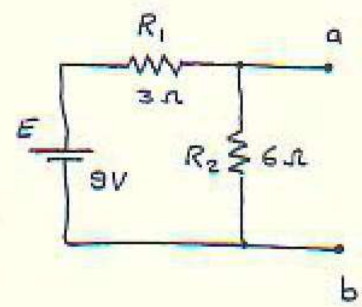
$E_{th} = I_N R_N$

$I_N = \dfrac{E_{th}}{R_{th}}$

Example: For the circuit shown, find the Norton equivalent circuit for the network to the left of (a-b).

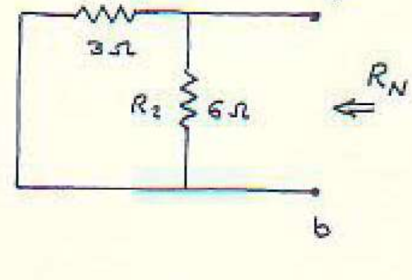


Solution: steps 1 and 2

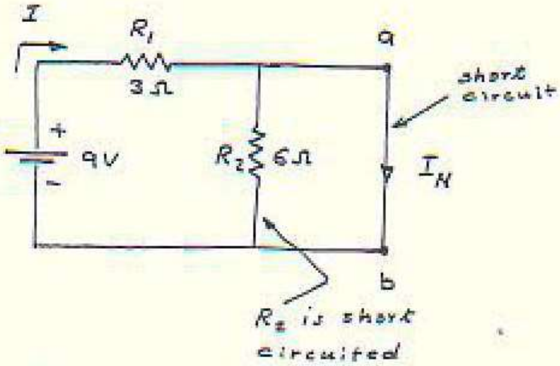


step 3: $R_N = ?$

$$R_N = R_1 \parallel R_2 = \frac{3(6)}{3+6} = 2\Omega$$

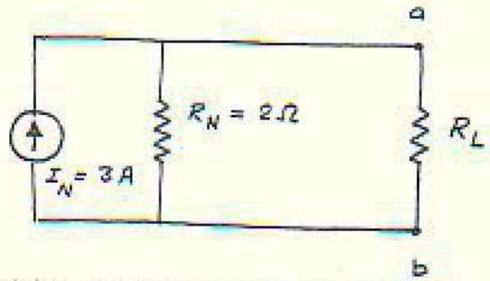


STEP 4 $\quad I_N = ?$



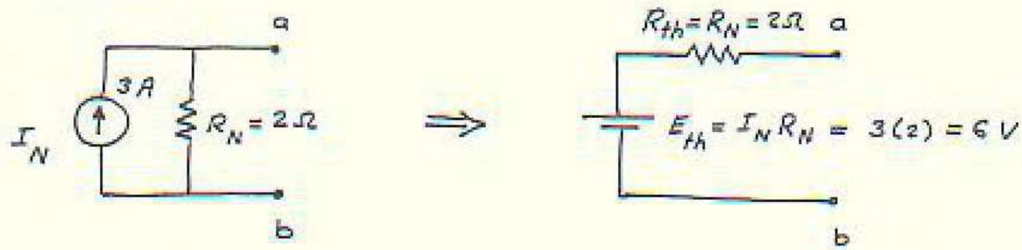
$$I_N = I = \frac{E}{R_1} = \frac{9}{3}$$
$$= 3\ A$$

∴ step 5 :

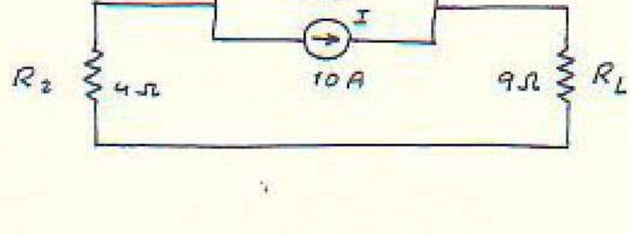


which is the Norton equivalent circuit of the network.

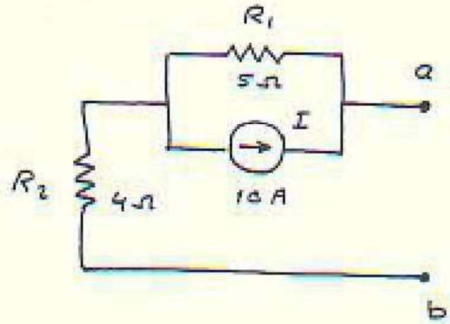
Note : Thevenin's theorem can be determined by Norton's theorem as shown :



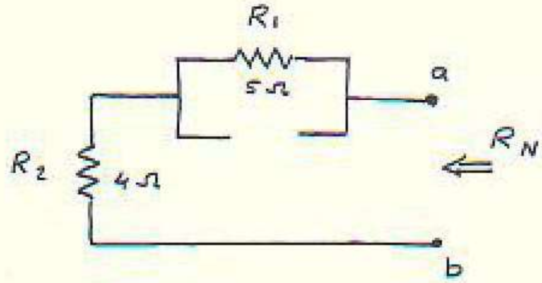
Example : Using Norton theorem find the current through the load resistor R_L in the network shown.



Solution: step 1 and 2

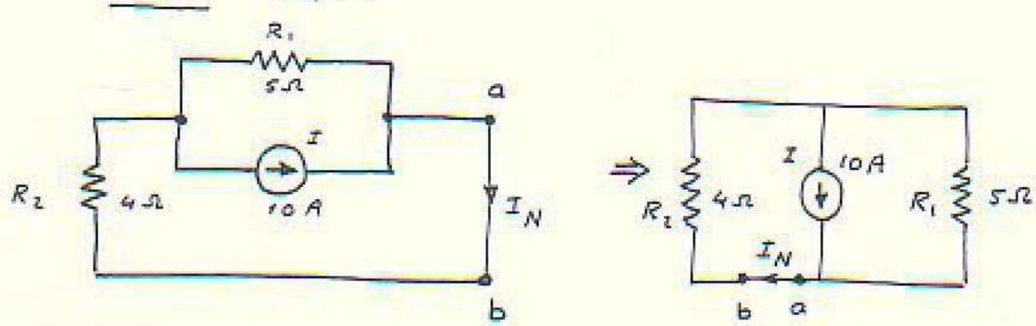


step 3 : $R_N = ?$



$$
\begin{aligned}
R_N &= R_1 + R_2 \\
&= 5 + 4 \\
&= 9\ \Omega
\end{aligned}
$$

step 4 $I_N = ?$



$$
\begin{aligned}
\therefore\ I_N &= I \cdot \frac{R_1}{R_1 + R_2} \\
&= 10\,\frac{5}{5+4} \\
&= 5.556\ A
\end{aligned}
$$

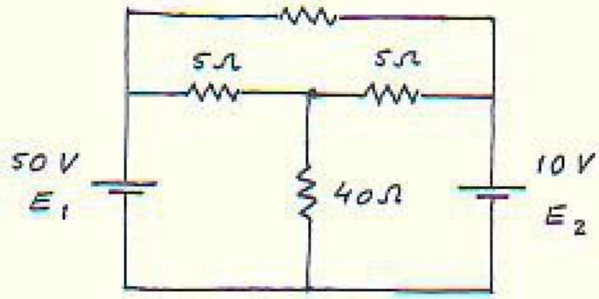
* step 5

$$\therefore\ I = \frac{I_N}{2} = 2.778\ A$$

Tutorial Sheet $N^{\underline{o}}$ 4 $\boxed{TS4}$

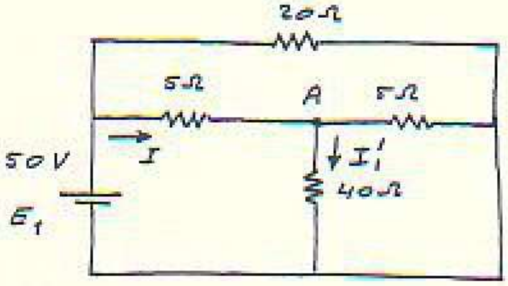
ملاحظة : جميع امثلة الكتاب وتمارينه واسئلته مطلوبة .

Example : Use the superposition theorem, find the current in the $40\,\Omega$ resistor of the circuit shown.

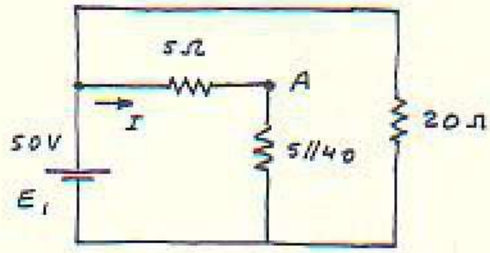


Solution :

* The effect of E_1



$\Rightarrow$



$5//40$ $\qquad$ $5//40 = 4.44\,\Omega$

$$\therefore\ I = \frac{50}{5 + 4.44} = 5.296\ A$$

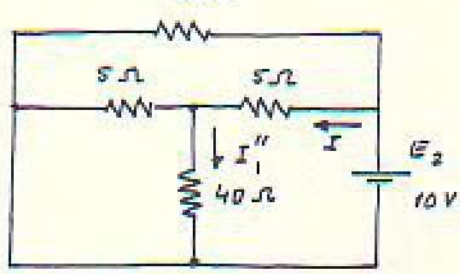
$$\therefore\ I_1' = I\,\frac{5}{5+40} = 5.296\ \frac{5}{45} = 0.589\ A$$

* The effect of E_2

$$I = \frac{10}{(5//40) + 5} = 1.059\ A$$

$$I_1'' = I\,\frac{5}{40+5} = 1.059\ \frac{5}{45}$$

$$= 0.118\ A$$

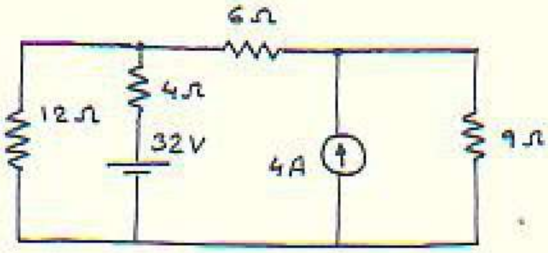


$$\therefore\ I_1 = I_1' + I_1'' = 0.589 + 0.118 = 0.707\ A.$$

Example

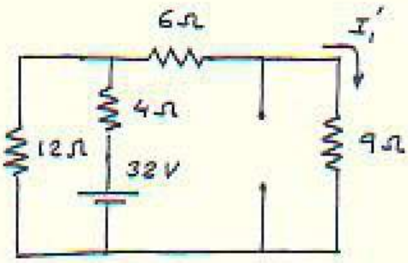
TS4

: For the circuits shown, calculate the current through the 9 Ω resistor using the superposition theorem.

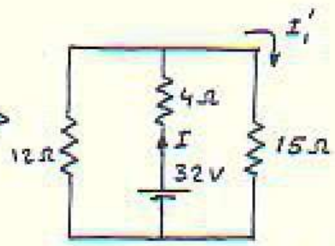


Solution

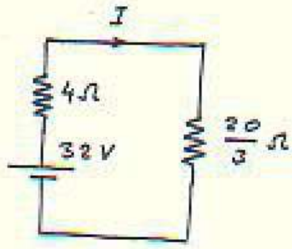
: * The effect of the voltage source



$6+9 = 15\,\Omega$



$12 // 15 = \frac{20}{3}\,\Omega$

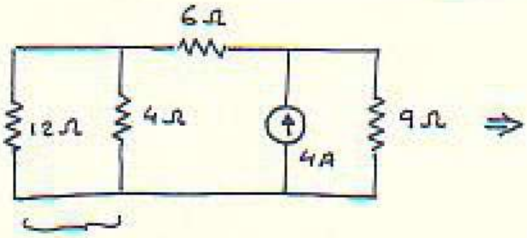


$$R_T = 4 + \frac{20}{3} = \frac{32}{3}\ \Omega$$

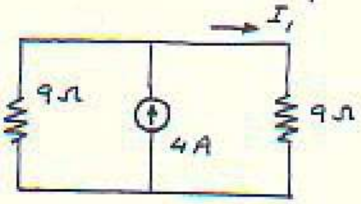
$$\therefore\ I = \frac{E}{R_T} = \frac{32}{(32/3)} = 3\ A$$

$$\therefore\ I_1' = I\,\frac{12}{12+15} = \frac{4}{3}\ A$$

* The effect of the current source :



$12 // 4 = 3\,\Omega$

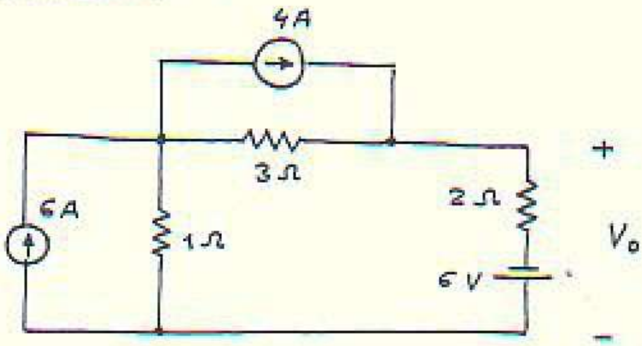


$$\therefore\ I_1'' = \frac{4}{2} = 2\ A$$

$$\therefore\ I_1 = I_1' + I_1'' = \frac{4}{3} + 2 = \frac{10}{3}\ A.$$

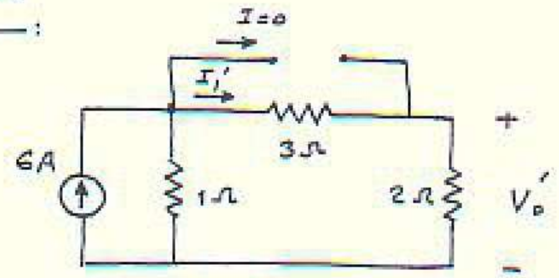
Example TS4

: Using the superposition theorem, find the value of the output voltage V_o in the circuit shown.



Solution

: + Effect 6A source :



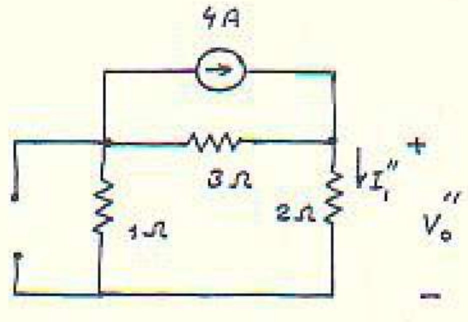
Using the current divider rule ;

$$I_1' = 6 \frac{1}{(1+2+3)}$$

$$= 1 \text{ A}$$

$$\therefore V_o' = I_1'(2) = 2 \text{ V}$$

+ Effect of 4A source



current divider rule

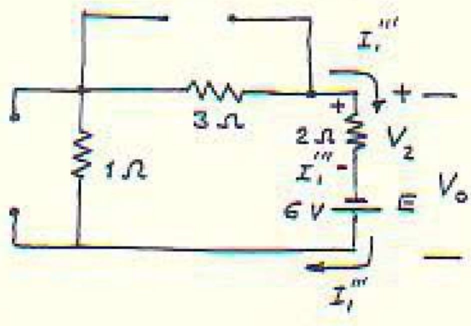
$$\therefore I_1'' = 4 \frac{3}{(1+2)+3}$$

$$= 2 \text{ A}$$

$$\therefore V_o'' = I_1''(2)$$

$$= 4 \text{ V}$$

+ Effect of 6V-source :



$$I_1''' = \frac{6}{1+3+2} = 1 \text{ A}$$

$$\therefore V_2 = I_1'''(2) = 2 \text{ V}$$

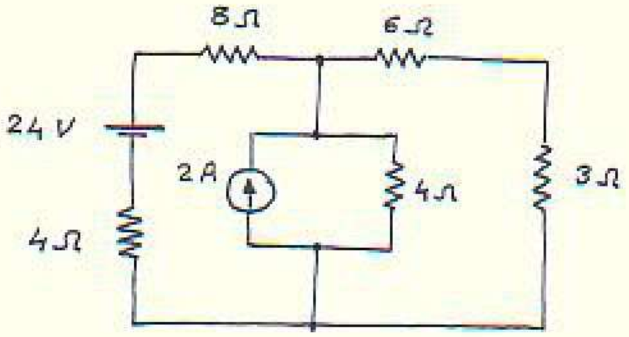
$$\therefore V_o''' = E - V_2 = -6 + 2 = -4 \text{ V}$$

$$\therefore V_o = V_o' + V_o'' - V_o''' = 2 + 4 - 4$$

$$= 2 \text{ V}$$

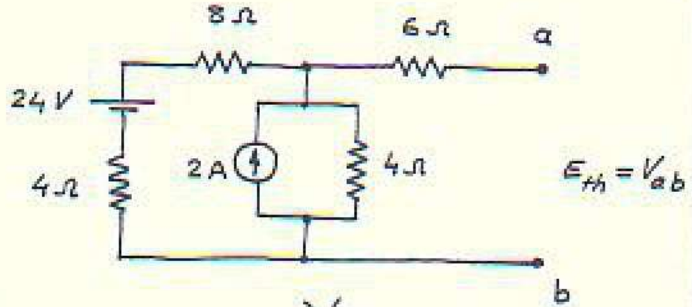
Example   TS4

: Use the Thevenin's theorem to find the current in the $3\,\Omega$ resistor in the network shown.

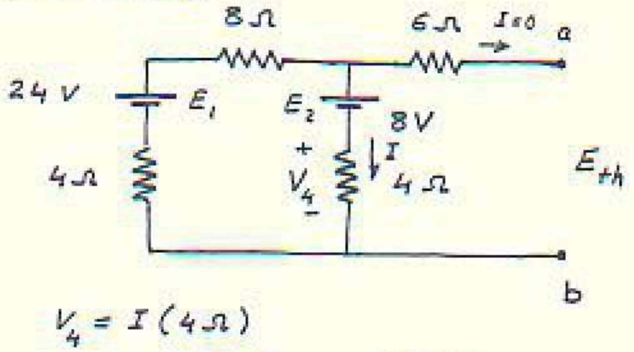


Solution

: $\underline{E_{th} = ?}$



* convert the current source to voltage source as shown



$\therefore\ E_{th} = E_2 + V_4$

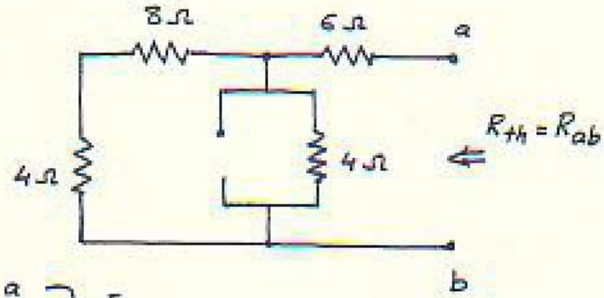
$$V_4 = I\,(4\,\Omega)$$

$$I = \frac{E_1 - E_2}{4+8+4} = \frac{24-8}{16} = 1\text{ A}$$

$$\therefore\ V_4 = (1)(?) = 4\text{ V}$$

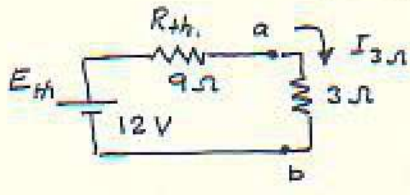
$\therefore\ E_{th} = 8 + 4 = 12\text{ V}$

$\underline{R_{th} = ?}$



$$R_{th} = \left[(8+4)\,//\,4\right] + 6$$

$$= 9\,\Omega$$



$$\therefore\ I_{3\Omega} = \frac{12}{9+3}$$

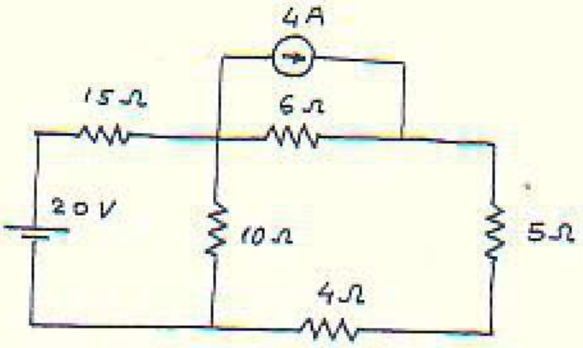
$$= 1\text{ A}$$

Note: Repeat this example to find the value of R_L for max. power transfer and compute $P_{L_{max}}$

TS 4

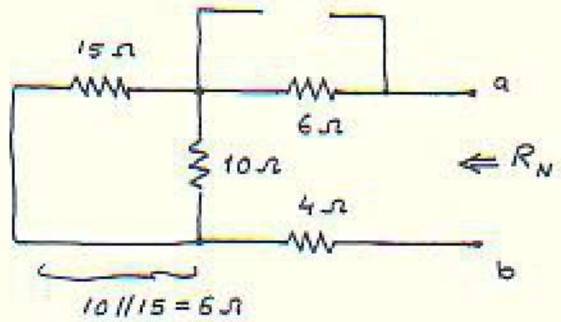
Example

For the circuit shown, find the value of the current passing through the 5 Ω resistor using Norton's theorem. Calculate the power absorbed by this resistor.



Solution

$R_N = ?$

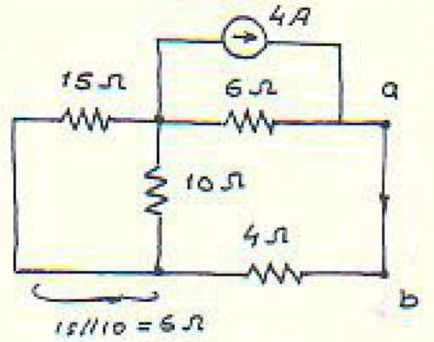


$$R_N = (15 // 10) + 6 + 4$$
$$= 16\ \Omega$$

$I_{sc} = ?$

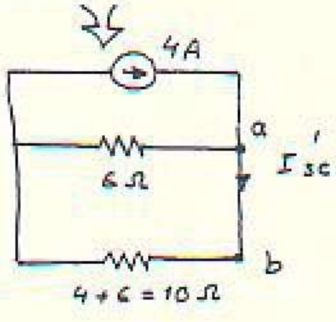
We have two sources; we can use superposition theorem to find the resulting I_{sc}.

Effect of 4 A source

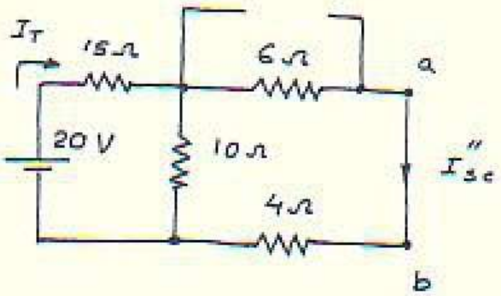


current divider rule

$$I'_{sc} = 4 \, \frac{6}{6+16} = \frac{4(6)}{16}$$
$$= \frac{3}{2} = 1.5\ A$$



Effect of 20V source



$$I_T = \frac{E}{R_T}$$

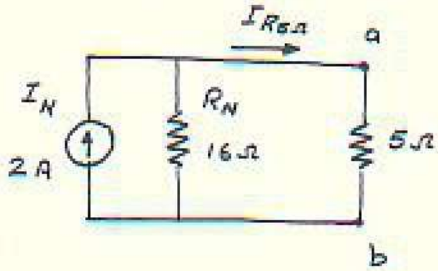
$$R_T = \Big[\overbrace{(6+4)}^{10} // 10\Big] + 15$$

$$= 20\ \Omega$$

$$\therefore\ I_T = \frac{20}{20} = 1\ A$$

$$\therefore\ I''_{sc} = \frac{I_T}{2} = \frac{1}{2} = 0.5\ A$$

$$\therefore\ I_N = I'_{sc} + I''_{sc} = 1.5 + 0.5 = \underline{2\ A}$$



$$\therefore\ I_{R_{5\Omega}} = I_N \frac{R_N}{R_N + R_{5\Omega}}$$

$$= 2\ \frac{16}{16+5} = 1.52\ A$$

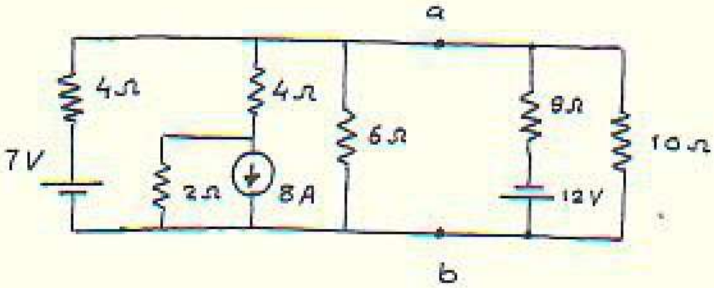
and the power $P = I^2_{R_{5\Omega}} \cdot R_{5\Omega}$

$$= (1.52)^2 \cdot (5) = 11.6\ W$$

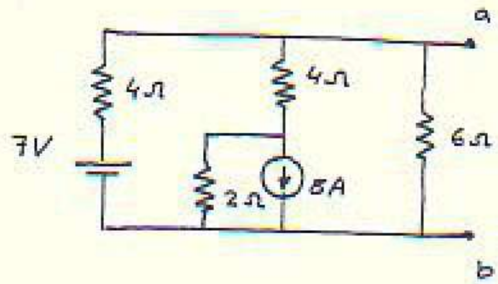
ملاحظة : من الممكن استخراج E_{th} ومن ثم تحويلها الى دائرة نورتن المكافئة ومن ثم الحصول على التيار .

