Electrical Engineering Fundamentals

First class

AC & DC Examples

University: ***Tikrit***
College: ***Petroleum Processes Engineering***
Department: ***Petroleum Systems Control Engineering***
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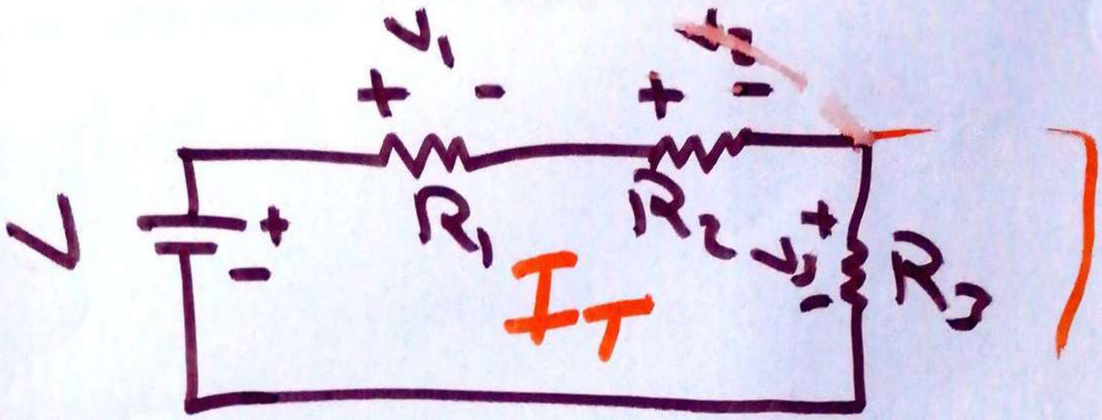
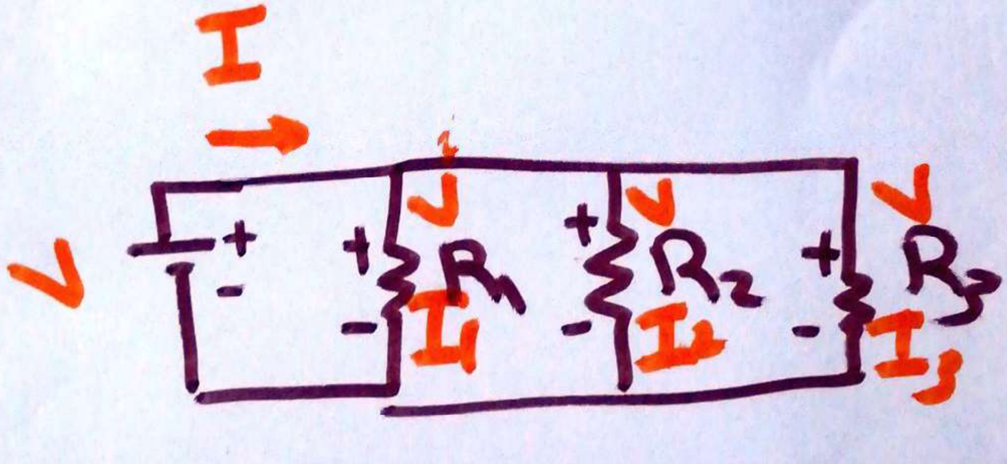
2024-2025

$$\left.\begin{aligned} V &= IR \\ P &= IV \end{aligned}\right\} \text{ohm Law}$$



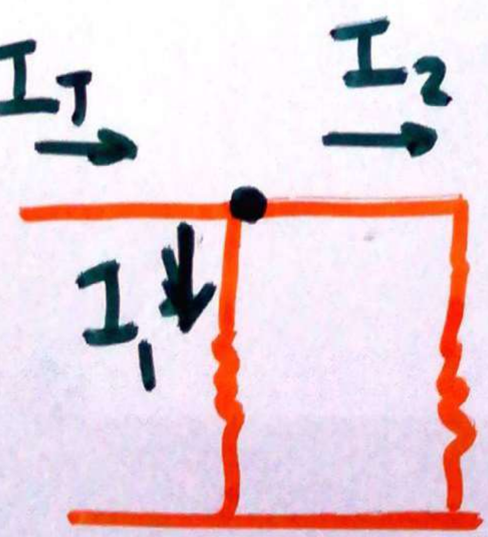


Kirchoff's Law

① Kirchoff's current Law (KCL)

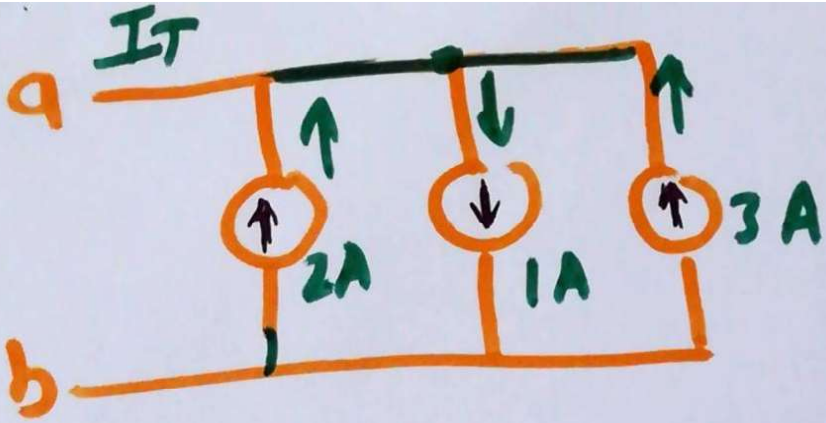
$$\sum I_n = 0$$

$$\sum_{m=1}^{M} I_{mi} = \sum_{n=1}^{N} I_n$$

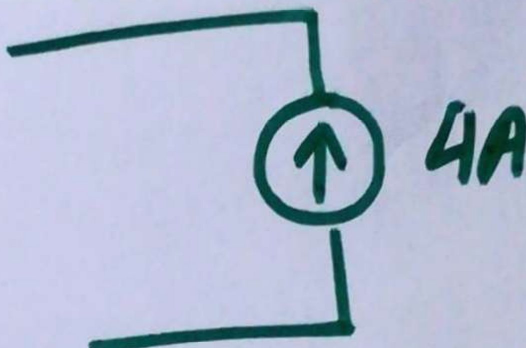


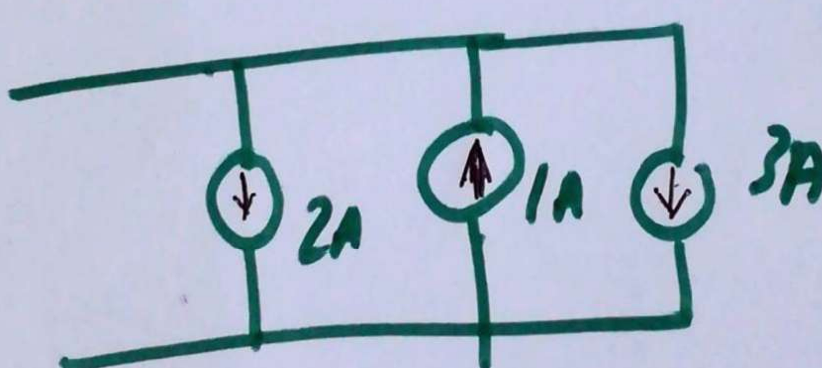
$$I_T - I_1 - I_2 = 0$$

$$I_T = I_1 + I_2$$

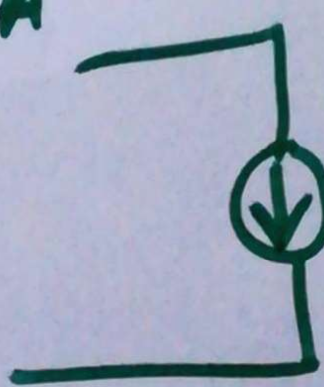


$$I_T = 2 - 1 + 3 = 4$$





$$2 - 1 + 3 = 4\,A$$

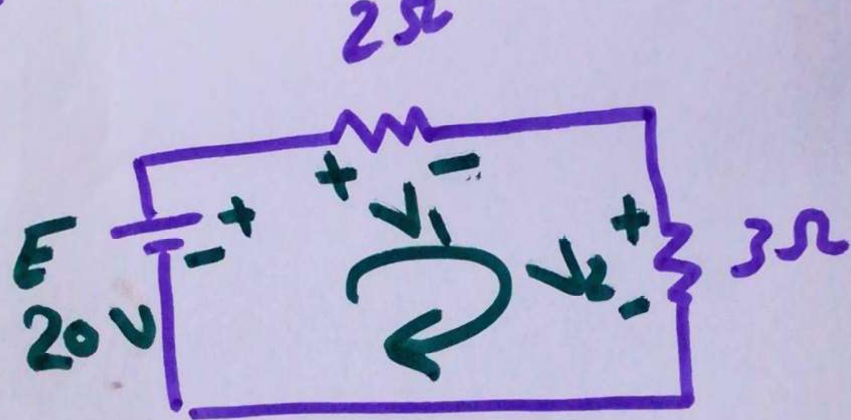


$$-2 + 1 - 3 = -4$$

② Kirchoff's Voltage Law (KVL)

$$\sum_{m=1}^{M} V_m = 0$$

EX:



$$+E - V_1 - V_2 = 0$$

$$\boxed{20 - V_1 - V_2 = 0}$$

$$V = IR$$

$$V_1 = IR_1 = I(2) = 2I \quad \text{—} ①$$

$$V_2 = IR_2 = 3I \quad \text{—} ②$$

$$20 - 2I - 3I = 0$$

$20 - 5I = 0$

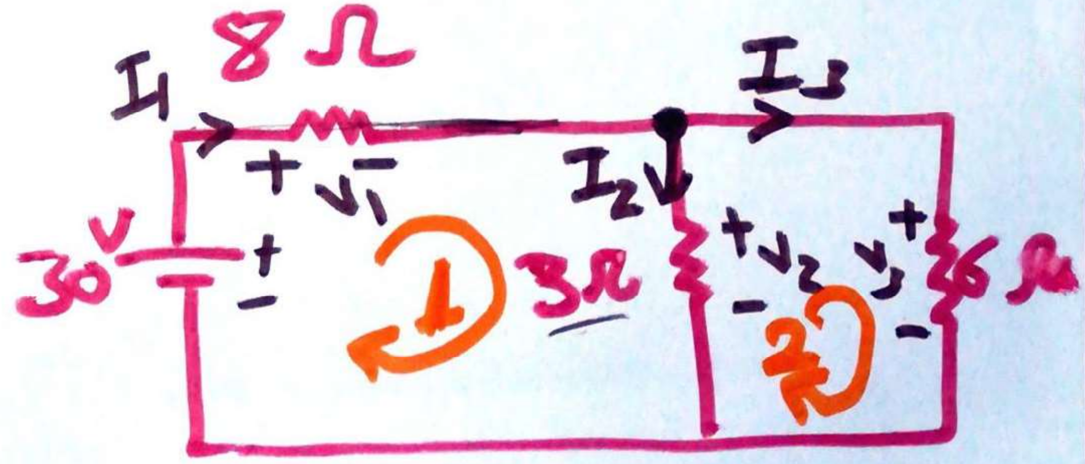
$I = 4$

$V_1 = 4 * 2 = 8\ V$

$V_2 = 4 * 3 = 12\ V$

$20 - 8 - 12 = 0$

Example: Calculate the current and voltage



KCL

KVL

$$v_1 = I_1 R_1 = I_1 8$$

$$v_2 = I_2 R_2 = 3 I_2$$

$$v_3 = I_3 R_3 = 6 I_3$$

$$I_1 = I_2 + I_3$$

$$I_1 - I_2 - I_3 = 0 \quad — (1)$$

Loop$_1$

$$E - v_1 - v_2 = 0$$

$$30 - 8 I_1 - 3 I_2 = 0$$

$$\boxed{I_1 = \frac{30 - 3 I_2}{8}} \quad — (2)$$

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$+V_2 - V_3 = 0$

$3I_2 - 6I_3 = 0$

$$\boxed{I_3 = \frac{3I_2}{6}} \quad - \quad 3$$

Sub in equ 2 and 3

& in 1

$I_1 - I_2 - I_3 = 0$

$$\frac{30 - 3I_2}{8} - I_2 - \frac{3I_2}{6} = 0$$

$$\frac{30 - 3I_2}{8} - \frac{(6I_2 + 3I_2)}{6} = 0$$

$I_2 = 2A$

$$I_1 = \frac{30 - 3I_2}{8}$$

$$= \frac{30 - 6}{8}$$

$I_1 = 3A$

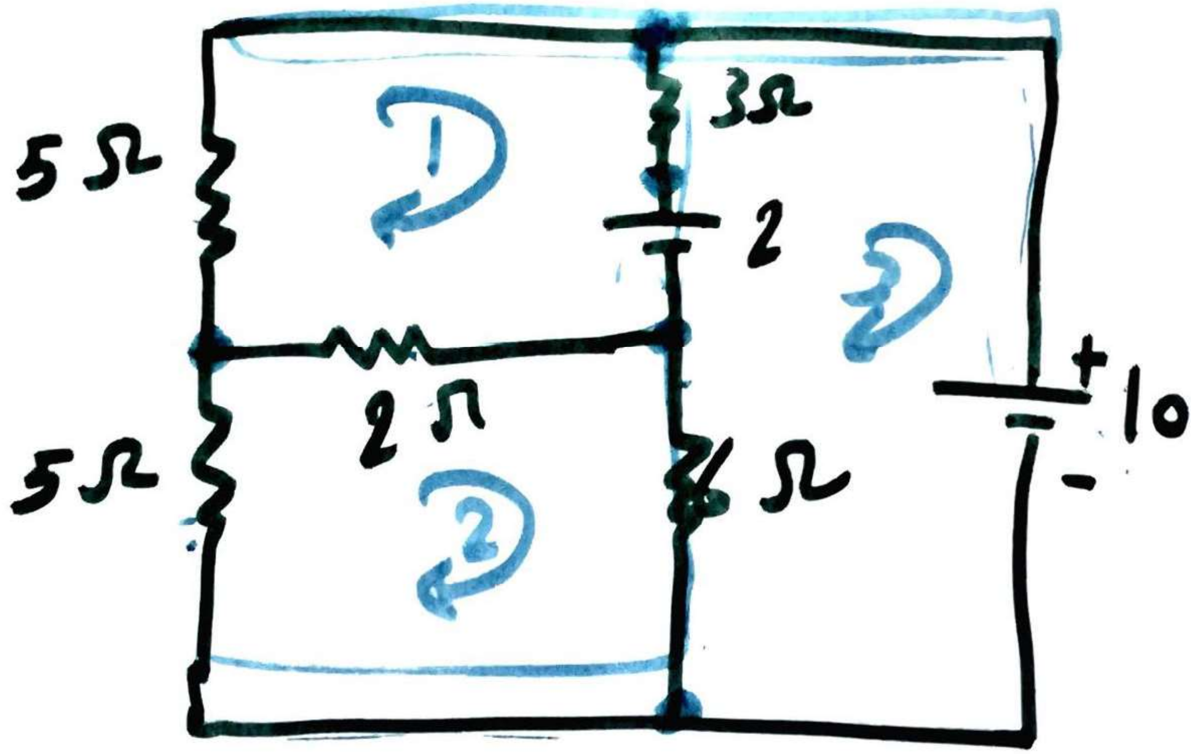
$$I_3 = \frac{3*2}{6} = 1$$

$$V_1 = 3 \times 8 = 24V$$

$$V_2 = 3 I_2 = 3*2 = 6V$$

$$V_3 = 6*1 = 6V$$

$$3A - 2A - 1A = 0$$

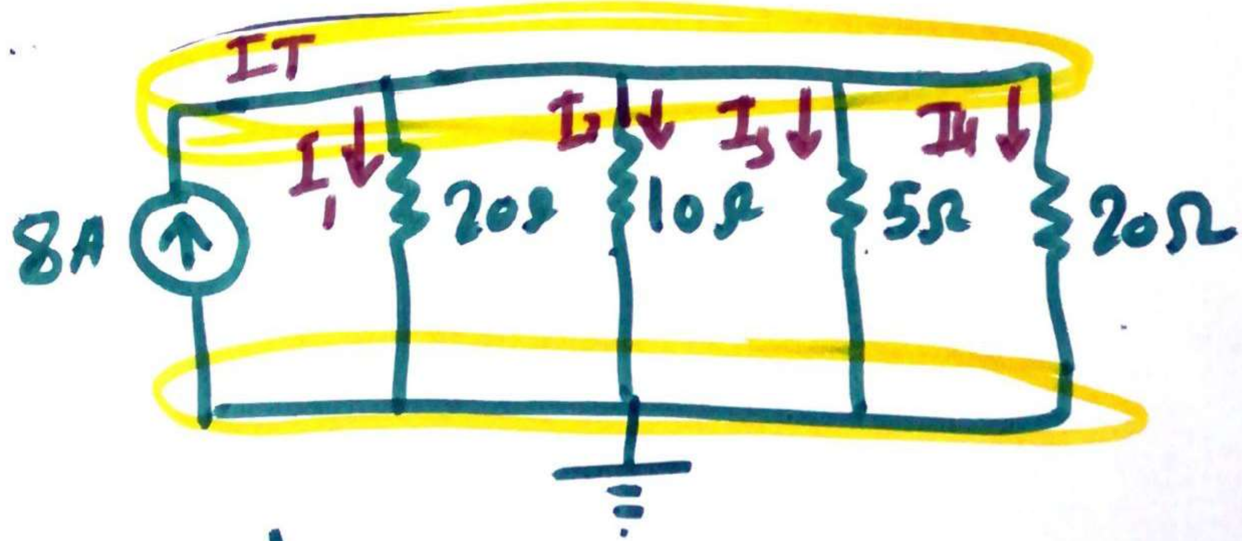


branches(b) = 7

nodes(n) = 5

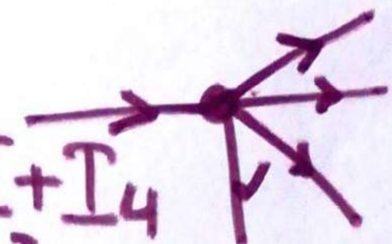
Loop(l) = 3

$$b = l + n - 1$$
$$= 3 + 5 - 1 = 7$$



KCL

$$I_T = I_1 + I_2 + I_3 + I_4$$



~~V=20~~ $V = IR$

$$V = I_1(20) \Rightarrow I_1 = \frac{V}{20}$$

$$V = I_2(10) \Rightarrow I_2 = \frac{V}{10}$$

$$V = I_3(5) \Rightarrow I_3 = \frac{V}{5}$$

$$V = I_4(20) \Rightarrow I_4 = \frac{V}{20}$$

$$8 = \frac{V}{20} + \frac{V}{10} + \frac{V}{5} + \frac{V}{20}$$

$$V = 20V$$

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$$I_1 = \frac{20}{20} = 1\ A$$

$$I_2 = \frac{20}{10} = 2\ A$$

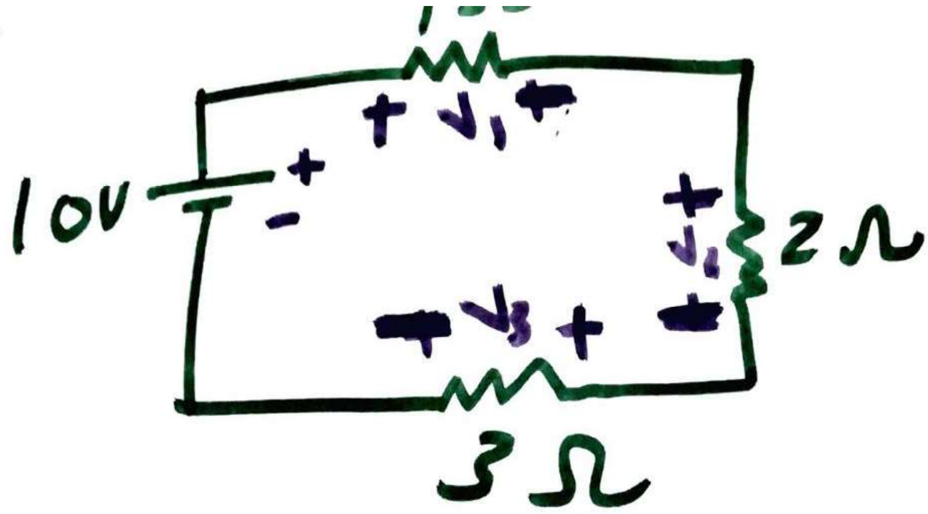
$$I_3 = \frac{20}{5} = 4\ A$$

$$I_4 = \frac{20}{20} = 1\ A$$

$$I_T = I_1 + I_2 + I_3 + I_4$$

$$= 1 + 2 + 4 + 1$$

$$I_T = 8$$



$$I,\ V = ?$$

$$E - V_1 - V_2 - V_3 = 0 \quad \text{—} \textcircled{1}$$

$$V_1 = IR_1 = 1I \quad \text{—} \textcircled{2}$$

$$V_2 = IR_2 = 2I \quad \text{—} \textcircled{3}$$

$$V_3 = IR_3 = 3I \quad \text{—} \textcircled{4}$$

$$10 - I - 2I - 3I = 0$$

$$10 - 6I = 0$$

$$I = \frac{10}{6} A$$

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$$V_1 = \frac{10}{6} \times 1 = \frac{10}{6} V$$

$$V_2 = \frac{10}{6} \times 2 = \frac{10}{3} V$$

$$V_3 = \frac{10}{\cancel{6}} \times \cancel{3} = \frac{10}{2} = 5V$$

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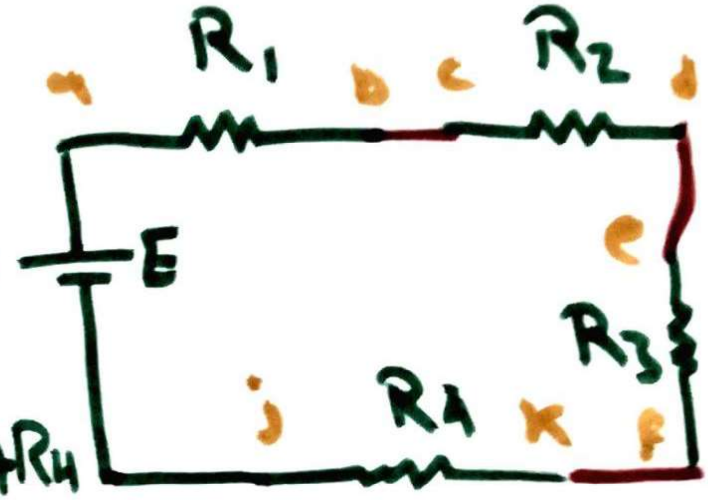
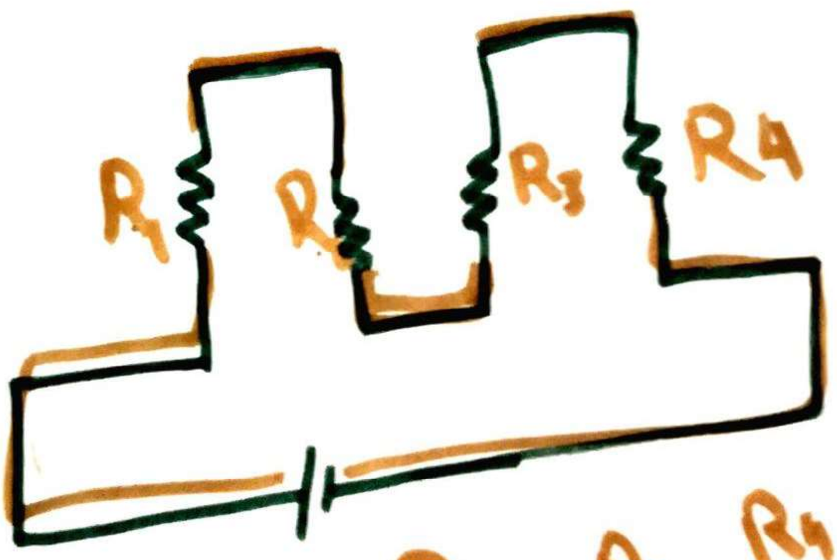
Series Circuits