Example: Find the Norton equivalent circuit for the portion of the network to the left of (a - b) in the circuit shown.

T54



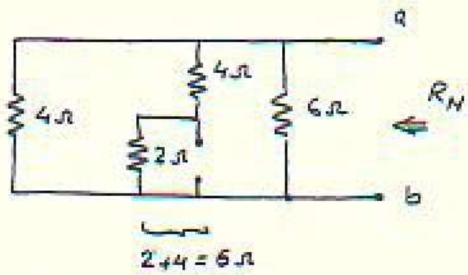
Solution:



$R_N = ?$:

$R_N = 6 // 6 // 4$

$= 1.714\ \Omega$

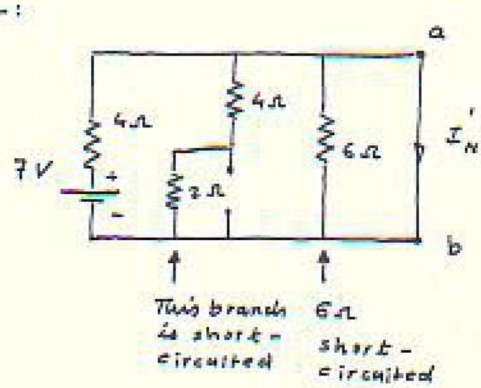


$I_N = ?$: We have 2 sources, it is recommended to use the superposition theorem to find I_N

ملاحظة : علينا اتباع اي
طريقة ، مثلاً للحصول على I_N

– Effect of 7 V source :

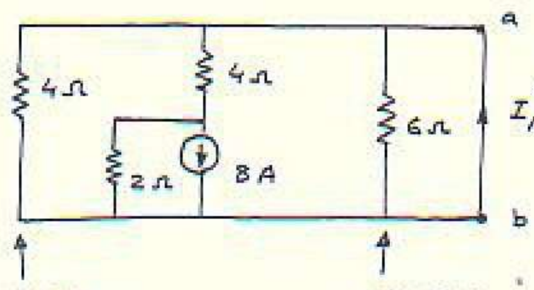
$$\therefore\ I_N' = \frac{7}{4} = 1.75\ A$$



- Effect of 8A source :

current divider rule

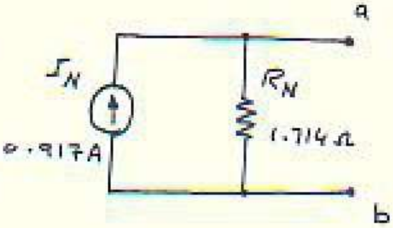
$$\therefore I_N'' = 8 \frac{2}{2+4}$$
$$= 2.667 \text{ A}$$



short cct short cct.

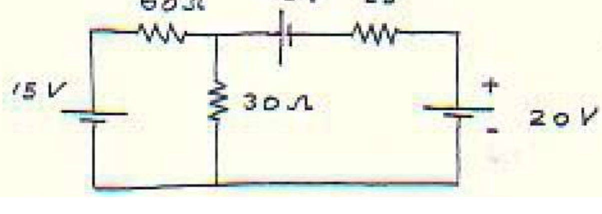
$$\therefore I_N = I_N' - I_N'' = 2.667 - 1.75$$
$$= 0.917 \text{ A}$$ (in the direction of I_N'').

∴ The Norton equivalent cct of the portion of the network to the left of (a-b) is :



ملحوظة : من الممكن الحصول على دائرة نورتن من دائرة (Thevenin).

Example : For the circuit shown, find the current through the 20V voltage source using Thevenin's theorem.



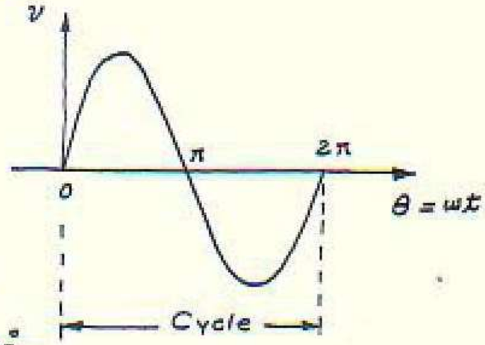
AC Circuit

Definitions Related to AC Waveforms

EEE

* **The Cycle**

: One complete set of positive and negative values of an alternating quantity is called a (cycle). Complete cycle is said to spread over 360° or 2π radians



$$\therefore \; 360^\circ = 2\pi \text{ radians}$$

$$\Rightarrow 1 \text{ rad.} = \frac{360^\circ}{2\pi} = \frac{180^\circ}{\pi}$$

$$\therefore \; 1 \text{ rad.} = 57.3^\circ$$

- To convert from degrees to radians

$$\boxed{\text{Radians} = \left(\frac{\pi}{180}\right) \times \text{degrees}}$$

- To convert from radians to degrees

$$\boxed{\text{Degrees} = \left(\frac{180}{\pi}\right) \times \text{radians}}$$

For examples

$$90^\circ \rightarrow \text{rad.} = \frac{\pi}{180} \times 90^\circ = \frac{\pi}{2}$$

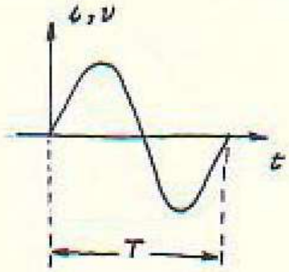
$$30^\circ \rightarrow \text{rad.} = \frac{\pi}{180} \times 30 = \frac{\pi}{6}$$

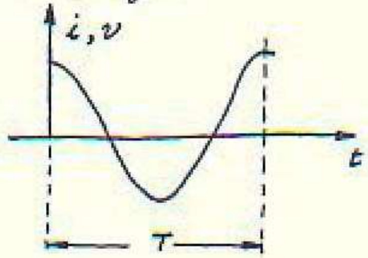
$$\frac{\pi}{3} \rightarrow \text{deg.} = \frac{180}{\pi} \times \frac{\pi}{3} = 60^\circ$$

$$\frac{3\pi}{2} \rightarrow \text{deg.} = \frac{180}{\pi} \times \frac{3\pi}{2} = 270^\circ$$

* **Time Period (T)**

: It is the time taken by an alternating quantity to complete one cycle.





$$T = \frac{1}{f} \qquad \Rightarrow f \text{ is the frequency.}$$

* *Instantaneous Value*

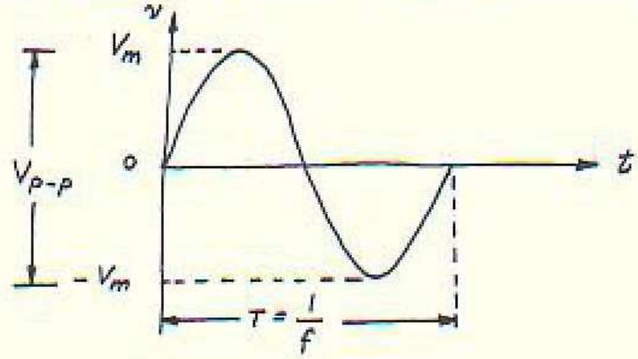
: The magnitude of a waveform (alternating voltage or current) at any instant of time. It is denoted by small letters such as v, e, e_1, e_2, i_1, i_2, p ...etc.

* *Amplitude (or Peak Value or Maximum Value)*

: The maximum value (positive or negative) of an alternating quantity is called amplitude. It is denoted by capital letters, such as E_m, V_m, I_m, ... etc.

* *Frequency*

: It is the number of cycles that occur in one second



* *Peak to Peak Value*

: It is denoted by E_{p-p} or V_{p-p} and represents the full voltage between positive and negative peaks of the waveform (see the figure above.).

In general form for the sinusoidal voltage or current

$$\boxed{e = E_m \sin\theta} \qquad \theta \text{ in deg.}$$

$$\boxed{e = E_m \sin\omega t} \qquad \theta = \omega t$$

$$\boxed{e = E_m \sin 2\pi f t} \qquad \omega = 2\pi f$$

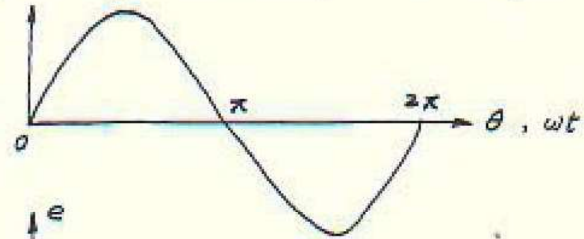
$$\boxed{e = E_m \sin \frac{2\pi}{T} t} \qquad f = \frac{1}{T}$$

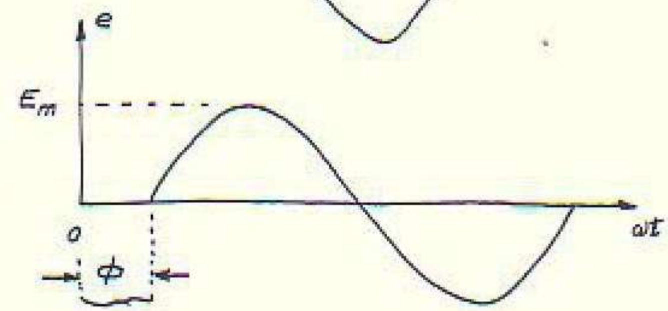
6.4 Phase Relations

________________ : A sinusoidal waveform may start from zero as had been shown earlier, or it may be shifted to the left or to the right as shown.

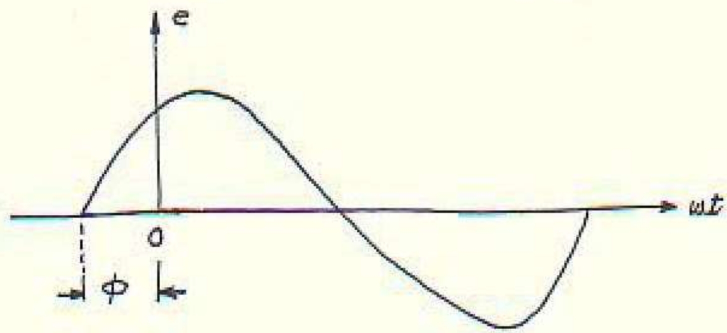
$$\boxed{e = E_m \sin \omega t}$$

* it has zero phase shift.





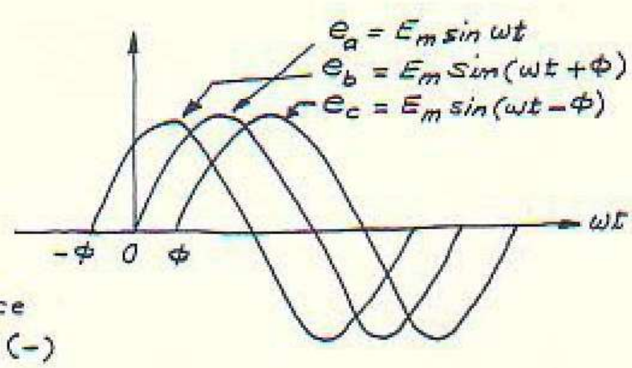
$$\boxed{e = E_m \sin(\omega t - \phi)}$$



$$\boxed{e = E_m \sin(\omega t + \phi)}$$

In the figure shown, three sinusoidal waveform are plotted with different phases:

$$e_a = E_m \sin \omega t$$
$$e_b = E_m \sin(\omega t + \phi)$$
$$e_c = E_m \sin(\omega t - \phi)$$



* [A plus (+) sign when used in connection with phase difference denotes (lead) whereas minus (-) sign denotes (lag).

Some Useful Relations
____

$$\sin(\omega t + 90^\circ) = \sin(\omega t + \tfrac{\pi}{2}) = \cos(\omega t)$$
$$\text{cos(wt-90)} = \cos(\omega t - \tfrac{\pi}{2}) = \sin \omega t$$
$$\sin(-\theta) = -\sin\theta$$
$$\cos(-\theta) = \cos\theta$$
$$-\sin(\theta) = \sin(\theta \pm 180^\circ)$$
$$-\cos(\theta) = \cos(\theta \pm 180^\circ)$$

Examples

___: What is the phase difference between the following sets of voltages and currents ?

(a). $v = 10 \sin(\omega t + 30^\circ)$
$i = 5 \sin(\omega t + 70^\circ)$

(b). $v = 10 \sin(\omega t - 20^\circ)$
$i = 15 \sin(\omega t + 60^\circ)$

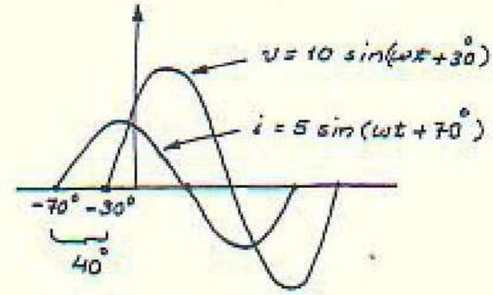
(c). $i = 2 \cos(\omega t + 10^\circ)$
$v = 3 \sin(\omega t - 10^\circ)$

(d). $i = -2 \cos(\omega t - 60^\circ)$
$v = 3 \sin(\omega t - 150^\circ)$

(e). $i = -\sin(\omega t + 30^\circ)$
$v = 2 \sin(\omega t + 10^\circ)$

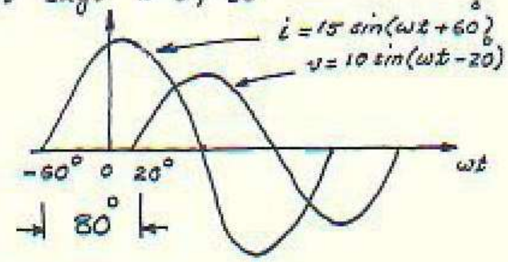
Solutions ___:

(a).



The phase difference = 40°
i leads v by 40°
$\boxed{\text{or}}$ v lags i by 40°

(b). The phase difference $60^\circ + 20^\circ = 80^\circ$
$\therefore$ i leads v by 80°
or v lags i by 80°



Similarly you can find the results of c, d and e, and the results are:

(c).

$i = 2\cos(\omega t + 10^\circ) = 2 \sin(\omega t + 10^\circ + 90^\circ) = 2 \sin(\omega t + 100^\circ)$

$\therefore \left.\begin{array}{l} i = 2 \sin(\omega t + 100^\circ) \\ v = 3 \sin(\omega t - 10^\circ) \end{array}\right\} \Rightarrow$ The phase difference $= (100 + 10) = 110^\circ$

$\therefore$ i lead v by 110°

(e) $i = -\sin(\omega t + 30^\circ) = \sin(\omega t + 30^\circ \overset{\pm}{-} 180^\circ) = \sin(\omega t - 150^\circ)$

$\therefore \left.\begin{array}{l} i = \sin(\omega t - 150^\circ) \\ v = 2 \sin(\omega t + 10^\circ) \end{array}\right\} \Rightarrow$ The phase difference $= (150^\circ + 10^\circ) = 160^\circ$

$\therefore$ v leads i by 160°

or $i = -\sin(\omega t + 30^\circ) = \sin(\omega t + 30^\circ + 180^\circ) = \sin(\omega t + 210^\circ)$

$\therefore \left.\begin{array}{l} i = \sin(\omega t + 210^\circ) \\ v = 2 \sin(\omega t + 10^\circ) \end{array}\right\} \Rightarrow$ The phase difference is 200°

$\therefore$ i lead v by 200°

(d) $i = -2\cos(\omega t - 60^\circ) = 2\cos(\omega t - 60^\circ - 180^\circ) = 2\cos(\omega t - 240^\circ)$

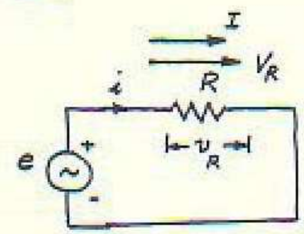
$= 2 \sin(\omega t - 240^\circ + 90^\circ) = 2 \sin(\omega t - 150^\circ)$

$v = 3 \sin(\omega t - 150^\circ)$ $\Rightarrow$ The phase diff. $= 0^\circ$ } V and i are in phase

Circuit Elements in the Phasor Domain :

⊕ AC Through Pure Ohmic Resistor Alone

In the cct. shown, let the applied voltage be given by:



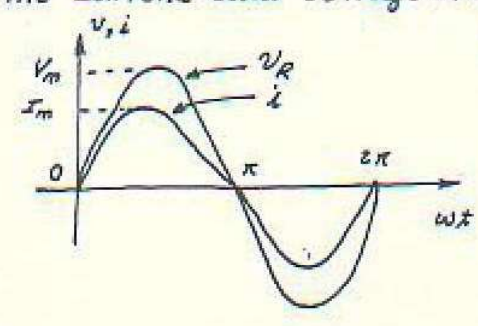
$$e = E_m \sin \omega t = E_m \sin \theta$$

$$\therefore i = \frac{e}{R} = \frac{E_m}{R} \sin \omega t$$

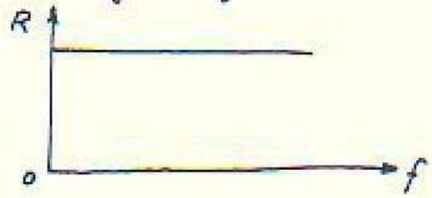
$$\therefore \boxed{i = I_m \sin \omega t}$$

and $\boxed{v_R = iR = I_m R \sin \omega t = V_m \sin \omega t}$

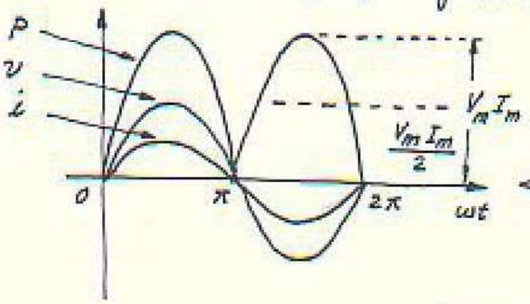
⊕ In resistors, the current and voltage are $\boxed{\text{in phase}}$



* The frequency response of the resistive load is uniform, ie, the value of the resistance doesn't change as the frequency changes.



* The Average Power (Real Power): The instantaneous power to the resistive load is given by:



$$p = vi$$
$$= V_m I_m \sin^2 \omega t = V_m I_m \frac{(1-\cos 2\omega t)}{2}$$

$$\therefore \ p = \underbrace{\frac{V_m I_m}{2}}_{\text{constant term}} - \underbrace{\frac{V_m I_m}{2} \cos 2\omega t}_{\text{Time varying term with average value} = 0}$$

$$\boxed{\therefore \ P_{av} = P = \frac{V_m I_m}{2}} = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}}$$

$$= V_{rms} I_{rms} = V_{eff} I_{eff}$$

$$\boxed{\text{or} \ \ P_{av} = P = V_{rms} I_{rms}}$$

* The Power factor is defined as the cosine of the phase angle between the voltage and current, ie,

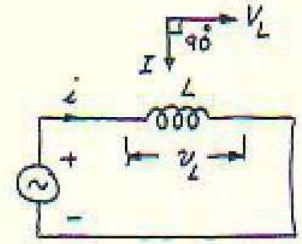
$$\text{Power factor} = P.f = F_p = \cos \phi$$

where ϕ is the phase difference angle between i and v. Since the voltage and current are in phase (ie, the phase difference = 0), then; $\phi = 0$

$$\therefore \cos\phi = \cos 0^\circ = 1$$

$\Rightarrow$ power factor = 1 $\Rightarrow$ For resistive load only.

(*) AC Through Pure Inductance Alone



$$v_L = L\frac{di}{dt}$$

If the current i is given by:

$$i = I_m \sin\omega t$$

$$\Rightarrow v_L = L.\frac{d}{dt}(I_m \sin\omega t) = \omega L I_m \cos\omega t$$

or $\quad v_L = V_m \cos\omega t = V_m \sin(\omega t + 90^\circ) \quad \Leftarrow V_m = \omega L I_m$

From the equations of i and v_L, it is clear that v_L lead i by an angle of (90°), or the current i lags v_L by (90°).

- Let us define the reactance of an inductor, in a way similar to Ohm's law:

$$\text{Reactance} = \frac{\text{Cause}}{\text{Effect}} = \frac{\text{Voltage}}{\text{Current}}$$

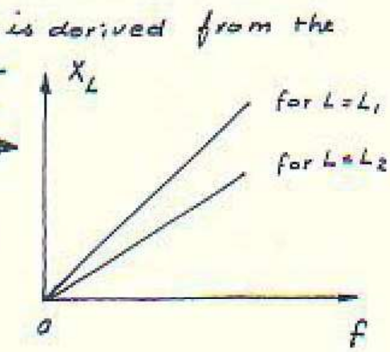
$$\therefore \underset{\text{of an inductor}}{\text{Reactance}} = X_L = \frac{V_m}{I_m} = \frac{\omega L I_m}{I_m} = \omega L$$

$$\boxed{\therefore X_L = \omega L}$$

* The frequency response of the pure inductor is derived from the relation:

$$\boxed{X_L = \omega L = 2\pi f L}$$

and is shown in the figure. X_L increases as the frequency is increased in a linear relationship.



Power factor: The power factor $p.f = \cos\phi$

since $\phi = 90^\circ$ $\Rightarrow$ $\boxed{\therefore p.f = \cos 90^\circ = 0}$

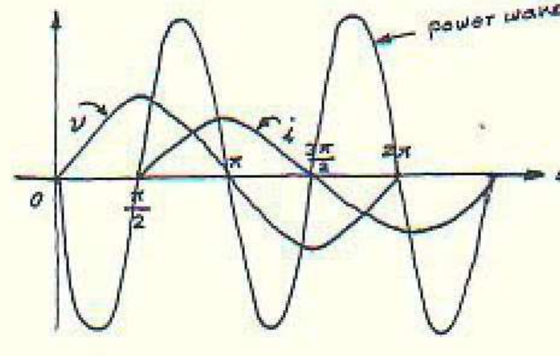
The average power: The instantaneous power for the pure inductive circuit is:

$$p = v_L i$$

$$\Rightarrow p = v_L \cdot i = V_m I_m \sin\omega t \cdot \sin(\omega t + 90^\circ)$$

$$\therefore p = \frac{V_m I_m}{2} \sin 2\omega t$$

The average value of p is zero $\Rightarrow$ $\boxed{P_{av} = 0}$

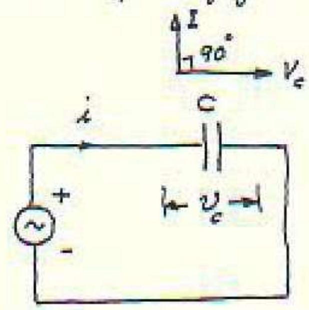


⊛ AC Through a Pure Capacitor

: For the capacitor of the figure shown:

$$i_c = C\frac{dV_c}{dt}$$

let $v_c = V_m \sin\omega t$



$$\therefore i_c = C\frac{d}{dt}(V_m \sin\omega t)$$

$$\Rightarrow i_c = \omega C V_m \cos\omega t$$
$$= \omega C V_m \sin(\omega t + 90^\circ) = I_m \sin(\omega t + 90^\circ)$$

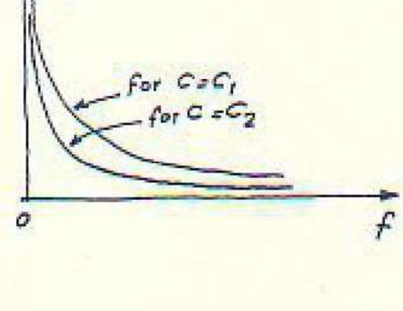
It is clear, from the equations of i_c and v_c, that i_c <u>leads</u> v_c by an angle of 90° or v_c <u>lags</u> i_c by 90°

* The capacitive reactance (X_c) is :

$$X_c = \frac{V_m}{I_m} = \frac{V_m}{\omega C V_m} = \frac{1}{\omega C}$$

$$\boxed{\therefore X_c = \frac{1}{\omega C}}$$

* The frequency response of a pure capacitor is derived from the relation above and is shown in the figure.
X_c is decreasing as the frequency is increased in a <u>non-linear</u> behaviour.



* Power factor

Since the phase difference between i_c and V_c is 90°, this means that $\phi = 90^\circ$, then:

$$\boxed{p.f = \cos\phi = \cos 90^\circ = 0}$$

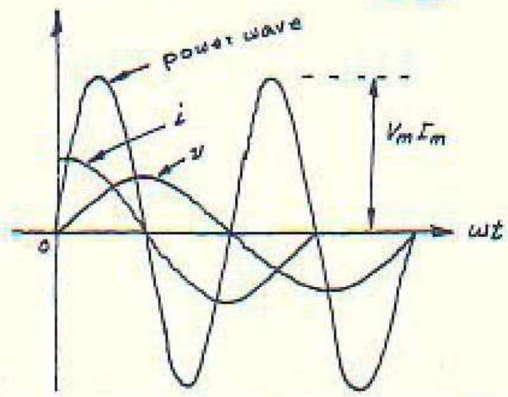
* The average power

The instantaneous power p is given as:

$$p = v_c i_c = V_m I_m \sin\omega t . \sin(\omega t + 90^\circ)$$

$$\Rightarrow p = \underbrace{\frac{V_m I_m}{2} \sin 2\omega t}_{\text{This quantity has an average value of zero}}$$

$\therefore$ The average power of a pure capacitive load is zero.