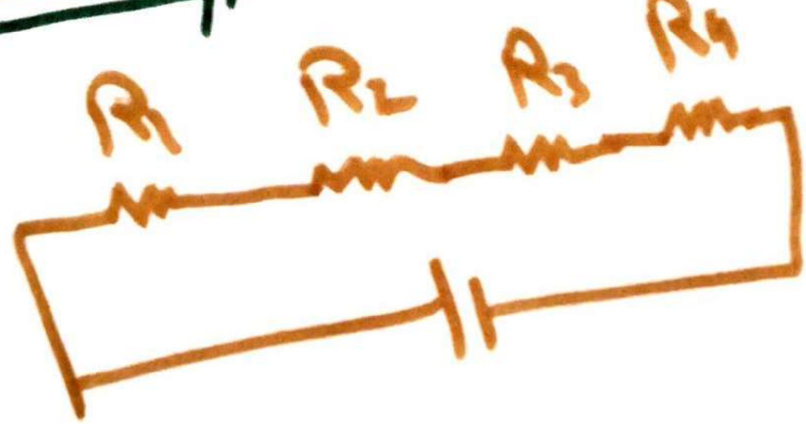
$$I = I_1 = I_2 = I_3 = I_4$$

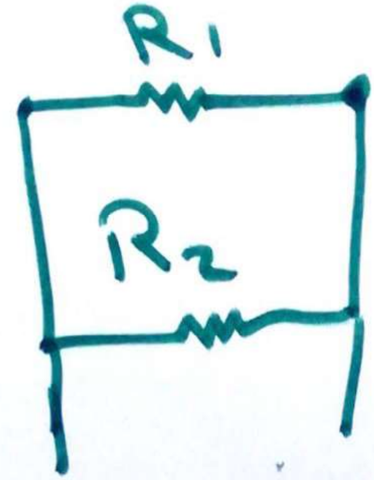
$$E = V_1 + V_2 + V_3 + V_4$$

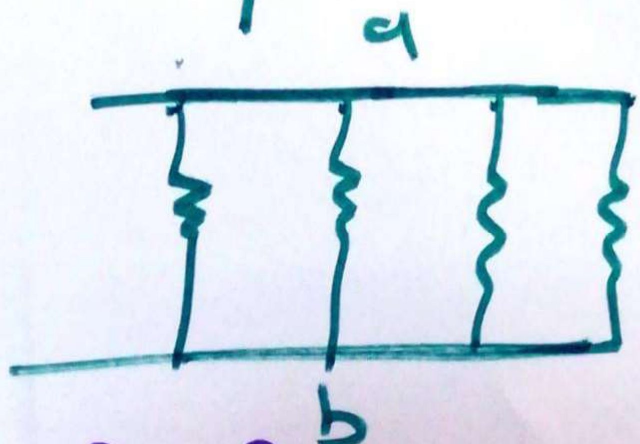
$$R_T = R_1 + R_2 + R_3 + R_4$$



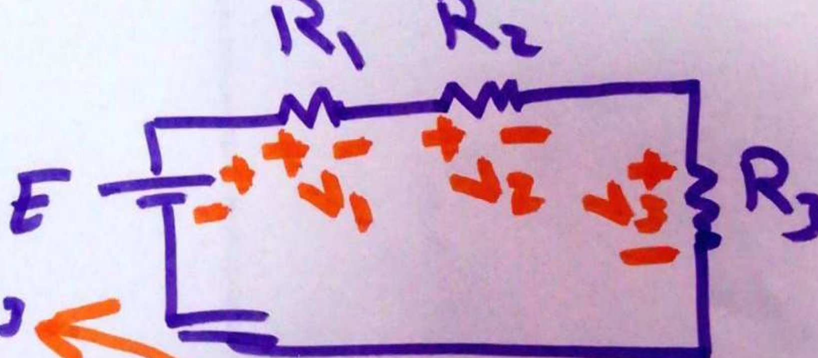








$V = IR$



$I = I_1 = I_2 = I_3$

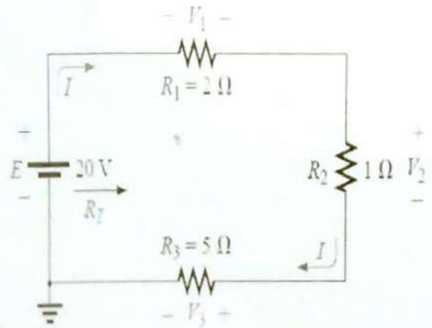
$R_T = R_1 + R_2 + R_3$

$E = V_1 + V_2 + V_3$

$E = IR_1 + IR_2 + IR_3$

$E = I(R_1 + R_2 + R_3)$

$E = IR_T$ $\qquad I = \frac{E}{R_T}$



EXAMPLE

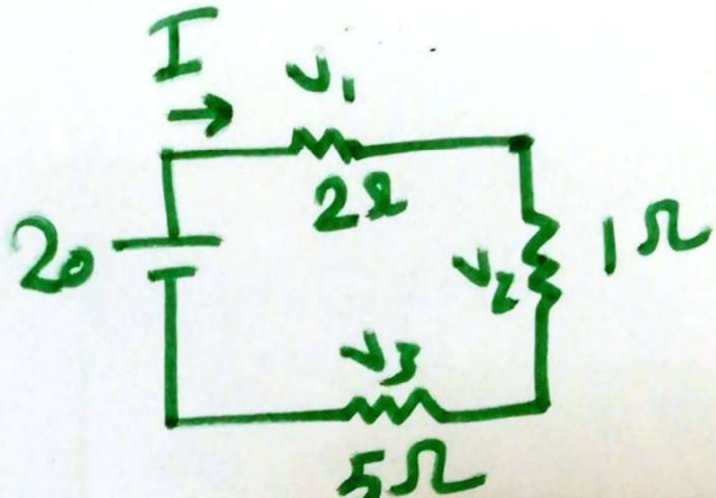
a. Find the total resistance for the series circuit of Fig. 5.7.

b. Calculate the source current I_s.

c. Determine the voltages V_1, V_2, and V_3.

d. Calculate the power dissipated by R_1, R_2, and R_3.

e. Determine the power delivered by the source, and compare it to the sum of the power levels of part (d).



$R_T = R_1 + R_2 + R_3$

$R_T = 2 + 1 + 5$

$R_T = 8\,\Omega$

$I = \frac{E}{R_T} = \frac{20}{8} = 2.5A$

KVL

$V = IR$

$V_1 = IR_1 = 2.5 * 2 = 5V$

$V_2 = IR_2 = 2.5 * 1 = 2.5V$

$V_3 = IR_3 = 2.5 * 5 = 12.5V$

$\cancel{5+} E = V_1 + V_2 + V_3 = 5 + 2.5 + 12.5 = 20$

$$P = VI$$

$$P_1 = V_1 I = 5 * 2.5 = 12.5$$

$$P_2 = V_2 I = 2.5 * 2.5 = 6.25$$

$$P_3 = V_3 I = \cancel{2.5} \; 12.5 * 2.5 = 31.25$$

$$P_T = E I = 20 * 2.5 = \underline{\underline{50}}$$

$$P = P_1 + P_2 + P_3 = 12.5 + 6.25 + 31.25$$

$$= \underline{\underline{50}}$$

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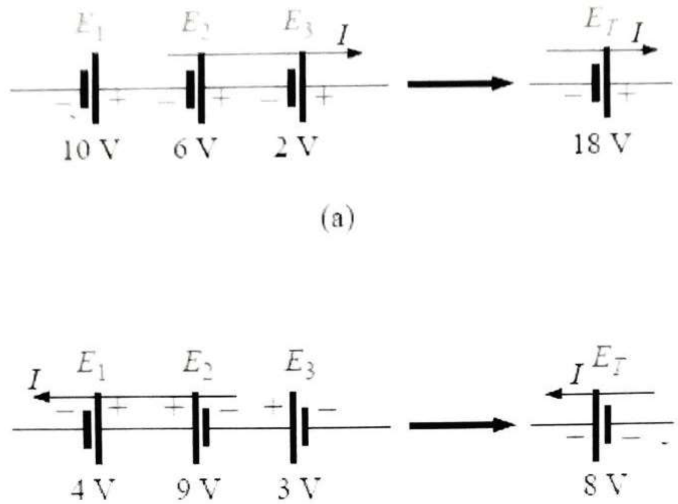
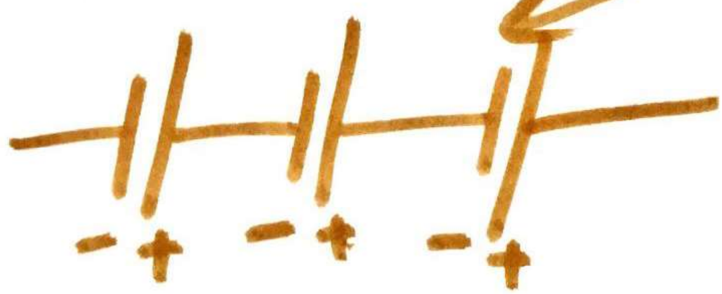
VOLTAGE SOURCES IN SERIES

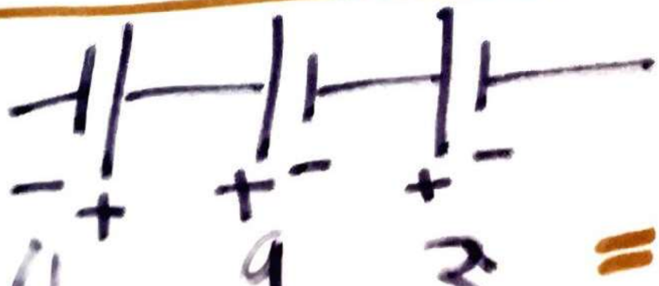


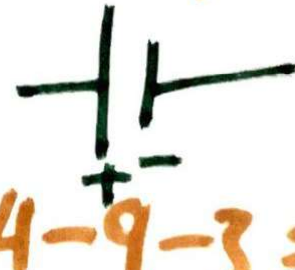
(a)

(b)



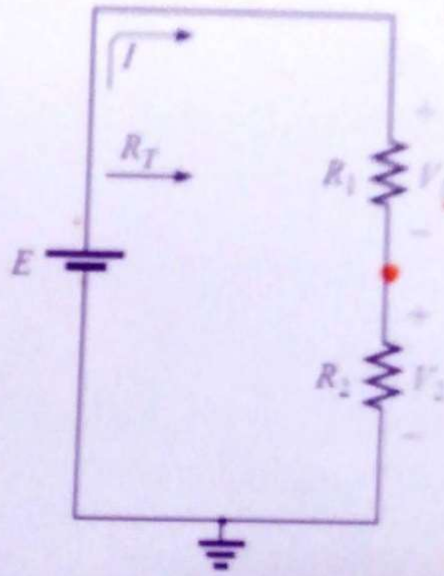
$+10+6+2=+18$





$=+4-9-3=-8$

VOLTAGE DIVIDER RULE



$R_T = R_1 + R_2$

$E = V_1 + V_2$

$E = IR_1 + IR_2$

$E = I(R_1 + R_2)$

$\boxed{E = I R_T}$

$I = \frac{E}{R_T}$

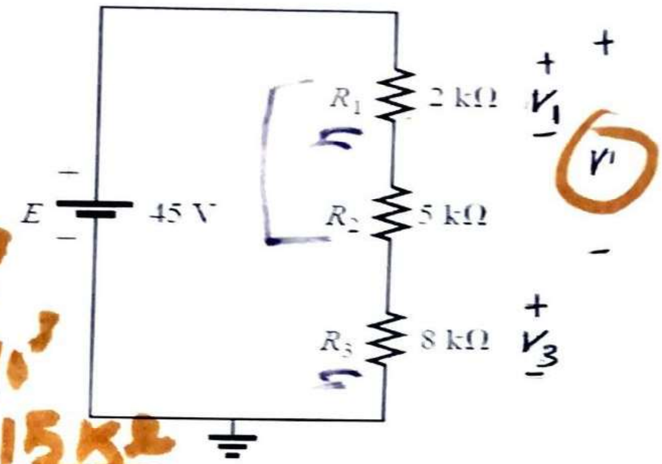
$V_1 = I R_1$

$\boxed{V_1 = \frac{E}{R_T} * R_1} = \frac{E R_1}{R_T}$

$V_2 = \frac{E R_2}{R_T}$

$V_x = \frac{E R_x}{R_T}$

EXAMPLE Using the voltage divider rule, determine the voltages V_1 , V' and V_3 for the series circuit of Fig.



$R_T = 2*10^3 + 5*10^3 + 8*10^3 = 15k\Omega$

$V_x = \frac{E\,R_x}{R}$

$V_1 = \frac{E\,R_1}{R_T} = \frac{45*2*10^3}{15*10^3} = 6V$

$V_3 = \frac{E\,R_3}{R_T} = \frac{45*8*10^3}{15*10^3} = 24V$

~~$R' = 2*10^3 +$~~

$R' = R_1 + R_2 = 2*10^3 + 5*10^3 = 7k\Omega$

$V' = \frac{E*R'}{R_T} = \frac{45*7*10^3}{15*10^3} = 21V$

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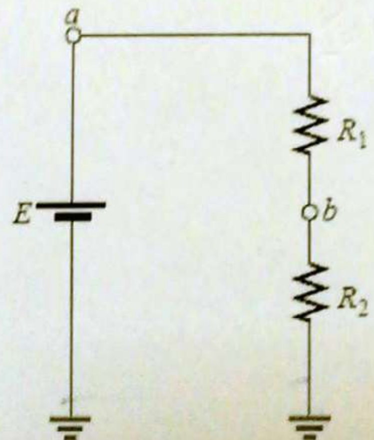
$E = V_1 + V_2 + V_3$

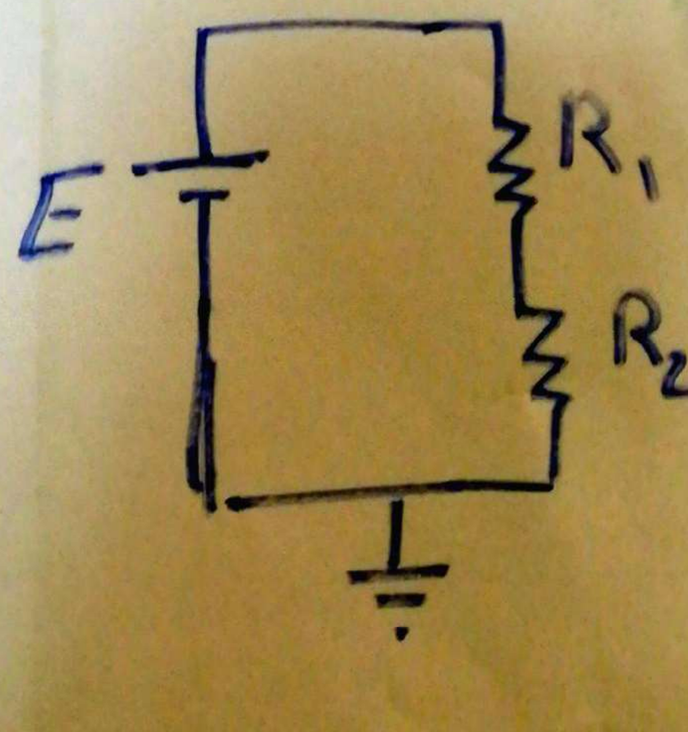
$= \overline{V}_1 + V_3$

$= 21 + 24 = 45V$

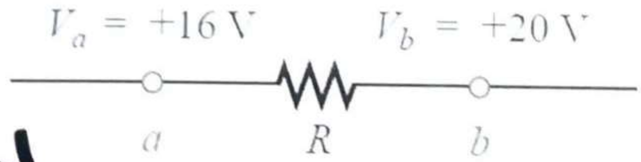
Voltage Sources and Ground

V=0





EXAMPLE 5.14 **Find the voltage V_{ab} for the conditions of Fig. 5.38.** V_{ba}

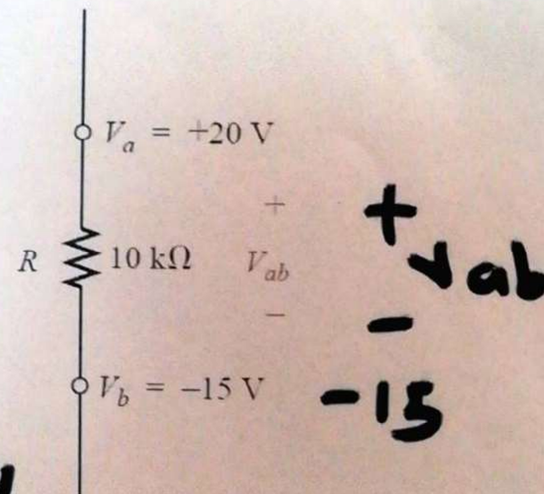


$V_{ab} = V_a - V_b = 16 - 20 = -4V$

$V_{ba} = V_b - V_a = 20 - 16 = +4V$

EXAMPLE 5.15 **Find the voltage V_a for the configuration of Fig. 5.39.**

~~$V_{ab} = 20 - ($~~

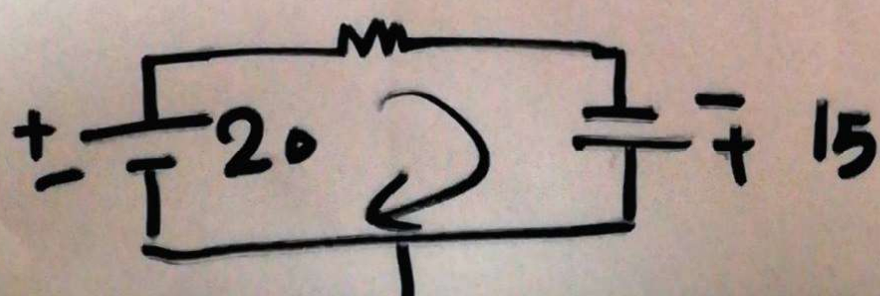


$V_{ab} = V_a - V_b$

$V_{ab} = 20 - (-15)$

$V_{ab} = 20 + 15 = 35V$

$V_{ba} = V_b - V_a = -15 - 20 = -35V$



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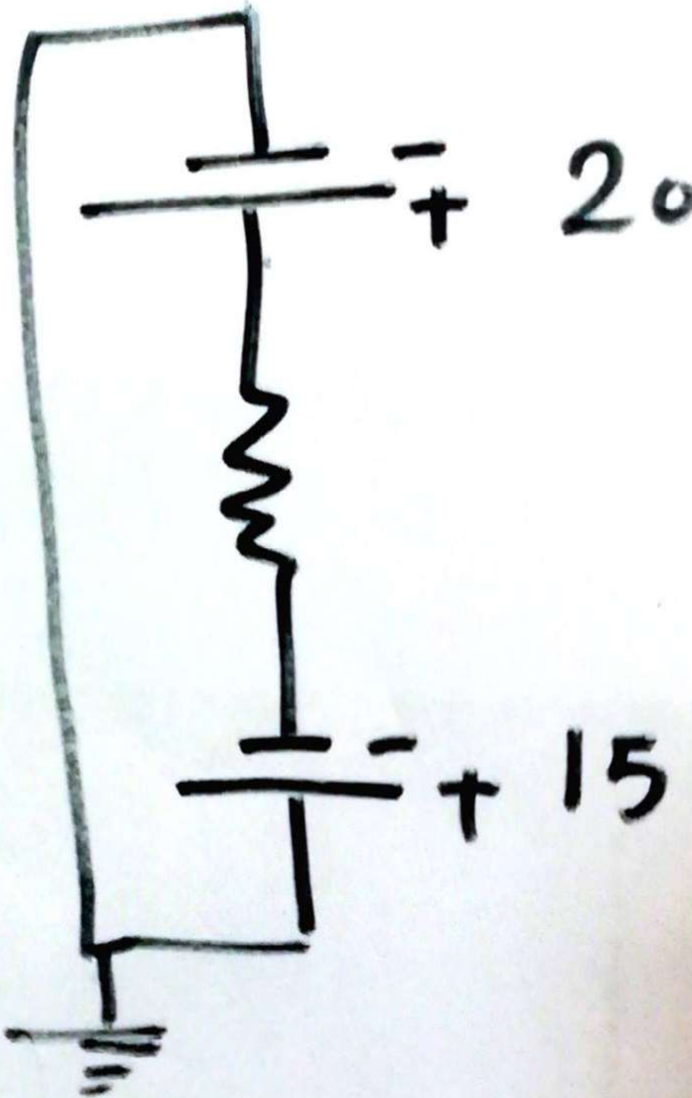
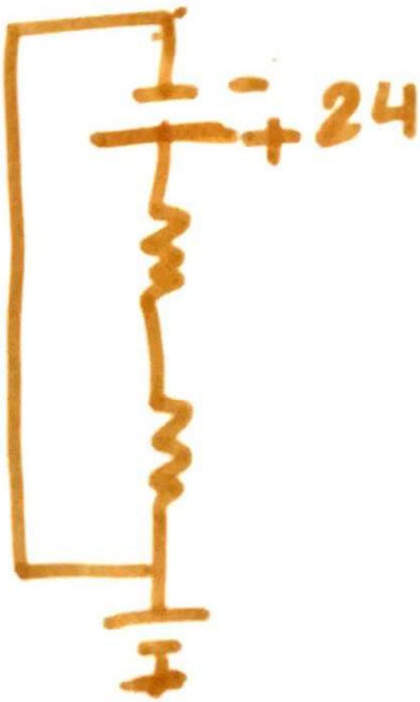
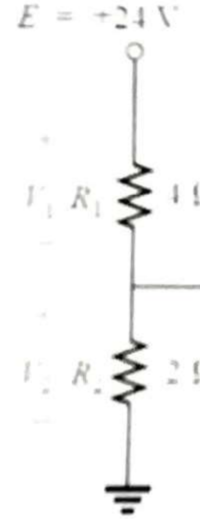


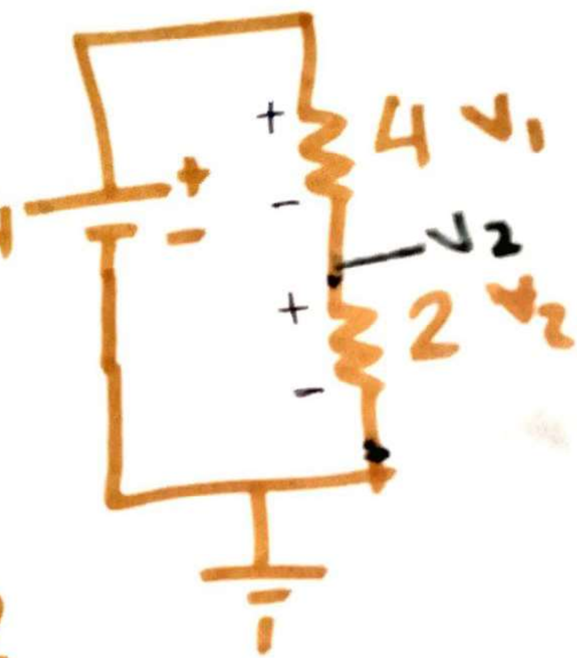
- + 20
- + 15

EXAMPLE Using the voltage divider rule, determine the voltages V_1 and V_2 of Fig.





$R_T = R_1 + R_2 = 4 + 2 = 6$



$$V_1 = \frac{E R_1}{R_T} = \frac{24 * 4}{6} = 16\,V$$

$$V_2 = \frac{E R_2}{R_T} = \frac{24 * 2}{6} = 8\,V$$

KVL $E = V_1 + V_2$

$= 16 + 8 = 24$

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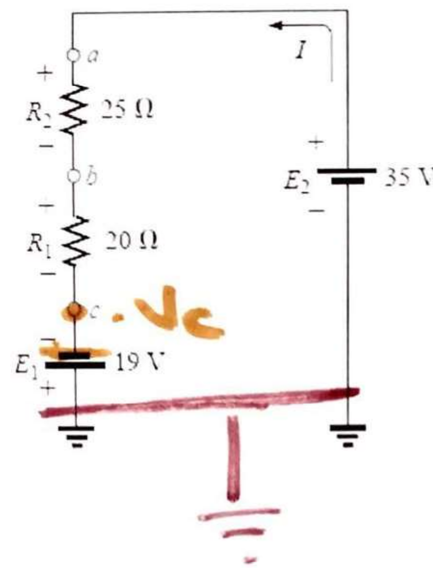
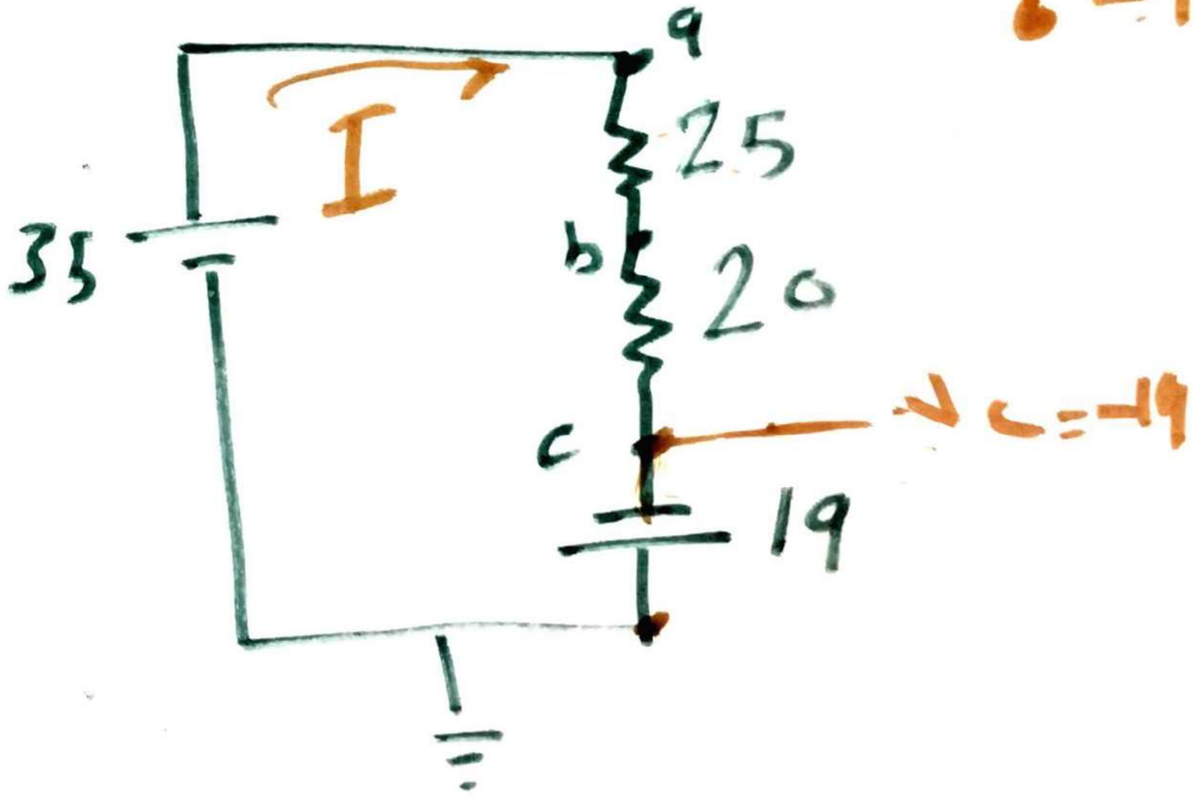
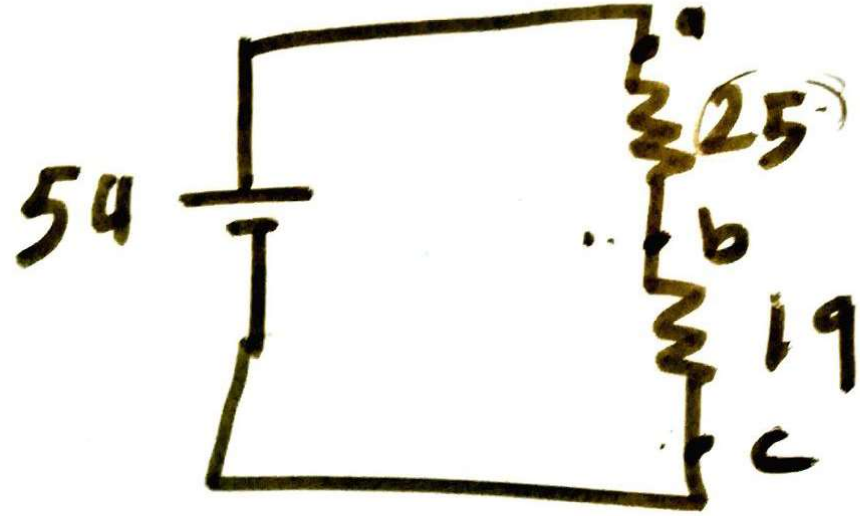
EXAMPLE **Determine V_{ab}, V_{cb}, and V_c for the network of Fig.**







$$V_{ab} = V_{25} = \frac{E \times 25}{44} = \frac{54 \times 25}{44}$$

$$= 30V$$

V_{cb} $\quad - V_{bc}$

$$\cancel{V_{bc}} = \cancel{V_{cb}} = -\frac{E\, R_{19}}{R_T}$$

$$= -\frac{54 \times 19}{44} = -24$$

$$V_c = -19 - 0 = -19$$

$$I = \frac{54}{45} = 1.2$$

$$V_{25} = V_{ab} = I R_x = 1.2 * 25 = 30^v$$

$$V_{cb} = -V_{bc} = -I R_{20} = -1.2 * 20$$

$$= -24V$$

$$V_c = -19$$

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<table>
<tr><th>No.</th><th>Example</th></tr>
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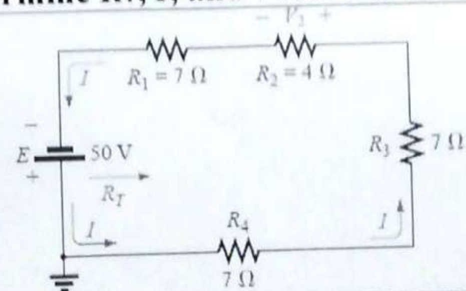
EXAMPLE 5.2 **Determine R_T, I, and V_2 for the circuit of Fig. 5.8.**



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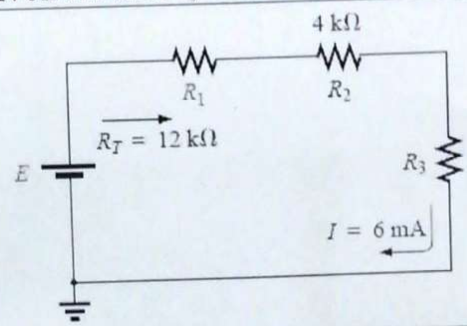
EXAMPLE 5.3 **Given R_T and I, calculate R_1 and E for the circuit of**



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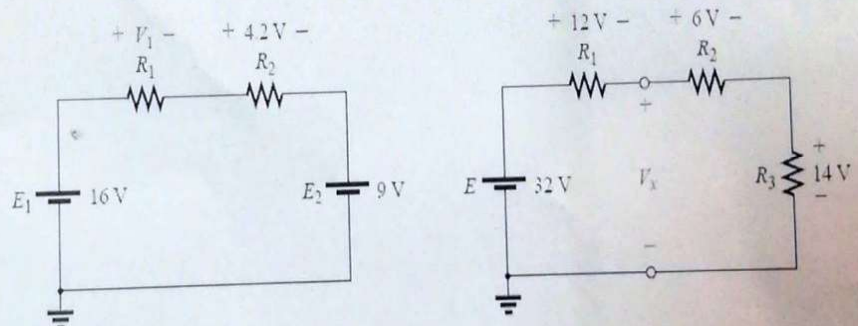
EXAMPLE 5.4 **Determine the unknown voltages for the networks of Fig. 5.14.**



(a) (b)

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EXAMPLE 5.7 **For the circuit of Fig. 5.17:**

a. Find R_T.

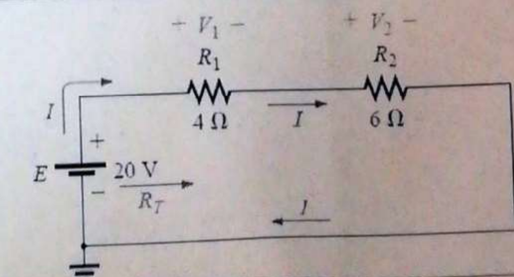
b. Find I.

c. Find V_1 and V_2.

d. Find the power to the 4Ω and 6Ω resistors.

e. Find the power delivered by the battery, and compare it to that dissipated by the 4Ω and 6Ω resistors combined.

f. Verify Kirchhoff's voltage law (clockwise direction).



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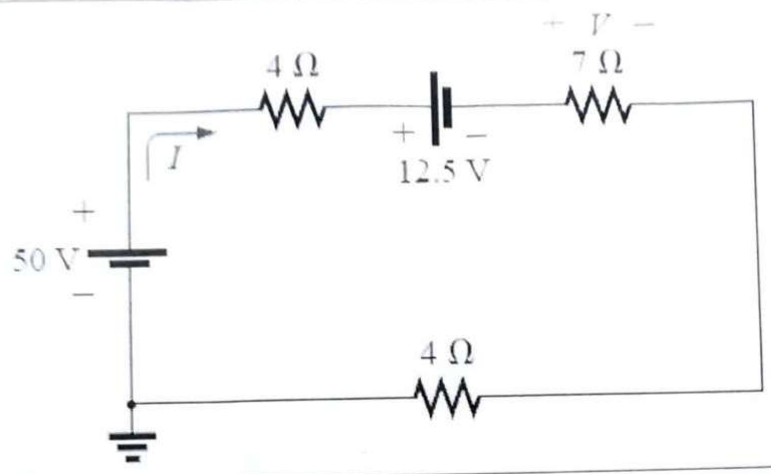
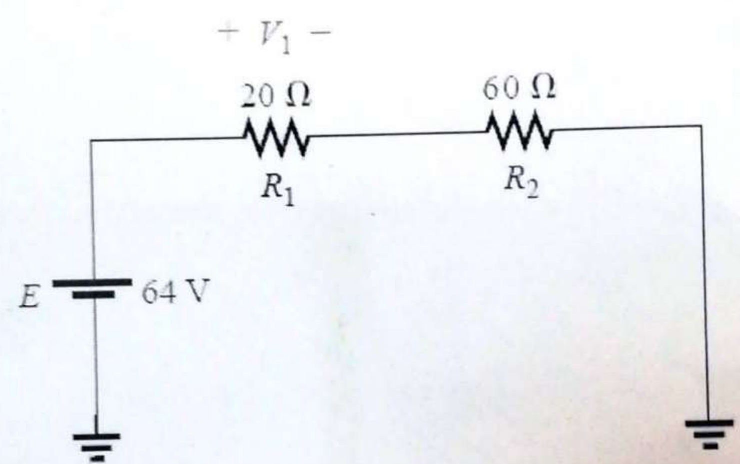
EXAMPLE 5.9 Determine I and the voltage across the 7Ω resistor for the network of Fig.



5

EXAMPLE 5.10 Determine the voltage V_1 for the network of Fig. 5.27.



8

EXAMPLE 5.20 For the network of Fig. 5.50:

a. Calculate Vab.

b. Determine Vb.

c. Calculate Vc.

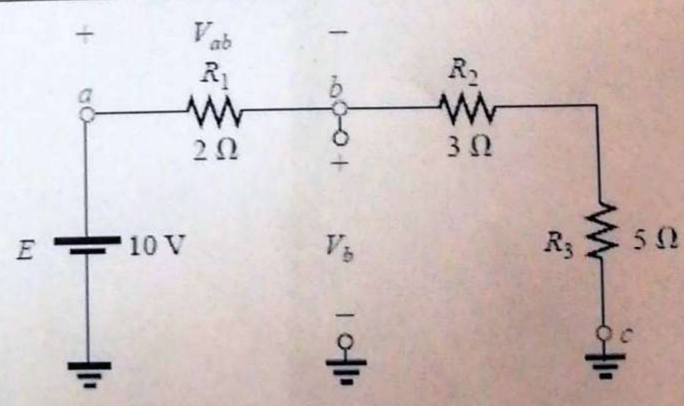


FIG. 5.50

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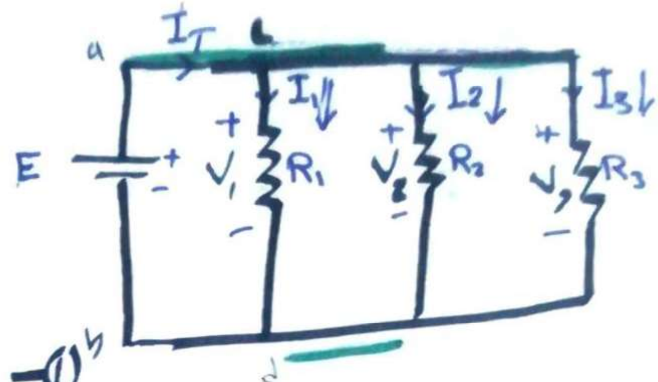
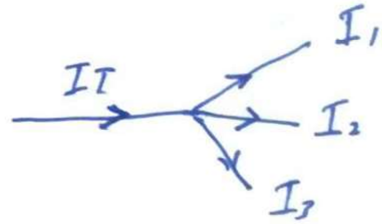
Parallel Circuits

KVL

KCL



$$I_T = I_1 + I_2 + I_3 \quad (1)$$



$$V = IR \Rightarrow I = \frac{V}{R} \quad (2)$$

$$\frac{E}{R_T} = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}$$

$$E = V_1 = V_2 = V_3 = V$$

$$\frac{V}{R_T} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

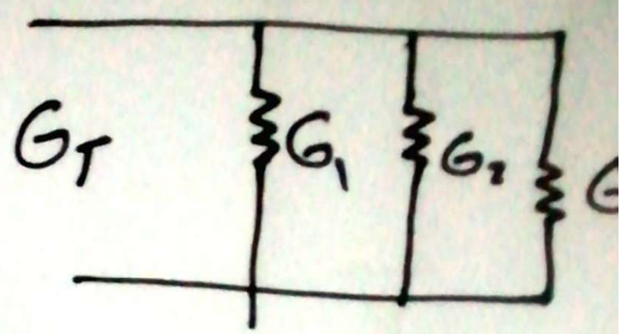
$$\frac{\cancel{V}}{R_T} = \cancel{V}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \text{-----} \frac{1}{R_n}$$

$$G = \frac{1}{R}$$

$$G_T = G_1 + G_2 + G_3$$



Special cases

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots\cdots \frac{1}{R_N}$$

* **Case 1 :**

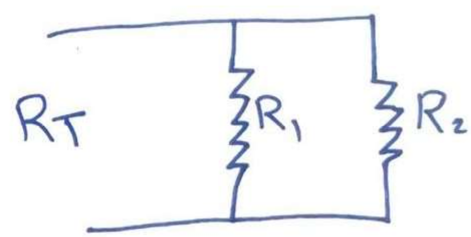
$$R_1 = R_2 = R_3 = \cdots\cdots R_N = R$$

$$\frac{1}{R_T} = \frac{1}{R} + \frac{1}{R} + \frac{1}{R} + \cdots + \frac{1}{R}$$

$$\frac{1}{R_T} = N \times \frac{1}{R}$$

$$\frac{1}{R_T} = \frac{1}{R} + \frac{1}{R} + \frac{1}{R} = \frac{1+1+1}{R} = \frac{3}{R}$$

* **Case 2 :**



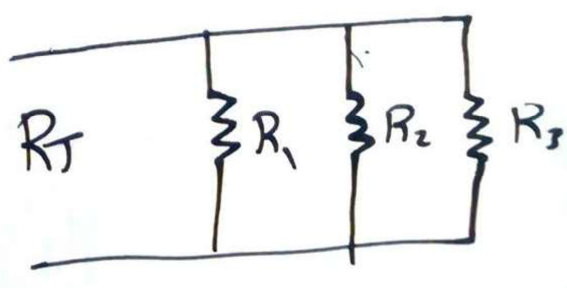
$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{R_T} = \frac{R_2 + R_1}{R_1 R_2}$$

$$R_T = \frac{R_1 R_2}{R_1 + R_2}$$

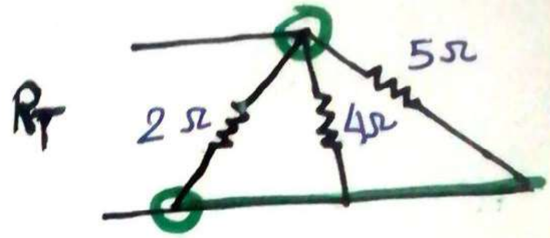
Case 3 :

$$R_T = \frac{R_1 R_2 R_3}{R_1 R_2 + R_2 R_3 + R_3 R_1}$$



$\boxed{\text{H.W}}$

Example :- Determine the total resistance for the network shown.



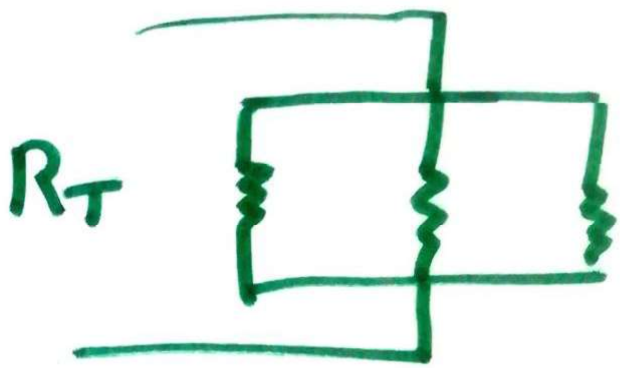
$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

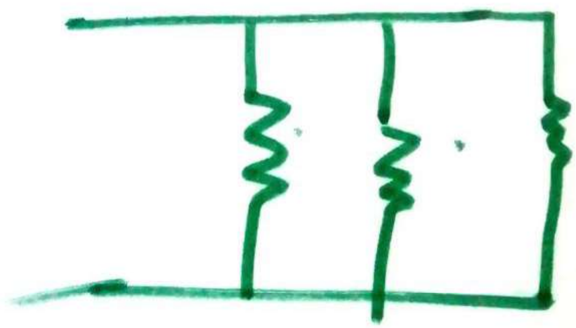
$$\frac{1}{R_T} = \frac{1}{2} + \frac{1}{4} + \frac{1}{5}$$

$$\frac{1}{R_T} = 0.95$$

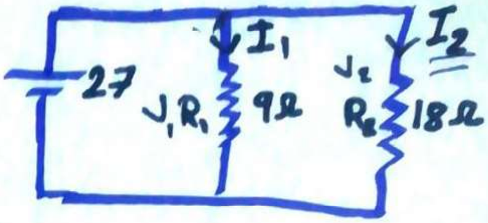
$$R_T = \frac{1}{0.95} = 1.053\,\Omega$$

$$R_T = \frac{2*4*5}{2*4 + 2*5 + 4*5}$$





Example :- for the parallel network shown:



a- Calculate R_T.

b- Determine I_T.

c- Calculate I_1 and I_2.

d- Determine the power to each resistive load.

e- Determine the power delivered by the source and compare it with total power dissipated by the resistive elements.

Solution

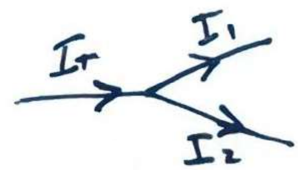
a

$$R_T = \frac{R_1 R_2}{R_1 + R_2} = \frac{9 * 18}{9 + 18} = 6\,\Omega$$

b $I_T = ?$

$$V = IR \Rightarrow I = \frac{V}{R} \Rightarrow I_T = \frac{E}{R_T}$$

$$I_T = \frac{27}{6} = \underline{\underline{4.5\,A}}$$



c I_1 & I_2 ?

$$E = V_1 = V_2$$

$$I_1 = \frac{V_1}{R_1} = \frac{E}{R_1} = \frac{27}{9} = \underline{\underline{3\,A}}$$

$$I_2 = \frac{V_2}{R_2} = \frac{E}{R_2} = \frac{27}{18} = 1.5\,A$$

$$I_T = I_1 + I_2$$
$$= 3 + 1.5 = 4.5\,A$$

(d) $P = IV$

$P_1 = I_1 V_1 = I_1 * E = 3 * 27 = \underline{\underline{81\ W}}$

$P_2 = I_2 V_2 = I_2 * E = 1.5 * 27 = \underline{\underline{40.5}}\ W$

(e) $P_T = IV = I_T E = 4.5 * 27 = \underline{\underline{121.5\ W}}$

$\cancel{P_T} = P_1 + P_2 = 81 + 40.5 = \underline{\underline{121.5}}$

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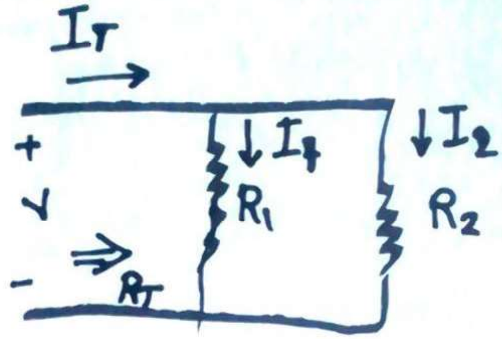
Current Divider Rule

$$V = I_T R_T \Rightarrow I = \frac{V}{R_T}$$



$$R_T = \frac{R_1 * R_2}{R_1 + R_2} \quad \text{—— ①}$$

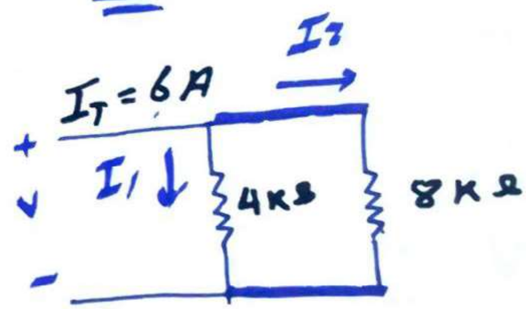
$$I_1 = \frac{V}{R_1} = \frac{I R_T}{R_1} \quad \text{—— ②}$$

$$I_1 = \frac{I \frac{R_1 * R_2}{R_1 + R_2}}{R_1} = \frac{I R_2}{R_1 + R_2}$$

$$I_2 = \frac{I R_1}{R_1 + R_2}$$

Example : Determine the current I_2 for the network

$$R_T = \frac{R_1 * R_2}{R_1 + R_2}$$



$$I_2 = \frac{6 * 4*10^3}{4*10^3 + 8*10^3} = 2A$$

$$I_1 = \frac{I_T * R_2}{R_1 + R_2} = \frac{6 * 8*10^3}{4*10^3 + 8*10^3} = 4A$$

$$I_1 + I_2 = 4 + 2 = 6A$$

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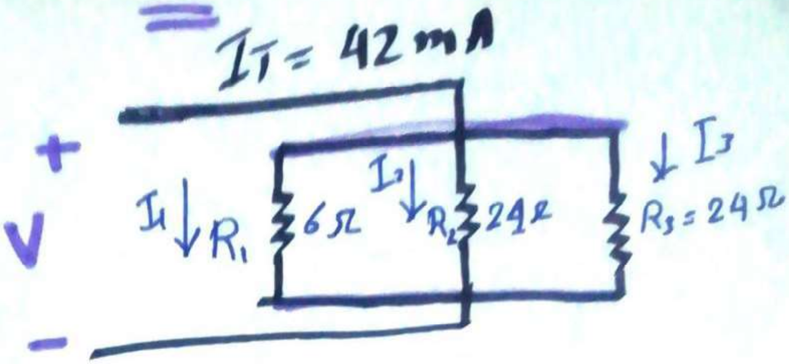
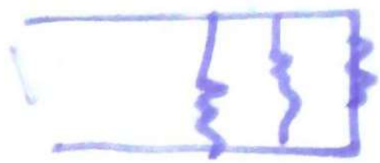
Example :- Find the current I_1 for the network





$$I_1 = \frac{V}{R_1} \quad \text{—— ①}$$

$$V = I_T R_T \quad \text{—— ②}$$

$$R_T = \frac{R_1 R_2 R_3}{R_1 R_2 + R_2 R_3 + R_1 R_3} \quad \text{—— ③}$$

$$I_1 = \frac{I_T R_T}{R_1} = \frac{I_T \dfrac{\cancel{R_1} R_2 R_3}{R_2 R_1 + R_2 R_3 + R_1 R_3}}{\cancel{R_1}}$$

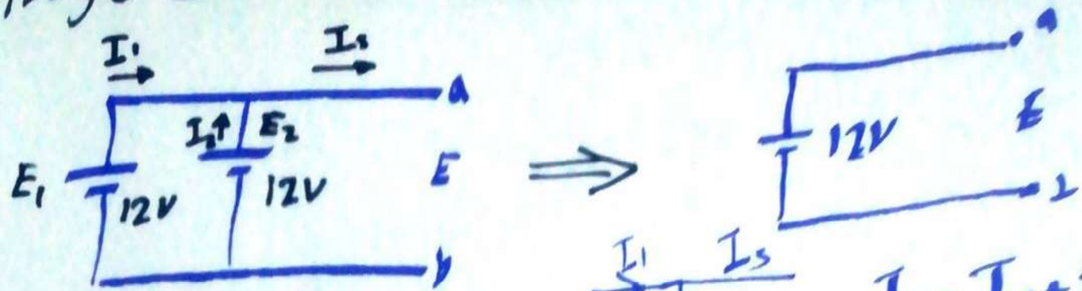
$$I_1 = \frac{I_T R_2 R_3}{R_1 R_2 + R_2 R_3 + R_3 R_1} = \frac{43*10^{-3} * 24 * 24}{6*24 + 24*24 + 24\,6} = 0.028\,A = 28\,mA$$

① $R_T = 4\,\Omega$

② $V = I_T R_T = 42*10^{-3} * 4 = 0.168\,V$

③ $I_1 = \frac{V}{R_1} = \frac{0.168}{6} = 0.025\,A = 25*10^{-3}\,A = 28\,mA$

Voltage Source in parallel

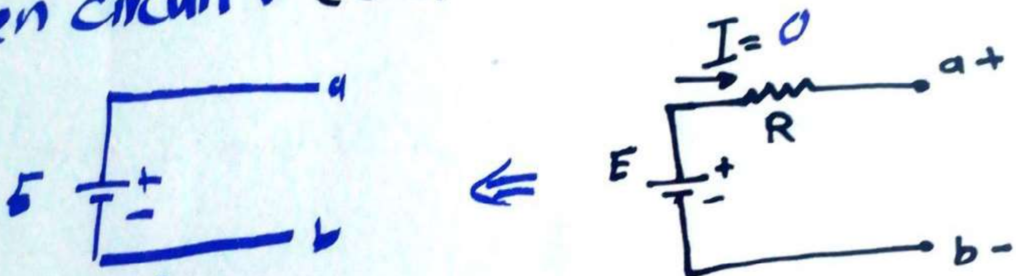


$E_1 = E_2 = E$

$I_s = I_1 + I_2$

Open and short circuits

* Open circuit : (O.C)

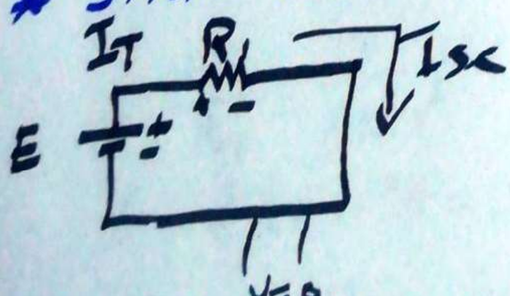


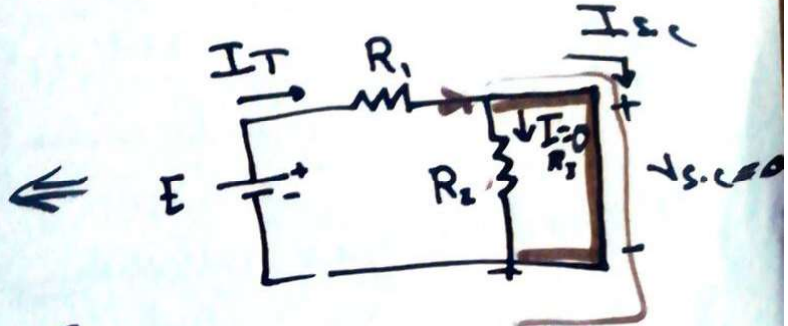
$V_{ab} = E$

$E = V_{ab}$

$V_{open\ circuit} = V_{O.C} = E = V_{ab}$

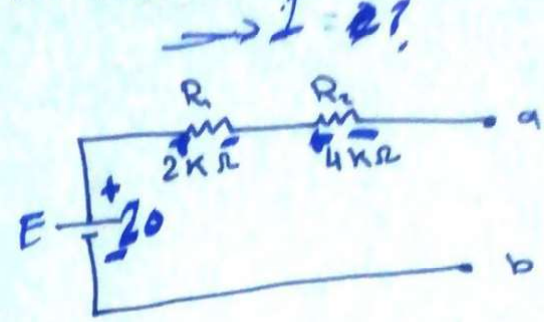
* Short circuit : (S.C)