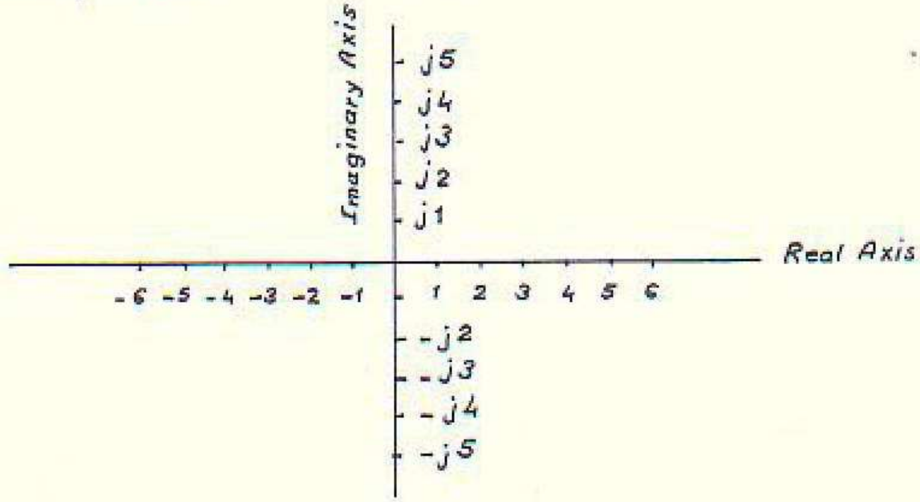


Summary of AC paramters for R, L & C

Element / Paramter	R	L	C
Power factor $\cos\phi$	1	0	0
Average Power $P_{av} = P$	$\frac{V_m I_m}{2} = V_{rms} I_{rms}$	0	0
Impedance Z	R	$X_L = \omega L$	$X_c = \frac{1}{\omega C}$
Phase difference ϕ between v & i	0	90° v_L leads i or i lags v_L	90° v_c lags i_c or i_c leads v_c
Frequency Response	Uniform (constant)	Linear (Increasing)	Non-linear (Decreasing)

Complex Numbers

_______________ : A complex number is a number that represents a point in a two dimensional plane located with reference to two distinct axes. It defines a vector drawn from the origin to that point. The plane used to represent complex numbers is called the complex plane; the two axes are called the real and imaginary. It is important that the scale on the axis of imaginaries must be the same as that on the axis of reals.



The complex plane

Complex numbers can be represented in the following forms:

1. Rectangular form $\Rightarrow$ $\bar{E} = a + jb$
2. Polar form $\Rightarrow$ $\bar{E} = E_m \angle \phi$
3. Trigonometric form $\Rightarrow$ $\bar{E} = E_m (\cos\phi + j \sin\phi)$
4. Exponential form $\Rightarrow$ $\bar{E} = E_m e^{j\phi}$

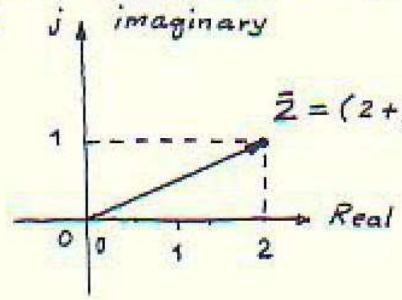
* Rectangular form

: It is customary in this form to denoted the complex numbers as:

$$\boxed{\bar{Z} = R \mp jX}$$

where; $\bar{Z}$ is the complex number.
R is the real part.
X is the imaginary part.
j is an operator $= \sqrt{-1}$ and is equivalent to 90° phase angle.

For example, the complex number $\bar{Z} = 2 + j1$ is represented in the complex plane as shown:



Mathematical Operations in the Rectangular Form:

* *Equality*: If we have two complex numbers $\bar{Z} = x + jy$ and $\bar{W} = u + jv$, then if $\bar{Z} = \bar{W}$, then it follows that

$$x = u \quad \text{and} \quad v = y$$

Example: Given that $\bar{Z}_1 = 5 + j10$, and $Z_2 = 5 + jX$. If $\bar{Z}_1 = \bar{Z}_2$, find the value of X

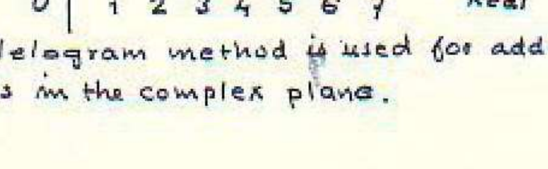
Solution: $X = 10$

* *Addition and Subtraction*: The sum of two complex numbers has a real number equal to the sum of the real components and an imaginary number equal to the sum of the imaginary components.

Example: Given $\bar{Z}_1 = (2 + j3)$, and $\bar{Z}_2 = (4 + j1)$. Find $\bar{Z}_T$, if $\bar{Z}_T = \bar{Z}_1 + \bar{Z}_2$.

Solution:

$$\bar{Z}_T = \bar{Z}_1 + \bar{Z}_2$$
$$= 2 + j3 + 4 + j1 = 6 + j4$$



The parallelogram method is used for adding and subtraction of quantities in the complex plane.

* Multiplication

- The product of a real and imaginary number is imaginary ; Thus:

$$2(j3) = j6$$

- The product of two positive imaginary numbers is real and negative, thus:

$$(j2)(j3) = -6$$

and

$$(j3)(-j4) = +12$$

since $(j)(j) = -1$

Complex numbers are multiplied by the ordinary rules of algebra. As an example ;

$$(2+j3)(4+j1) = (2)(4) + (2)(j1) + (j3)(4) + (j3)(j1)$$
$$= 8 + j2 + j12 - 3 = 5 + j14$$

In general ;

$$\boxed{(a+jb)(c+jd) = (ac-bd) + j(ad+bc)}$$

* Division

By way of illustration , let us consider the division of $\bar{V} = (5+j10)$ by $\bar{I} = (2+j1)$.

$$\Rightarrow \frac{5+j10}{2+j1}$$

* The first step is to multiply both numarator and denominator by $(2-j1)$ which is called complex conjugate of the denominator and usually denoted by asterisk,

$$\bar{I} = 2 + j1$$
$$\bar{I}^* = 2 - j1 \leftarrow \text{complex conjugate of } \bar{I}.$$

$$\boxed{(\bar{I})(\bar{I}^*) = \text{Real value}}$$

then ; for $\dfrac{5+j10}{2+j1} = \dfrac{(5+j10)(2-j1)}{(2+j1)(2-j1)} = \dfrac{20+j15}{5}$

$$\therefore \frac{5+j10}{2+j1} = 4 + j3$$

In general :

$$\boxed{\frac{a+jb}{c+jd} = \frac{ac+bd}{c^2+d^2} + j\,\frac{bc-ad}{c^2+d^2}}$$

* The j operator

$$j = \sqrt{-1} \Rightarrow 90^\circ \text{ ccw rotation}$$

$$j^2 = -1 \Rightarrow 180^\circ \text{ ccw rotation}$$

$$j^3 = j^2 \cdot j = -j \Rightarrow 270^\circ \text{ ccw rotation}$$

$$j^4 = j^2 \cdot j^2 = +1 \Rightarrow 360^\circ \text{ ccw rotation}$$

and we have also:

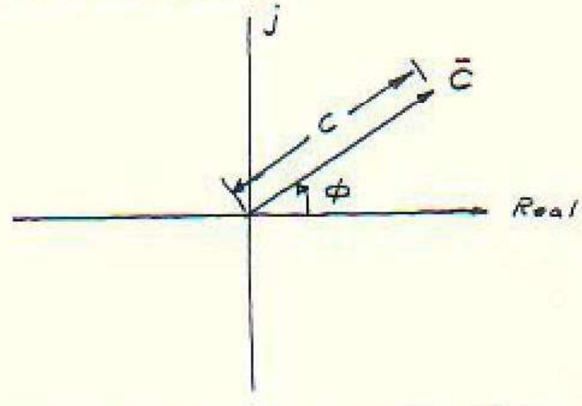
$$\boxed{\frac{1}{j} = -j}$$

Polar Form

In this form, the complex number is represented as

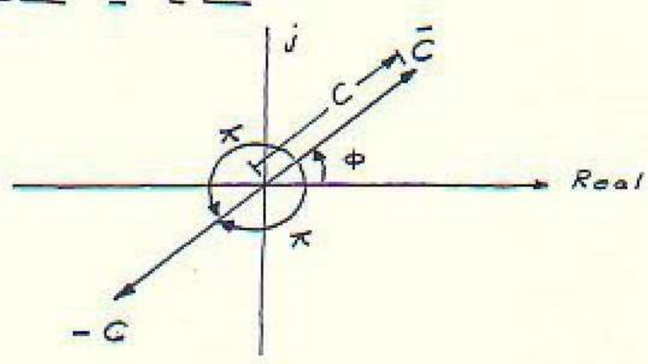
$$\bar{C} = C \angle \phi$$

where C is the magnitude only, and ϕ is always measured counter-clockwise



* A negative sign has the effect shown in the figure

$$-\bar{C} = -C\angle\phi = C\angle\phi \mp \pi$$



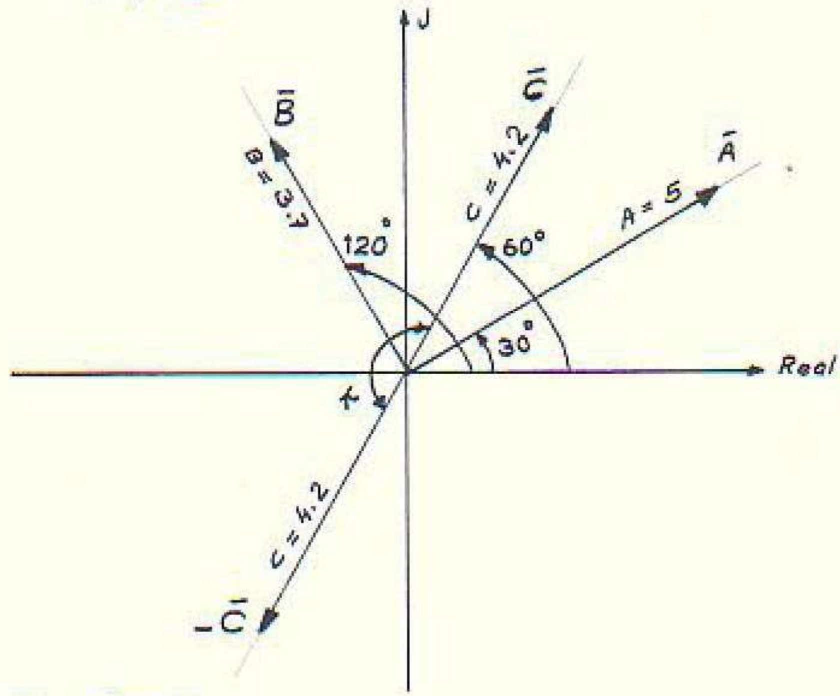
Example: Sketch the following complex numbers in the complex plane.

a. $\bar{A} = 5\angle 30^\circ$

$\bar{B} = 3.7\angle 120^\circ$

$\bar{C} = -4.2\angle 60^\circ$

Solution:



Conversion Between Forms

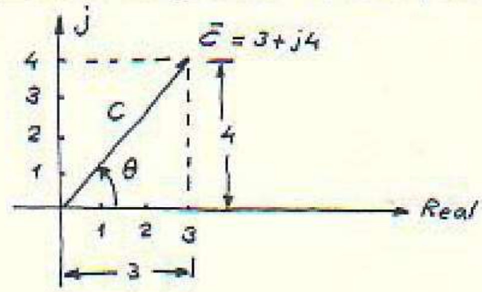
* Rectangular to Polar:

Example: Convert the following from rectangular to polar form.

$\bar{C} = 3 + j4$

The magnitude $C = \sqrt{3^2 + 4^2}$
$= 5$

The angle $\phi = \tan^{-1}\left(\frac{4}{3}\right)$
$= 53.13^\circ$



$\therefore \bar{C}$ in the polar form is $\bar{C} = 5\angle 53.13^\circ$

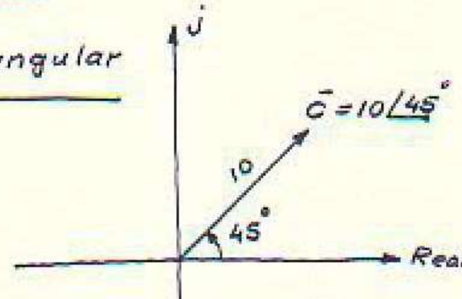
* Polar to Rectangular

Example: convert the following from polar to rectangular form:

$\bar{C} = 10\angle 45^\circ$

$\bar{C} = \text{Real} + j\,\text{Imaginary}$

$\therefore \bar{C} = 7.07 + j\,7.07$



$\text{Real} = 10\cos 45^\circ = 7.07$

$\text{Imaginary} = 10\sin 45 = 7.07$

Example: Convert the following from rectangular to polar form.

$$\bar{C} = -6 + j3$$

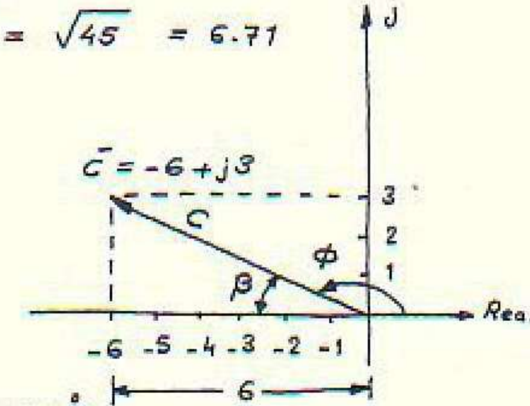
Solution:

$$C = \sqrt{(-6)^2 + (3)^2} = \sqrt{45} = 6.71$$

$$\theta = \tan^{-1}\left(\frac{3}{6}\right)$$

$$= 26.57^\circ$$

$$\Rightarrow \phi = 180^\circ - 26.57^\circ$$
$$= 153.43^\circ$$



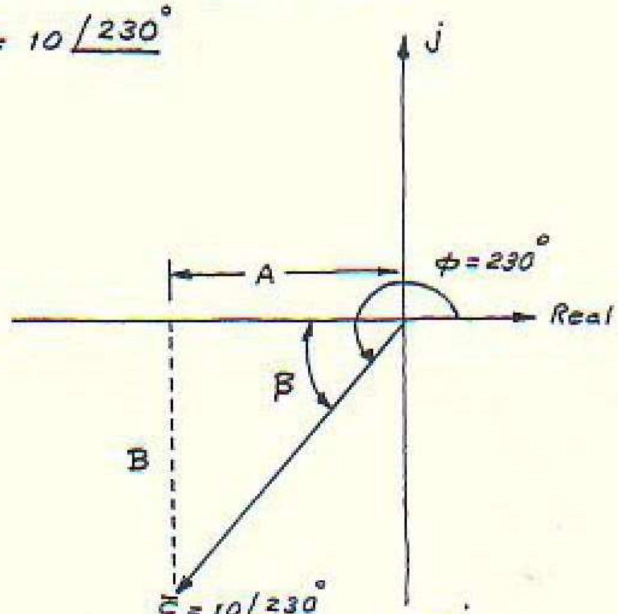
$\therefore$ $\bar{C}$ in polar form is:

$$\bar{C} = C\angle\phi = 6.71\angle 153.43^\circ$$

Example: Convert from polar to rectangular form.

$$\bar{C} = 10\angle 230^\circ$$

Solution:



$$A = 10\cos\beta$$
$$= 10\cos(230^\circ - 180^\circ)$$
$$= 10\cos 50^\circ$$
$$= 6.428$$

$$B = 10\sin\beta$$
$$= 10\sin(230 - 180^\circ)$$
$$= 10\sin 50^\circ$$
$$= 7.66$$

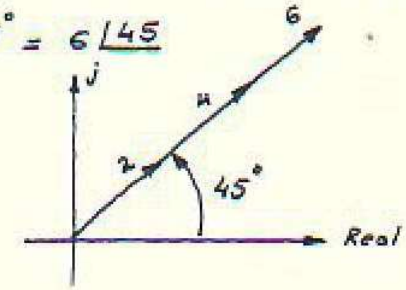
$\therefore$ $\bar{C}$ in rectangular form is

$$\bar{C} = -6.428 - j\,7.66$$

Mathematical Operations in the Polar Form :

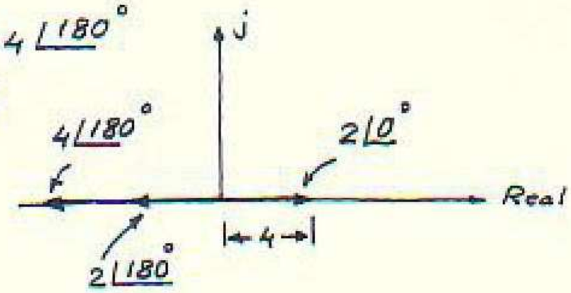
* **Addition and Subtraction** : Addition or subtraction cannot be performed in polar form <u>unless</u> the complex numbers have the same angle ϕ or differ only by multiples of $180°$

Ex: $2\angle 45° + 4\angle 45° = 6\angle 45$



Ex: $2\angle 0° + 4\angle 180°$

$= 2\angle 180°$



* **Multiplication** If we have two complex numbers

$$\bar{C}_1 = C_1\angle \phi_1$$
$$\bar{C}_2 = C_2\angle \phi_2$$

Then $\bar{C}_1 . \bar{C}_2 = C_1 C_2 \angle \phi_1 + \phi_2$

Ex: Find $\bar{C}_1 C_2$, if $\bar{C}_1 = 5\angle 20°$, and $\bar{C}_2 = -10\angle 30°$

$$\bar{C}_1 . \bar{C}_2 = (5)(-10)\angle 20° + 30° = \underline{-50\angle 50°}$$

* **Division** : If we have two complex numbers $\bar{C}_1 = C_1\angle \phi_1$ and $\bar{C}_2 = C_2\angle \phi_2$,

Then $\dfrac{\bar{C}_1}{\bar{C}_2} = \dfrac{C_1}{C_2} \angle \phi_1 - \phi_2$

Ex: Given $\bar{C}_1 = 15\angle 10°$ and $\bar{C}_2 = 2\angle 7°$, find $\dfrac{\bar{C}_1}{C_2}$

$$\frac{C_1}{C_2} = \frac{15}{2} \angle 10° - 7° = \underline{7.5\angle 3°}$$

Series & Parallel AC Circuits

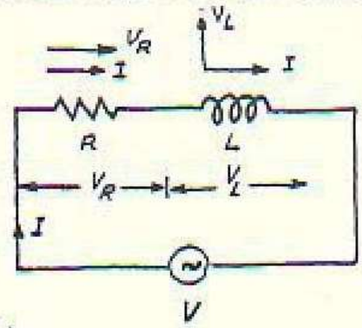
1 Series AC Circuits :

1.1 AC Through R and L

: Consider the circuit shown below;

V = the rms value of the applied voltage.

I = the rms value of the resultant current.



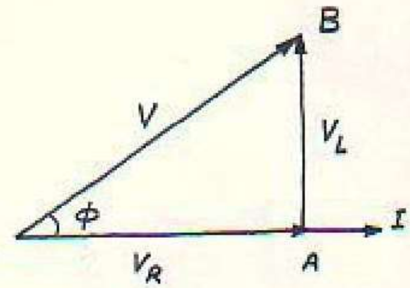
$$\bar{V} = \bar{V}_R + \bar{V}_L$$

$\Rightarrow$ $V_R = IR$ (in phase with I)

$V_L = IX_L$ (leading I by $90°$)

The vector diagram for these voltage drops can be obtained as:

$$\Rightarrow V = \sqrt{V_R^2 + V_L^2}$$



voltage triangle

$$= \sqrt{(IR)^2 + (IX_L)^2}$$

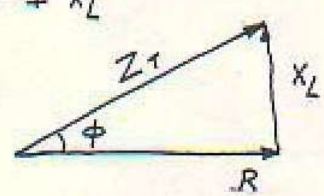
$$= I\sqrt{R^2 + X_L^2}$$

$$\Rightarrow I = \frac{V}{\sqrt{R^2 + X_L^2}} = \frac{V}{Z_T}$$

$\Rightarrow$ Z is known as the impedance of the circuit

$$Z_T = \sqrt{R^2 + X_L^2}$$

$$Z_T^2 = R^2 + X_L^2$$



Impedance traingle

* The phase difference angle ϕ can be determined as:

$$\tan\phi = \frac{V_L}{V_R} = \frac{IX_L}{IR} = \frac{\omega L}{R}$$

ملاحظة

$$\cos\phi = \frac{R}{Z_T}$$

$$\therefore \boxed{\phi = \tan^{-1}\frac{X_L}{R}}$$

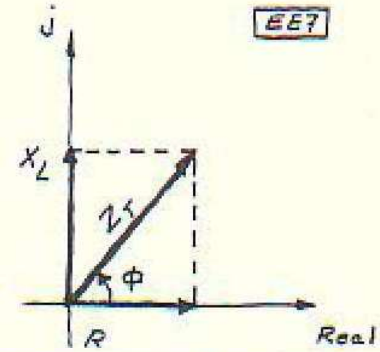
It is clear that the current I lags behind the applied voltage V by an angle (ϕ).

* In phasor notation

$$\bar{Z}_T = \bar{Z}_1 + \bar{Z}_2$$

$$= R\angle 0 + X_L \angle 90$$

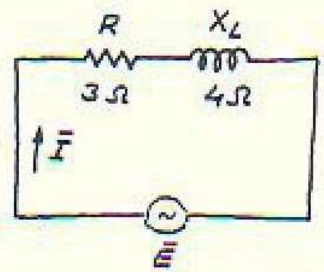
$$\Rightarrow \therefore \bar{Z}_T = R + jX_L$$



Impedance diagram

Example

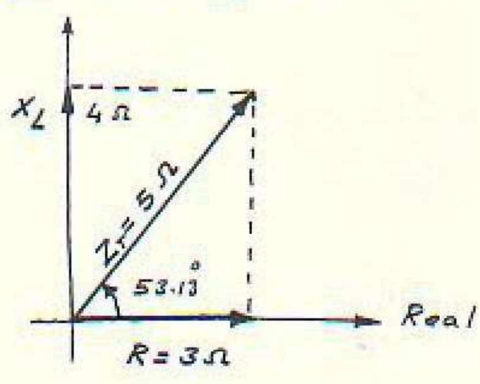
For the circuit shown, determine the total impedance and draw the impedance diagram



Solution : $\bar{Z}_T = \bar{Z}_1 + \bar{Z}_2 = R\angle 0 + X_L\angle 90^\circ = R + jX_L$

$$= 3 + j4$$

$$\Rightarrow \therefore \bar{Z}_T = 5\angle 53.13^\circ$$



Note that, the angle ϕ ($= 53.13^\circ$ in this example) is always positive in the impedance diagram.

$$\phi = \tan^{-1}\frac{X_L}{R} = \tan^{-1}\frac{4}{3}$$

$$= 53.13^\circ$$

* Active, Reactive and Apparent Power (The Power Triangle):

* The Apparent Power:

It is the product of the rms values of the applied voltage and the circuit current

$$\Rightarrow \text{Apparent power} = S = VI = (IZ).I = I^2 Z$$

وحدة قياس

volt-ampere (VA)

* The Active Power:

It is the power which is actually dissipated in the circuit resistance.

$$\Rightarrow \text{Active power} = P = I^2 R = VI\cos\phi \qquad \text{watts (W)}$$

* The Reactive Power:

It is the power developed in the <u>inductive</u> reactance of the circuit.

$$\Rightarrow \text{Reactive Power} = Q = I^2 X_L = I^2 (Z\sin\phi)$$

$$\therefore Q = VI\sin\phi$$

وحدة قياس

Volt-amper reactive (VAR)

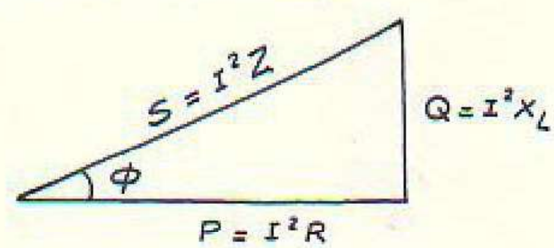
These three powers are shown in the power triangle:

From the power triangle:

$$S^2 = P^2 + Q^2$$

or

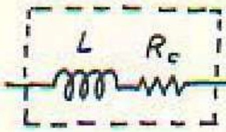
$$S = \sqrt{P^2 + Q^2}$$



The power triangle

* The quality factor of the Coil:

It is defined as the reciprocal of the power factor of the coil. Hence:



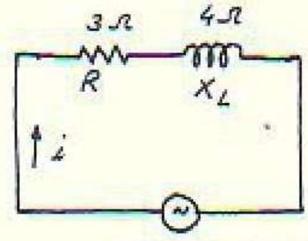
coil

$$\Rightarrow Q_{factor} = \frac{1}{\text{power factor}} = \frac{1}{\cos\phi} = \frac{Z}{R_c} = \frac{\sqrt{R_c^2 + X_L^2}}{R_c}$$

if R_c is too small compared with X_L, then:

$$\boxed{Q_{factor} = \frac{X_L}{R_c}}$$

Example: For the circuit shown, draw the phasor diagram of the voltages across each element and the applied voltage, and determine :- The power factor.
- The active and reactive power.
- The apparent power.