$I_{S.C} = \frac{E}{R_1} = I_T = I$

Example:- (a) For the network shown, determine V_{ab}

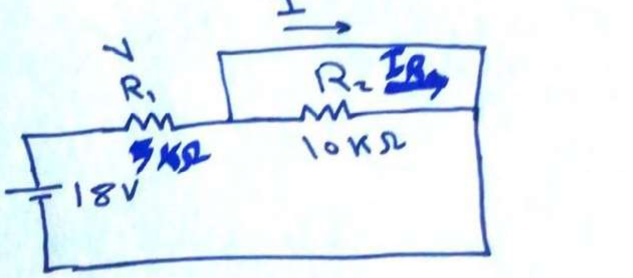


Solution:-

$V_{ab} = E = 20$

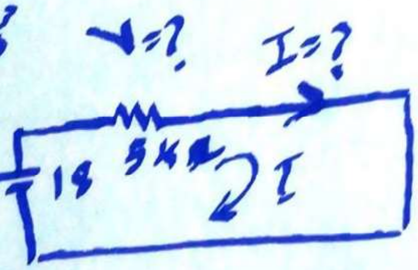
$I = 0$

(b) Calculate I and V for the network shown, I_{R_2}



$R_T = R_1 + R_2 = 5\times10^3 + 0$

Solution:- $R_T = R_1 = 5\times10^3$



$I = \frac{E}{R_T} = \frac{18}{5\times10^3} = 3.6mA$

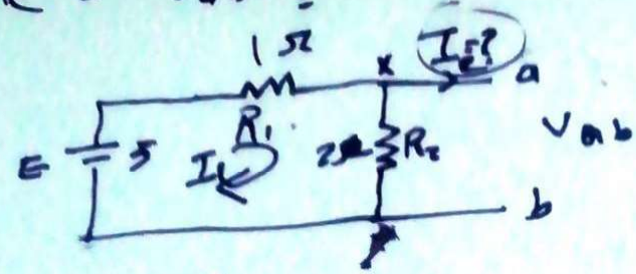
$I = 3.6\times10^{-3}A = 3mA$

$V_{R_1} = IR = \cancel{3\times10^{-3}\times5\times10} = 3.6\times10^{-3}\times5\times10^3$

$V_{R_1} = 18V$

$I_{R_2} = 0$

Example = $V_{ab} = ?$



$$\underline{V_{ab} = V_{R_2}}$$

$$V = IR \Rightarrow I = \frac{V}{R} \Rightarrow I_T = \frac{E}{R_T}$$

$$R_T = R_1 + R_2 = 1 + 2 = 3\,\Omega$$

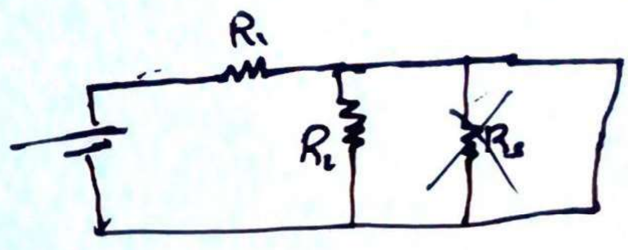
$$I_T = \frac{5}{3} = 1.666\ A$$

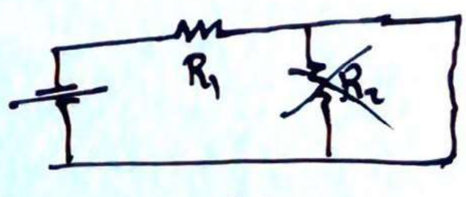
$$V_{R_2} = I * R_2 = 1.666 * 2 = 3.333\,V$$

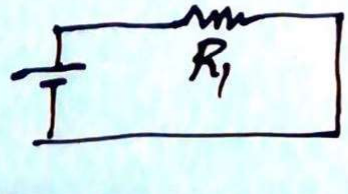
$$\therefore\ V_{ab} = V_{R_2} = 3.333\ V$$

$$I_2 = I_{o.s} = 0$$

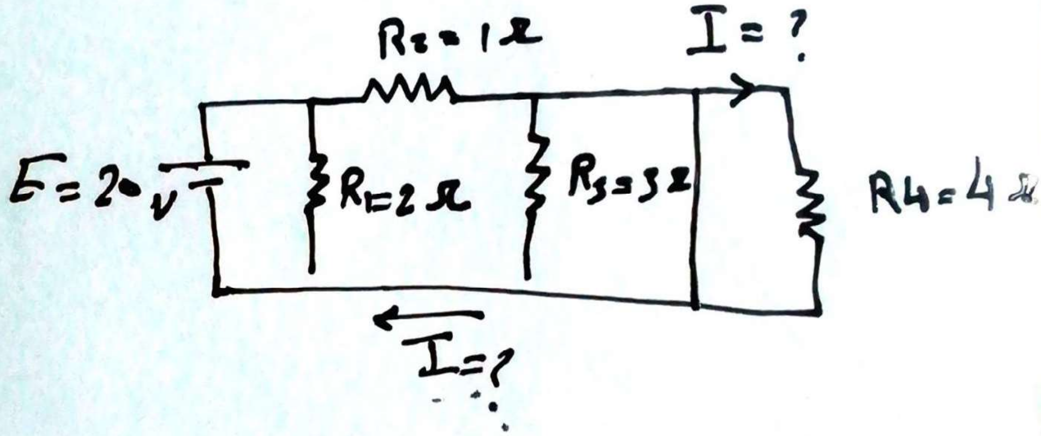
Example:







H . W



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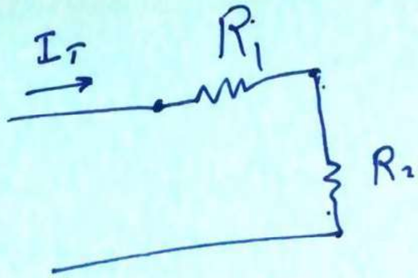
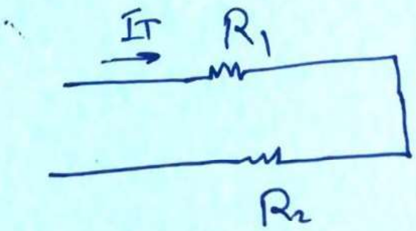
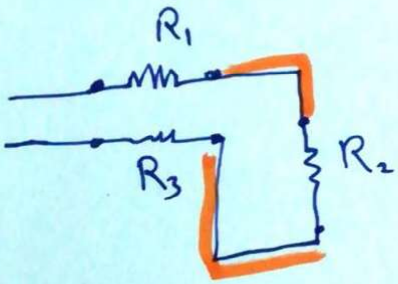
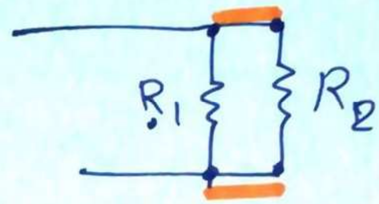
Series and Parallel circuits



$$I_T = I_{R_1} = I_{R_2}$$

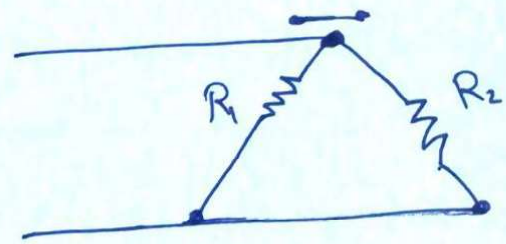


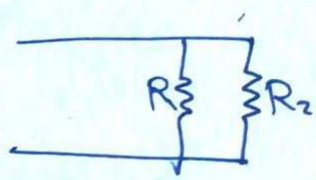


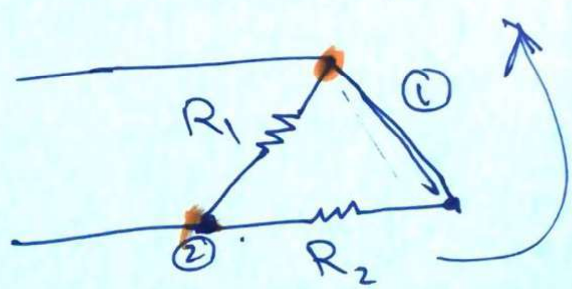


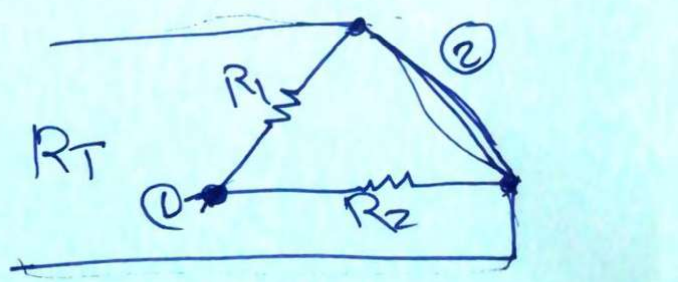
$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$$

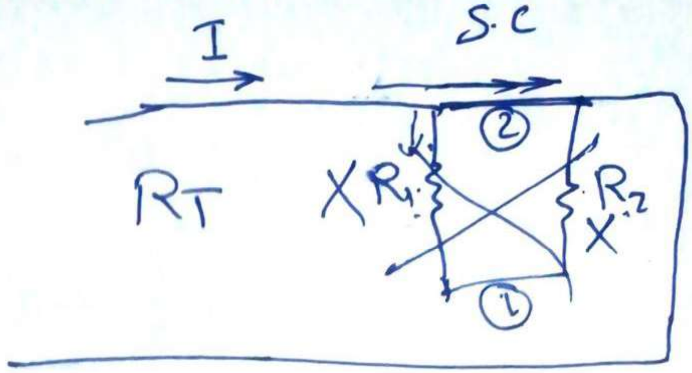
$$R_T = \frac{R_1 * R_2}{R_1 + R_2}$$



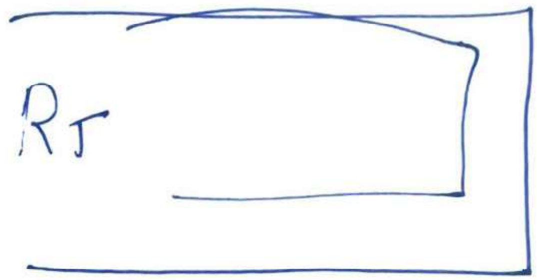


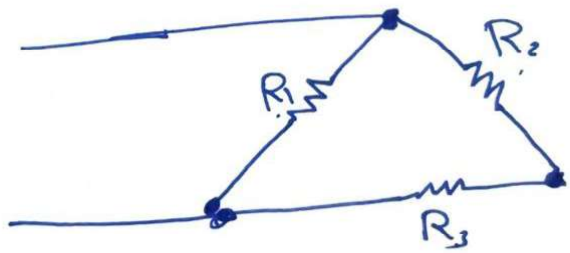




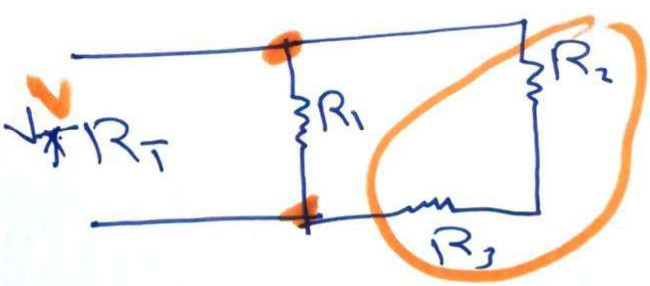


$R_T = 0$

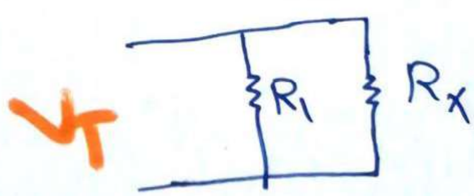


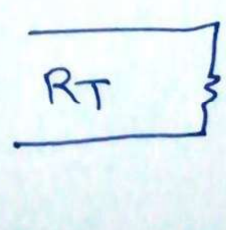


$R_X = R_1 + R_2$

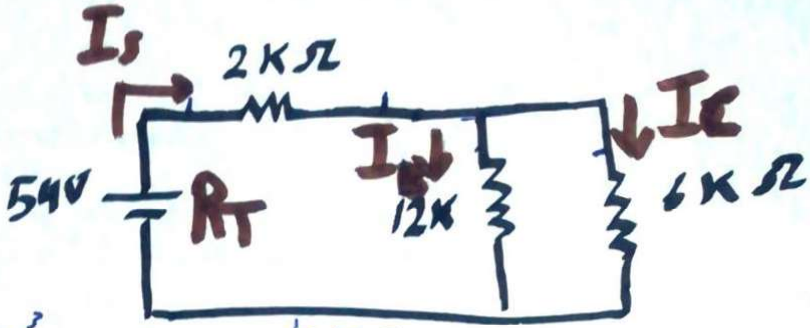


$$R_T = \frac{R_1 R_X}{R_1 + R_X}$$

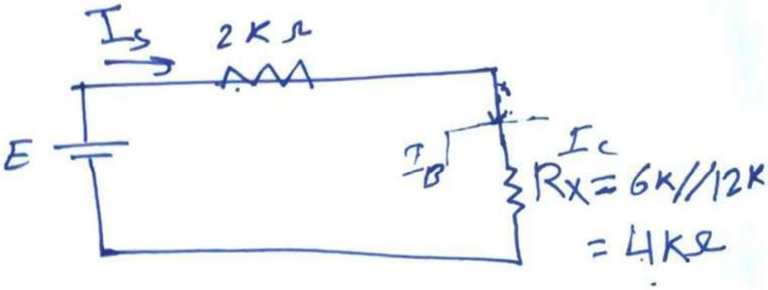




Example ①: Calculate the indicated current.



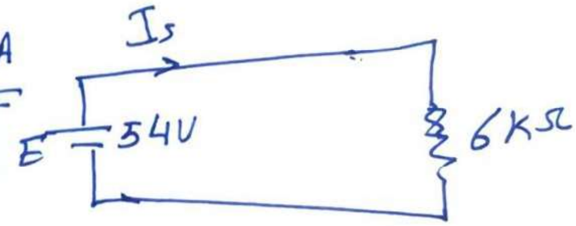
$$6k // 12k = \frac{12*10^3 * 6*10^3}{12*10^3 + 6*10^3} = 4000 = 4K\Omega$$



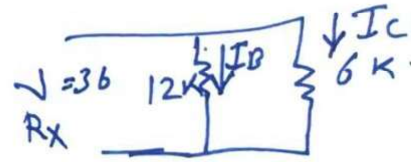
$$\underline{\underline{R_T}} = 2*10^3 + 4*10^3 = 6K\Omega$$

$$V = IR$$

$$I_s = I_T = \frac{E}{R_T} = \frac{54}{6*10^3} = \underline{\underline{9mA}}$$

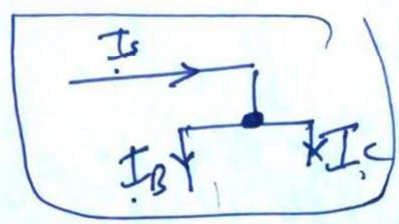


$$V_{Rx} = 9*10^{-3} * 4*10^3 = 36\,V$$



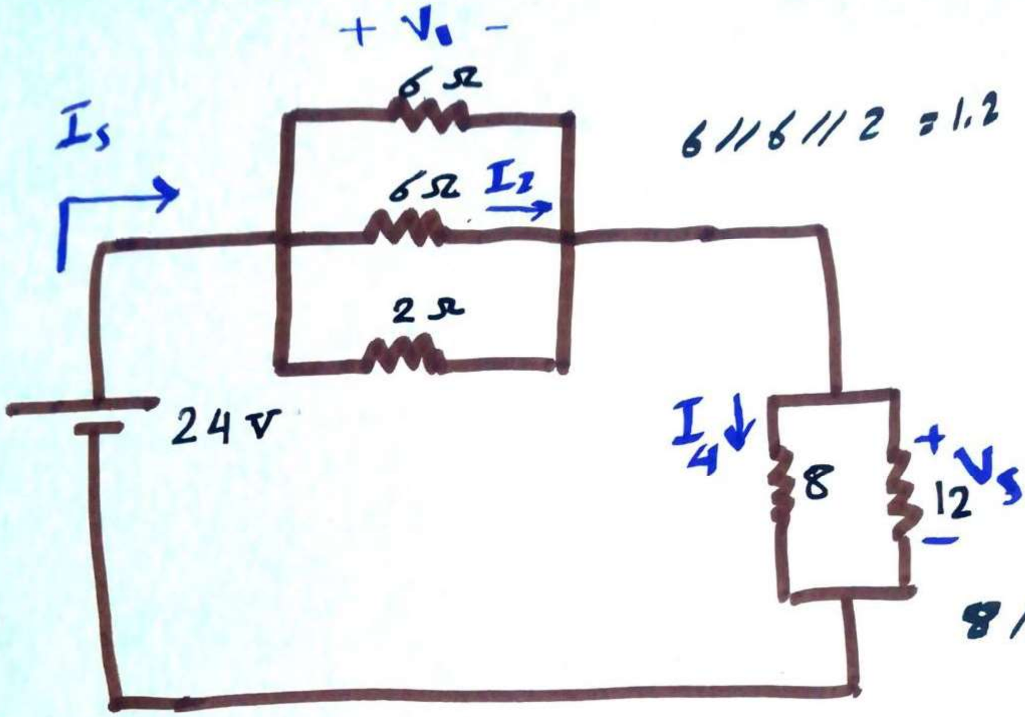
$$I_B = \frac{V_{12K}}{R_{12K}} = \frac{36}{12*10^3} = 3mA$$

$$I_C = \frac{V_{6K}}{R_{6K}} = \frac{36}{6*10^3} = 6mA$$



$$I_T = 3*10^{-3} + 6*10^{-3} = \underline{\underline{9mA}} = I_s$$

Example ③: Find the indicated current and the Voltage for the network.



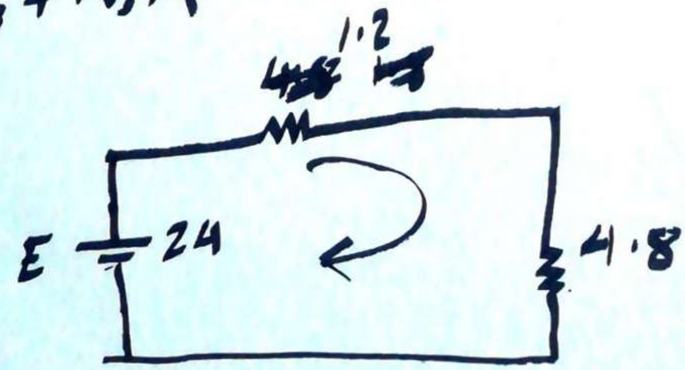
$6 // 6 // 2 = 1.2$

$8 // 12 = 4.8$

$V = IR \quad \therefore I = \frac{V}{R_T}$

$$R = \frac{R_1 R_2}{R_1 + R_2} = \frac{8 * 12}{8 + 12} = 4.8\ \Omega$$

$$R = \frac{R_1 R_2 R_3}{R_1 R_2 + R_2 R_3 + R_3 R_1} = \frac{6 * 6 * 2}{6*6 + 6*2 + 2*6} = 1.2\ \Omega$$



$R_T = 1.2 + 4.8 = 6$

$I_T = \frac{24}{6} = 4\ A$

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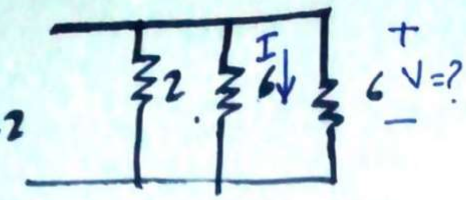
$V = 4.2\,V$

$R_T = 1.2$



$V_{1.2\Omega} = IR = 4 * 1.2 = 4.8\,V$

$V_{4.8} = IR = 4 * 4.8 = 19.2\,V$

$V_1 = V_{1.2} = 4.2\,V$

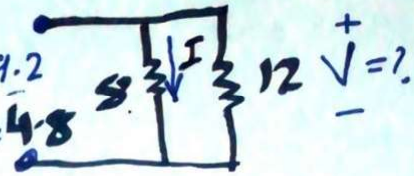
$I_2 = ?$ $\quad R = 6$

$V = 4.2$

$I_2 = \frac{V}{R} = \frac{4.2}{6} = 0.7A$

$V = 19.2$

$R_T = 4.8$



$= 4.8 + 19.2 = \underline{\underline{24}}$

$V_{12} = V_8 = V_{1.2} = 19.2\,V$

$I4 = ?$

$R = 8 \quad V = 19.2\,V$

$I = \frac{V}{R} = \frac{19.2}{8} = 2.4A$

OR

$$\boxed{I_2 = \frac{4 * 4.8}{8} = 2.4}$$

$= \frac{I_T * R_T}{R_8}$

$I_T = \frac{19.2}{4.8} = 4$

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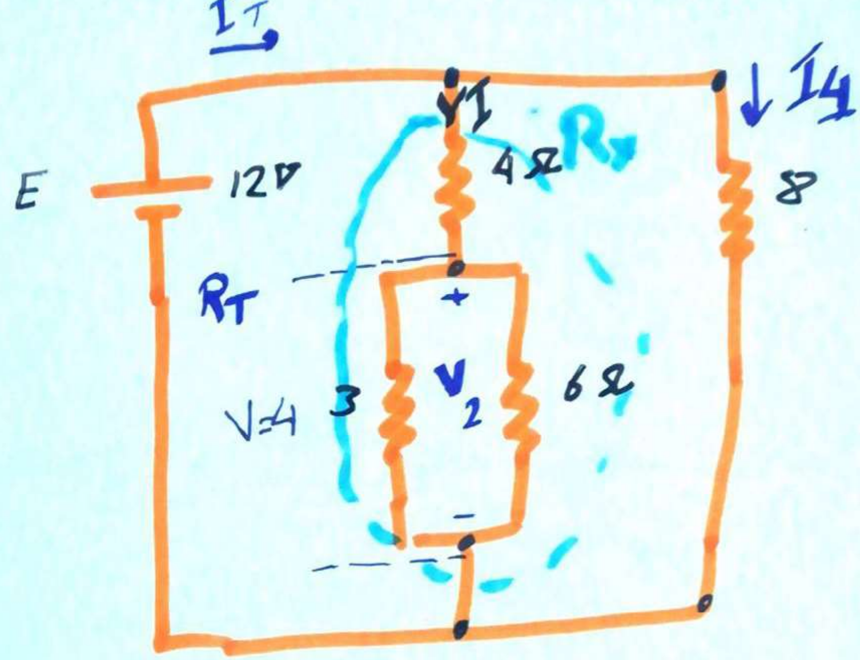
EXAMPLE ④: Find the current I_4 and the Voltage V_2 for the network. R_T

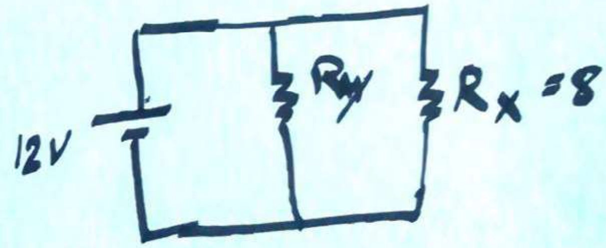


$I_4 = ?$

$R = 8$

$V_{R_x} = 12V$

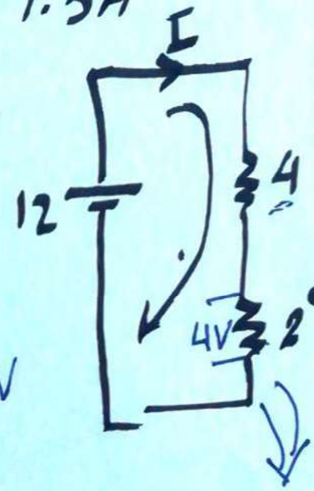
$$I_4 = \frac{V}{R} = \frac{12}{8} = 1.5A$$

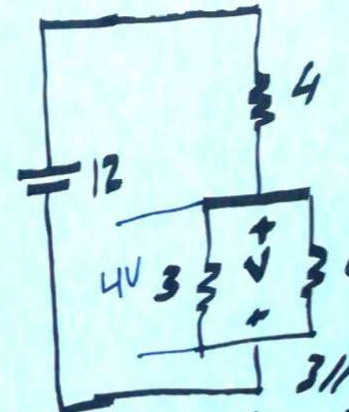


$$I = \frac{V}{R} = \frac{12}{6} = \underline{\underline{2A}}$$

$$V_2 = I * R = 2 * 2 = 4V$$

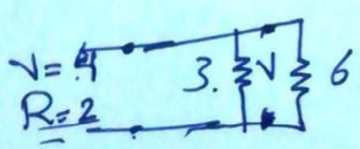
$$V_2 = 4V$$



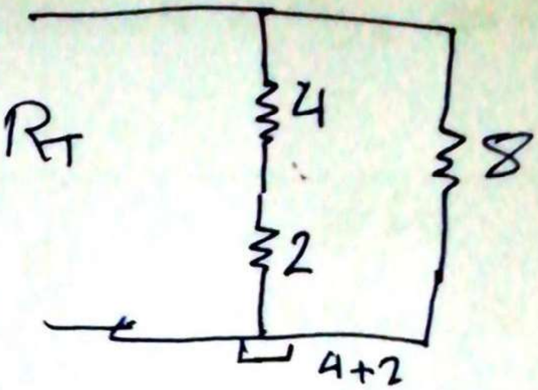


$3 // 6$

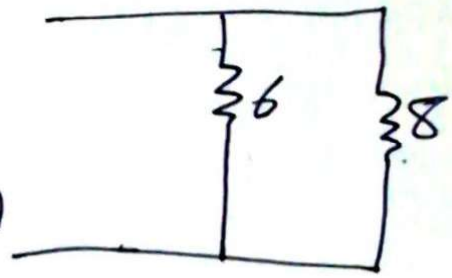
$$R = \frac{3*6}{3+6} = \frac{18}{9} = 2$$