$v = 141.4 \sin\omega t$

Solution:

* $\because v = 141.4 \sin\omega t \Rightarrow \bar{V} = 100\angle 0^\circ$

* $\bar{Z}_T = \bar{Z}_1 + \bar{Z}_2 = R + jX_L$
$= 3 + j4$
$= 5\angle 53.13^\circ$

* The current $\bar{I}$ $\Rightarrow \bar{I} = \dfrac{\bar{V}}{\bar{Z}_T}$

$\therefore \bar{I} = \dfrac{100\angle 0^\circ}{5\angle 53.13^\circ} = 20\angle -53.13^\circ$

* The voltage drops $\bar{V}_R$ and $\bar{V}_L$:

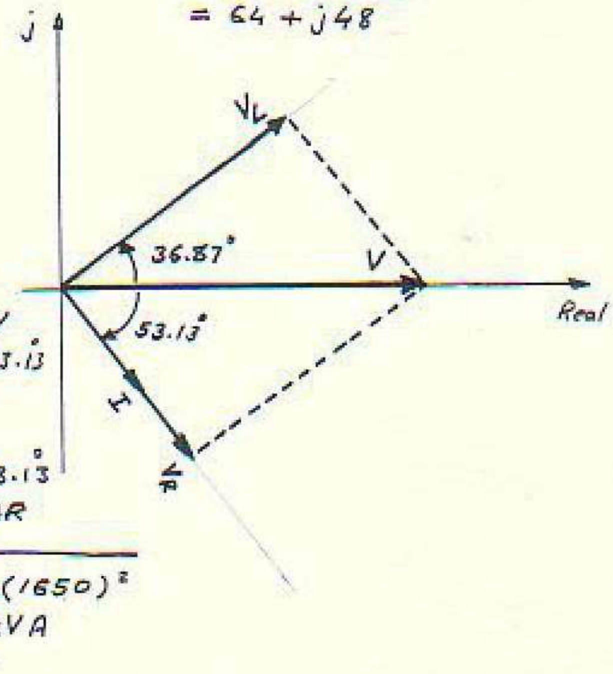
$\bar{V} = \bar{V}_R + \bar{V}_L$ $\Rightarrow \bar{V}_R = \bar{I}R = (20\angle -53.13^\circ)(3)$
$= 60\angle -53.13^\circ$
$= 36 - j48$

$V_L = \bar{I}\bar{X}_L = (20\angle -53.13)(4\angle 90^\circ)$
$= 80\angle 36.87^\circ$
$= 64 + j48$

$\therefore \bar{V} = 36 - j48 + 64 - j48$
$= 100$
$= 100\angle 0^\circ$

في الخطوة دي

* The phasor diagram



* Powers

active power (real) (average) $P = I^2R = (20)^2(3) = 1200\,W$
or $P = VI\cos\phi = (100)(20)\cos 53.13^\circ$
$= 1200\,W = 1.2\,kW$

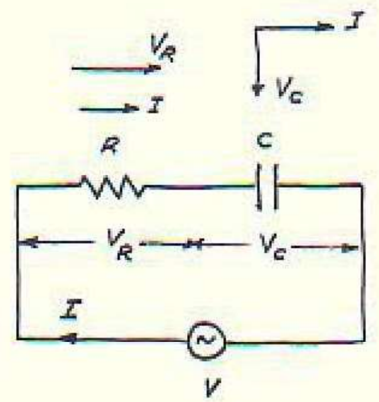
reactive power $Q = VI\sin\phi = (100)(20)\sin 53.13^\circ$
$= 1600\,VAR = 1.6\,kVAR$

$S = \sqrt{P^2 + Q^2} = \sqrt{(1200)^2 + (1650)^2}$
$= 1968\,VA = 1.968\,kVA$

* The power factor: $P.f = \cos\phi = \cos 53.13^\circ = \underline{0.6 \text{ lagging}}$

اعتماداً على التيار بالنسبة للفولتية

1.2 AC Through R and C



Consider the circuit shown, where

V = rms value of the applied voltage.
I = rms value of the resultant current.

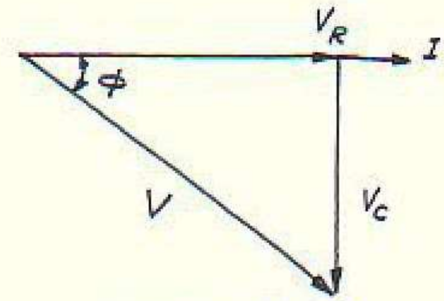
$$V_R = IR \quad \text{(in phase with } I\text{)}$$
$$V_L = IX_C \quad \text{(lagging } I \text{ by } 90^\circ\text{)}$$

* In Vector Notations:

$$\bar{V} = \sqrt{\bar{V}_R^2 + \bar{V}_c^2}$$
$$= \sqrt{I^2R + I^2X_c^2}$$
$$= I\sqrt{R^2 + X_c^2}$$

or $$I = \frac{V}{\sqrt{R^2 + X_c^2}} = \frac{V}{Z}$$

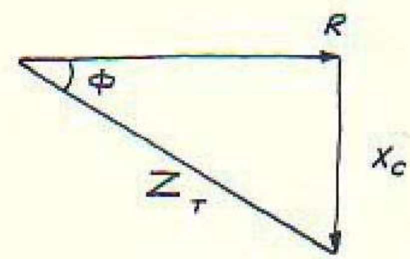
$$Z = \sqrt{R^2 + X_c^2}$$



voltage triangle

* $$\phi = \tan^{-1}\frac{X_c}{R}$$

$$\therefore \text{p.f} = \cos\phi = \frac{R}{Z}$$



impedance triangle

It is clear that I lead V by an angle ϕ. Hence if:

$$v = V_m \sin \omega t$$

then:

$$i = I_m \sin(\omega t + \phi)$$

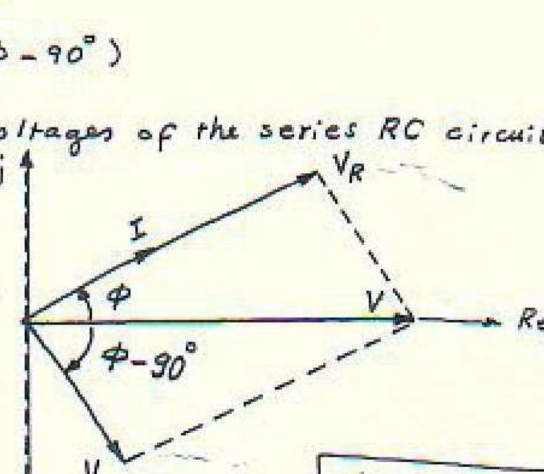
so that the current i lead the applied voltage v by an angle ϕ, and

$$v_R = I_m R \sin(\omega t + \phi)$$

$$v_c = I_m X_c \sin(\omega t + \phi - 90^\circ)$$

* The phasor diagram of the voltages of the series RC circuit can be as shown:

$$\bar{V} = V\angle 0$$
$$\bar{I} = I\angle\phi$$
$$\bar{V}_R = V_R\angle\phi$$
$$\bar{V}_c = V_c\angle\phi - 90^\circ$$

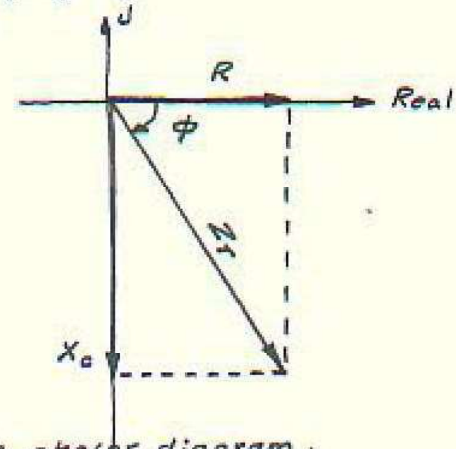


phasor diagram

* **The impedance diagram**

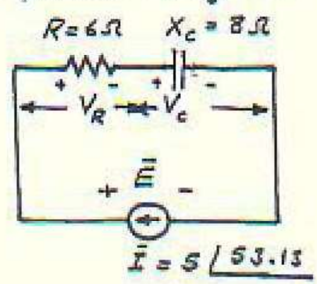
in phasor notation $\quad \bar{Z}_T = \bar{Z}_1 + \bar{Z}_2 = R\angle 0^\circ + X_c\angle -90^\circ$
$= R - jX_c$

$$\therefore \bar{Z}_T = \sqrt{R^2 + X_c^2} = Z_T \angle \phi$$



⇒ Note that ϕ is always negative for RC circuits.

Example: For the circuit shown, draw the phasor diagram.



Solution:

* $\bar{Z}_T = \bar{Z}_1 + \bar{Z}_2$
$= 6\angle 0^\circ + 8\angle -90^\circ$
$= 6 - j8 = 10\angle -53.13^\circ$

* $\bar{E}$?

$\bar{E} = \bar{I}\bar{Z}_T = (5\angle 53.13^\circ)(10\angle -53.13^\circ)$
$= \underline{50\angle 0}$

$\bar{V}_R$?

$\bar{V}_R = \bar{I}R = (5\angle 53.13^\circ)(6)$
$= \underline{30\angle 53.13^\circ}$

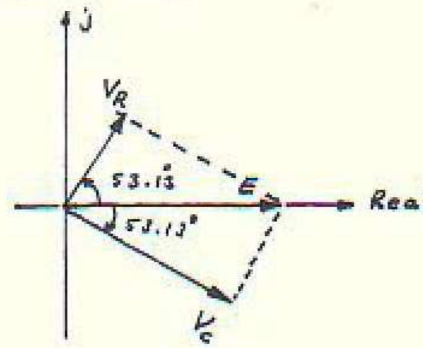
$\bar{V}_c$?

$\bar{V}_c = \bar{I}\bar{X}_c = (5\angle 53.13^\circ)(8\angle -90^\circ)$
$= 40\angle -36.87^\circ$

* you can find that:

$\bar{E} = \bar{V}_R + \bar{V}_c$

using the above values.



* Similarly, as in the case of series RL circuit, the active (average or true) power, reactive power can be determined.

* The active power P is

$$P = I^2 R$$

$$\boxed{P = VI\cos\phi}$$

* The reactive power Q is:

$$Q = I^2 X_c$$

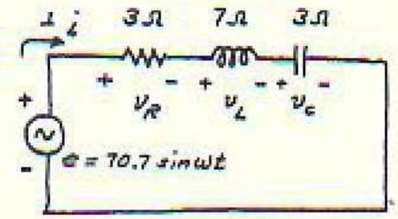
$$\boxed{Q = VI\sin\phi}$$

* and the apparent power $S = \sqrt{P^2 + Q^2}$

Example

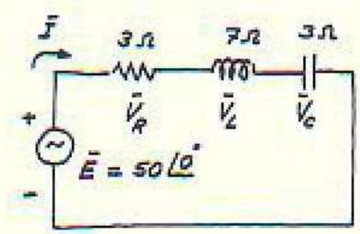
: For the circuit shown, determine:

- $\bar{Z}_T$, and draw the impedance diagram.
- $\bar{I}$, $\bar{V}_R$, $\bar{V}_L$, $\bar{V}_c$ in the phasor domain, and draw the phasor diagram.
- i, v_R, v_L, v_C in the time domain.
- The power factor of the circuit.
- The active, reactive and the apparent powers.



Solution

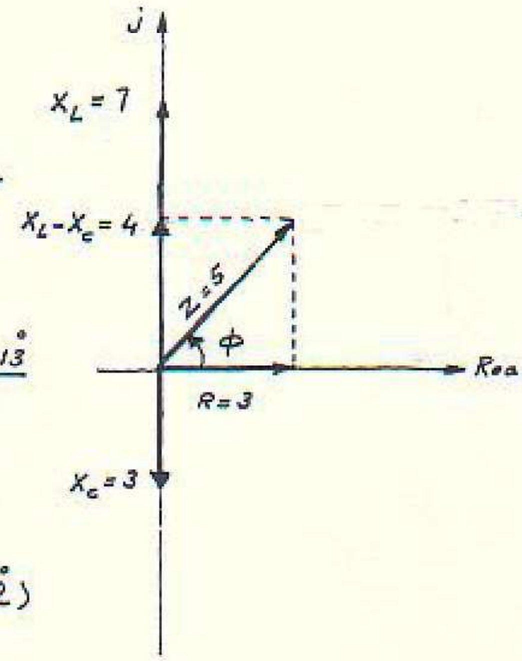
: In phasor notation the circuit is redrawn as:



$$\bar{Z}_T = \bar{Z}_1 + \bar{Z}_2 + \bar{Z}_3$$
$$= R\angle 0 + X_L\angle 90^\circ + X_c\angle -90^\circ$$
$$= 3 + j7 - j3$$
$$= 3 + j4$$

$$\therefore \bar{Z}_T = 5\angle 53.13^\circ$$

- The impedance diagram is ⇒



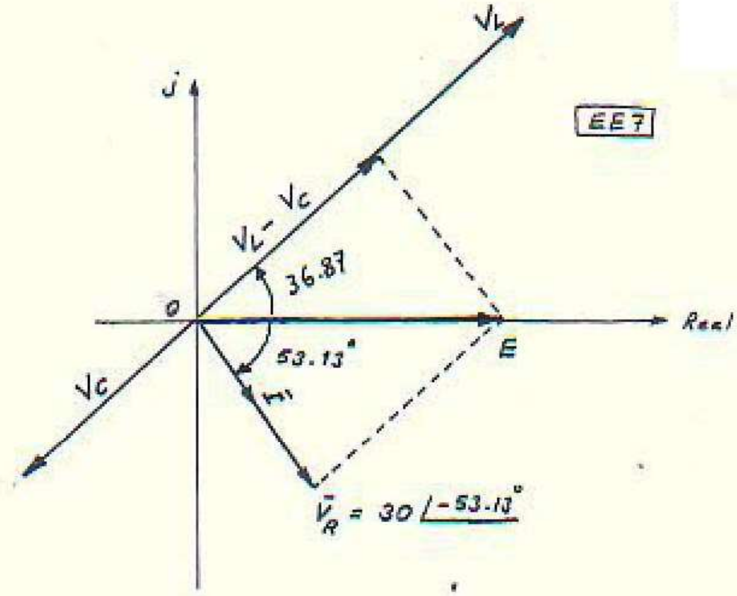
$$\bar{I} = \frac{\bar{E}}{\bar{Z}} = \frac{50\angle 0}{5\angle 53.13^\circ} = 10\angle -53.13^\circ$$

$$\bar{V}_R = \bar{I}R = (10\angle -53.13^\circ)(3\angle 0^\circ)$$
$$= 30\angle -53.13$$

$$V_L = \bar{I}\bar{X}_L = (10\angle -53.13)(7\angle 90^\circ)$$
$$= 70\angle 36.87^\circ$$

$$V_c = \bar{I}\bar{X}_c = (10\angle -53.13^\circ)(3\angle -90)$$
$$= 30\angle -143.13^\circ$$

* **The phasor diagram**



* The time domain

$$i = \sqrt{2}\,(10)\sin(\omega t - 53.13°) = 14.14\sin(\omega t - 53.13°)$$
$$v_R = \sqrt{2}\,(30)\sin(\omega t - 53.13°) = 42.42\sin(\omega t - 53.13)$$
$$v_L = \sqrt{2}\,(70)\sin(\omega t + 36.87°) = 98.98\sin(\omega t + 36.87°)$$
$$v_c = \sqrt{2}\,(30)\sin(\omega t - 143.13°) = 42.42\sin(\omega t - 143.13°)$$

* **The total power**

$$P_T = VI\cos\phi = (50)(10)\cos 53.13° = \underline{300\ W}$$

$\boxed{\text{or}}$ $P_T = I^2 R = (10)^2(3) = \underline{300\ W}$

from the voltage phasor diagram ⇔ * **The power factor**:

$$p.f = \cos\phi = \cos 53.13° = 0.6\ \underline{\text{lagging}}$$

$$\text{or } p.f = \cos\phi = \frac{R}{Z_T} = \frac{3}{5} = 0.6 \text{ lagging}$$

$P \Rightarrow$ * Active power = true power = $VI\cos\phi = (50)(10)\cos 53.13° = 300\ W$

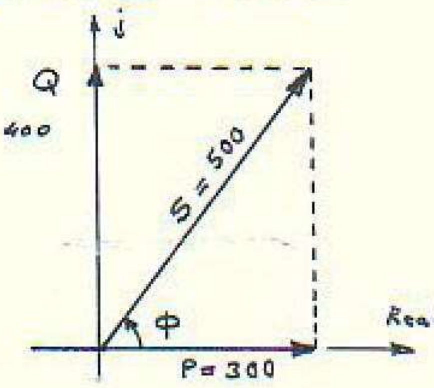
$Q \Rightarrow$ Reactive power = $Q = VI\sin\phi = (50)(10)\sin 53.13° = 400\ VAR$

$S \Rightarrow$ Apparent power = $S = \sqrt{P^2 + Q^2} = \sqrt{(300)^2 + (400)^2} = 500\ VA$

∴ **The power triangle**

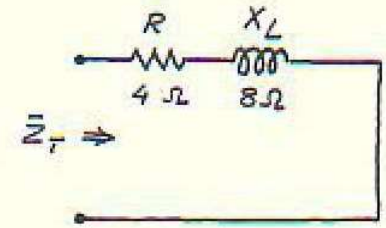
$$\boxed{\bar{S} = P + jQ}$$

$\bar{S} \Rightarrow$ Complex apparent power.



Tutorial Sheet Nº 7

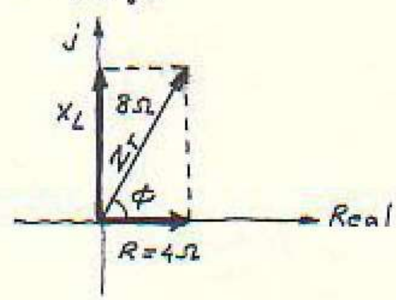
Example: Draw the impedance diagram for the circuit shown and find the total impedance.



Solution: $\bar{Z}_T = \bar{Z}_1 + Z_2$

$$= R\angle 0 + X_L\angle 90^\circ = R + jX_L = 4 + j8$$

$$\therefore \bar{Z}_T = 8.944\angle 63.43^\circ \ \Omega$$



Example: Determine the input impedance to the series network shown

Solution:

$$\bar{Z}_T = \bar{Z}_1 + \bar{Z}_2 + \bar{Z}_3$$

$$= R\angle 0 + X_L\angle 90 + X_C\angle -90$$

$$= R + jX_L - jX_C$$

$$= R + j(X_L - X_C)$$

$$= 6 + j(10 - 12)$$

$$= 6 - j2$$

$$\Rightarrow \bar{Z}_T = 6.325\angle -18.43^\circ \ \Omega$$

Example

: A 60 Hz sinusoidal voltage ($v = 141 \sin\omega t$) is applied to a series R-L circuit. The values of the resistance and the inductance are 3 Ω and 0.0106 H respectively.

(a). Compute the rms value of the current in the circuit and its phase angle with respect to the voltage.

(b). Write the expression for the instantaneous current in the circuit.

(c). Find the average power dissipated by the circuit.

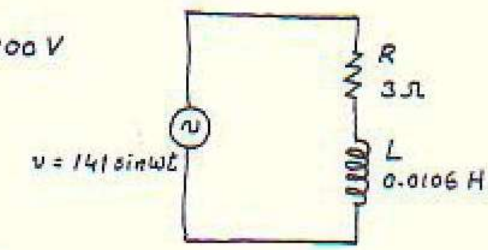
(d). Calculate the p.f of the circuit.

Solution

: We have ; $v = V_m \sin\omega t$

$$\Rightarrow V = \frac{V_m}{\sqrt{2}} = \frac{141}{\sqrt{2}} = 100\ V$$

$$\therefore \bar{V} = 100\underline{/0}$$



(a). $\bar{I} = \dfrac{\bar{V}}{\bar{Z}}$

$$\bar{Z} = R + jX_L$$
$$= R + j(2\pi f L)$$
$$= 3 + j(2\pi \times 60 \times 0.0106)$$
$$= 3 + j4$$
$$\therefore \bar{Z} = 5\,\underline{/53.1^\circ}$$

$$\Rightarrow \bar{I} = \frac{100\underline{/0^\circ}}{20\,\underline{/53.1^\circ}}$$

$\therefore \bar{I} = 20\underline{/-53.1}$ $\Rightarrow$ the current lags the voltage by 53.1°

(b). $i = I_m \sin(\omega t - 53.1^\circ)$

$= \sqrt{2}(20)\sin(\omega t - 53.1^\circ) = \underline{28.28 \sin(\omega t - 53.1^\circ)}$

(c). $P = VI\cos\phi$

$= (100)(20)\cos 53.1^\circ = 1200\ W$

or $P = I^2 R = (20)^2(3) = 1200\ W$

(d). $p.f = \cos\phi$

$= \cos 53.1^\circ$

$= 0.6$ lagging

Example

: A two elements series circuit is connected across an ac circuit having a source ($e = \sqrt{2}(200)\sin(\omega t + 20^\circ)$ V. The current in the circuit is then found to be $i = \sqrt{2}(10)\cos(341t - 25^\circ)$. Determine the parameters of the circuit.

Solution

The applied voltage is :

$$v = \sqrt{2}(200)\sin(\omega t + 20^\circ)$$

$$\Rightarrow \bar{V} = 200\angle 20^\circ$$

The current is :

$$i = \sqrt{2}\,(10)\cos(\omega t - 25^\circ)$$
$$= \sqrt{2}\,(10)\sin(\omega t - 25^\circ - 90^\circ)$$
$$\therefore\ i = \sqrt{2}\,(10)\sin(\omega t + 65^\circ)$$

$$\Rightarrow \bar{I} = 10\angle 65^\circ$$

$$\Rightarrow \bar{Z} = \frac{\bar{V}}{\bar{I}} = \frac{200\angle 20^\circ}{10\angle 65^\circ} = 20\angle -45^\circ$$

Note $\phi = -45^\circ$ (leading)

$$= 14.14 - j\,14.14$$

This impedance represents a series circuit with $R = 14.14\ \Omega$ and a capacitive reactance (because of the $-j$) of $X_C = 14.14\,\Omega$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

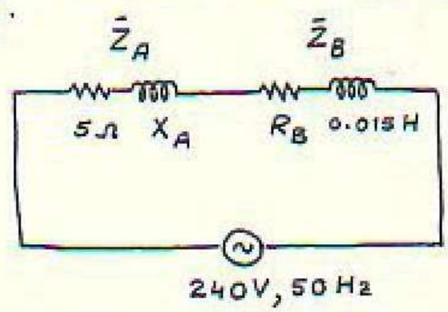
* من معادلة التيار :
$\omega = 314$ rad/sec.

$$\therefore\ 14.14 = \frac{1}{314C} \Rightarrow C = \frac{1}{(14.14)(314)} = 2.25\times 10^{-4}\ F$$

$\therefore$ The circuit has $R = 14.14\ \Omega$
and $C = 225\ \mu F$

Example: Two coils A and B are connected in series across 240 V, 50 Hz supply. The resistance of A is $5\,\Omega$ and the inductance of B is 0.015 H. If the input supply is 3 kW and 2 kVAR, find the resistance of B and the inductance of A. Calculate the voltage across each coil.

Solution:



* From the power triangle, and the circuit shown,

$$S = \sqrt{P^2 + Q^2} = \sqrt{3^2 + 2^2}$$
$$= 3.606\ KVA$$

$$S = VI \Rightarrow I = \frac{S}{V} = \frac{3606}{240} = 15.025\ A$$

But, $P = 3kW = 300\,W$
$$= I^2 R_T = I^2(R_A + R_B)$$
$$\therefore\ 3000 = (15.025)^2(R_A + R_B)$$
$$\Rightarrow R_A + R_B = 13.3\ \Omega$$

$\therefore$ Since $R_A = 5\,\Omega \Rightarrow R_B = 13.3 - 5 = \underline{8.3\ \Omega}$

Similarly, we have:

ملاحظة: من الممكن حل هذا المثال بأكثر من طريقة، والوصول إلى النتائج نفسها.

$$Q = 2\ kVAR = 2000\ VAR$$
$$= I^2 X_{L_T} = (15.03)^2 X_{L_T}$$

$$\therefore\ X_{L_T} = \frac{2000}{(15.03)^2} = 8.85\ \Omega$$

$$X_{L_T} = X_A + X_B \Rightarrow X_A = 8.85 - X_B$$
$$= 8.85 - (2\pi f L_B)$$
$$= 8.85 - (2\pi \times 50 \times 0.015)$$
$$= 8.85 - 4.713 = 4.13\ \Omega$$

$$\therefore\ \bar{Z}_A = R_A + jX_A = 5 + j\,4.13 = 6.48 \angle 39.57^\circ$$

and $\bar{Z}_B = R_B + jX_B = 8.3 + j\,4.713 = 9.54 \angle 29.59^\circ$ $X_B = 2\pi f L_B$ $= 2\pi \times 50 \times 0.015$

$$\therefore\ \bar{Z}_T = \bar{Z}_1 + \bar{Z}_2 = 5 + j4.13 + 8.3 + j4.713$$
$$=$$

$$\therefore\ \bar{V}_A = \bar{I}\,\bar{Z}_A = \checkmark$$

$$\bar{V}_B = \bar{I}\,\bar{Z}_B = \checkmark$$

or $$\bar{V}_A = \frac{\bar{V}\,\bar{Z}_A}{\bar{Z}_A + \bar{Z}_B} = \checkmark$$

The results must be the same.

and

$$\bar{V}_B = \frac{\bar{V}\,\bar{Z}_B}{\bar{Z}_A + \bar{Z}_B} = \checkmark$$

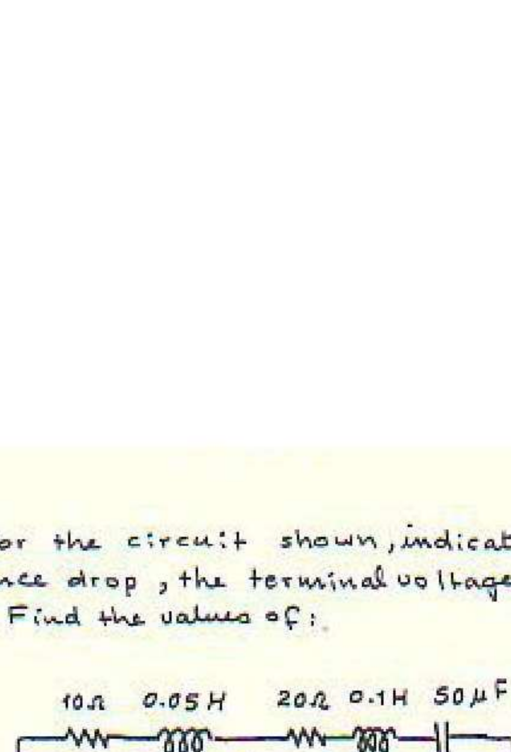
Example: Draw the phasor diagram for the circuit shown, indicating the resistance and the reactance drop, the terminal voltages $\bar{V}_1$ and $\bar{V}_2$ and the current. Find the values of:

(a). The current $\bar{I}$.

(b). $\bar{V}_1$

(c). $\bar{V}_2$

(d). The power factor



Solution:

$$R_T = 10 + 20 = 30\ \Omega$$

$$L_T = 0.05 + 0.1 = 0.15\ H$$

$$\Rightarrow X_L = \omega L = 2\pi f L_T = 2\pi(50)(0.15)$$
$$= 47.1\ \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{2\pi(50)(50\times10^{-6})} = 63.7\ \Omega$$

$$\therefore\ \bar{Z}_T = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(30)^2 + (47.1 - 63.7)^2}$$

$$= 34.3\ \underline{\lfloor -28.96^\circ}$$

(a). $$\therefore\ \bar{I} = \frac{\bar{V}}{\bar{Z}} = \frac{200\ \underline{\lfloor 0^\circ}}{34.3\ \underline{\lfloor -28.96}} = 5.83\ \underline{\lfloor 28.96} \quad \underline{\text{leading}}$$

(b). $\bar{V}_1 = ?$

$$\bar{V}_1 = \bar{I}\,\bar{Z}_1 \qquad \Rightarrow \bar{Z}_1 = 10 + jX_{L_1}$$

$$= 10 + j\,(2\pi f L_1)$$

$$= 10 + j\,(2\pi(50)\,0.05)$$

$$= 10 + j\,15.7$$

$$\therefore \bar{Z}_1 = 18.6 \angle 57.5$$

$$\therefore \bar{V}_1 = (5.83\angle 28.96)(18.6\angle 57.5)$$

$$= 108.4 \angle 86.46^\circ$$

(c). $\bar{V}_2 = ?$

$$\bar{V}_2 = \bar{I}\,\bar{Z}_2 \qquad \Rightarrow \bar{Z}_2 = 20 + jX_{L_2} - jX_{C_2}$$

$$= 20 + j\,(2\pi f L_2) - \frac{1}{2\pi f C_2}$$

$$= 20 + j\,31.4 - j\,63.7$$

$$= 20 - j\,32.3$$

$$\therefore \bar{Z}_2 = 37.74 \angle -58.2^\circ$$

$$\therefore \bar{V}_2 = \bar{I}\,\bar{Z}_2 = (5.83\angle 28.96^\circ)(37.74\angle -58.2^\circ)$$

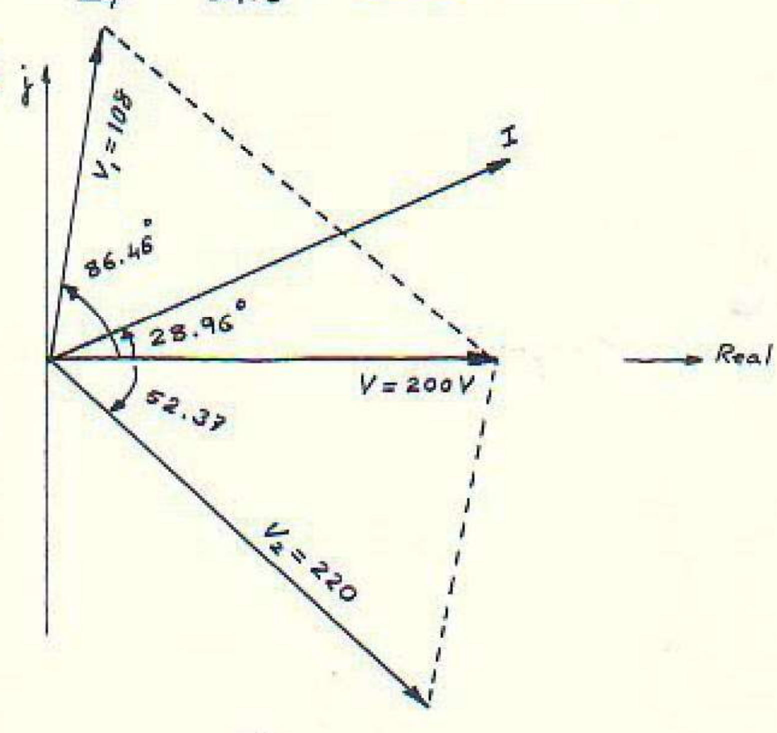
$$= 220.1 \angle -52.37$$

(d). The combined (overall) power factor of the circuit :

- from part (a) $\Rightarrow$ $p.f = \cos\phi = \cos 28.96^\circ$
  $= 0.87$ leading

- or $\quad p.f = \dfrac{R}{Z_T} = \dfrac{30}{34.3} = 0.87$ (leading)

The phasor diagram



7.2 Parallel AC Circuits

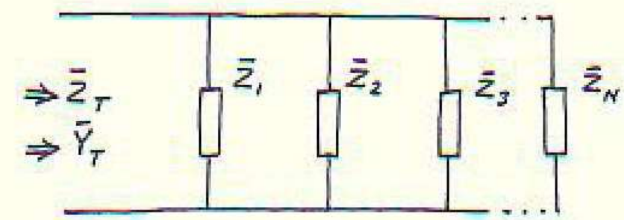
+ Admittance and Susceptance

- In the dc circuit analysis, we had used the term conductance to represent the reciprocal of the resistance R; ie:

$$G = \frac{1}{R} \qquad \text{where } G \text{ is the conductance}$$

The total conductance of the paralled circuit is then found by adding the conductance of eachb branch.

- In AC circuit analysis, we define the admittance ($\bar{Y}$) as equal to $1/\bar{Z}$. For the parallel circuit shown:



+ The total admittance Y_T:

$$\bar{Y}_T = \bar{Y}_1 + \bar{Y}_2 + \bar{Y}_3 + \dots \bar{Y}_N$$

then since $\bar{Y} = \frac{1}{\bar{Z}}$; so the total impedance $\bar{Z}_T$:

$$\frac{1}{\bar{Z}_T} = \frac{1}{\bar{Z}_1} + \frac{1}{\bar{Z}_2} + \frac{1}{\bar{Z}_3} + \dots \frac{1}{\bar{Z}_N}$$

- As mentioned earlier, for 2 branches parallel ac circuit, then:

$$\bar{Z}_T = \frac{\bar{Z}_1 \bar{Z}_2}{\bar{Z}_1 + \bar{Z}_2}$$

- Also for 3 parallel branches;

$$\bar{Z}_T = \frac{\bar{Z}_1 \bar{Z}_2 \bar{Z}_3}{\bar{Z}_1 \bar{Z}_2 + \bar{Z}_2 \bar{Z}_3 + \bar{Z}_1 \bar{Z}_3}$$

+ INGENERAL, we have: $\bar{Z}_T = R \mp jX$, the

$$\bar{Y}_T = \frac{1}{R} \mp \frac{1}{jX} = \boxed{G \pm jB}$$

where: $G \Rightarrow$ Conductance $= \frac{1}{R}$ (Siemens, S) تقاس بوحدة

$B \Rightarrow$ Susceptance $= \frac{1}{X}$ (Siemens, S)

TS7

$\therefore \quad \bar{Z}_T = \bar{Z}_1 + \bar{Z}_{23}$

$$\bar{Z}_{23} = \frac{\bar{Z}_2 \bar{Z}_3}{\bar{Z}_2 + \bar{Z}_3}$$

$$= \frac{(10 - j12)(6 + j10)}{(16 - j2)} = 10.9 + j3.1$$

$$= 11.3 \angle 15.9^\circ \ \Omega$$

$\Rightarrow \bar{Z}_T = (4 + j\,6) + (10.9 + j\,3.1)$

$= 14.9 + j\,9.1 \quad = 17.5 \angle 31.4^\circ \quad \Omega$

* Now, finding $\bar{I} = ?!$

$$\bar{I} = \frac{\bar{V}}{\bar{Z}} = \frac{200 \angle 0^\circ}{17.5 \angle 31.4^\circ} = 11.4 \angle -31.4^\circ \quad A$$

* To draw the phasor diagram, we have (till now) $\vec{V}$ and $\bar{I}$, then we have to find the following quantities:

* $\bar{V}_{AB} = \bar{I}\bar{Z}_1 = (11.4 \angle -31.4^\circ)(7.2 \angle 56.3^\circ)$

$= 82.2 \angle 24.9^\circ$ volts

* $\bar{V}_{BC} = \bar{I}\bar{Z}_{123} = (11.4 \angle -31.4^\circ)(11.3 \angle 15.9^\circ)$

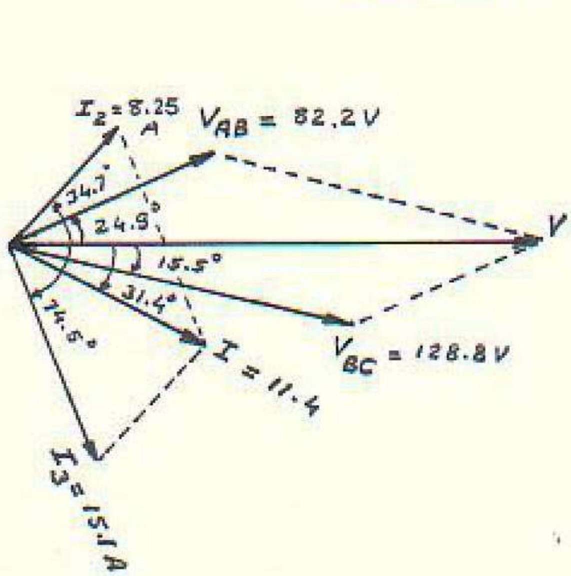
$= 128.8 \angle -15.5^\circ$ volts

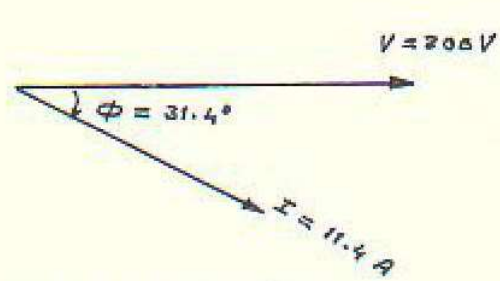
* $\bar{I}_2 = \dfrac{\bar{V}_{BC}}{\bar{Z}_2} = \dfrac{128.8 \angle -15.5^\circ}{15.6 \angle -50.2^\circ}$

$= 8.25 \angle 34.7^\circ$ A

* $\bar{I}_3 = \dfrac{128.8 \angle -15.5^\circ}{11.7 \angle 58^\circ}$

$= 15.1 \angle -74.5^\circ$ A





The power factor is

$\cos\phi = \cos 31.4^\circ$

$= \underline{0.85}$ lagging

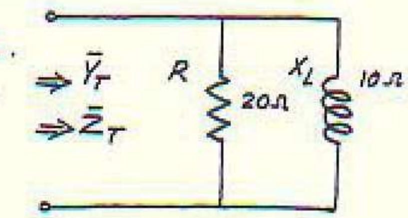
Example: For the circuit shown; EE7

a. Determine the admittance of each branch.

b. Find the input admittance.

c. Calculate the input impedance.

d. Draw the admittance diagram.



Solution:

ⓐ:

- $\bar{Y}_1 = \bar{G} = G\angle 0 = \frac{1}{R}\angle 0 = \frac{1}{20}\angle 0 = 0.05\angle 0$
  $= 0.05 + j0$
  $= 0.05$

- $\bar{Y}_2 = \bar{B}_L = \frac{1}{X_L}\angle -90^\circ = \frac{1}{10}\angle -90^\circ = 0.1\angle -90^\circ$
  $= 0 - j0.1$
  $= -j0.1$

ⓑ: $\bar{Y}_T = \bar{Y}_1 + \bar{Y}_2 = 0.05 - j0.1 = G - jB_L$

ⓒ: $\bar{Z}_T = \frac{1}{\bar{Y}_T} = \frac{1}{0.05 - j0.1} = \frac{1}{0.112\angle -63.43^\circ}$
$= 8.93\angle 63.43^\circ$

OR

$$\bar{Z}_T = \frac{\bar{Z}_1 \bar{Z}_2}{\bar{Z}_1 + \bar{Z}_2} = \frac{(20\angle 0)(10\angle 90^\circ)}{20 + j10}$$

$$= \frac{200\angle 90^\circ}{22\angle 26.57^\circ}$$

$= 8.93\angle 63.43^\circ$ ⟸ which is the same as calculated above.

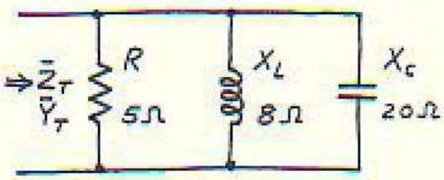
ⓓ: The admittance diagram:

EE7

Example

: For the circuit show;

a. Determine the admittance of each branch.
b. Find the input admittance.
c. Calculate the input impedance.
d. Draw the admittance diagram.



Solution

: (a).

$$\bar{Y}_1 = \bar{G} = \frac{1}{R\angle 0} = \frac{1}{5}\angle 0 = 0.2\angle 0^\circ = 0.2 + j0 = 0.2$$

$$\bar{Y}_2 = \bar{B}_L = \frac{1}{X_L\angle 90^\circ} = \frac{1}{X_L}\angle -90^\circ = \frac{1}{8}\angle -90^\circ = 0.125\angle -90^\circ = -j0.125$$

$$\bar{Y}_3 = \bar{B}_c = \frac{1}{X_c\angle -90^\circ} = \frac{1}{X_c}\angle 90^\circ = \frac{1}{20}\angle 90^\circ = 0.05\angle 90^\circ = +j0.05$$

(b).

$$\begin{aligned}\bar{Y}_T &= \bar{Y}_1 + \bar{Y}_2 + \bar{Y}_3 \\ &= 0.2 - j0.125 + j0.05 \\ &= 0.2 - j0.075 \\ &= 0.2136\angle -20.56^\circ\end{aligned}$$

(c).

$$\begin{aligned}\bar{Z}_T &= \frac{1}{\bar{Y}_T} = \frac{1}{0.2136\angle -20.56^\circ} \\ &= 4.68\angle 20.56^\circ\end{aligned}$$

or

$$\bar{Z}_T = \frac{\bar{Z}_1\bar{Z}_2\bar{Z}_3}{\bar{Z}_1\bar{Z}_2 + \bar{Z}_2\bar{Z}_3 + \bar{Z}_1 Z_3}$$

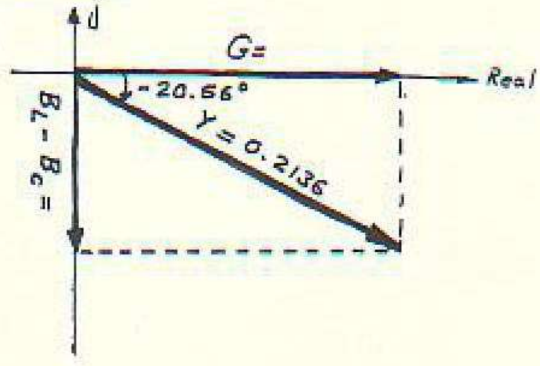
$$= \frac{(5\angle 0^\circ)(8\angle 90^\circ)(20\angle -90^\circ)}{(5\angle 0^\circ)(8\angle 90^\circ) + (8\angle 90^\circ)(20\angle -90^\circ) + (5\angle 0^\circ)(20\angle -90^\circ)}$$

$$= \frac{800\angle 0^\circ}{40\angle 90^\circ + 160\angle 0^\circ + 100\angle -90^\circ}$$

$$= \frac{800\angle 0^\circ}{j40 + 160 - j100} = \frac{800\angle 0^\circ}{160 - j60}$$

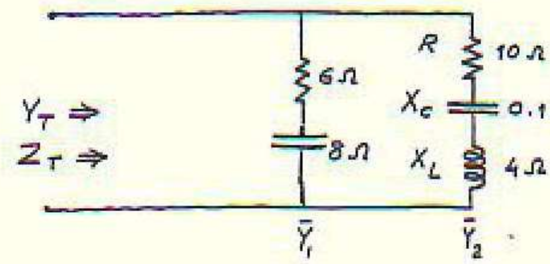
$$= \frac{800\angle 0^\circ}{170.88\angle -20.56^\circ} = \underline{\underline{4.68\angle 20.56^\circ}}$$

(d). *The Admittance Diagram*



EE7

Example: Find the admittance of the circuit shown



Solution: $\bar{Z}_1 = 6 - j8$

$$\Rightarrow \bar{Y}_1 = \frac{1}{\bar{Z}_1} = \frac{1}{6-j8} = \frac{6+j8}{6^2+8^2} = \frac{6}{100} + j\frac{8}{100}$$

$$= 0.06 + j0.08$$

$$\bar{Z}_2 = 10 + j4 - j0.1$$

$$= 10 + j3.9$$

$$\Rightarrow \bar{Y}_2 = \frac{1}{\bar{Z}_2} = \frac{1}{10+j3.9} = \frac{10-j3.9}{10^2+3.9^2} = \frac{10}{115.21} - j\frac{3.9}{115.21}$$

$$= 0.087 - j0.034$$

$$\therefore \bar{Y}_T = \bar{Y}_1 + \bar{Y}_2$$

$$= 0.06 + j0.08 + 0.087 - j0.034$$

$$= 0.147 + j0.046$$

$$= 0.154 \angle 17.3762^\circ$$

* $\Rightarrow$ OR you can try again to get $\bar{Y}_T$ as follows:

$$\bar{Z}_T = \frac{\bar{Z}_1 \bar{Z}_2}{\bar{Z}_1 + Z_2}$$

$$= \frac{(6-j8)(10+j3.9)}{(6-j8)+(10+j3.9)}$$

and proceed to get $\bar{Z}_T \Rightarrow Y_T$ must be the same value

$$\bar{Y}_T = \frac{1}{\bar{Z}_T} = 0.154 \angle 17.3762^\circ$$

Illustrative Examples on R-L , R-C , and R.L.C Prallel AC Circuits

EE7

* R-L parallel ac circuits

Example :

For the circuit shown ;

a. $\bar{Z}_T$

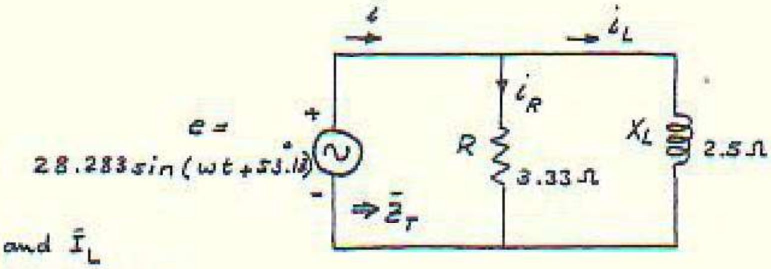
b. Draw the admittance diagram

c. The currents $\bar{I}$, $\bar{I}_R$, and $\bar{I}_L$

d. Draw the current phasor diagram.

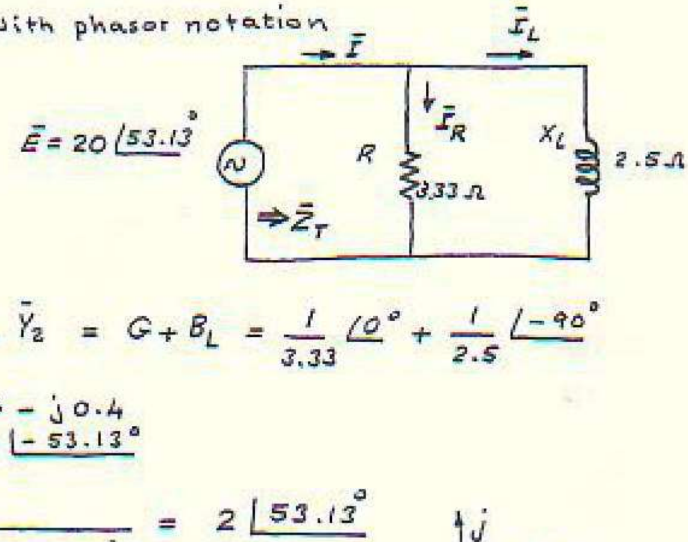
e. Calculate the active , reactive , and complex apparent powers.

f. Determine the power factor.



Solution

Draw the circuit with phasor notation



ⓐ. $\bar{Z}_T$:

$$\bar{Y}_T = \bar{Y}_1 + \bar{Y}_2 = G + B_L = \frac{1}{3.33}\angle 0^\circ + \frac{1}{2.5}\angle -90^\circ$$

$$= 0.3 - j0.4$$

$$= 0.5\angle -53.13^\circ$$

$$\therefore \bar{Z}_T = \frac{1}{\bar{Y}_T} = \frac{1}{0.5\angle -53.13^\circ} = 2\angle 53.13^\circ$$

ⓑ. The admittance diagram $\Rightarrow$

(c). $\bar{I} = \dfrac{\bar{E}}{\bar{Z}_T} = \dfrac{20\angle 53.13^\circ}{2\angle 53.13^\circ} = 10\angle 0^\circ$

$\bar{I}_R = \dfrac{\bar{E}}{R} = \dfrac{20\angle 53.13^\circ}{3.33\angle 0^\circ} = 6\angle 53.13^\circ$

$\bar{I}_L = \dfrac{\bar{E}}{\bar{X}_L} = \dfrac{20\angle 53.13^\circ}{2.5\angle 90^\circ} = 8\angle -36.87^\circ$

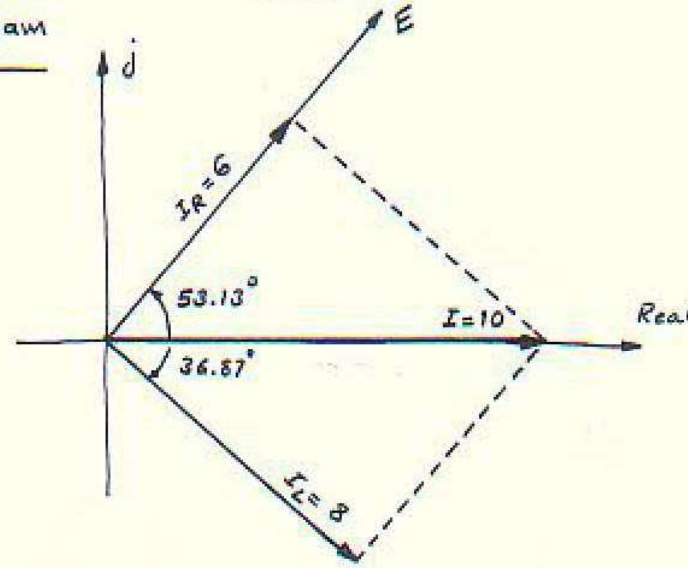
* For check

using KCL $\Rightarrow \bar{I} = \bar{I}_R + I_L$

$\Rightarrow \bar{I} = 6\angle 53.13^\circ + 8\angle -36.87$
$= (3.6 + j4.8) + (6.4 - j4.8)$
$= 10 + j0$
$= 10\angle 0^\circ$

which is the same as calculated above

(d). The current phasor diagram



(e). Active power $= P = EI\cos\phi = (20)(10)\cos 53.13^\circ$
$= 120$ W

Reactive power $= Q = EI\sin\phi = (20)(10)\sin 53.13^\circ$
$= 160$ VAR

$\therefore$ Complex apparent power $= \bar{S} = P + jQ = 120 + j160$

$\therefore \quad \bar{S} = P^2 + Q^2 = 200$ VA

from the phasor dig.