R_T



$$R_T = \frac{6 * 8}{6+8} = 3.4$$



$$I = \frac{E}{R} = \frac{12}{3.4} = 3.5A$$

6//8

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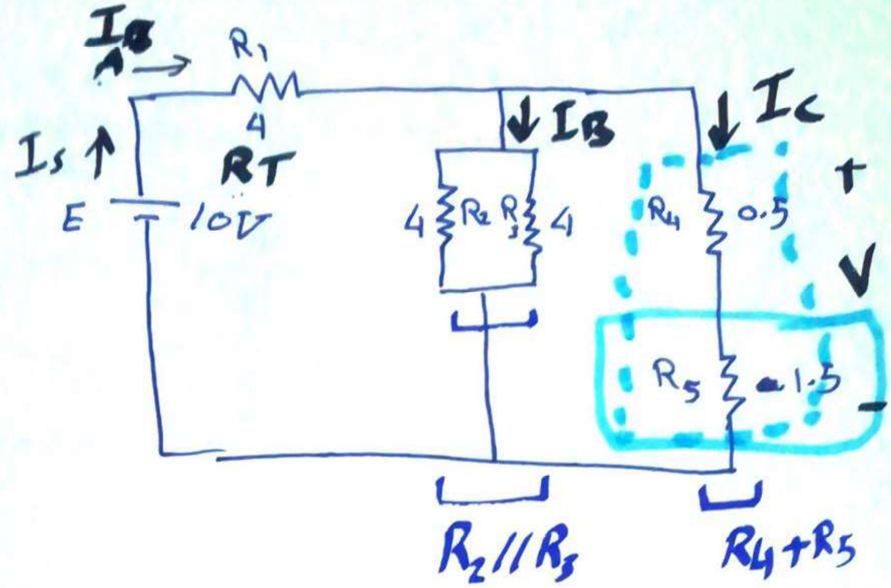
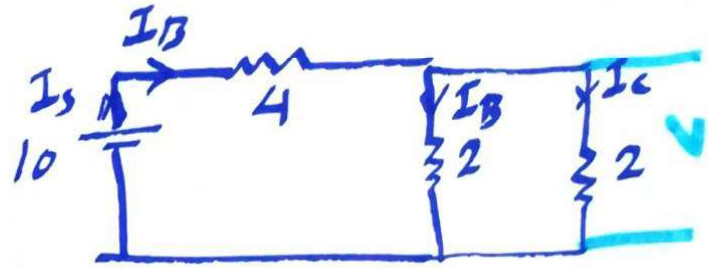
Example 3: Calculate the indicate Voltage and current.
5

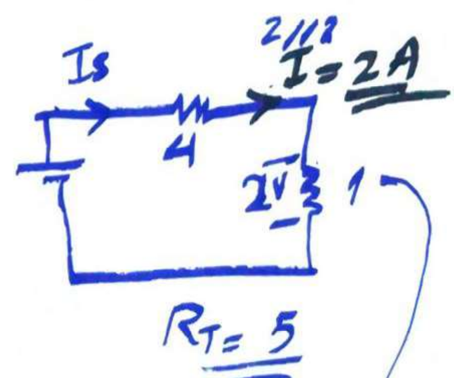


$V = 2V$

H.W

$V_{1.5} = ?$





1. $R_T = 5$

2. $I_s = \frac{V}{R_T} = \frac{10}{5} = 2A$

3. $I_A = I_s = 2A$

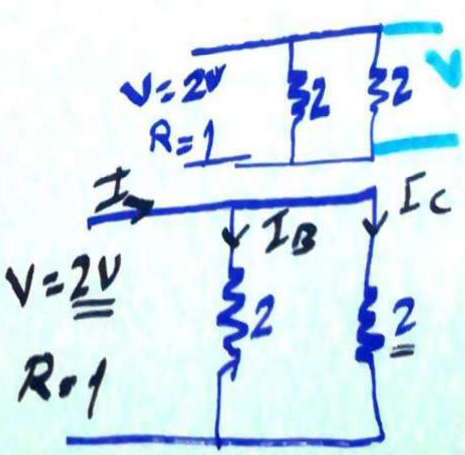
$V_{1\Omega} = I * R = 2 * 1 = 2$

$I_B = \frac{V}{R} = \frac{2}{2} = 1$

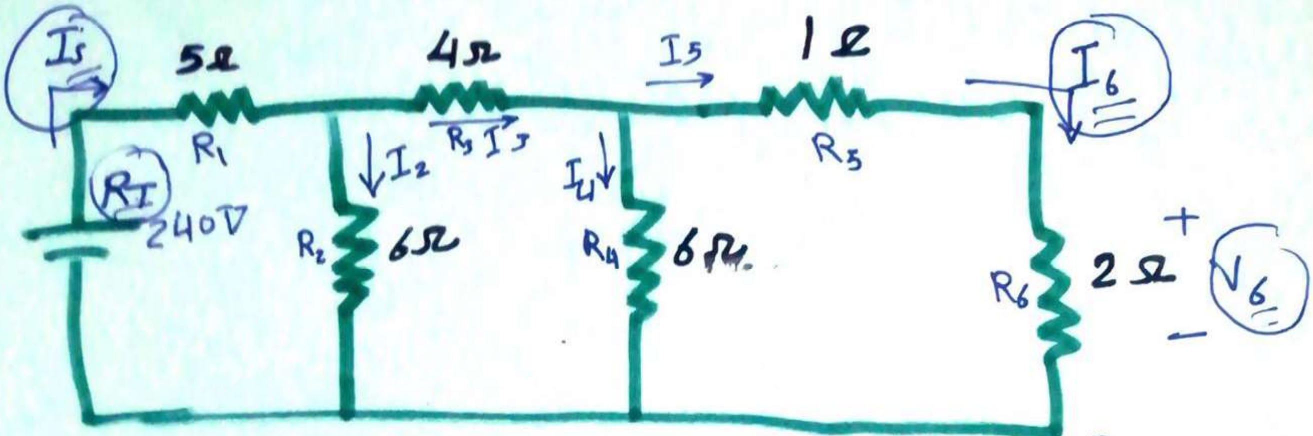
KCL

$I_c = \frac{V}{R} = \frac{2}{2} = 1$

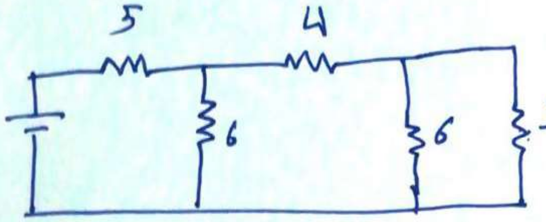
$I = I_B + I_c = 1 + 1 = 2A$

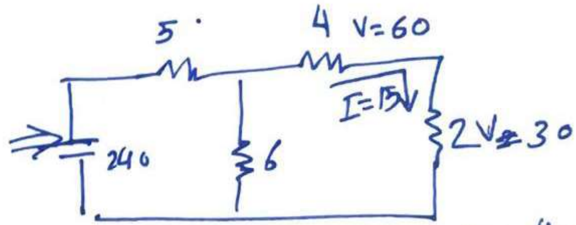


Example ②: Calculate the indicated current and the voltage.



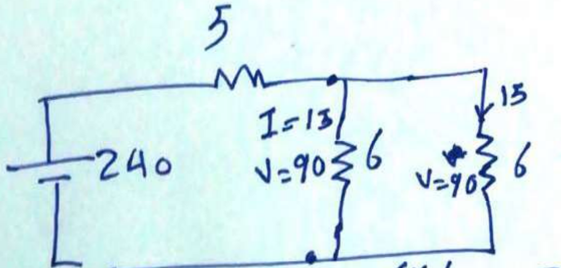
$R = 1 + 2 = 3$



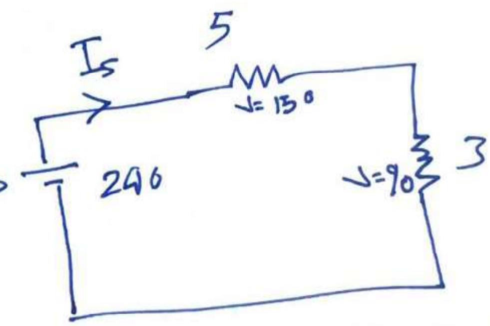


$R = 2 + 4 = 6$

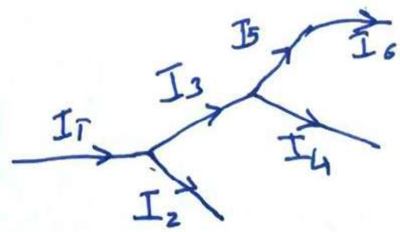
$$\frac{3 * 6}{3 + 6} = \frac{18}{9} = 2$$

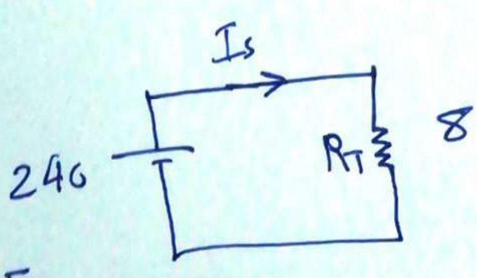


$$R = \frac{6 * 6}{6 + 6} = 3$$



$3 + 5 = 8$





$$I_s = \frac{E}{R_T} = \frac{240}{8} = 30A$$

$V_{R_3} = I * 5 = 30 * 5 = 150$

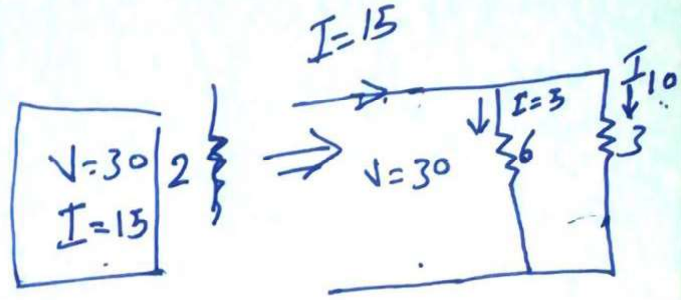
$V = 30 * 3 = 90V$

$I_6 = \frac{90}{6} = 15A$

$I = \frac{90}{6} = 15A$

$V_4 = 15 * 4 = 60V$

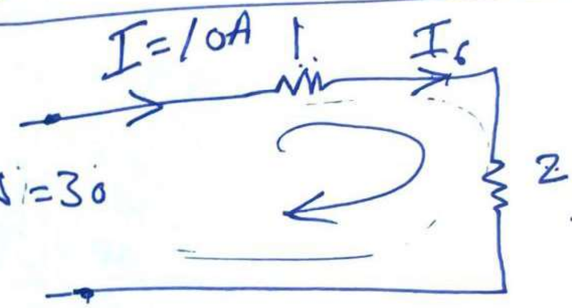
$V_2 = 15 * 2 = 30$



$I_6 = \frac{V}{R} = \frac{30}{6} = 5A$

$I_3 = \frac{V}{R} = \frac{30}{3} = 10A$

$\left.\begin{array}{l} I = 10A \\ V = 30 \end{array}\right\} 3\Omega$



$V = IR = 10 * 1 = 10V$

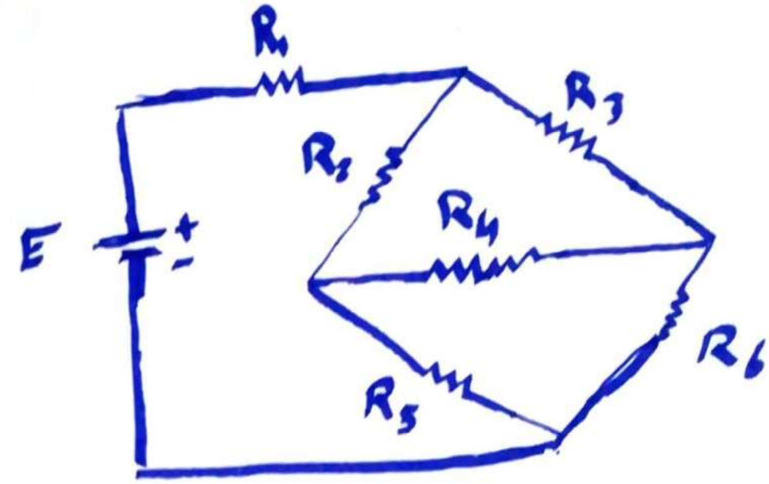
$V_{2\Omega} = IR = 10 * 2 = 20V$

$V_T = 10 + 20 = 30V$

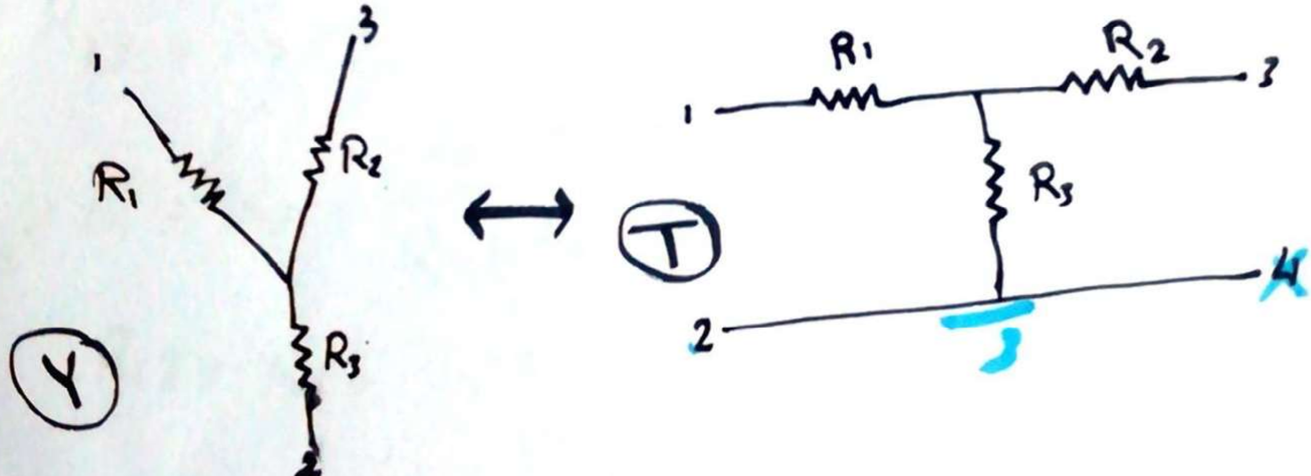
$I_6 = 10A$

$V_6 = 20V$

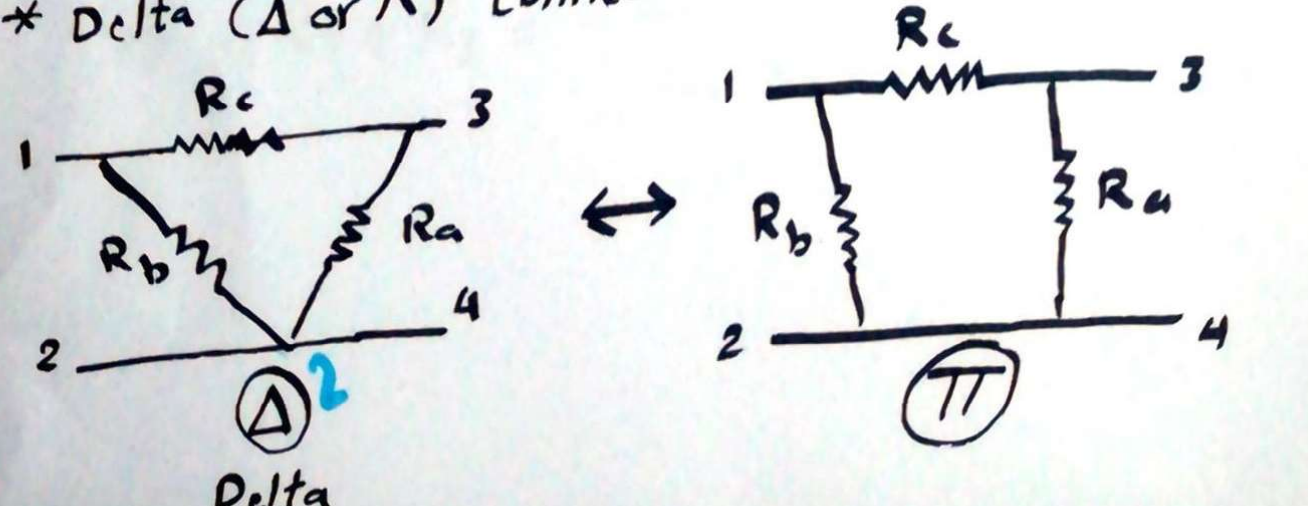
Delta – star transformation



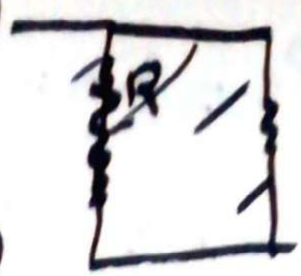
~~why~~ star (Y or star) connection
T



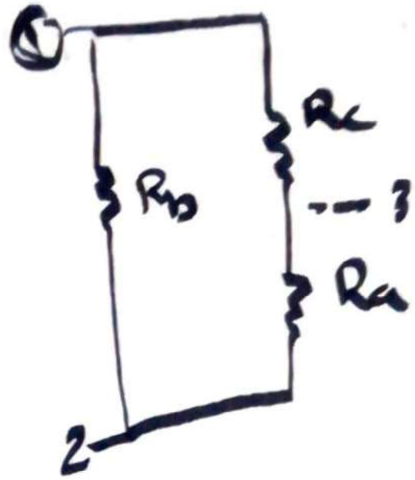
* Delta (Δ or π) connect



$$R_{12} = \frac{R_b(R_c+R_a)}{R_b+(R_c+R_a)} \quad (1)$$



$$R_{13} = \frac{R_c \ast (R_a+R_b)}{R_c+R_b+R_a} \quad (2)$$



$$R_{23} = \frac{R_a(R_c+R_b)}{R_a+R_b+R_c} \quad (3)$$

$$R_{12} = R_1 + R_3 \quad (4)$$

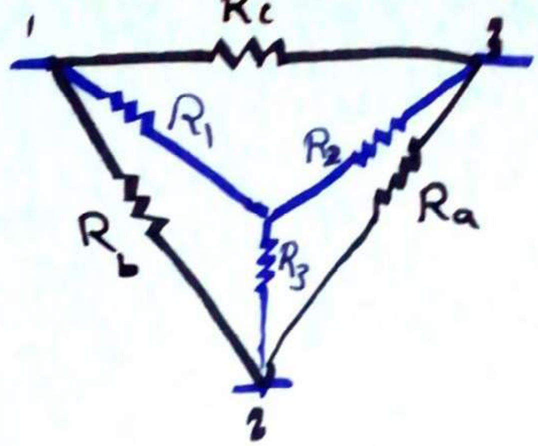
$$R_{13} = R_1 + R_2 \quad (5)$$

$$R_{23} = R_2 + R_3 \quad (6)$$

$$R_{12} = R_1 + R_3 = \frac{R_b(R_c+R_a)}{R_b+R_c+R} \quad (1)$$

$$R_{13} = R_1 + R_2 = \frac{R_c \ast (R_a+R_b)}{R_c+R_b+R_a} \quad (2)$$

$$R_{23} = R_2 + R_3 = \frac{R_a(R_c+R_b)}{R_a+R_b+R_c} \quad (3)$$



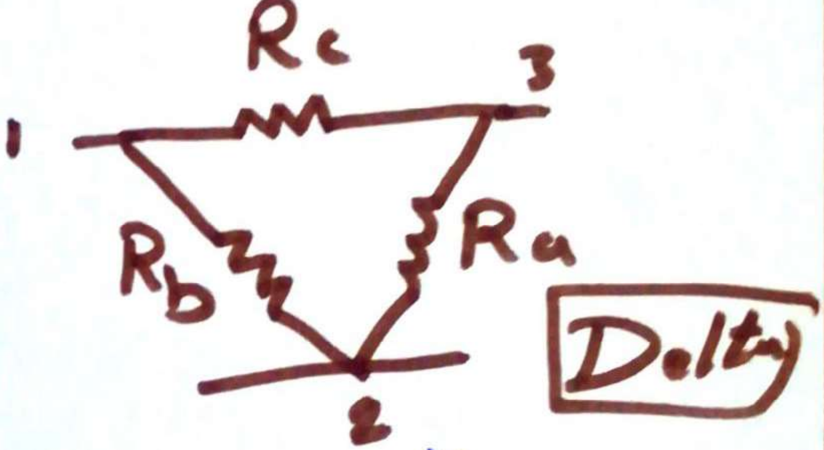
Δ Y

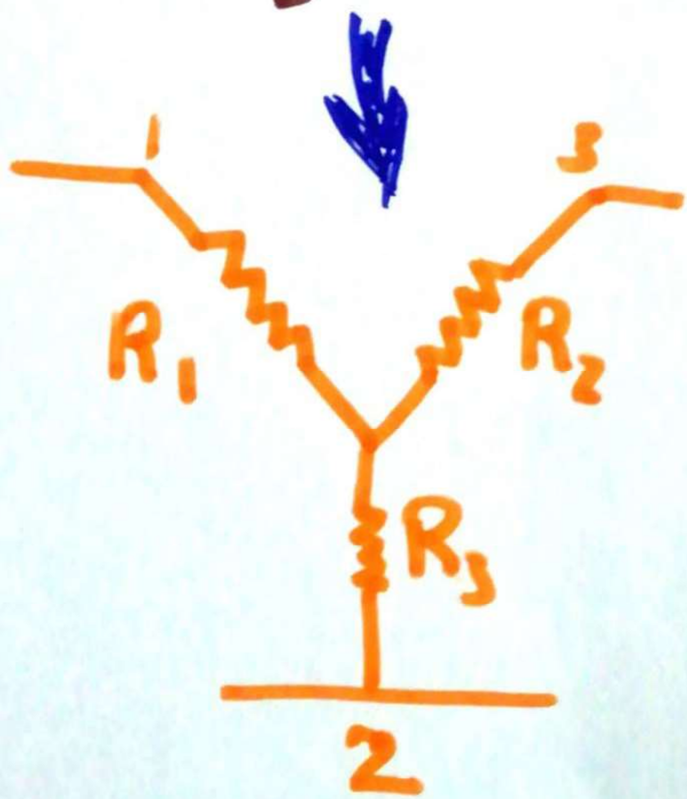
Delta to Wye transformation

$$\boxed{R_1 = \frac{R_c * R_b}{R_a + R_b + R_c}}$$

$$R_2 = \frac{R_c \times R_a}{R_a + R_b + R_c}$$

$$\boxed{R_3 = \frac{R_a \ R_b}{R_a + R_b + R_c}}$$





$$R_c = R_b = R_a = R_\Delta$$

$$R_1 = R_2 = R_3 = R_Y$$

$$R_Y = \frac{R_\Delta}{3}$$

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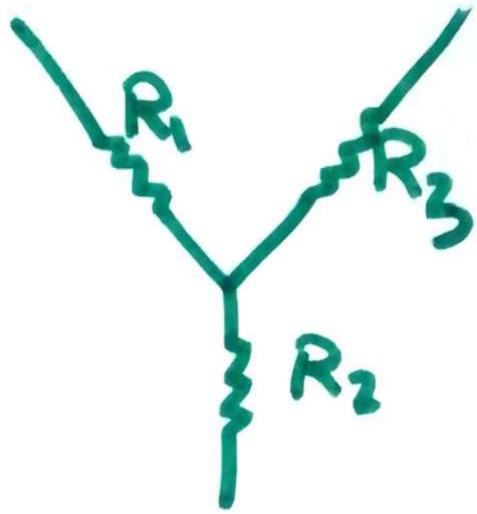
Wye to Delta Transformation

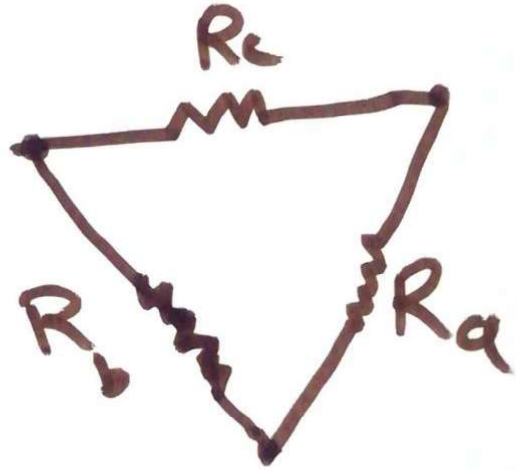
(Y) (Δ)

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

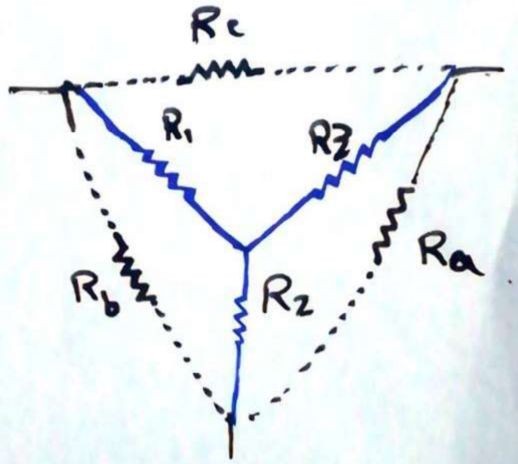




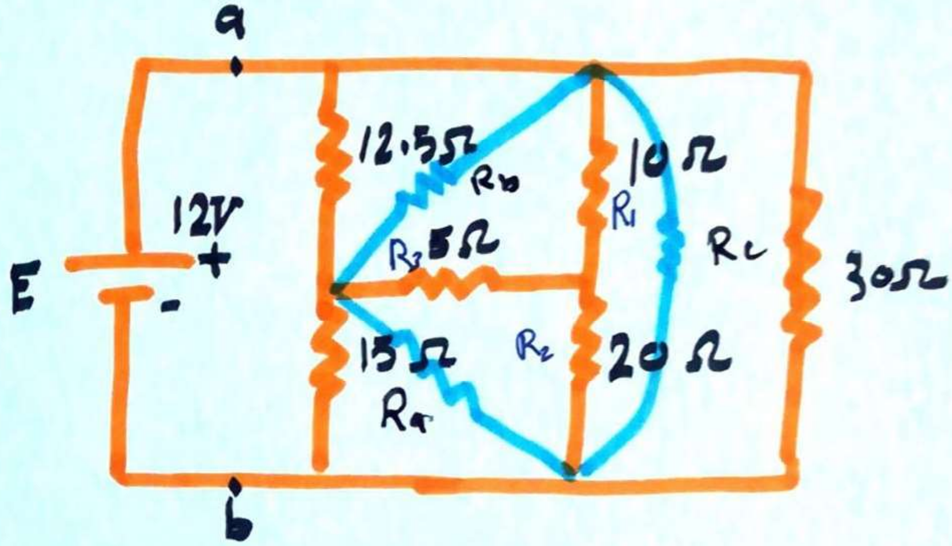
$$R_1 = R_2 = R_3 = R_Y$$

$$R_a = R_b = R_c = R_\Delta$$

$$R_\Delta = 3R_Y$$



Example : obtain the equivalent resistance R_{ab} for the circuit shown and used it to find the current I.