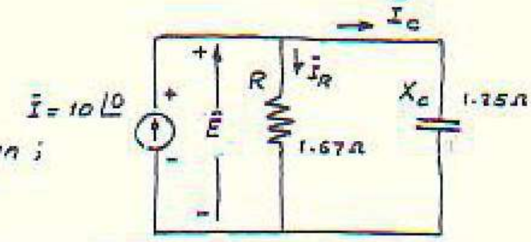
(f). The power factor $P.f = \cos\phi = \cos 53.13^\circ = 0.6$ lagging

or $\cos\phi = \dfrac{P}{EI} = \dfrac{E^2/R}{EI} = \dfrac{EG}{I} = \dfrac{G}{Y_T}$

ملاحظة : يمكن حساب $\bar{Z}_T$ حسب العلاقة ⟸ $\dfrac{\bar{Z}_1 \bar{Z}_2}{\bar{Z}_1 + \bar{Z}_2}$ والحصول على النتيجة نفسها .

EE7

* R-C parallel ac circuit



Example: For the circuit shown ;

a. Determine $\bar{Z}_T$.

b. Draw the admittance diagram.

c. Calculate $\bar{E}$, I_R, and $\bar{I}_c$, and draw the current phasor diagram.

d. Active, reactive, and apparent powers.

e. Determine the power factor for the circuit.

Solution: (a)

$$\bar{Y}_T = \bar{Y}_1 + \bar{Y}_2 = \frac{1}{R\angle 0} + \frac{1}{X_c\angle -90^\circ} = \frac{1}{1.67}\angle 0^\circ + \frac{1}{1.25}\angle 90$$

$$= 0.6 + j0.8 = 1\angle 53.13^\circ$$

Also: $\bar{Z}_T = \dfrac{\bar{Z}_1\bar{Z}_2}{\bar{Z}_1 + \bar{Z}_2}$

$$\Rightarrow \bar{Z}_T = \frac{1}{Y_T} = \frac{1}{1\angle 53.13^\circ} = 1\angle -53.13^\circ$$

(b).

$$\bar{E} = \frac{\bar{I}}{Y_T} = \frac{10\angle 0^\circ}{1\angle 53.13^\circ} \quad \text{or} \quad \bar{E} = \bar{I}\bar{Z}_T$$

$$= 10\angle -53.13^\circ$$

$$\bar{I}_R = \bar{E}\bar{G} \quad \text{or} \quad \bar{I}_R = \frac{\bar{E}}{R}$$

$$= (10\angle -53.13^\circ)(0.6)$$

$$= 6\angle -53.13^\circ$$

$$I_c = \bar{E}\bar{B}_c \quad \text{or} \quad \bar{I}_c = \frac{\bar{E}}{\bar{X}_c}$$

$$= (10\angle -53.13^\circ)(0.8\angle 90)$$

$$= 8\angle 36.87^\circ$$

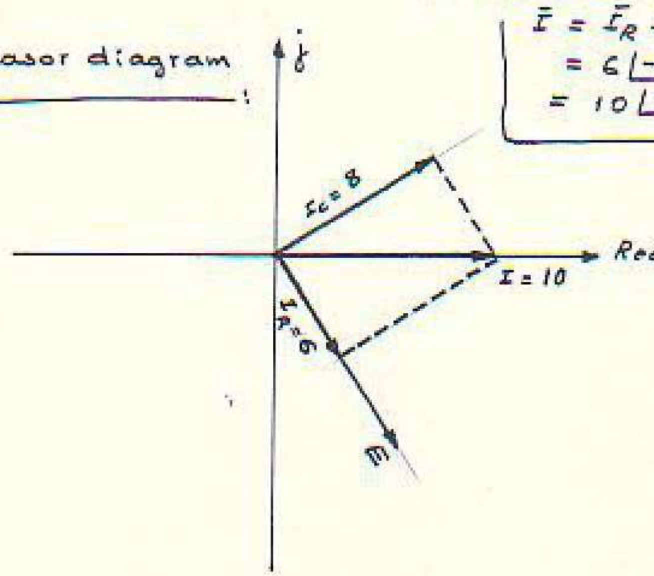
$\Rightarrow$ As a check ?

$$\bar{I} = \bar{I}_R + \bar{I}_c$$

$$= 6\angle -53.13 + 8\angle 36.87^\circ$$

$$= 10\angle 0^\circ$$

(c). The phasor diagram :



ⓓ. Active power $= P = EI\cos\phi$

$= (10)(10)\cos 53.13^\circ$

$= 60\ W$

* or $P = E^2 G$

$= (10)^2 (0.6)$

$= 60\ W$

Reactive power $= Q = EI\sin\phi$

$= (10)(10)\sin 53.13^\circ$

$= 80\ VAR$

$\therefore$ Apparent power $S = \sqrt{P^2 + Q^2} = \sqrt{(60)^2 + (80)^2}$

$= 100\ VA$

ⓔ. The power factor

$p.f = \cos 53.13$

$= 0.6$ <u>leading</u> ⇐ from the phasor diagram $\bar{I}$ leads $\bar{E}$

* <u>R-L-C parallel AC circuit</u> :

<u>Example</u> : For the circuit shown;

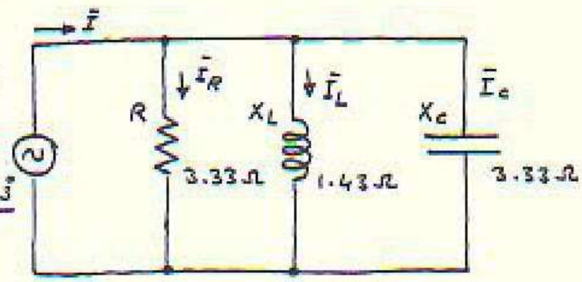
a. Determine $\bar{Z}_T$.

b. Calculate $\bar{I}$, $\bar{I}_R$, $\bar{I}_L$ & $\bar{I}_c$

c. Draw the phasor diag. $\bar{E} = 100\angle 53.13^\circ$

d. Calculate the active (real) power

e. Determine the power factor



<u>Solution</u> : ⓐ. $\bar{Z}_T = ?$ ⇒ $\bar{Y}_T = \bar{Y}_1 + \bar{Y}_2 + \bar{Y}_3$

$= \frac{1}{R}\angle 0^\circ + \frac{1}{X_L}\angle -90^\circ + \frac{1}{X_c}\angle 90^\circ$

$= \frac{1}{3.33}\angle 0^\circ + \frac{1}{1.43}\angle -90^\circ + \frac{1}{3.33}\angle 90^\circ$

$= 0.3 - j0.7 + j0.3$

$= 0.3 - j0.4$

$= 0.5\angle -53.13^\circ$

$\therefore \bar{Z}_T = \frac{1}{\bar{Y}_T} = \frac{1}{0.5\angle -53.13^\circ} = 2\angle 53.13^\circ$

ⓑ. $\bar{I} = \frac{\bar{E}}{\bar{Z}_T} = \frac{100\angle 53.13^\circ}{2\angle 53.13^\circ} = 50\angle 0^\circ$

$I_R = \bar{E}\bar{G} = (100\angle 53.13^\circ)(0.3\angle 0^\circ)$

$= 30\angle 53.13^\circ$

$\bar{I}_R = \frac{\bar{E}}{R} = \checkmark$ ممكن كده ⇐

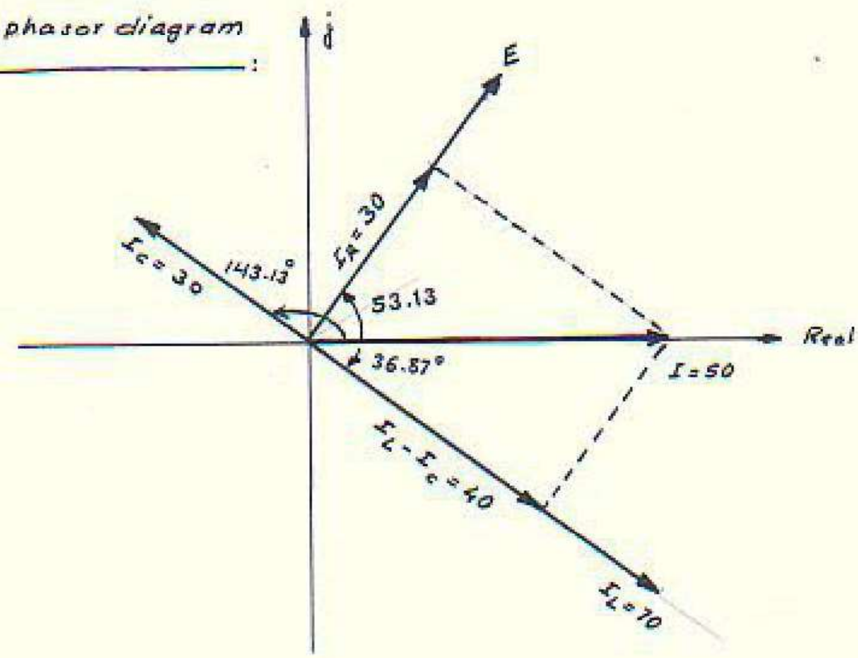
$\Rightarrow \bar{I}_L = \frac{\bar{E}}{\bar{X}_L}$ $\boxed{EE7}$

$$\bar{I}_L = \bar{E}\bar{B}_L = (100\angle 53.13^\circ)(0.7\angle -90) = 70\angle -36.87^\circ$$

$$I_c = \bar{E}\bar{B}_c = (100\angle 53.13^\circ)(0.3\angle 90) = 30\angle 143.13^\circ$$

$\boxed{\text{Prove that : } \bar{I} = \bar{I}_R + \bar{I}_L + \bar{I}_c}$

(c). The current phasor diagram :



(d). Active power = P = $EI\cos\phi$
(Real power)

$$= (100)(50)\cos 53.13^\circ$$
$$= 3000 \text{ W}$$
$$= 3.0 \text{ kW}$$

$\Rightarrow$ or $P = E^2 G$
$$= (100^2)(0.3)$$
$$= 3.0 \text{ kW}$$

(e). The power factor $\Rightarrow$ p.f $= \cos\phi$
$$= \cos 53.13$$
$$= 0.6 \text{ lagging}$$
$\Leftarrow$ from the phasor diagram.

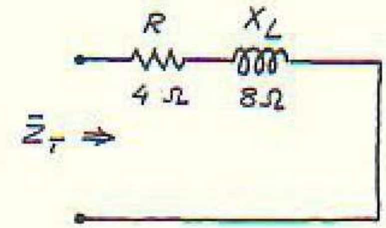
$$\bar{Z}_T = \frac{\bar{Z}_1\bar{Z}_2\bar{Z}_3}{\bar{Z}_1\bar{Z}_2 + \bar{Z}_2\bar{Z}_3 + \bar{Z}_1\bar{Z}_3}$$

$\Leftarrow$ ملاحظة : من الممكن كذلك حساب Z_T
والحصول على النتيجة نفسها .

Tutorial Sheet Nº 7

TS7

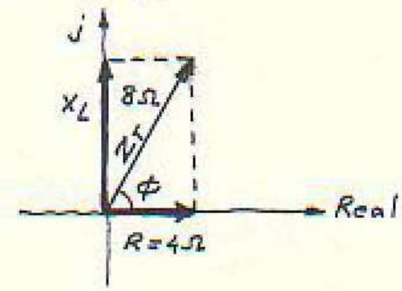
Example: Draw the impedance diagram for the circuit shown and find the total impedance.



Solution: $\bar{Z}_T = \bar{Z}_1 + Z_2$

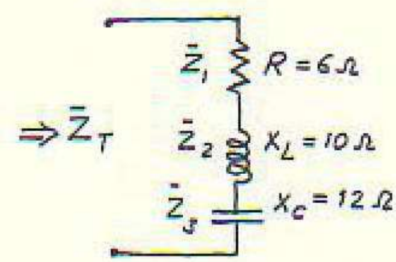
$$= R\angle 0 + X_L \angle 90^\circ = R + jX_L = 4 + j8$$

$$\therefore \bar{Z}_T = 8.944 \angle 63.43^\circ \ \Omega$$



Example: Determine the input impedance to the series network shown

Solution:



$$\bar{Z}_T = \bar{Z}_1 + \bar{Z}_2 + \bar{Z}_3$$

$$= R\angle 0 + X_L \angle 90 + X_c \angle -90$$

$$= R + jX_L - jX_c$$

$$= R + j(X_L - X_c)$$

$$= 6 + j(10 - 12)$$

$$= 6 - j2$$

$$\Rightarrow \bar{Z}_T = 6.325 \angle -18.43^\circ \ \Omega$$

Example

: A 60 Hz sinusoidal voltage ($v = 141 \sin\omega t$) is applied to a series R-L circuit. The values of the resistance and the inductance are $3\,\Omega$ and 0.0106 H respectively.

(a). Compute the rms value of the current in the circuit and its phase angle with respect to the voltage.

(b). Write the expression for the instantaneous current in the circuit.

(c). Find the average power dissipated by the circuit.

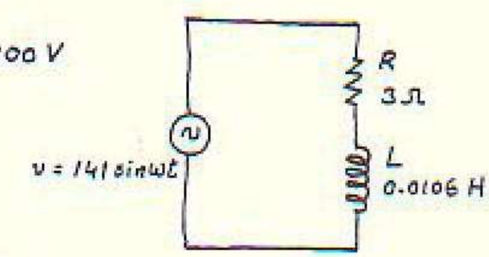
(d). Calculate the p.f of the circuit.

Solution

: We have ; $v = V_m \sin\omega t$

$$\Rightarrow V = \frac{V_m}{\sqrt{2}} = \frac{141}{\sqrt{2}} = 100\text{ V}$$

$$\therefore \bar{V} = 100\angle 0$$



(a). $\bar{I} = \dfrac{\bar{V}}{\bar{Z}}$

$$\bar{Z} = R + jX_L$$
$$= R + j(2\pi f L)$$
$$= 3 + j(2\pi \times 60 \times 0.0106)$$
$$= 3 + j4$$
$$\therefore \bar{Z} = 5\angle 53.1^\circ$$

$$\Rightarrow \bar{I} = \frac{100\angle 0^\circ}{20\angle 53.1^\circ}$$

$$\therefore \bar{I} = 20\angle -53.1$$ $\Rightarrow$ the current lags the voltage by 53.1°

(b). $i = I_m \sin(\omega t - 53.1^\circ)$

$$= \sqrt{2}(20)\sin(\omega t - 53.1^\circ) = \underline{28.28 \sin(\omega t - 53.1^\circ)}$$

(c). $P = VI\cos\phi$

$$= (100)(20)\cos 53.1^\circ = 1200\text{ W}$$

or $P = I^2 R = (20)^2(3) = 1200$ W

(d). $p.f = \cos\phi$

$$= \cos 53.1^\circ$$
$$= 0.6 \quad \underline{\text{lagging}}$$

Example

: A two elements series circuit is connected across an ac circuit having a source ($e = \sqrt{2}(200)\sin(\omega t + 20^\circ)$ V. The current in the circuit is then found to be $i = \sqrt{2}(10)\cos(341t - 25^\circ)$. Determine the parameters of the circuit.

TST

Solution

: The applied voltage is :

$$v = \sqrt{2}(200)\sin(\omega t + 20^\circ)$$

$$\Rightarrow \bar{V} = 200\angle 20^\circ$$

The current is :

$$i = \sqrt{2}\,(10)\cos(\omega t - 25^\circ)$$
$$= \sqrt{2}\,(10)\sin(\omega t - 25^\circ - 90^\circ)$$
$$\therefore\ i = \sqrt{2}\,(10)\sin(\omega t + 65^\circ)$$

$$\Rightarrow \bar{I} = 10\angle 65^\circ$$

$$\Rightarrow \bar{Z} = \frac{\bar{V}}{\bar{I}} = \frac{200\angle 20^\circ}{10\angle 65^\circ} = 20\angle -65^\circ$$

Note $\phi = -45^\circ$ (leading)

$$= 14.14 - j\,14.14$$

This impedance represents a series circuit with $R = 14.14\ \Omega$ and a capacitive reactance (because of the $-j$) of $X_c = 14.14\,\Omega$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

* من معادلة التيار :
$\omega = 314$ rad/sec.

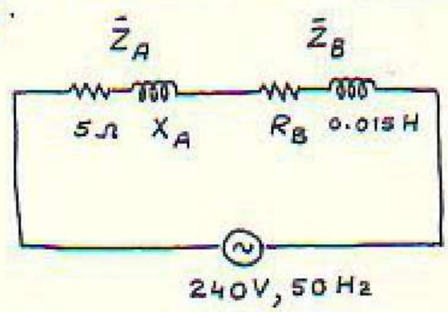
$$\therefore\ 14.14 = \frac{1}{314C} \quad \Rightarrow C = \frac{1}{(14.14)(314)} = 2.25 \times 10^{-4}\ F$$

$\therefore$ The circuit has $R = 14.14\ \Omega$
and $C = 225\ \mu F$

Example

: *Two coils A and B are connected in series across 240 V, 50 Hz supply. The resistance of A is $5\,\Omega$ and the inductance of B is 0.015 H. If the input supply is 3 kW and 2 kVAR, find the resistance of B and the inductance of A. Calculate the voltage across each coil.*

Solution



: * From the power triangle, and the circuit shown,

$$S = \sqrt{P^2 + Q^2} = \sqrt{3^2 + 2^2}$$
$$= 3.606\ KVA$$

$$S = VI \quad \Rightarrow I = \frac{S}{V} = \frac{3606}{240} = 15.025\ A$$

$$\text{But,}\quad P = 3\,kW = 300\,W$$
$$= I^2 R_T = I^2 (R_A + R_B)$$
$$\therefore\ 3000 = (15.025)^2 (R_A + R_B)$$

$$\Rightarrow R_A + R_B = 13.3\ \Omega$$

$$\therefore\ \text{Since } R_A = 5\,\Omega \quad \Rightarrow R_B = 13.3 - 5 = \underline{8.3\ \Omega}$$

T57

Similarly, we have:

ملاحظة : من الممكن حل هذا المثال بأكثر من طريقة والوصول إلى النتائج نفسها .

$$Q = 2\ kVAR = 2000\ VAR$$
$$= I^2 X_{L_T} = (15.03)^2 X_{L_T}$$

$$\therefore X_{L_T} = \frac{2000}{(15.03)^2} = 8.85\ \Omega$$

$$X_{L_T} = X_A + X_B \Rightarrow X_A = 8.85 - X_B$$
$$= 8.85 - (2\pi f L_B)$$
$$= 8.85 - (2\pi \times 50 \times 0.015)$$
$$= 8.85 - 4.713 = 4.13\ \Omega$$

$$\therefore \bar{Z}_A = R_A + jX_A = 5 + j\,4.13 = 6.48 \underline{|39.57^\circ}$$

and $\bar{Z}_B = R_B + j X_B = 8.3 + j\,4.713 = 9.54 \underline{|29.59^\circ}$ $X_B = 2\pi f L_B$ $= 2\pi \times 50 \times 0.015$

$$\therefore \bar{Z}_T = \bar{Z}_1 + \bar{Z}_2 = 5 + j\,4.13 + 8.3 + j\,4.713$$
$$=$$

$$\therefore \bar{V}_A = \bar{I}\bar{Z}_A = \checkmark$$

$$\bar{V}_B = \bar{I}\bar{Z}_B = \checkmark$$

or $\bar{V}_A = \dfrac{\bar{V}\bar{Z}_A}{\bar{Z}_A + \bar{Z}_B} = \checkmark$ The results must be the same.

and

$$\bar{V}_B = \frac{\bar{V} Z_B}{\bar{Z}_A + \bar{Z}_B} = \checkmark$$

Example:

A 240 V, 50 Hz series R-C circuit takes an rms current of 20 A. The maximum value for the current occurs 1/900 seconds before the maximum value of the voltage. Calculate:

(a). The power factor.

(b). Average power.

(c). The parameters of the circuit.

Solution:

The time duration of the voltage $(T) = 0.02 = 0.02$ sec.

$$\therefore 0.05\ \text{sec.} \Rightarrow 360^\circ, \text{ then}$$

$$(1/900)\ \text{sec} \Rightarrow \left[\frac{360^\circ (1/900)}{0.02}\right]^\circ = \text{The phase shift angle } (\phi)$$

$$\therefore \phi = 20^\circ$$

(a). $\therefore$ $p.f = \cos\phi = \cos 20^\circ$

$= 0.9397$ (leading)

(b). Average power $= P = VI\cos\phi$

$= 240(20)\cos 20^\circ$

$= 4510$ W

$= 4.510$ kW

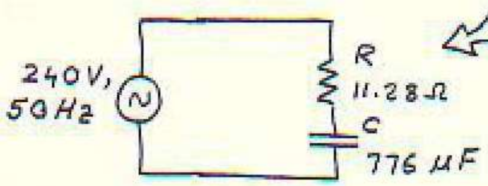
(c). $\bar{Z} = \dfrac{\bar{V}}{\bar{I}} = \dfrac{240\angle 0^\circ}{20\angle 20^\circ} = 12\angle -20^\circ$

$= 20\cos(-20) + j\,12\sin(-20)$

$= 11.28 - j\,4.1$

$\therefore$ $\bar{Z}$ is composed of $R = 11.28\ \Omega$, and

$X_c = 4.1\ \Omega$



$X_c = \dfrac{1}{\omega C} = \dfrac{1}{2\pi f C}$

$\therefore C = \dfrac{1}{2\pi f X_c} = 7.76\times10^{-4}$ F

$= 776\ \mu F$

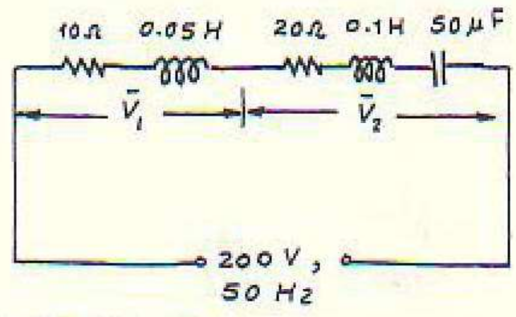
Example: Draw the phasor diagram for the circuit shown, indicating the resistance and the reactance drop, the terminal voltages $\bar{V}_1$ and $\bar{V}_2$ and the current. Find the values of:

(a). The current $\bar{I}$.

(b). $\bar{V}_1$

(c). $\bar{V}_2$

(d). The power factor



Solution:

$R_T = 10 + 20 = 30\ \Omega$

$L_T = 0.05 + 0.1 = 0.15$ H

$\Rightarrow X_L = \omega L = 2\pi f L_T = 2\pi(50)(0.15)$

$= 47.1\ \Omega$

$$X_c = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{2\pi(50)(50\times10^{-6})} = 63.7\ \Omega$$

$$\therefore \bar{Z}_T = \sqrt{R^2 + (X_L - X_c)^2} = \sqrt{(30)^2 + (47.1 - 63.7)^2}$$

$$= 34.3\angle -28.96^\circ$$

(a). $\therefore \bar{I} = \dfrac{\bar{V}}{\bar{Z}} = \dfrac{200\angle 0^\circ}{34.3\angle -28.96} = 5.83\angle 28.96$ leading

TST

(b). $\bar{V}_1 = ?$

$$\bar{V}_1 = \bar{I}\,\bar{Z}_1 \qquad \Rightarrow \bar{Z}_1 = 10 + jX_{L_1}$$

$$= 10 + j\,(2\pi f L_1)$$

$$= 10 + j\,(2\pi(50)\,0.05)$$

$$= 10 + j\,15.7$$

$$\therefore \bar{Z}_1 = 18.6\,\underline{|57.5}$$

$$\therefore \bar{V}_1 = (5.83\,\underline{|28.96})(18.6\,\underline{|57.5})$$

$$= 108.4\,\underline{|86.46^\circ}$$

(c). $\bar{V}_2 = ?$

$$\bar{V}_2 = \bar{I}\,\bar{Z}_2 \qquad \Rightarrow \bar{Z}_2 = 20 + jX_{L_2} - jX_{C_2}$$

$$= 20 + j\,(2\pi f L_2) - \frac{1}{2\pi f C_2}$$

$$= 20 + j\,31.4 - j\,63.7$$

$$= 20 - j\,32.3$$

$$\therefore \bar{Z}_2 = 37.74\,\underline{|-58.2^\circ}$$

$$\therefore \bar{V}_2 = \bar{I}\,\bar{Z}_2 = (5.83\,\underline{|28.96^\circ})(37.74\,\underline{|-58.2^\circ})$$

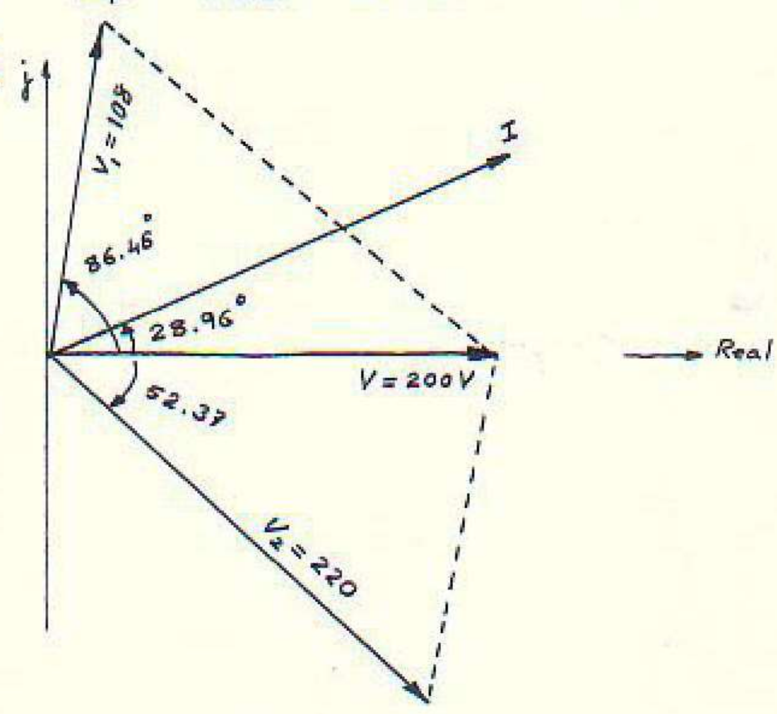
$$= 220.1\,\underline{|-52.37}$$

(d). The combined (overall) power factor of the circuit:

- from part (a) $\Rightarrow$ $p.f = \cos\phi = \cos 28.96^\circ$
  $= 0.87$ leading

- or $\quad p.f = \dfrac{R}{Z_T} = \dfrac{30}{34.3} = \underline{0.87}$ (leading)

The phasor diagram



TS7

Example: In a circuit it is found that the applied voltage is to <u>lag</u> the current by $30°$.

(a). Is the power factor lagging or leading ?
(b). What is the value of the power factor ?
(c). Is the circuit inductive or capacitive ?

Solution:

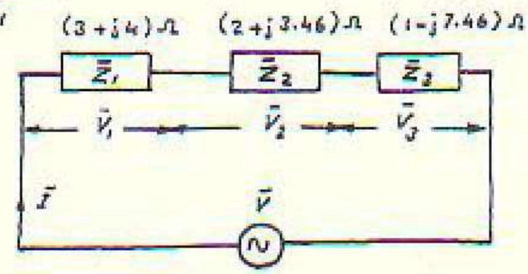
(a). The power factor is <u>leading</u>, since the <u>current leads</u> the voltage.

(b). The power factor is
$$p.f = \cos\phi = \cos 30 = 0.866 \text{ (lead)}$$

(c) The circuit is <u>capacitive</u>.

Example: In the circuit diagram of the Fig. shown, the voltage drop across $\bar{Z}_1$ is $(10 + j0)$ volts. Find out :

(a). The current in the circuit.
(b). The voltage drop across $\bar{Z}_2$ and $\bar{Z}_3$
(c). The voltage of the source.



Solution: (a). $\bar{I} = \dfrac{\bar{V}}{\bar{Z}_T}$ $\Rightarrow$ $\bar{Z}_T = ✓$, $\bar{V} = ?$

$$\text{or } I = \frac{\bar{V}_1}{\bar{Z}_1} = \frac{10 + j0}{3 + j4} = \frac{10\angle 0°}{5\angle 53.1°} = 2\angle -53.1°$$
$$= 2(\cos 53.1° - j\sin 53.1°) = \underline{1.2 - j1.6}$$

(b). $\bar{V}_2 = \bar{I}\bar{Z}_2 = (1.2 - j1.6)(2 + j3.46) = \underline{7.936 + j0.952}$ volts

$\bar{V}_3 = \bar{I}\bar{Z}_3 = (1.2 - j1.6)(1 - j7.46) = -10.74 - j10.55$ volts

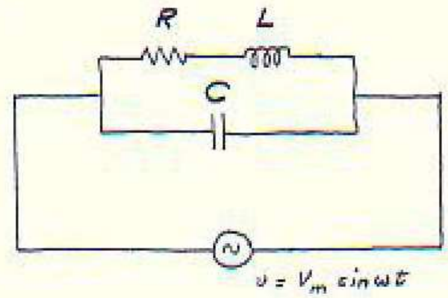
(c).
$$\bar{V} = \bar{V}_1 + \bar{V}_2 + \bar{V}_3$$
$$= (10 + j0) + (7.936 + j0.952) + (-10.74 - j10.55)$$
$$= 7.2 - j9.6 = 12\angle -53.1° \text{ volts}$$

نلاحظ ان الفولتية الكلية والتيار المار في الدائرة كلاهما بنفس الطور والسبب في ذلك ان $\bar{Z}_T$ في هذه الدائرة لا تمثل عدداً مركباً بل قيمة حقيقية فقط (real part) اي انها تمثل مقاومة فقط والتي بدورها لا تؤثر في الطور بين الفولتية والتيار .

TST

Example (HW)

: Derive expressions for the equivalent impedance and (or) the admittance for the circuit shown.



Answer

:

$$\bar{Z}_T = \frac{R + j\omega\left[L(1-\omega^2 LC^2) - CR^2\right]}{(1-\omega^2 LC)^2 + \omega^2 C^2 R^2}$$

and

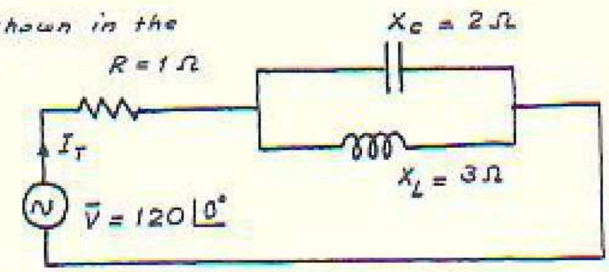
$$\bar{Y}_T = \frac{R - j\omega\left[L(1-\omega^2 LC) - CR^2\right]}{R^2 + \omega^2 L^2}$$

* Please check the answers.

Example

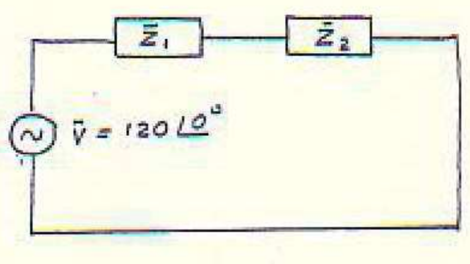
: For the circuit shown in the Fig., determine :

(a). $\bar{Z}_T$
(b). $\bar{I}_T$
(c). I_c
(d). $\bar{V}_R$
(e). $\bar{V}_c$
(f). Average power
(g). The power factor of the circuit.



Solution

: Redraw the circuit (option) :



Let $\bar{Z}_1 = R = 1\angle 0^\circ\ \Omega$

& $Z_2 = X_c \parallel X_L$

$$= \frac{X_c X_L}{X_c + X_L} = \frac{(-j2)(j3)}{-j2 + j3}$$

$$= \frac{6\angle 0^\circ}{j1} = \frac{6\angle 0}{1\angle 90^\circ}$$

$$= 6\angle -90^\circ = -j6\ \Omega$$

(a). $\therefore \quad \bar{Z}_T = \bar{Z}_1 + \bar{Z}_2 = 1 - j6 = 6.08 \angle{-80.54^\circ} \quad \Omega$ [T37]

(b). $\bar{I}_T = \dfrac{\bar{V}}{\bar{Z}} = \dfrac{120 \angle{0^\circ}}{6.08 \angle{-80.54^\circ}} = 19.74 \angle{80.54^\circ} \quad A$

(c). $I_c = ?$

Using the current divider rule, I_c can be calculated as:

$$\bar{I}_c = I_T \frac{X_L}{X_L + X_c} = 19.74 \angle{80.54^\circ} \cdot \frac{3 \angle{90^\circ}}{1 \angle{90^\circ}}$$

$$= 59.22 \angle{80.54^\circ} \quad A.$$

(d). $\bar{V}_R = ?$

$$\bar{V}_R = \bar{I}_T \bar{Z}_1 = (19.74 \angle{80.54^\circ})(1 \angle{0})$$

$$= 19.74 \angle{80.54^\circ} \quad V$$

(e) $\bar{V}_c = ?$

$$\bar{V}_c = \bar{I}_T \bar{Z}_2 = (19.74 \angle{80.54^\circ})(6 \angle{-90^\circ})$$

$$= 118.44 \angle{-9.46^\circ} \quad V$$

$\boxed{OR}$ $\quad \bar{V}_c = \bar{I}_c \bar{X}_c = (59.22 \angle{80.54^\circ})(2 \angle{-90^\circ})$

$$= 118.44 \angle{-9.46^\circ}$$

(f). Average power = Active power

$$P = I_T^2 R = (19.74)^2 \times 1 = 389.67 \text{ W}$$

or $\quad P = VI \cos\phi = 389.67 \text{ W}$

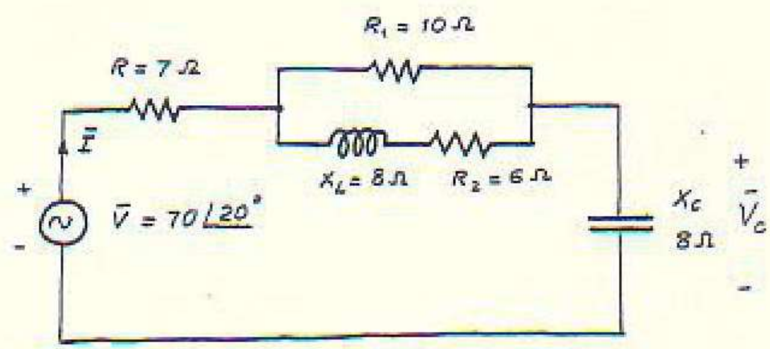
(g). Power factor $= \cos\phi \qquad \phi = 80.54^\circ$

$$\therefore \text{p.f} = \cos 80.54^\circ = 0.164 \quad \underline{\text{leading}}$$

Example (HW):

(a). Calculate the voltage V_c using the voltage divider rule.

(b). Calculate the current I



Answer:

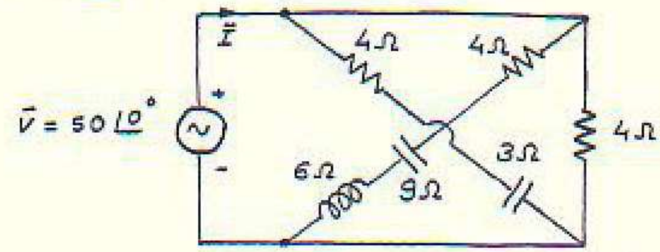
(a). $\bar{V}_c = 42.42 \angle{-45.38^\circ}$ volts

(b). $\bar{I} = 5.3 \angle{44.62^\circ} \quad A$

TS7

Example

: Find the current I in the circuit shown.



Solution

: Let $\bar{Z}_1 = 4 - j3$

$$\bar{Z}_2 = 4 + j6 - j9 = 4 - j3$$

and

$$\bar{Z}_3 = 4 + j0 = 4\ \Omega$$

لوحظ ان $\bar{Z}_1$ ، $\bar{Z}_2$ ، $\bar{Z}_3$ مربوطة على التوازي

لوحظ ان $\bar{Z}_1 = \bar{Z}_2$

$$\bar{Z}_T = \bar{Z}_1 \,//\, Z_2 \,//\, Z_3$$

since $\bar{Z}_1 = Z_2 \Rightarrow Z_{12} = \frac{Z_1}{2} = \frac{4-j3}{2} = \underline{2 - j1.5\ \Omega}$

$$\therefore \bar{Z}_T = \bar{Z}_{12} \,//\, \bar{Z}_3 = \frac{\bar{Z}_{12}\bar{Z}_3}{\bar{Z}_{12} + \bar{Z}_3} = \frac{4(2-j1.5)}{4 + 2 - j1.5}$$

$$\therefore \bar{Z}_T = \frac{8-j6}{6-j1.5} = \frac{10\angle -36.87^\circ}{6.18\angle -14.04^\circ} = 1.62\angle -22.83^\circ$$

$$\therefore \bar{I}_T = \frac{\bar{V}}{\bar{Z}_T} = \frac{50\angle 0^\circ}{1.62\angle -22.83^\circ} = \underline{30.86\angle 22.83^\circ}$$

Note

: Z_T can be calculated from:

$$\bar{Z}_T = \frac{\bar{Z}_1\bar{Z}_2\bar{Z}_3}{\bar{Z}_1\bar{Z}_2 + \bar{Z}_2\bar{Z}_3 + \bar{Z}_1\bar{Z}_3}$$

or $\bar{Z}_T = \frac{1}{\bar{Y}_T}$ where $\bar{Y}_T = Y_1 + Y_2 + Y_3$

$$\bar{Y}_1 = \frac{1}{\bar{Z}_1}, \quad \bar{Y}_2 = \frac{1}{\bar{Z}_2}, \quad \bar{Y}_3 = \frac{1}{\bar{Z}_3}$$

والحصول على النتيجة نفسها .

Example

: For the circuit shown,

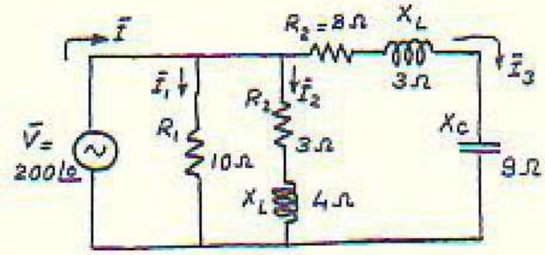
(a). Compute $\bar{I}$.

(b). Find $\bar{I}_1$, $\bar{I}_2$ and $\bar{I}_3$

(c). Verify Kirchhoff's Current law by showing that :

$$\bar{I} = \bar{I}_1 + \bar{I}_2 + \bar{I}_3$$

(d). Find the total impedance of the circuit.



Solution

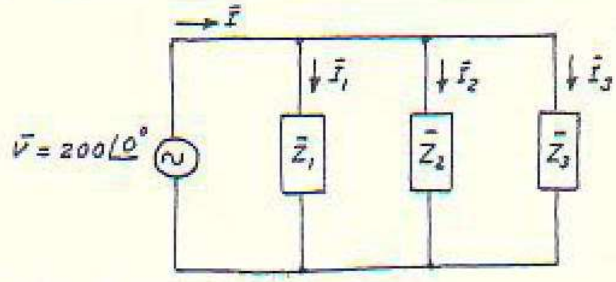
: Redraw the circuit as shown in the Fig. below;

where;

$$\bar{Z}_1 = 10\angle 0^\circ = 10\,\Omega$$

$$\bar{Z}_2 = 3 + j4$$

$$\bar{Z}_3 = 8 + j3 - j9 = 8 - j6$$



$$\bar{Y}_T = \bar{Y}_1 + \bar{Y}_2 + \bar{Y}_3 = \frac{1}{\bar{Z}_1} + \frac{1}{\bar{Z}_2} + \frac{1}{\bar{Z}_3}$$

$$= \frac{1}{10} + \frac{1}{3+j4} + \frac{1}{8-j6}$$

$$= \left(\frac{1}{10}\right) + \frac{3}{9+16} - j\frac{4}{9+16} + \frac{8}{64+36} + j\frac{6}{64+36}$$

$$= \frac{1}{10} + \frac{3}{25} + \frac{8}{100} - j\frac{4}{25} + j\frac{6}{100}$$

$$\therefore \bar{Y}_T = 0.3 - j0.1 \quad (S)$$

(a). $\bar{I} = ?$

$$\bar{I} = \bar{V}.\bar{Y}_T = 200\angle 0^\circ\,(0.3 - j0.1)$$

$$= \underline{60 - j20}$$

(b).

$$\bar{I}_1 = \frac{\bar{V}}{\bar{Z}_1} = \frac{200\angle 0^\circ}{10\angle 0^\circ} = 20\angle 0^\circ \quad A$$

$$I_2 = \frac{\bar{V}}{\bar{Z}_2} = \frac{200\angle 0^\circ}{5\angle 53.13^\circ} = 40\angle -53.13^\circ \quad A$$

and

$$\bar{I}_3 = \frac{\bar{V}}{\bar{Z}_3} = \frac{200\angle 0^\circ}{10\angle -36.87^\circ} = 20\angle 36.87^\circ$$

(c)

$$\bar{I} = \bar{I}_1 + \bar{I}_2 + I_3$$

$$60 - j20 = 20\angle 0^\circ + 40\angle -53.13^\circ + 20\angle 36.87^\circ$$

$$= (20 + j0) + (24 - j32) + (16 + j12)$$

$$\therefore \underline{60 - j20} = \underline{60 - j20}$$

TS7

(d). $\bar{Z}_T = ?$

$$\bar{Z}_T = \frac{1}{\bar{Y}_T} = \frac{1}{0.3 - j0.1} = \frac{0.3}{(0.3)^2 + (0.1)^2} + j\frac{0.1}{(0.3)^2+(0.1)^2}$$

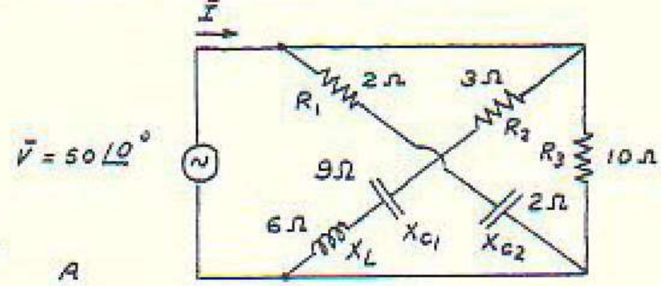
$$\therefore \bar{Z}_T = 3 + j1$$

For check

$$\begin{aligned}\bar{V} &= \bar{I}\,\bar{Z}_T \\ &= (60 - j20)(3 + j1) \\ &= 180 - j60 + j60 + 20 \\ &= 200 + j0 = 200\angle 0^\circ\end{aligned}$$

Example (HW)

: Find the current I in the circuit shown in the Fig.



Answer

$\bar{I} = 33.201\angle 38.89^\circ$ A

Example

: The load taken from a supply consists of :(a). lamp load of 10 kW at unity power factor , (b). motor load of 80 kVA at 0.8 power factor (lag) , and (c). motor load of 40 kVA at 0.7 power factor leading.

Calculate the total load taken from the supply in kW and in kVA and the power factor of the combined load.

Solution

:

ملاحظة : من المناسب لحل هذا المثال ان يكون بشكل جدول يضم انواع القدرة المختلفة ومركبات القدرة وكما يأتي :

Load	kVA	$\cos\phi$	$\sin\phi$	kW	kVAR	
(a)	10	1	0	10	0	(p.f = 1)
(b)	80	0.8	0.6	64	− 48	(p.f lag)
(c)	40	0.7	0.714	28	+ 28.6	(p.f lead)
			TOTAL →	102	− 19.4	

⇒

T S 7

$\therefore$ Total kW = 102 $\Rightarrow P_T$

Total kVAR = − 19.4 (lagging) $\Rightarrow Q_T$

$\therefore$ Total kVA taken from the supply S_T

$$S_T = \sqrt{P_T^2 + Q_T^2}$$

$$= \sqrt{(102)^2 + (-19.4)^2} = 103.9 \text{ kVA}$$

and the power factor $= \cos\phi = \dfrac{P_T}{S_T} = \dfrac{102}{103.9} = 0.982$

ويمكن حل المثال أكثر وضوحاً كالآتي

$$S_T = S_1 + S_2 + S_3$$
$$= P_1 + jQ_1 + P_2 + jQ_2 + P_3 + jQ_3$$
$$= [10\cos 0 + j0 + 80\cos\phi_2 + j\,80\sin\phi_2 + 40\cos\phi_3 + j\,40\sin\phi_3] \times 10^3$$
$$= [(10 + 64 + 28) + j(0 - 48 + 28.6)] \times 10^3$$
$$\therefore \bar{S}_T = 102 - j\,19.4 \quad \text{kVA}$$
$$= 103.9 \underline{/-10.77} \quad \text{kVA}$$

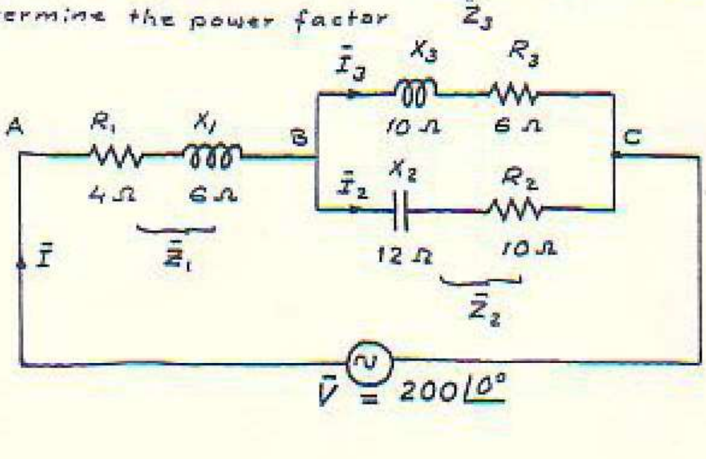
$\therefore S = 103.9$ kVA

p.f $= \cos(-10.77)$

$= 0.982$ (lagging)

Example

: Determine the current drawn by the following circuit, where a voltage of 200 V is applied across its terminals. Draw the phasor diagram, and determine the power factor of the circuit.



Solution

: $\bar{Z}_1 = 4 + j6 = 7.2 \underline{/56.3^\circ} \quad \Omega$

$\bar{Z}_2 = 10 - j12 = 15.6 \underline{/-50.2^\circ} \quad \Omega$

$Z_3 = 6 + j10 = 11.7 \underline{/58^\circ} \quad \Omega$

AC Network Analysis

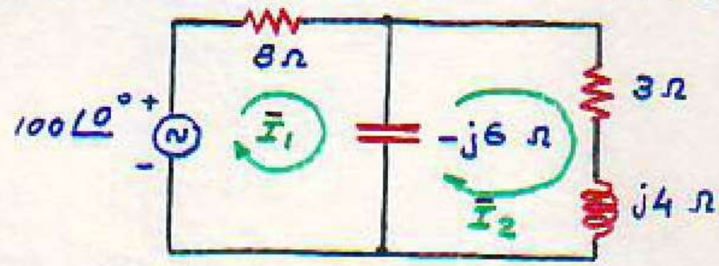
EE3

TS3

Demonstrative Examples :

* Mesh (Loop) Analysis :

Example : Find the power output of the voltage source in the circuit shown; prove that this power equals the power in the circuit resistors. Use mesh analysis in your solution.



Solution : * Loop 1 :

$$100\angle 0^\circ = \bar{I}_1(8-j6) - \bar{I}_2(-j6)$$

$$\Rightarrow 100\angle 0^\circ = \bar{I}_1(8-j6) + \bar{I}_2(j6) \quad \text{—— (1)}$$

* Loop 2 :

$$0 = \bar{I}_2(3+j4-j6) - \bar{I}_1(-j6)$$

$$= \bar{I}_1(j6) + \bar{I}_2(3-j2) \quad \text{—— (2)}$$

$$\bar{I}_1 = \frac{\Delta_1}{\Delta} \quad \text{and} \quad \bar{I}_2 = \frac{\Delta_2}{\Delta}$$

$$\Delta = \begin{vmatrix} (8-j6) & j6 \\ j6 & (3-j2) \end{vmatrix} = (8-j6)(3-j2) - (j6)^2$$

$$\Rightarrow \Delta = 58.703\angle -35.33^\circ$$

Similarly

$$\Delta_1 = \begin{vmatrix} 100\angle 0^\circ & j6 \\ 0 & (3-j2) \end{vmatrix} = (300 - j2) = 360\angle -33.69^\circ$$

and

$$\Delta_2 = \begin{vmatrix} (8-j6) & 100\angle 0^\circ \\ j6 & 0 \end{vmatrix} = j600 = 600\angle 90^\circ$$

$$\therefore\ \bar{I}_1 = \frac{\Delta_1}{\Delta} = \frac{360\angle -33.69^\circ}{58.703\angle -35.33^\circ} = 6.133\angle 1.64^\circ$$

$$\bar{I}_2 = \frac{\Delta_2}{\Delta} = \frac{600\angle 90^\circ}{58.703\angle -35.33^\circ} = 10.22\angle 125.33^\circ$$

$\therefore$ Total Power output = total power absorbed by resistors

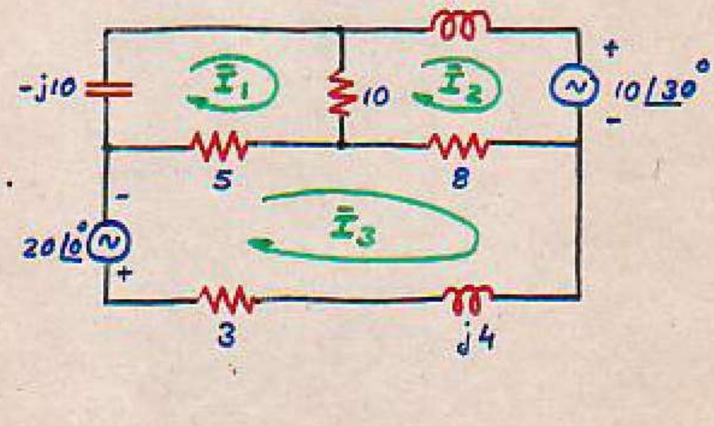
$$\therefore\ V I_1 = I_1^2(8) + I_2^2(3)$$

$$\boxed{(100)(6.133)} = \boxed{300.4} + \boxed{313.3}$$

$$\therefore\ \underline{613.3 \text{ W} \doteq 613.7 \text{ W}}$$

Example:

Write the mesh current equations for the circuit shown.