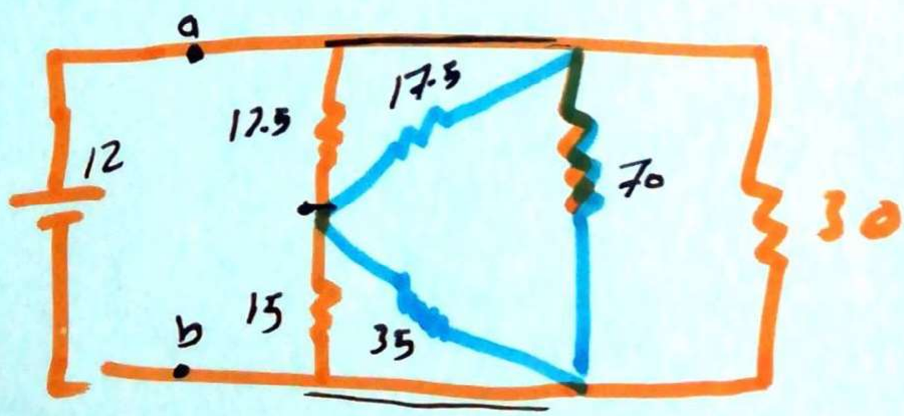


Solution :

$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1} = \frac{10*20 + 20*5 + 5*10}{10} = 35$$

$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2} = \frac{10*20 + 20*5 + 5*10}{20} = 17.5$$

$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3} = \frac{10*20 + 20*5 + 5*10}{5} = 70$$



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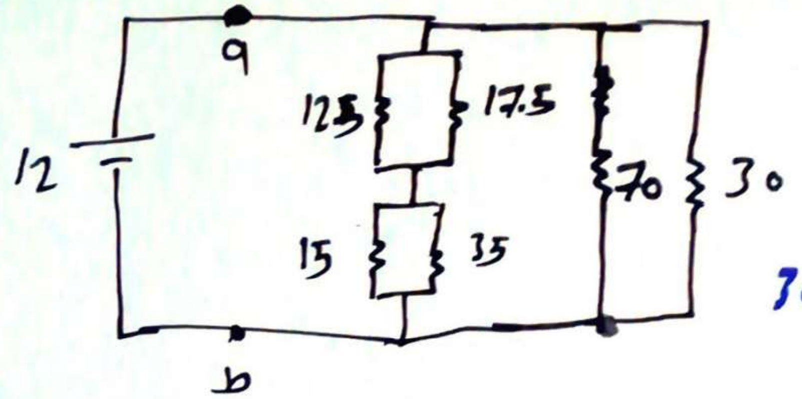
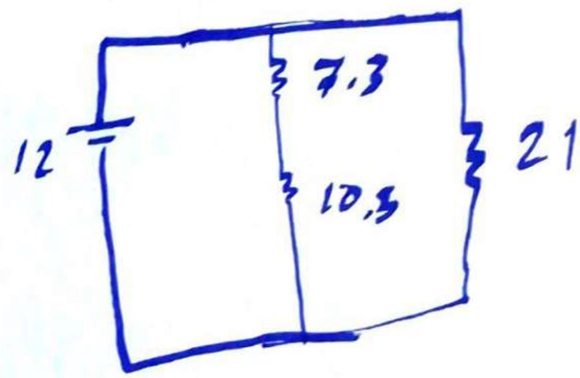


30//70

$30 \parallel 70 = \frac{30 * 70}{30+70} = 21\,\Omega$

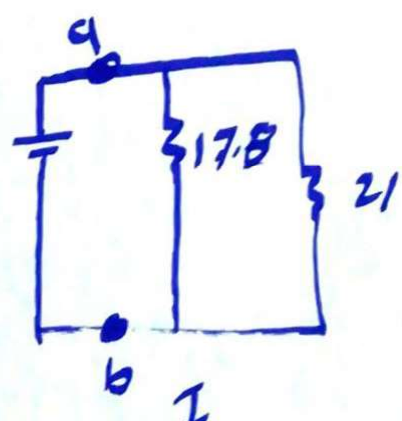
$17.5 \parallel 12.5 = \frac{17.5 * 12.5}{17.5+12.5} = 7.3\,\Omega$

$15 \parallel 35 = \frac{15 * 35}{15+35} = 10.5\,\Omega$

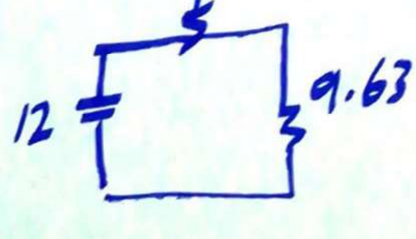


Rab

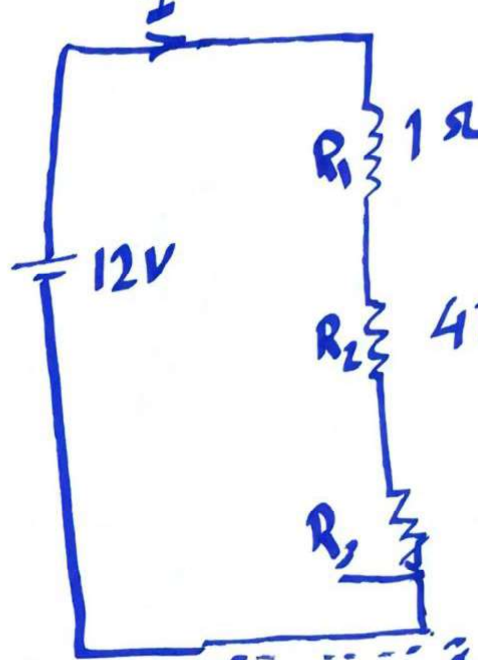
$= 17.5 \parallel 21 = \frac{17.8 * 21}{17.8+21} = 9.63\,\Omega$



$I_T = \frac{E}{R_T} = \frac{12}{9.63} = 1.246\,A$



Example =- ~~The~~ Three resistors are Connected in Series across a 12V battery. The first resistor has a value of 1Ω, the Second has a voltage drop of 4V, and the third has a power 12W. Calculat the Value of the circuit current.



Solution :

$R_1 = 1\Omega$

$P_{R_3} = 12$

$P_3 = I^2 R_3 = 12$ — ①

$V_2 = IR_2 = 4V$

$I = \dfrac{4}{R_2}$ — ②

$\left(\dfrac{4}{R_2}\right)^2 R_3 = 12 \Rightarrow R_3 = \dfrac{12}{16} R_2^2 \Rightarrow \boxed{R_3 = \dfrac{3}{4} R_2^2}$

KVL : $E = V_1 + V_2 + V_3$

$E = IR_1 + IR_2 + IR_3$

$12 = I\left(1 + R_2 + \dfrac{3}{4}R_2^2\right) \Rightarrow 12 = \dfrac{4}{R_2}\left(1 + R_2 + \dfrac{3}{4}R_2^2\right)$

$\dfrac{12R_2}{4} = 1 + R_2 + \dfrac{3}{4}R_2^2$

$3R_2 = 1 + R_2 + \dfrac{3}{4}R_2^2 \Rightarrow \left(\dfrac{3}{4}R_2^2 - 2R_2 + 1 = 0\right) \times 4$

$3R_2^2 - 8R_2 + 4 = 0$

(a) (b) (c)

$$R_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{8 \pm \sqrt{64 - 48}}{2 \times 3} =$$

$R_2 = 2\,\Omega$ OR $R_2 = \frac{2}{3}\,\Omega$

$$R_3 = \frac{3}{4} R_2^2 =$$

$R_3 = 3$

$R_3 = \frac{1}{3}$

①

$$I = \frac{V}{R_T} = \frac{12}{1+2+3} = \frac{12}{6} = 2\ A$$

②

$$I = \frac{V}{R_T} = \frac{12}{1 + \frac{2}{3} + \frac{1}{3}} = 6\ A$$

Solution ② :

$P_3 = 12W$ $\quad V_2 = 4$ $\quad R_1 = 1$ $\quad E = 12V$

$P_3 = I V_3$

$$P_3 = I V_3$$

$$V_3 = \frac{P_3}{I} = \frac{12}{I}$$

$$E = V_1 + V_2 + V_3$$

$$12 = IR_1 + 4 + \frac{12}{I}$$

$$8 = I + \frac{12}{I}$$

$$\frac{I^2 + 12}{I} = 8 \Rightarrow \frac{I^2 + 12}{I} = 8 \Rightarrow I^2 + 12 = 8I$$

$$I^2 - 8I + 12 = 0$$

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$a=1 \quad b=-8 \quad c=12$

$$I=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{8\pm\sqrt{64-48}}{2}=$$

$I=6A$

$I=2A$

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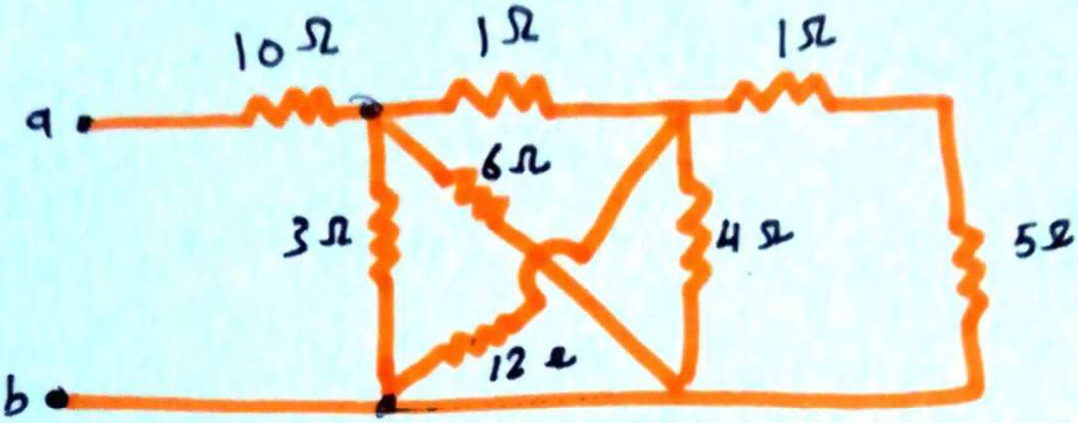
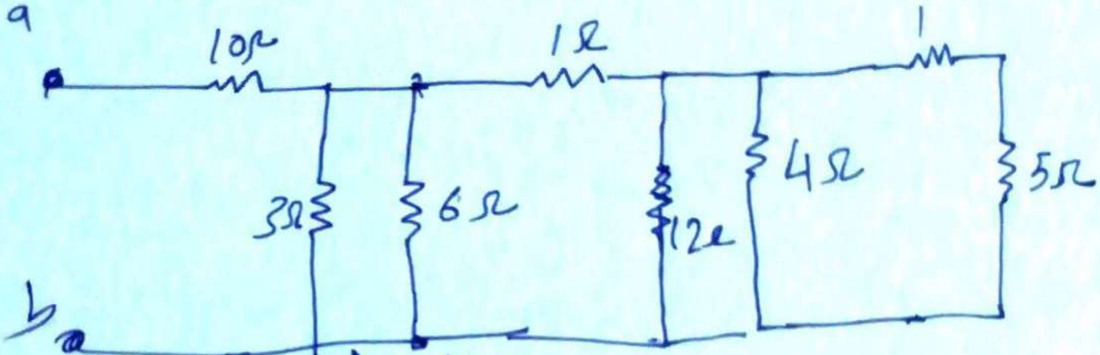
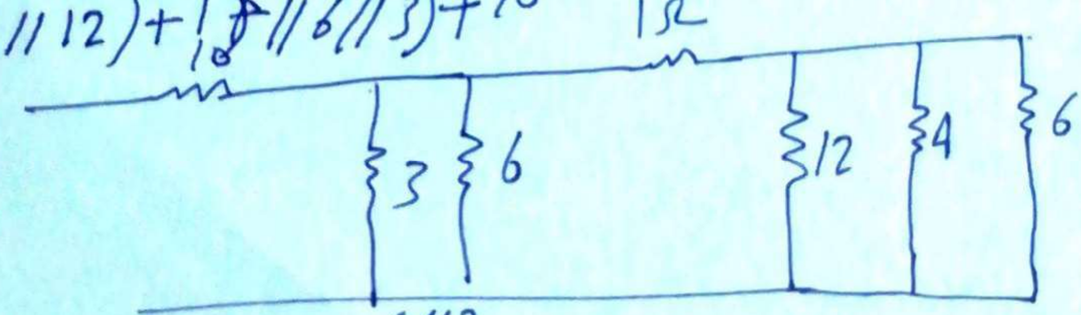
Calculate Rab





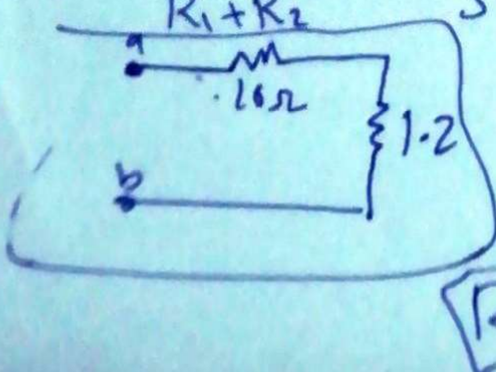
$((5+1)//4//12)+1)//6//3)+10$

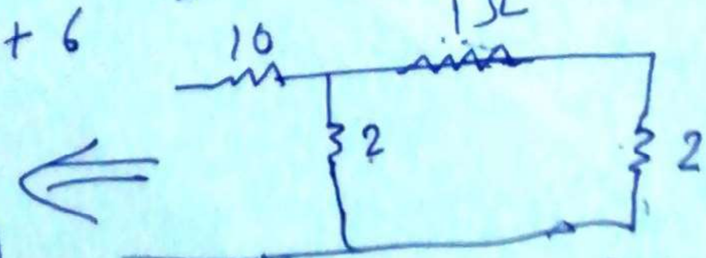


$6//3$ $\qquad$ $12//4//6$

$$R = \frac{R_1 R_2 R_3}{R_2 R_1 + R_2 R_3 + R_3 R_1} = \frac{12*4*6}{12*4+4*6+6*12} = 2$$

$$R = \frac{R_1 R_2}{R_1 + R_2} = \frac{3*6}{3+6} = 2$$





$2+1=3$

$3//2+10$

$R_{ab} = 11.2\,\Omega$

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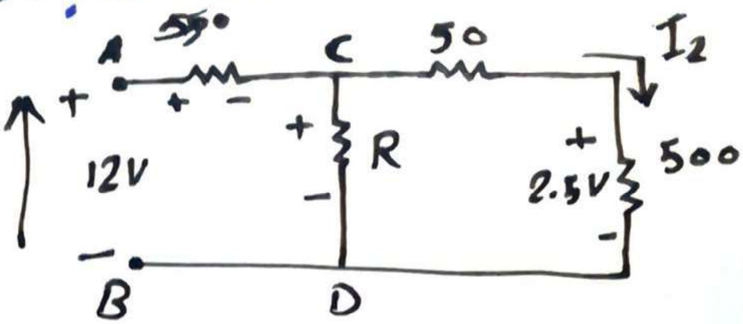
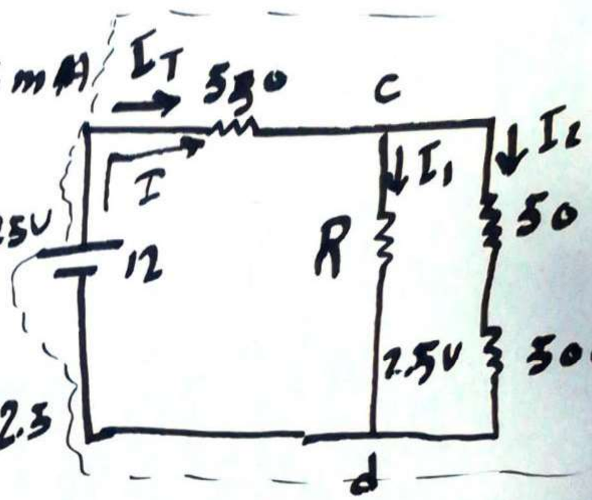
Example: what is the value of the unknown resistor R in the circuit shown, if the value voltage drop across the 500 Ω resistor is 2.5 V? All resistors are in ohms.



Solution:

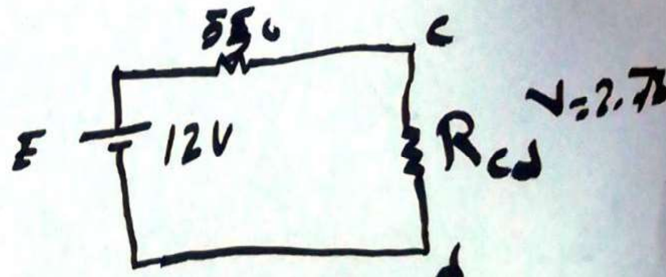


$$I_2 = \frac{V_{500}}{R} = \frac{2.5}{500} = 5*10^{-3} = 5\,mA$$

$$V_{50} = R_{50} * I_2 = 50*5*10^{-3} = 0.25V$$

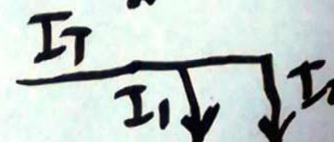
$$V_R = V_{50} + V_{500} = 0.25+2.5$$

$$V_R = 2.75V$$



$$V_{cd} = 2.75$$

$$E = V_{550} + V_{cd} \Rightarrow 12 = V_{550} + 2.75$$



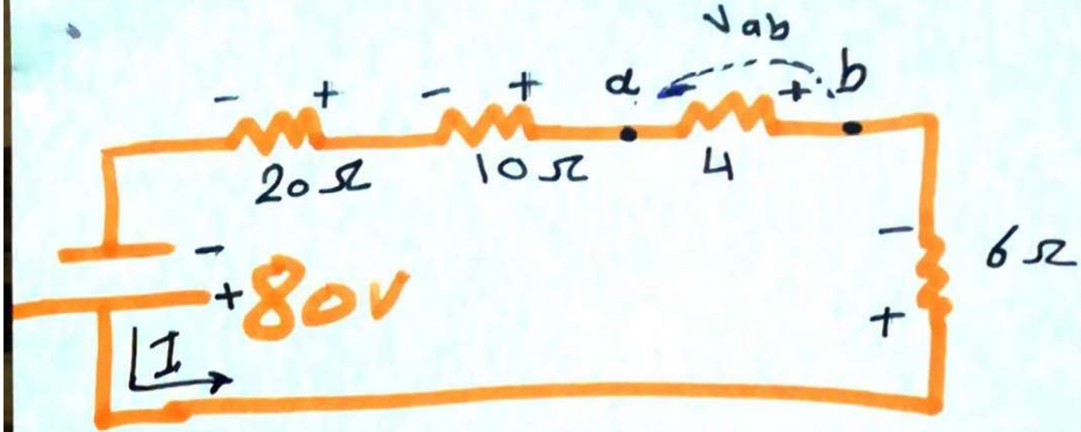
$$V_{5..} = 12 - 2.75 = 9.25V$$

$$I_{500} = I_T = \frac{V_{500}}{R_{500}} = \frac{9.25}{550} = 0.0168\,A$$

$$I_T = I_1 + I_2 \Rightarrow 0.0168 = I_1 + 5*10^{-3}$$

$$I_1 = 0.0118\,A \qquad R = \frac{V}{I_1} = \frac{2.75}{0.0118} = 233\,\Omega$$

Example: For the networks shown, find (V_{ab})



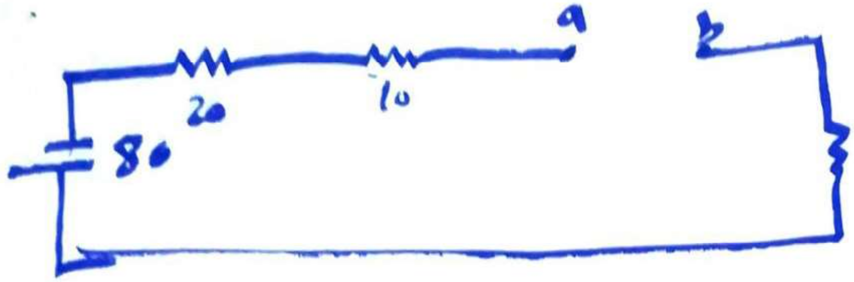
$$\underset{-\ +}{V_{ab}} = \frac{E * R_4}{R_T} = \frac{80 * 4}{40} = 8V$$

$$V_{ab} = V_a - V_b$$

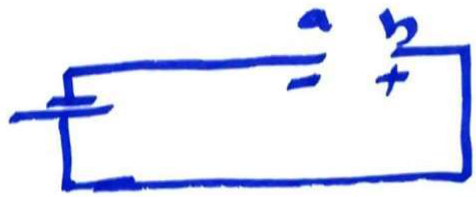
$$V_{ab} = -\frac{80 * 4}{40} = -8V$$

$$V_{ba} = -V_{ab}$$

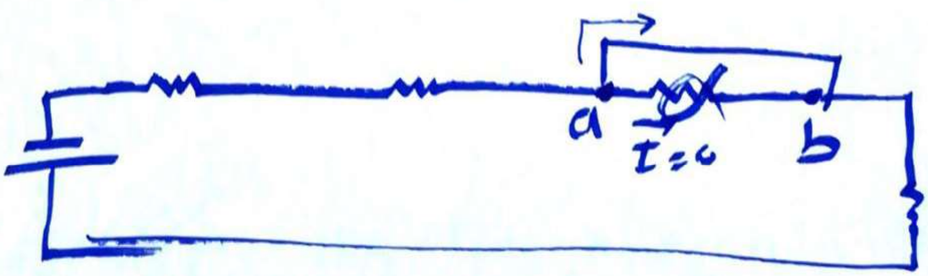
$$V_{ba} = V_b - V_a = -V_{ab} = -(-8) = 8$$



$V_{ab} = ?$



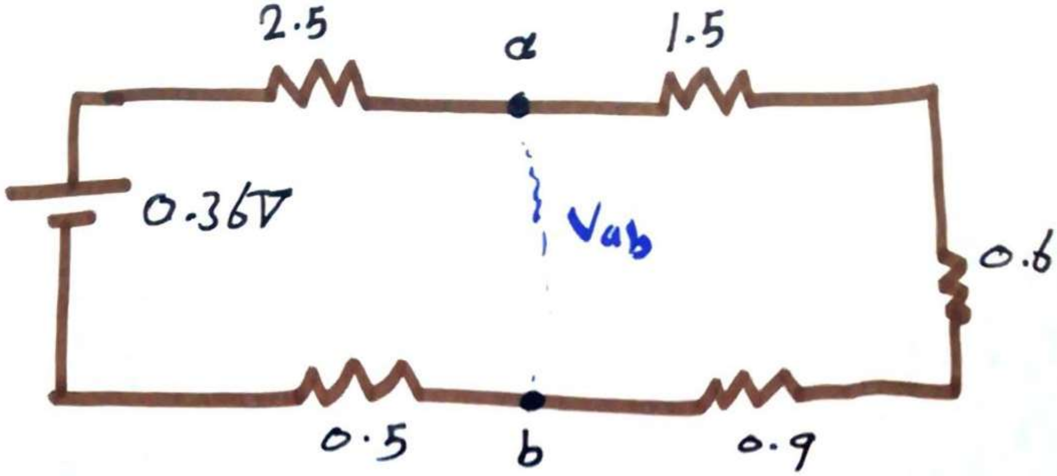
$V_{ab} = E = -80$



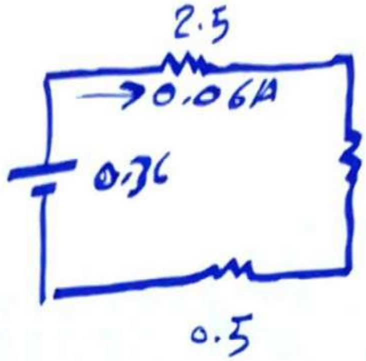
$V_{ab} = 0$

$V = 0$

Example: For the network shown, find V_{ab}



$$V_{ab} = \frac{E\,R_{ab}}{R_T}$$



$R_{ab} = 1.5 + 0.6 + 0.9$

$$V_{ab} = \frac{0.36 * 3}{2.5 + 0.5 + 3} =$$

$V_{ab} = \cancel{1.75}\ \cancel{1.8}\ 0.18\ V$

$$I = \frac{V}{R_T} = \frac{0.36}{6} = 0.06\ A$$

$$V_{ab} = V_{1.5} + V_{0.6} + V_{0.9} = 0.06*1.5 + 0.06*0.6 + 0.9*0.06 =$$

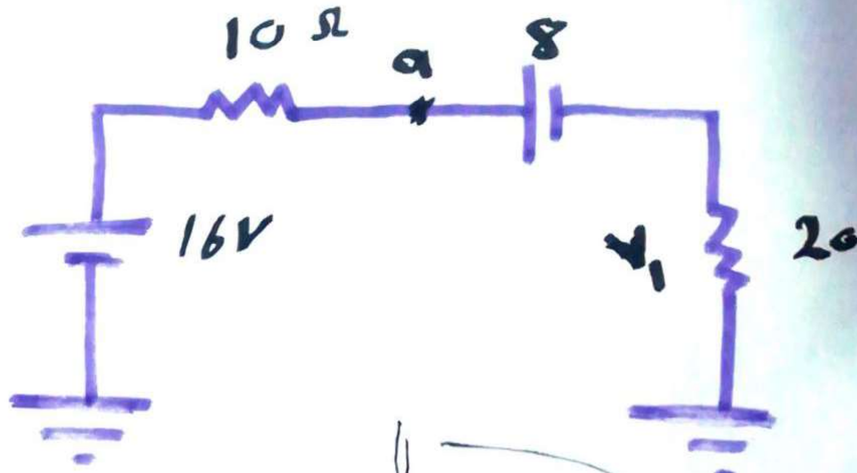
$V_{ab} = 0.18$

$$E = V_{2.5} + V_{0.5} + V_{ab}$$

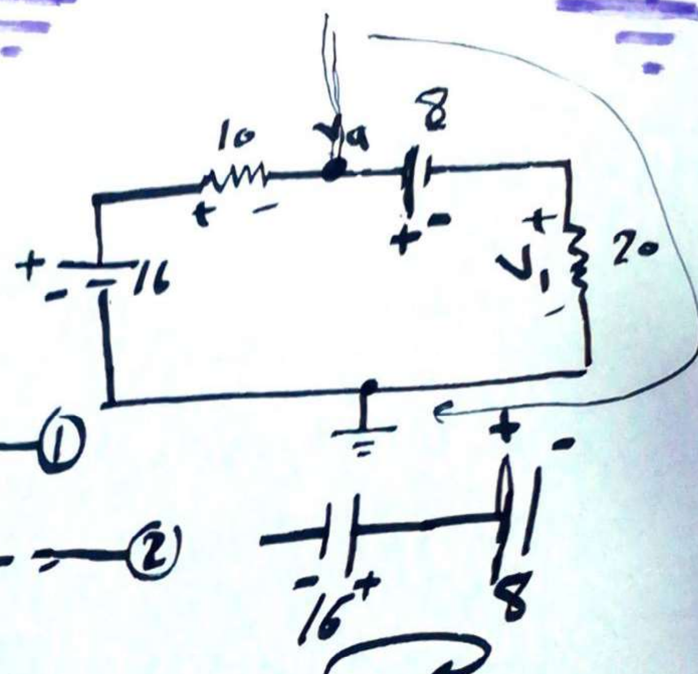
$$0.36 = 0.06*2.5 + 0.06*0.5 + V_{ab}$$

$$V_{ab} = 0.36 - 0.18 = 0.18\ V$$

Example: Determine the voltage V_a and V_1, for the network shown.



Solution:



$V_a = V_a - \cancel{V}0$

$V = V_{10\Omega} + V_a$

$16 = I(10) + V_a$ —(1)

$I = \frac{E}{R_T} = \frac{8}{30}$ —(2)

$+16 - 8 = 8$

$16 = \frac{8}{30} * 10 + V_a$

$V_a = 16 - \frac{8}{3} = \frac{40}{3}$

$V_1 = IR = \frac{8}{30} * 20 = \frac{16}{3}\,V$

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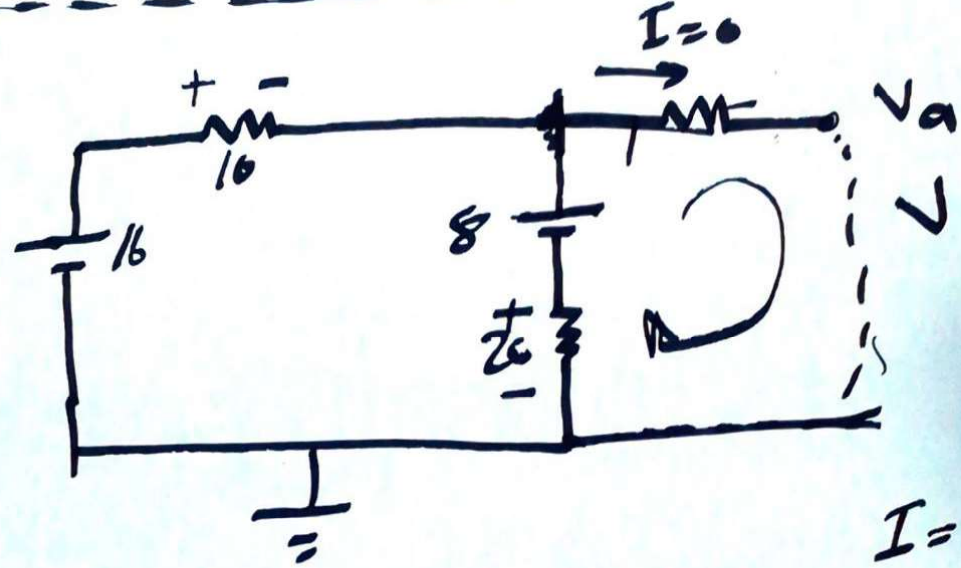
OR

$V_a = V_{8V} + V_{20}$

$V_a = 8 + IR_{20} = 8 + \frac{8}{30}20 = \frac{40}{3}V$



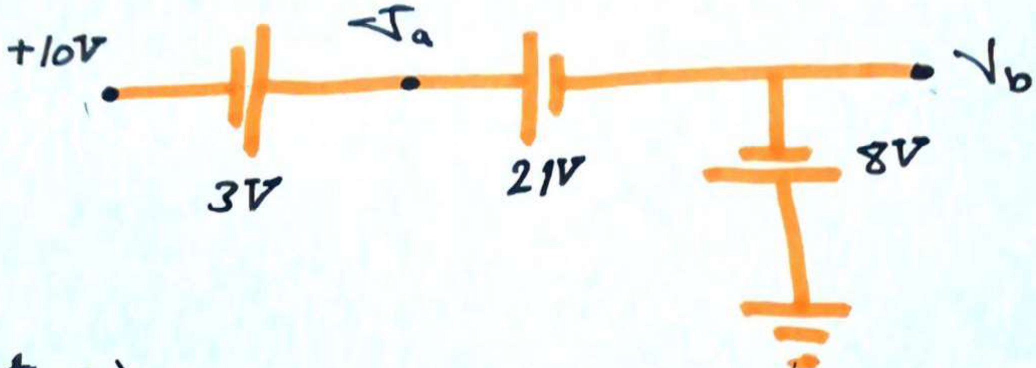
$I = \frac{8}{50}$

$V_a = V_a - 0 = V_a$

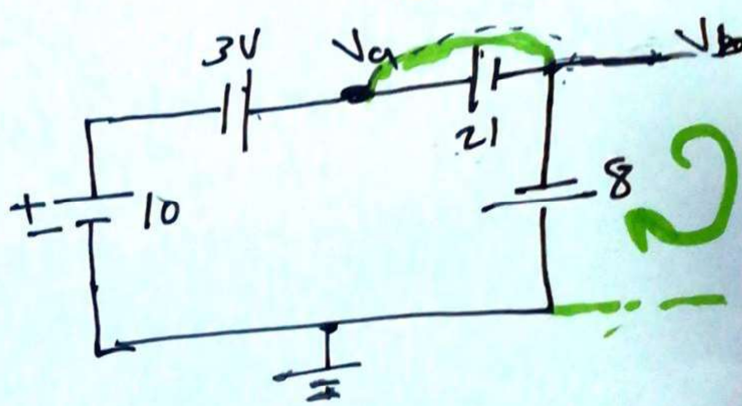
$V_a = 8 + V_{20} = 8 + I*20$

$= 8 + \frac{8}{30} * 20 = \frac{40}{3}$

Example: Determine the Voltage V_{ab} for the network shown.



Solution:



$V_{ab} = V_a - V_b$

$V_{ab} = 21$

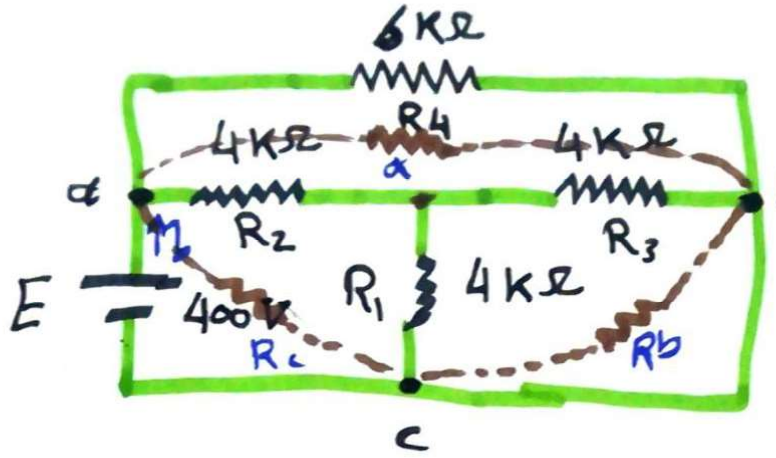
$V_a = V_a - 0$

$V_a = 21 - 8 = 13$

$V_b = V_b - 0 = -8$

$V_{ab} = V_a - V_b = 13 - (-8) = 21\,V$

Example: For the network, find the current I.



Solution:

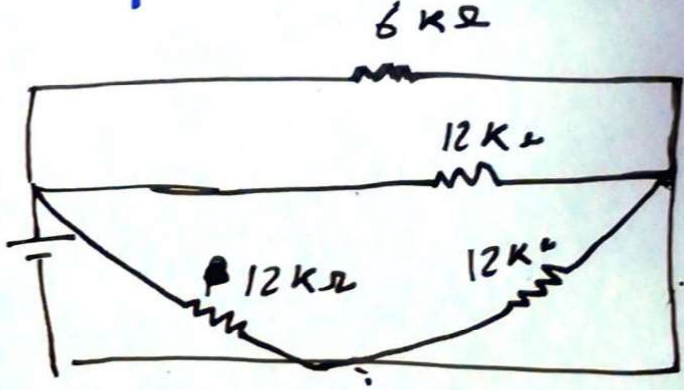
$$R_\alpha = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$

$V = I R_T$

$I = \frac{V}{R_T}$

$R_\Delta = 3 R_Y$

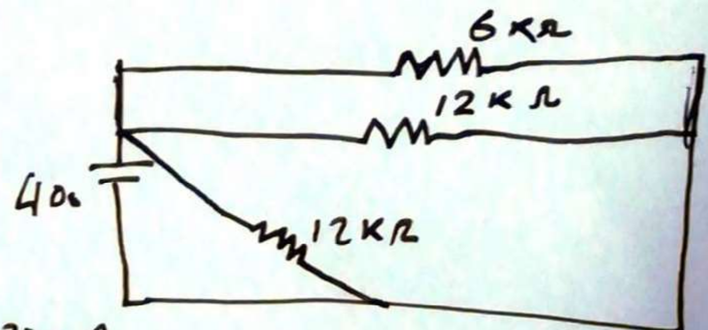
$R_a = R_b = R_c = R_\Delta = 3 R_Y = 3 * 4 * 10^3 = 12 K\Omega$



$R_T = 12K // 12k // 6K$

$$\frac{1}{R_T} = \frac{1}{12*10^3} + \frac{1}{6*10^{3}} + \frac{1}{12*10^{-3}}$$

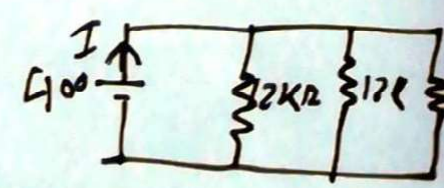
$R_T = 3K\Omega$



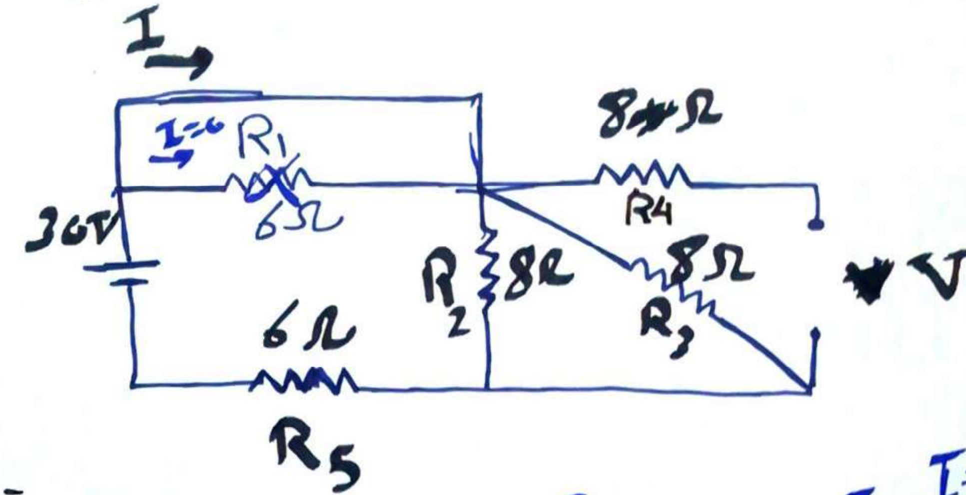
$$I = \frac{400}{3 \times 10^3} = \frac{2}{15}$$

$= 0.1333 A$

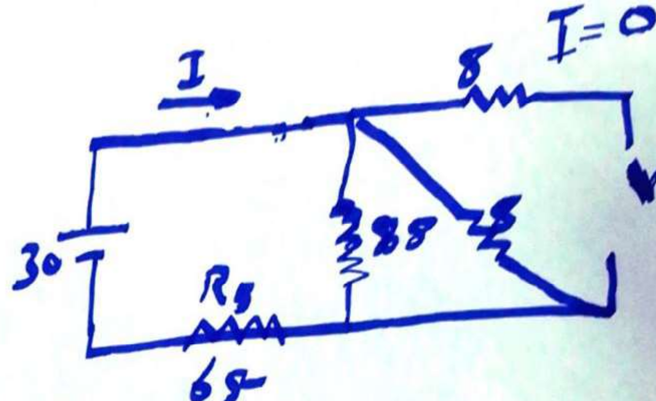
$= 133,333 mA$

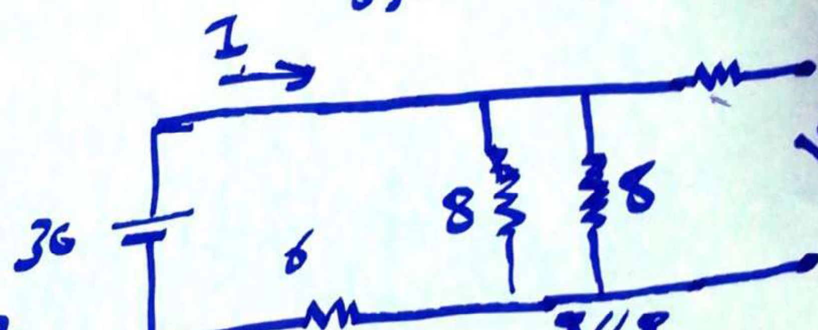


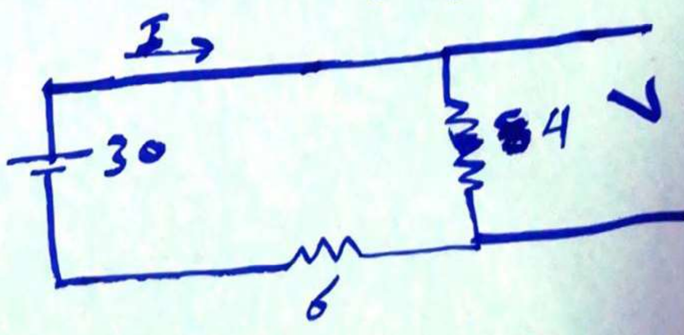
Example: for the circuit shown, calculate I and V.



Solution:







$I = \frac{E}{R_T} =$

$R_T = 4 + 6 = 10\,\Omega$

$I = \frac{30}{10} = 3\,A$

$V = V_{4\Omega} = \frac{30 * 4}{10} = 12\,V$

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Techniques of Circuit Analysis

$a_1 X + b_1 Y = c_1$ $\qquad$ $a_1, b_1, a_2, c_1, c_2 = $ num

$a_2 X + b_2 Y = c_2$ $\qquad$ 2×2

$$X = \frac{\Delta_1}{\Delta} = \frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} =$$

$$\Delta_1 = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix} = c_1 b_2 - b_1 * c_2$$

$$\Delta = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1 b_2 - a_2 b_1$$

$$Y = \frac{\Delta_2}{\Delta} = \frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

$$\Delta_2 = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix} = a_1 c_2 - a_2 c_1$$

$$\Delta = a_1 b_2 - a_2 b_1$$

Third order

$$a_1 x + b_1 y + c_1 z = d_1$$

$$a_2 x + b_2 y + c_2 z \neq d_2$$

$$a_3 x + b_3 y + c_3 z = d_3$$

$$x = \frac{\Delta_1}{\Delta} = \frac{\begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}}$$

$$x = \frac{\Delta_2}{\Delta} = \frac{\begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}}{\Delta}$$

$$z = \frac{\Delta_3}{\Delta} = \frac{\begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}}{\Delta}$$

$\Delta_1 =$

$$\Delta_1 = \left|\begin{matrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{matrix}\right| \begin{matrix} d_1 & b_1 \\ d_2 & b_2 \\ d_3 & b_3 \end{matrix}$$

$$\left[(d_1 * b_2 * c_3) + (b_1 c_2 d_3) + (c_1 d_2 b_3)\right] - \left[c_1 b_2 d_3 + d_1 c_2 b_3 + b_1 d_2 c_3\right]$$

$$\Delta = \left|\begin{matrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{matrix}\right| \begin{matrix} a_1 & b_1 \\ a_2 & b_2 \\ a_3 & b_3 \end{matrix}$$

$$\Delta = \left[a_1 b_2 c_3 + b_1 c_2 a_3 + c_1 a_2 b_3\right] - \left[c_1 b_2 a_3 + a_1 c_2 b_3 + b_1 a_2 c_3\right]$$

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Example: Find x & y.

$$-x + 2y = 3$$

$$3x - 2y = -2$$

$$x = \frac{\Delta_1}{\Delta} = \frac{\begin{vmatrix} 3 & 2 \\ -2 & -2 \end{vmatrix}}{\begin{vmatrix} -1 & 2 \\ 3 & -2 \end{vmatrix}} = \frac{3*(-2) - 2*(-2)}{(-1)*(-2) - 2*3}$$

$$= \frac{-2}{-4} = \boxed{\frac{1}{2}}$$

$$y = \frac{\Delta_2}{\Delta} = \frac{\begin{vmatrix} -1 & 3 \\ 3 & -2 \end{vmatrix}}{\begin{vmatrix} -1 & -2 \\ 3 & (-2) \end{vmatrix}} = \frac{(-1)(-2) - 3*3}{-4} =$$

$$= \frac{-7}{-4} = \boxed{\frac{7}{4}}$$

$$-x + 2y = 3$$

$$-\frac{1}{2} + 2*\frac{7}{4} = -\frac{1}{2} + \frac{7}{2} = \frac{-1+7}{2} = \frac{6}{2} = 3$$

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Example: find X, Y and Z

$$\begin{aligned} X - 2Z &= -1 \\ 3Y + Z &= 2 \\ X + 2Y + 3Z &= 0 \end{aligned} \quad\Rightarrow\quad \begin{aligned} X + 0Y - 2Z &= -1 \\ 0X + 3Y + Z &= 2 \\ X + 2Y + 3Z &= 0 \end{aligned}$$

$$X = \frac{}{} = \frac{\Delta_1}{\Delta}$$

$$X = \frac{\left|\begin{array}{ccc|cc} -1 & 0 & -2 & -1 & 0 \\ 2 & 3 & 1 & 2 & 3 \\ 0 & 2 & 3 & 0 & 2 \end{array}\right.}{\left|\begin{array}{ccc|cc} 1 & 0 & -2 & 1 & 0 \\ 0 & 3 & 1 & 0 & 3 \\ 1 & 2 & 3 & 1 & 2 \end{array}\right.}$$

$$= \frac{(-1*3*3) + \cancel{(0*1*0)}^{0} + (-2*2*2) - \cancel{(-2*3*0)}^{0} - (-1*1*2) - \cancel{(0*2*3)}}{(1*3*3) + \cancel{(0*1*1)}^{0} + \cancel{(-2*0*2)} - (-2*3*1) - (1*1*2) - \cancel{(0*0*3)}}$$

$$X = \frac{-9 + (-8) - (-2)}{(9) + (-6) - (2)} = \boxed{\frac{-15}{13}} =$$

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$Y = \frac{\Delta_2}{\Delta} =$

$\Delta = 13$

$$Y = \frac{\begin{vmatrix}1 & -1 & -2\\ 0 & 2 & 1\\ 1 & 0 & 3\end{vmatrix}\begin{matrix}1 & -1\\ 0 & 2\\ 1 & 0\end{matrix}}{13}$$

$$= \frac{(1*2*3)+(-1*1*1)+(-2*\cancel{0}*0)-(-2*2*1)-(1*1*\cancel{0})-(-1*\cancel{0}*3)}{13}$$

$$Y = \frac{6-1+4}{13} = \boxed{\frac{9}{13}}$$

$$Z = \frac{\Delta_3}{\Delta} = \frac{\begin{vmatrix}1 & 0 & -1\\ 0 & 3 & 2\\ 1 & 2 & 0\end{vmatrix}\begin{matrix}1 & 0\\ 0 & 3\\ 1 & 2\end{matrix}}{13}$$

$$Z = \frac{(1*3*\cancel{0})+(\cancel{0}*2*1)+(-1*\cancel{0}*2)-(-1*3*1)-(1*2*2)-(\cancel{0}*0*0)}{13}$$

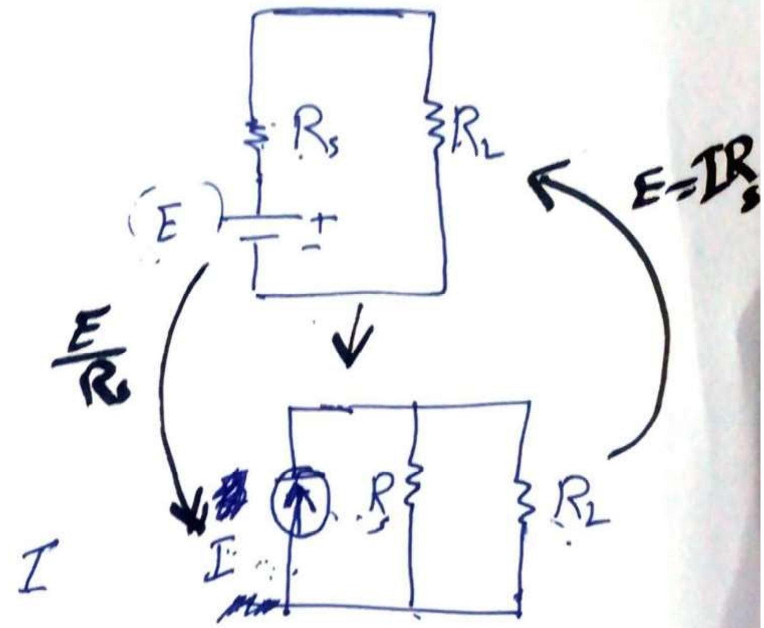
$$\frac{+3-4}{13} = \boxed{\frac{-1}{13}}$$

$$X = -\frac{15}{13} \qquad Y = \frac{9}{13}, \quad Z = \frac{-1}{13}$$

Source Transformation

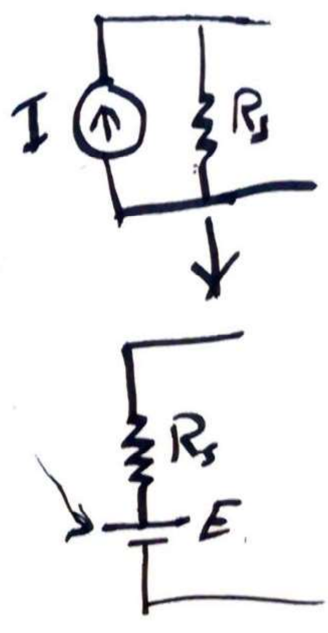
$V \rightarrow I$

$$I = \frac{E}{R_s}$$



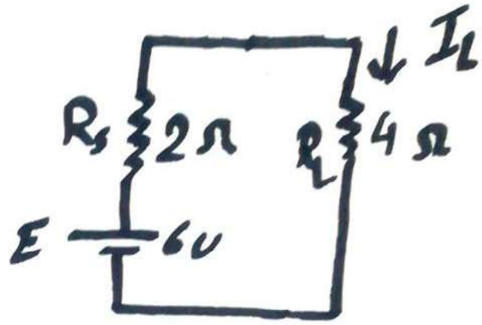
$I \rightarrow V$

$$E = I * R_s$$



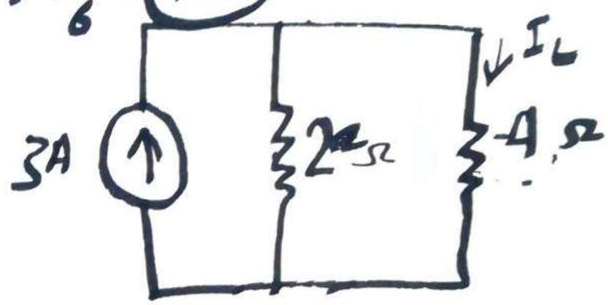
Example: Calculate the current through the load for each switch source.

$$I = \frac{E}{R_T} = \frac{6}{6} = 1A$$



$$I = \frac{E}{R_s} = \frac{6}{2} = 3$$

$$I_L = \frac{I \times R_s}{R_s + R_L} = \frac{3 \times 2}{6} = \frac{6}{6} = 1A$$