Solution

: The three mesh current equations are

* Loop 1

$$0 = (15-j10)\bar{I}_1 - 10\,\bar{I}_2 - 5\,\bar{I}_3$$

* Loop 2

$$-10\angle 30^\circ = -10\,\bar{I}_1 + (18+j5)\bar{I}_2 - 8\,\bar{I}_3$$

* Loop 3

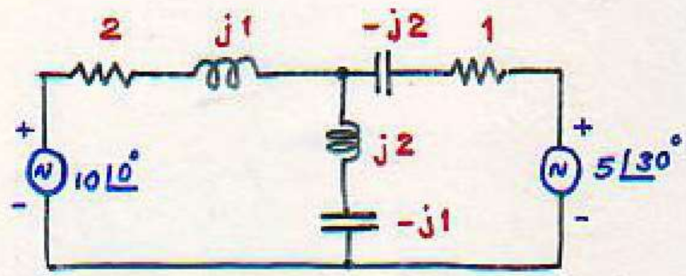
$$-20\angle 0^\circ = -5\,\bar{I}_1 - 8\,\bar{I}_2 + (16+j4)\bar{I}_3$$

$$\therefore\ \Delta = \begin{vmatrix} (15-j10) & -10 & -5 \\ -10 & (18+j5) & -8 \\ -5 & -8 & (16+j4) \end{vmatrix} = ?$$

Then you can find $\bar{I}_1 = \dfrac{\Delta_1}{\Delta}$, $\bar{I}_2 = \dfrac{\Delta_2}{\Delta}$ and $\bar{I}_3 = \dfrac{\Delta_3}{\Delta}$

Homework

: For the cct. shown, determine the branch voltages and currents and the power delivered by the source using mesh analysis.

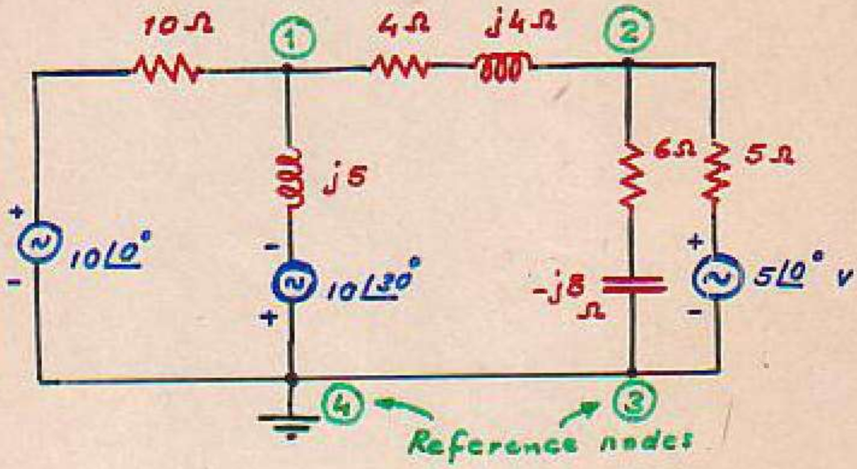


* All resistance and reactance values are in Ohms.

HW

* Nodal Analysis :

Example : Write the nodal equations for the circuit shown ;



Solution : We have two independent node 1 and 2 , so two equations have to be written :

Node 1 :

$$\bar{V}_1\left(\frac{1}{10}+\frac{1}{(4+j4)}+\frac{1}{j5}\right)-\bar{V}_2\left(\frac{1}{4+j4}\right)=\frac{10\angle 0^\circ}{10}-\frac{10\angle 30^\circ}{j5}$$

Node 2 :

$$\bar{V}_2\left(\frac{1}{4+j4}+\frac{1}{5}+\frac{1}{6-j8}\right)-\bar{V}_1\left(\frac{1}{4+j4}\right)=\frac{5\angle 0^\circ}{5}$$

From these two equations , the unknown voltages $\bar{V}_1$ and $\bar{V}_2$ must be determined.

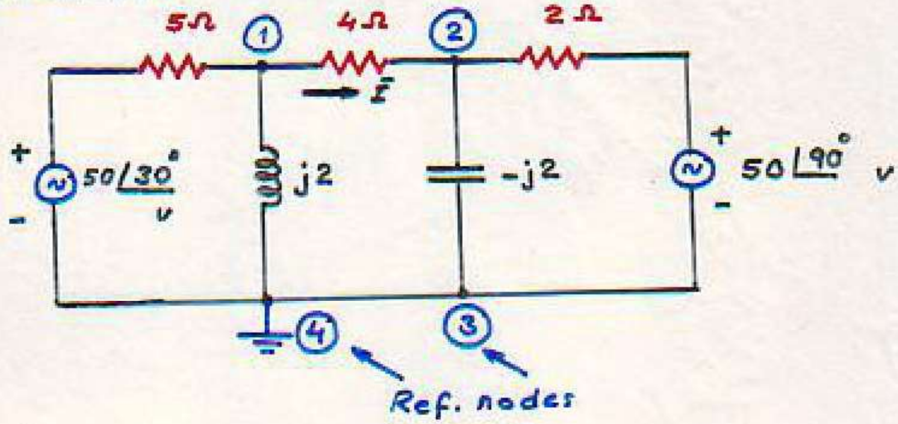
$\boxed{HW}$

Example: For the circuit shown in the Fig. below, determine the current flowing through the branch of 4-Ω resistance using :

(a). Nodal Analysis

(b). Thevenin's theorem.

(c). Mesh method



Solution

(a). By Nodal Analysis Method

Node 1

$$\bar{V}_1\left(\frac{1}{5}+\frac{1}{4}+\frac{1}{j2}\right)-\bar{V}_2\left(\frac{1}{4}\right)=\frac{50\angle 30^\circ}{5}$$

$$\therefore \Rightarrow \bar{V}_1(9-j10)-5\bar{V}_2 = 200\angle 30^\circ \quad \text{— (1)}$$

Node 2

$$\bar{V}_2\left(\frac{1}{4}+\frac{1}{2}+\frac{1}{-j2}\right)-\bar{V}_1\left(\frac{1}{4}\right)=\frac{50\angle 90^\circ}{2}$$

$$\therefore \Rightarrow \bar{V}_2(3+j2)-\bar{V}_1 = j100 \quad \text{— (2)}$$

Solving Equ. 1 and 2 ; we find:

$$\bar{V}_1 = j27.26 \quad \text{volts}$$

and

$$\bar{V}_2 = 19.58 + j29.36$$

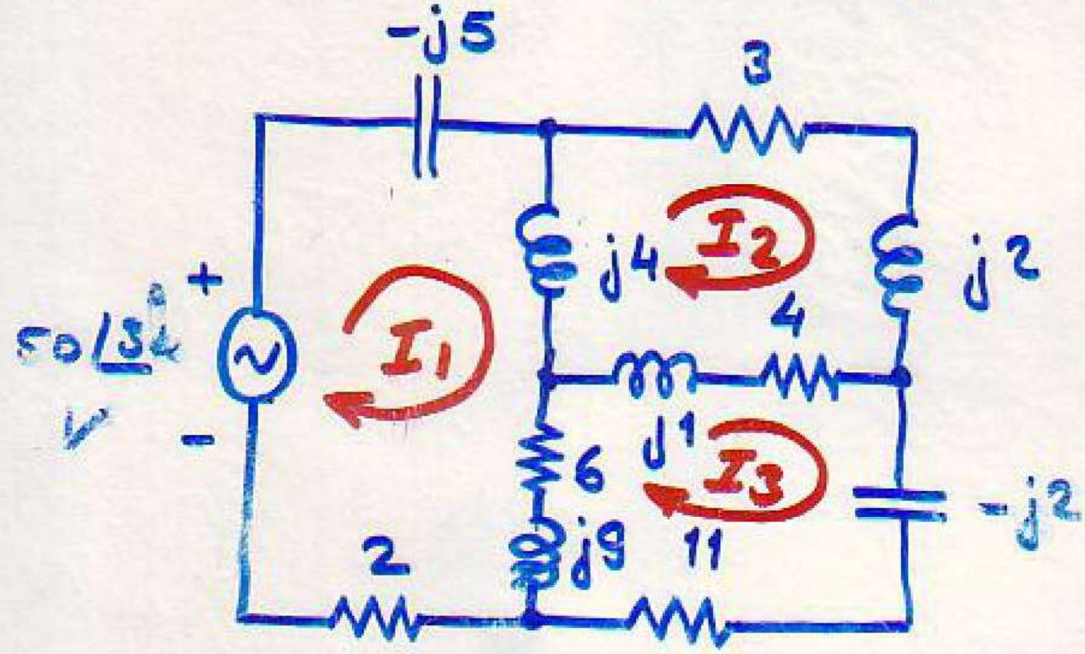
The current $\bar{I} = \dfrac{V_1 - V_2}{4} = \dfrac{j27.26 - 19.58 - j29.36}{4}$

$$= \frac{19.69\angle 186.12^\circ}{4} = 4.92\angle 186.12^\circ \ A$$

(*) Try to solve again using the methods of (b), and (c) and getting the same results.

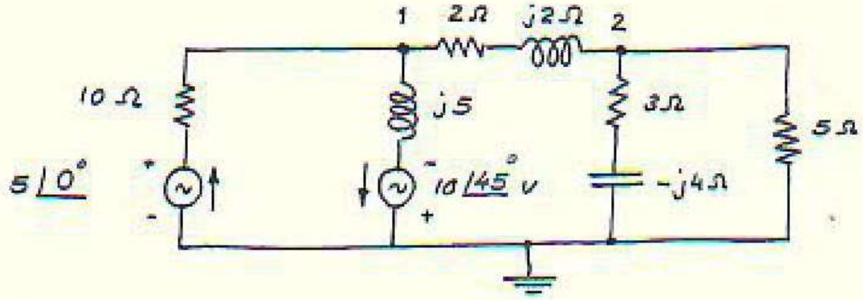
Homework (B)

For the cct. shown, find the current I_1 using the loop Method



TS9

Example : Write the nodal voltage equations for the circuit shown.



Solution : The nodal voltage equations are ;

For Node 1 :

$$\left(\frac{1}{10} - \frac{1}{j5} + \frac{1}{2+j2}\right)\bar{V}_1 - \left(\frac{1}{2+j2}\right)\bar{V}_2 = \frac{5\angle 0^\circ}{10} - \frac{10\angle 45^\circ}{j5}$$

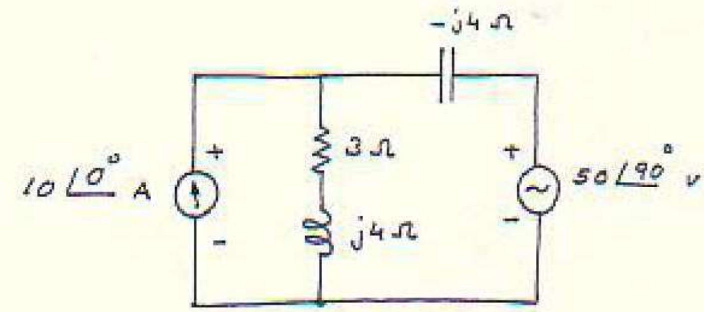
and

For Node 2 :

$$-\frac{1}{(2+j2)}\bar{V}_1 + \left(\frac{1}{2+j2} + \frac{1}{3-j4} + \frac{1}{5}\right)\bar{V}_2 = 0$$

Solving the above equations to determine $\bar{V}_1$ & $\bar{V}_2$.

Example : Apply the superposition theorem to determine the voltage drop across the $(3+j4)\ \Omega$ impedance, in the circuit shown.

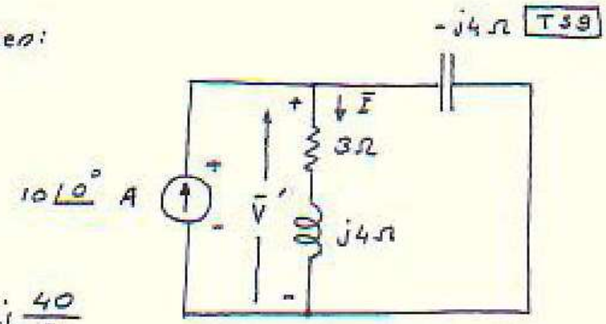


Solution :

* The effect of $(10\angle 0^\circ\ A)$ source alone ; Removing the $50\angle 90^\circ\ V$ source and substitute it by a short circuit, then :

⟹

* using the current divider rule, then:



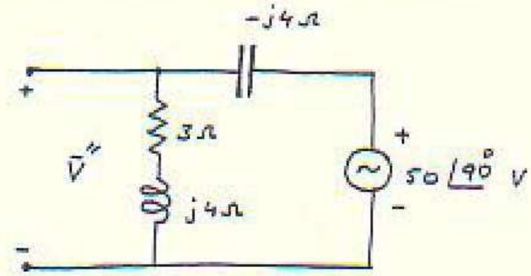
$$\bar{I} = \bar{I}_T \frac{\bar{Z}_1}{\bar{Z}_1 + \bar{Z}_2}$$

$$= (10\angle 0^\circ) \frac{(-j4)}{(-j4) + (3+j4)}$$

$$= (10\angle 0^\circ) \frac{(-j4)}{3} = -j\frac{40}{3}$$

$$\therefore \; V' = \bar{I}(3+j4) = -j\frac{40}{3}(3+j4) = \underline{53.3 - j40}$$

* The effect of ($50\angle 90^\circ$ V) voltage source; the circuit will be in this case as:



using the voltage divider rule, then:

$$\bar{V}'' = V_T \frac{\bar{Z}_2}{\bar{Z}_2 + \bar{Z}_1}$$

$$= (50\angle 90^\circ)\left(\frac{(3+j4)}{(3+j4)+(-j4)}\right)$$

$$= -66.7 + j50$$

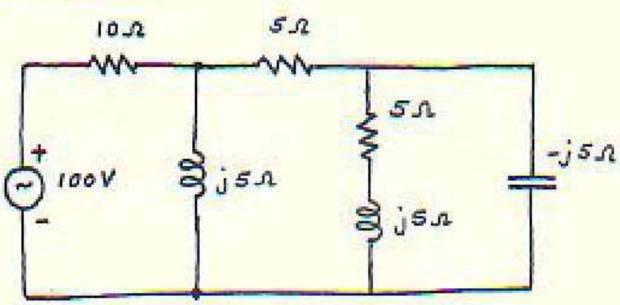
$$\therefore \; \bar{V} = \bar{V}' + \bar{V}'' = (53.3 - j40) + (-66.7 + j50)$$

$$= -13.4 + j10 = 16.72\angle 143.27^\circ$$

Example: For the network shown, determine the voltage across the capacitor, using:

(a). Thevenin's theorem.

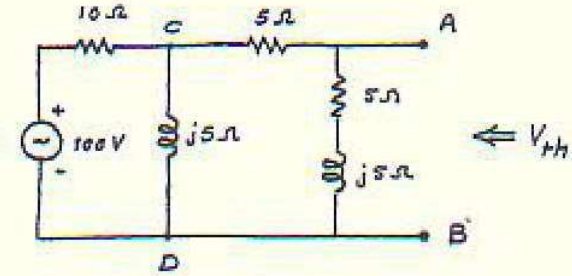
(b). Mesh current method.



⇒

Solution

: (a). Using Thevenin's theorem



⊛ $V_{th} = ?$

voltage divider rule ⇒

$$\bar{V}_{th} = \bar{V}_{AB}$$

$$= \bar{V}_{CD} \frac{(5+j5)}{(5+j5)+(5)}$$

$$= \bar{V}_{CD} \frac{(5+j5)}{(10+j5)}$$

$$\bar{V}_{CD} = V_T \frac{(j5)}{10+j5} = 100 \frac{(j5)}{10+j5}$$

$$\doteq 20 + j40$$

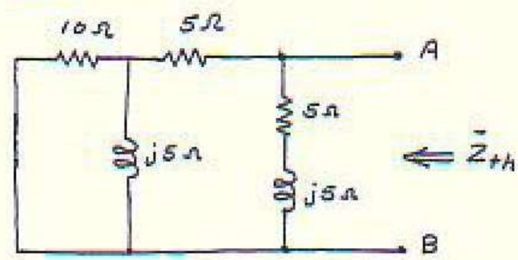
Check

$$\therefore \bar{V}_{th} = 21.1\angle 71.57^\circ \text{ V}$$

$$= 6.67 + j20$$

⊛ $\bar{Z}_{th} = ?$

when looking through terminals A and B, the voltage source removed; the equivalent impedance $\bar{Z}_{th}$ is determined from the circuit:

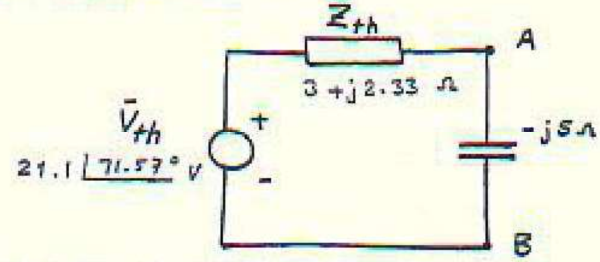


$$\bar{Z}_{th} = \left[(10 // j5) + 5\right] // (5+j5)$$

Check

$$\therefore \bar{Z}_{th} = 3 + j2.33 \ \Omega$$

⊛ Thevenin's equivalent circuit will be as shown:



∴ Total impedance

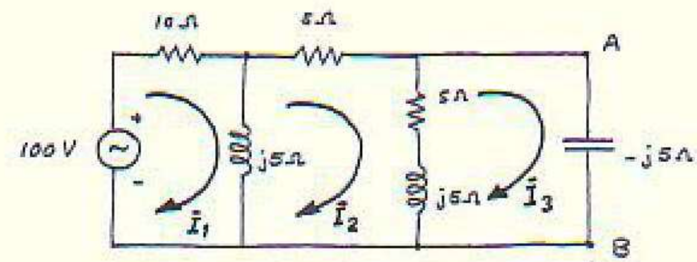
$$\bar{Z}_T = (3+j2.33) + (-j5)$$

$$= 3 - j2.67 = 4.02\angle -41.67^\circ \ \Omega$$

$$\therefore I = \frac{V_{th}}{\bar{Z}_T} = \frac{21.1\angle 71.57^\circ}{4.02\angle -41.67^\circ} = \underline{\underline{5.25\angle 113.24^\circ \text{ A}}}$$

TS9

(b). Using the mesh method



$\bar{I}_3 = \ ?$

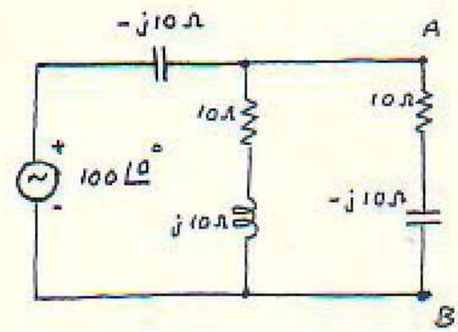
$$\bar{I}_3 = \frac{\Delta_3}{\Delta}$$

$$\therefore \ \bar{I}_3 = \frac{\begin{vmatrix} 10+j5 & -j5 & 100 \\ -j5 & 10+j10 & 0 \\ 0 & -(5+j5) & 0 \end{vmatrix}}{\begin{vmatrix} 10+j5 & -j5 & 0 \\ -j5 & 10+j10 & -(5+j5) \\ 0 & -(5+j5) & 5 \end{vmatrix}} = \frac{3535\angle 135^\circ}{673\angle 21.8^\circ}$$

$$\Rightarrow \quad \bar{I}_3 = 5.25\angle 113.2^\circ \ A$$

which is the same result.

Example: Use Norton's theorem to find the current in the load connected across terminals A and B of the circuit shown.

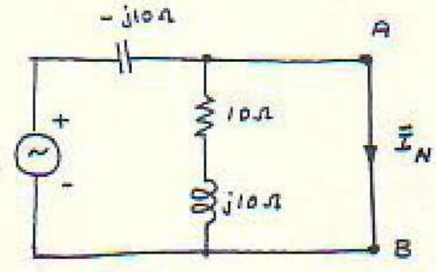


Solution: $\bar{I}_N = ? \Rightarrow$ the current in the short circuit terminals A & B

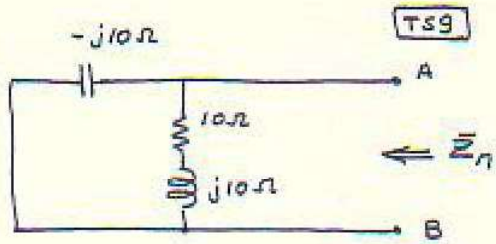
$$\therefore \ \bar{I}_N = \frac{\bar{V}_T}{\bar{Z}_T} = \frac{100\angle 0^\circ}{-j10}$$

$$= \frac{100\angle 0^\circ}{-10\angle 90} = \frac{100\angle 0^\circ}{10\angle -90^\circ}$$

$$= 10\angle 90 \ A$$



* $\bar{Z}_n = \bar{Z}_{th} = ?$



$$\bar{Z}_n = (-j10) // (10 + j10)$$

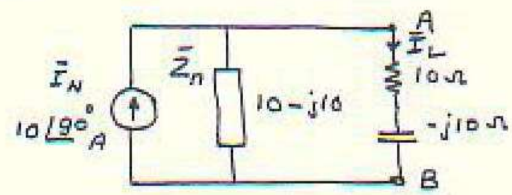
$$= \frac{(-j10)(10+j10)}{(-j10)+(10+j10)}$$

$$\therefore\ \bar{Z}_n = 10 - j10$$

* The Norton's equivalent circuit is :

$$\bar{I}_L = \frac{\bar{I}_N}{2} = \frac{10\angle 90}{2}$$

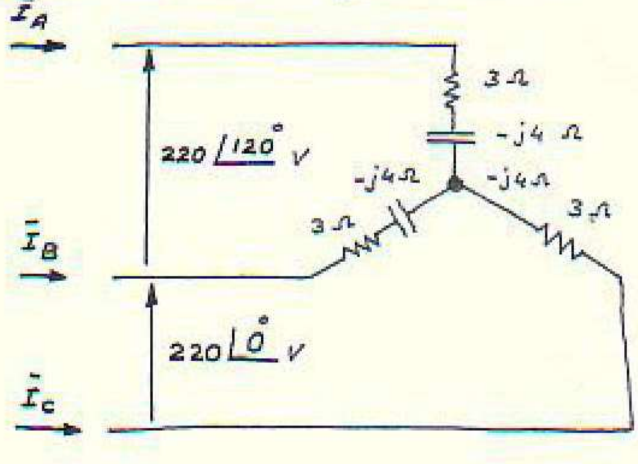
$$= 5\angle 90 \quad A$$



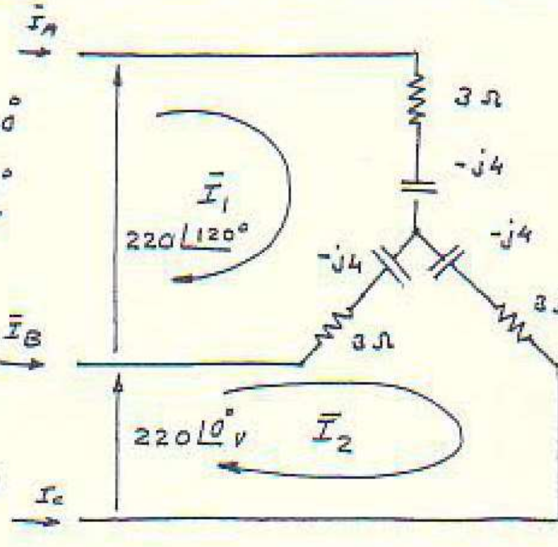
Since $\bar{I}_N$ is equally divided between the two equal impedances.

TS9

Example: For the circuit shown, determine the currents $\bar{I}_A$, $\bar{I}_B$ and $\bar{I}_C$, using the mesh current analysis method.



Solution: Using the mesh method, we have to determine the currents $\bar{I}_1$ and $\bar{I}_2$, then:



Mesh Equations

$$(6-j8)\bar{I}_1 - (3-j4)\bar{I}_2 = 220\angle 120^\circ$$

and

$$-(3-j4)\bar{I}_1 + (6-j8)\bar{I}_2 = 220\angle 0^\circ$$

$$\therefore \bar{I}_1 = \frac{\begin{vmatrix} 220\angle 120^\circ & -(3-j4) \\ 220\angle 0 & (6-j8) \end{vmatrix}}{\begin{vmatrix} (6-j8) & -(3-j4) \\ -(3-j4) & (6-j8) \end{vmatrix}}$$

$$= \frac{1905\angle 36.9^\circ}{75\angle -106.2^\circ} = 25.4\angle 143.1^\circ \text{ A}$$

and

$$\bar{I}_2 = \frac{\begin{vmatrix} (6-j8) & 220\angle 120^\circ \\ -(3-j4) & 220\angle 0^\circ \end{vmatrix}}{\Delta} = \frac{1905\angle -23.2^\circ}{75\angle -106.2} = 25.4\angle 83^\circ \text{ A}$$

$$\therefore \bar{I}_A = \bar{I}_1 = \underline{\underline{25.4\angle 143}} \text{ A},$$

$$\bar{I}_B = \bar{I}_2 - \bar{I}_1 = 25.4\angle 83^\circ - 25.4\angle 143.1^\circ = \underline{\underline{25.4\angle 23.1^\circ}} \text{ A}$$

$$\bar{I}_C = -\bar{I}_2 = -(25.4\angle 83^\circ) = \underline{\underline{25.4\angle -97^\circ}} \text{ A}$$