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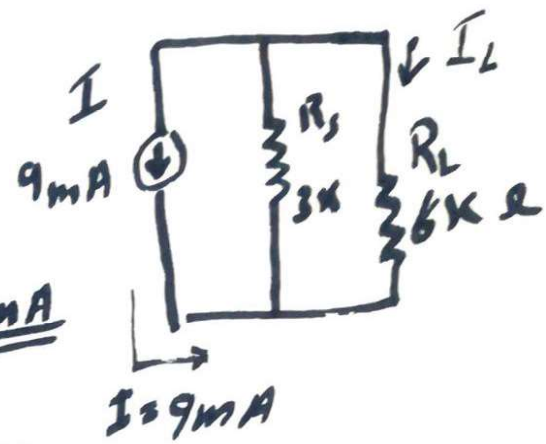
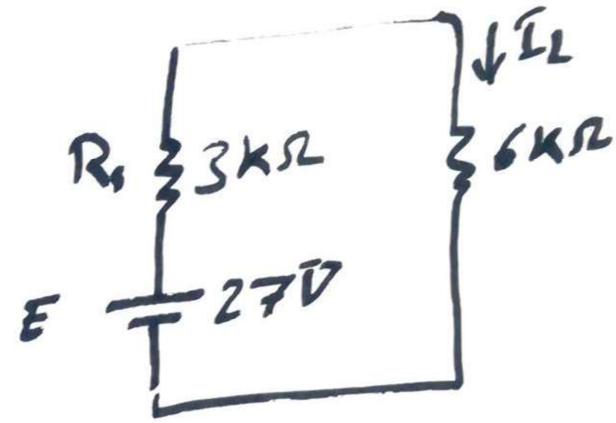
Example: determine I_L for the circuit.

$$I_L = \frac{I * R_s}{R_s + R_L} =$$

$$I = \frac{9*10^{-3} \times 3*10^{3}}{3*10^{3} + 6*10^{3}} = \frac{27}{9*10^{3}} = \underline{\underline{3\,mA}}$$





$$* \; I_L = \frac{27}{9\,K\Omega} = \underline{\underline{3\,mA}}$$

$E = I * R_s =$

$9*10^{-3} \times 3*10^{3} =$

$E = 27V$

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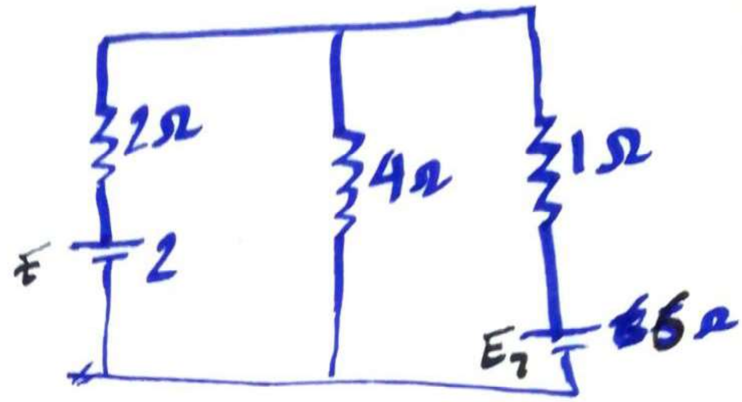
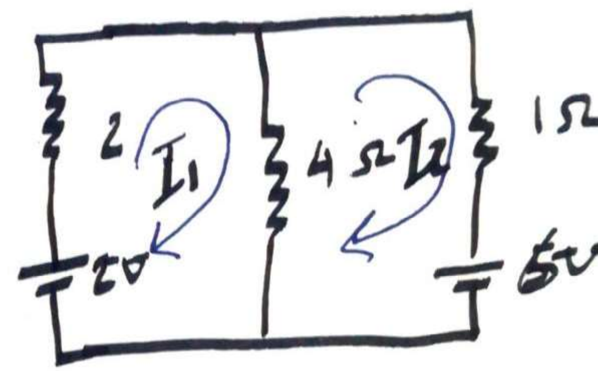
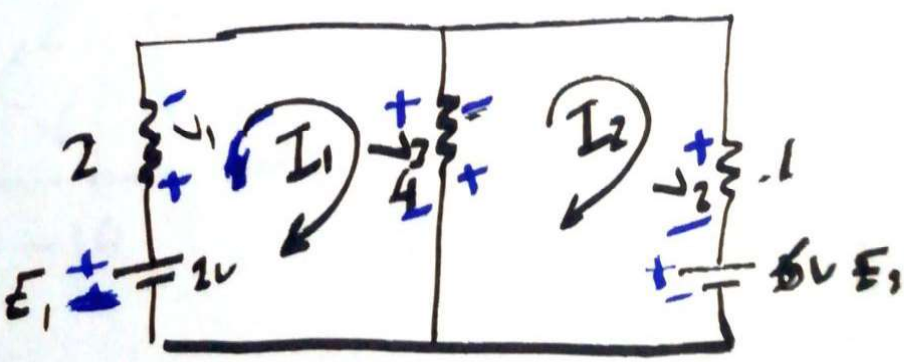
Loop(Mesh) Current Method



Step ①:



Step ②::



Step ③:: KVL

$V = IR$

Loop ①

$$E_1 - V_1 - V_3 = 0 \Rightarrow 2 - 2I_1 - 4(I_1 - I_2) = 0$$

$$2 - 2I_1 - 4I_1 + 4I_2 = 0 \Rightarrow (-6I_1 + 4I_2 = -2) \times -1$$

$$6I_1 - 4I_2 = 2 \quad \text{——} \; (1)$$

Loop 2

$$-E_2 - V_2 - V_3 = 0$$

$$-6 - I_2 - 4(I_2 - I_1) = 0$$

$$-6 - \underline{\underline{I_2}} - \underline{\underline{4I_2}} + 4I_1 = 0$$

$$4I_1 - 5I_2 = 6 \quad \text{—— } (1)$$

$$6I_1 - 4I_2 = 2$$

$$4I_1 - 5I_2 = 6$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{\begin{vmatrix} 2 & -4 \\ 6 & -5 \end{vmatrix}}{\begin{vmatrix} 6 & -4 \\ 4 & -5 \end{vmatrix}} = \frac{(2*-5)-(-4*6)}{(6*-5)-(-4*4)}$$

$$= \frac{-10+24}{-30+16}$$

$$= \frac{14}{-14} = \boxed{-1}$$

$$I_2 = \frac{\Delta_1}{\Delta} = \frac{\begin{vmatrix} 6 & 2 \\ 4 & 6 \end{vmatrix}}{-14} = \frac{6*6 - \cancel{4*6}\ 2*4}{-14}$$

$$= \frac{\overset{36}{\cancel{12}} - \overset{8}{\cancel{24}}}{-14} = \frac{28}{-14} = -2$$

$I_1 = -1$

$I_2 = -2$

$I_3 = I_1 - I_2 = -1-(-2) = 1A$

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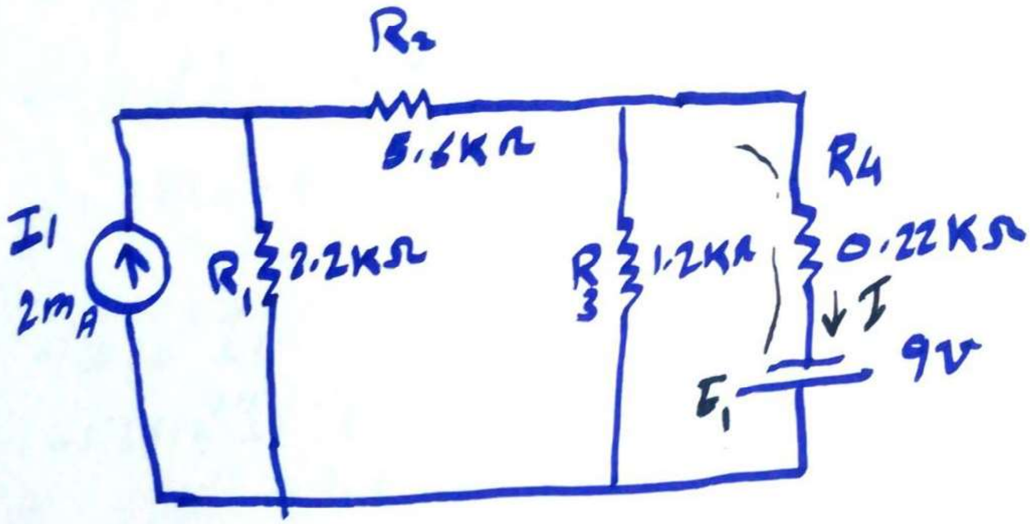
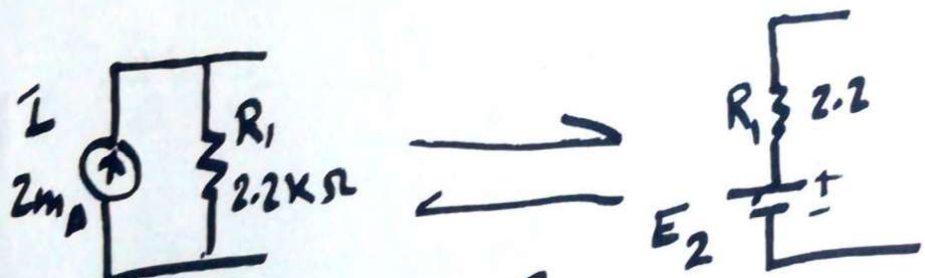
Example: using the mesh analysis, determine the current through 9V for the network.





$E = IR$

$= 2*10^{-3} * 2.2*10^{3}$

$= 4.4V$

Loop ①: KVL

$+E_2 - V_1 - V_2 - V_3 = 0$

$4.4 - 2.2*10^3 I_1 - 5.6*10^3 I_1 - 1.2*10^3 (I_1 - I_2) = 0$

$(-9*10^3 I_1 + 1.2*10^3 I_2 = -4.4) * -1$

$9*10^3 I_1 - 1.2*10^3 I_2 = 4.4$ —— ①

LOOP ②:.

$$+E_1 - V_3 - V_4 = 0$$

$$9 - 1.2*10^3(I_2 - I_1) - 0.22*10^3 I_2 = 0$$

$$(+1.2*10^3 I_1 - 1.42*10^3 I_2 = -9) * -1$$

$$-1.2*10^3 I_1 + 1.42*10^3 I_2 = 9 \quad \text{—— ②}$$

$$9*10^3 I_1 - 1.2*10^3 I_2 = 4.4$$

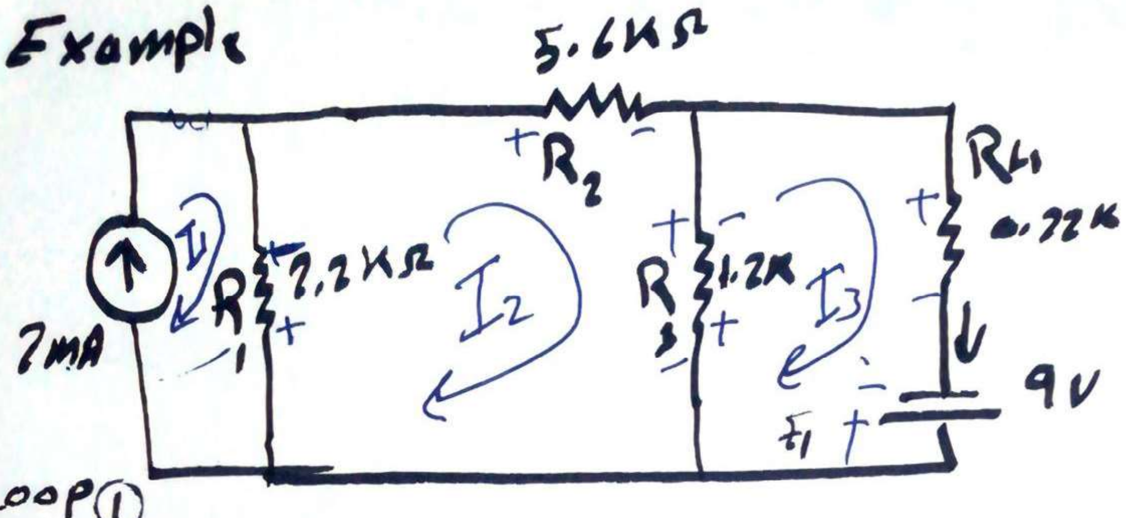
$$-1.2*10^3 I_1 + 1.42*10^3 I_2 = 9$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{\begin{vmatrix} 4.4 & -1.2*10^3 \\ 9 & 1.42*10^3 \end{vmatrix}}{\begin{vmatrix} 9*10^3 & -1.2*10^3 \\ -1.2*10^3 & 1.42*10^3 \end{vmatrix}}$$

$$I_1 = \frac{4.4 * 1.42*10^3 + 1.2*10^3*9}{9*10^3*1.42*10^3 - 1.2*10^3*1.2*10^3} = \frac{17048}{11340000}$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{\begin{vmatrix} 9*10^3 & 4.4 \\ -1.2*10^3 & 9 \end{vmatrix}}{11340000} = \frac{86280}{11340000} = 7.608 \text{ mA}$$

Example



LOOP(1)

$$I_1 = 2mA$$

LOOP(2)

$$-V_1 - V_2 - V_3 = 0$$

$$-2.2*10^3(I_2 - I_1) - 5.6*10^3 I_2 - 1.2*10^3(I_2 - I_3) = 0$$

$$-2.2*10^3 I_2 + 2.2*10^3 I_1 - 5.6*10^3 I_2 - 1.2*10^3 I_2 + 1.2*10^3 I_3 = 0$$

$$4.4 - 9*10^3 I_2 + 1.2 I_3 *10^3 = 0$$

$$(-9*10^3 I_2 + 1.2 I_3 *10^3 = -4.4) * -1$$

$$9*10^3 I_2 - 1.2 I_3 *10^3 = 4.4 \text{ — } (1)$$

LOOP 3:

$$+E_1 - V_3 - V_4 = 0$$

$$9 - 1.2*10^3(I_3 - I_2) - 0.22*10^3 I_3 = 0$$

$$\cancel{-1.42*10^3 I_2} + \cancel{1.2*10^3 I}$$

$$+1.2*10^3 I_2 - 1.42*10^3 I_3 = -9$$

$$-1.2*10^3 I_2 + 1.42*10^3 I_3 = 9 \text{ — } (2)$$

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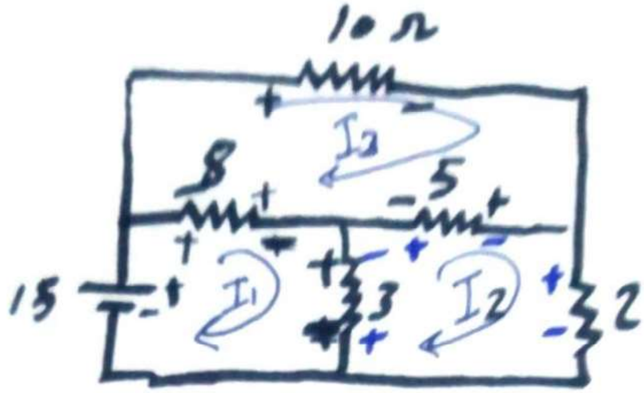
$$9*10^3 I_2 - 1.2\underset{*10^3}{I_3} = 4.4$$

$$-1.2*10^3 I_2 + 1.42*10^3 = 9$$

$$I_3 = \frac{\Delta 3}{\Delta} = \frac{\begin{vmatrix} 9*10^3 & 4.4 \\ -1.2*10^3 & 9 \end{vmatrix}}{\begin{vmatrix} 9*10^3 & -1.2*10^3 \\ -1.2*10^3 & 1.42*10^3 \end{vmatrix}} = \frac{86280}{11340000}$$

Example: find the current through the 10Ω resistor of the network shown.



Loop 1:

$$15 - 8(I_1 - I_3) - 3(I_1 - I_2) = 0$$

$$(-11I_1 + 3I_2 + 8I_3 = -15) \times -1$$

$$11I_1 - 3I_2 - 8I_3 = 15 \quad \text{—} (1)$$

Loop 2:

$$-3(I_2 - I_1) - 5(I_2 - I_3) - 2I_2 = 0$$

$$(3I_1 - 10I_2 + 5I_3 = 0) \times -1$$

$$-3I_1 + 10I_2 - 5I_3 = 0 \quad \text{—} (2)$$

Loop 3:

$$-8(I_3 - I_1) - 10I_3 - 5(I_3 - I_2) = 0$$

$$(8I_1 + 5I_2 - 23I_3 = 0) \times -1$$

$$-8I_1 - 5I_2 + 23I_3 = 0 \quad \text{—} (3)$$

$$11I_1 - 3I_2 - 8I_3 = 15$$

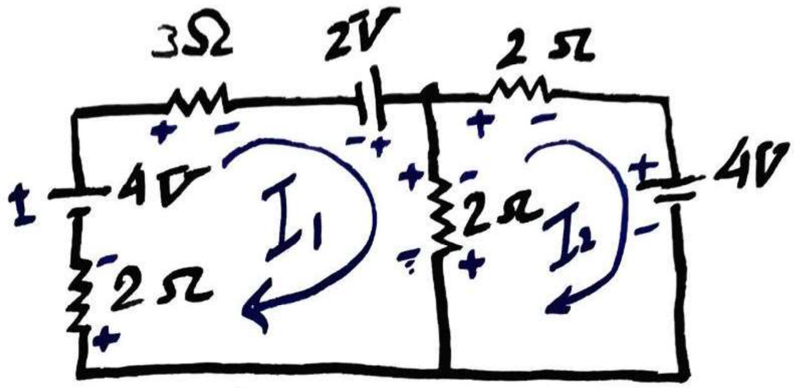
$$-3I_1 + 10I_2 - 5I_3 = 0$$

$$-8I_1 - 5I_2 + 23I_3 = 0$$

$$I_3 = \frac{\left|\begin{array}{ccc|cc} 11 & -3 & 15 & 11 & -3 \\ -3 & 10 & 0 & -3 & 10 \\ -8 & -5 & 0 & -8 & -5 \end{array}\right|}{\left|\begin{array}{ccc|cc} 11 & -3 & -8 & 11 & -3 \\ -3 & 10 & -5 & -3 & 10 \\ -8 & -5 & 23 & -8 & -5 \end{array}\right|} = 1.22\,A$$

$$= \frac{[(11 \times 10 \times 0) + (-3 \times 0 \times -8) + (15 \times -3 \times -5)] - [(15 \times 10 \times -8) + (11 \times 0 \times -5) + (-3 \ldots}{}$$

Example: for the circuit shown. find the current in the 3 Ω resistor using (a) Loop Method
(b) nodal method



Solution:

(a) Loop current method

$V = IR$

Loop 1:

$$4 - I_1 * 3 + 2 - 2(I_1 - I_2) - 2I_1 = 0$$

$$(-I_1(3+2+2) + 2I_2 = -6) * -1$$

$$7I_1 - 2I_2 = 6 \quad —— ①$$

Loop 2:

$$-4 - 2(I_2 - I_1) - 2I_2 = 0$$

$$(2I_1 - 4I_2 = 4) * -1$$

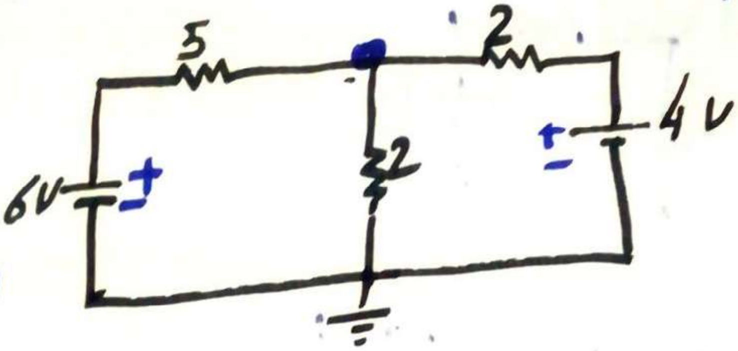
$$-2I_1 + 4I_2 = -4 \quad —— ②$$

$$I_1 = \frac{\begin{vmatrix} 6 & -2 \\ -4 & 4 \end{vmatrix}}{\begin{vmatrix} 7 & -2 \\ -2 & 4 \end{vmatrix}} = \frac{2}{3} A$$

$I_{3\Omega} = \cancel{-\frac{3}{2}} A$

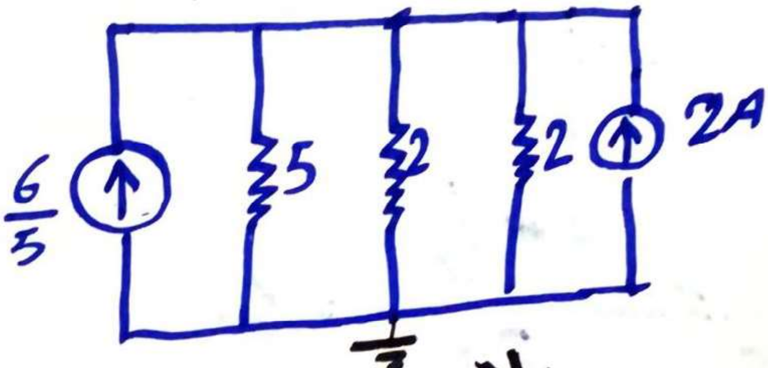
(b) Nodal voltage method

$I_{3\Omega} = ?$



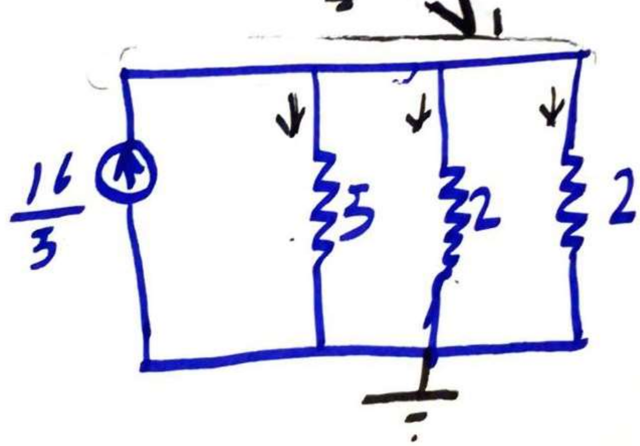
$I_1 = \frac{V}{R} = \frac{6}{5} A$

$I_2 = \frac{V}{R} = \frac{4}{2} = 2A$



$I_T = +\frac{6}{5} + 2 = +\frac{16}{5}$

$I = \frac{V}{R}$



$\frac{16}{5} = \frac{V_1}{5} + \frac{V_1}{2} + \frac{V_1}{2}$

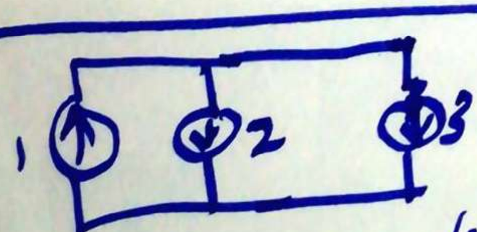
$\frac{16}{5} = V_1\left(\frac{1}{5} + \frac{1}{2} + \frac{1}{2}\right)$

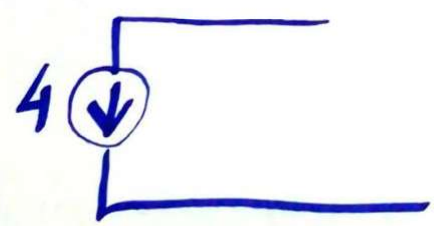
$\boxed{V_1 = \frac{8}{3}}$

$V_3 = \frac{10}{3}$

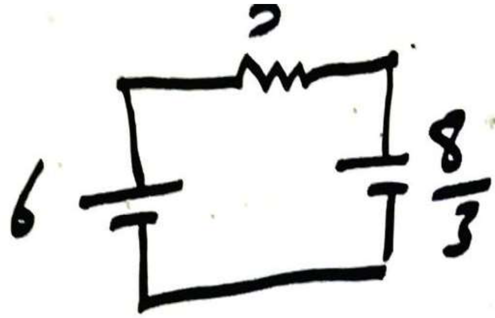
2+3

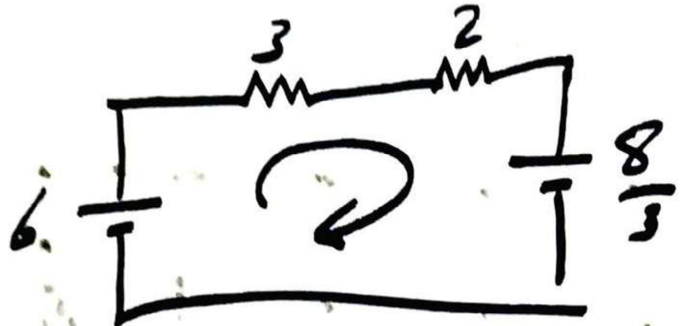
$I = \frac{V_5}{5\Omega} = \frac{\frac{10}{3}}{5} = \frac{2}{3}$





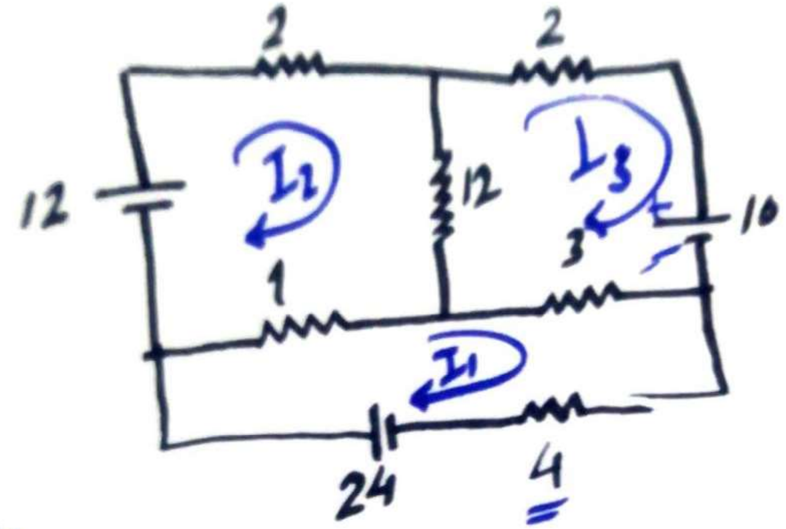
$+1-2-3 = -4 \downarrow$





$$I_{3\Omega} = \frac{6 - \frac{8}{3}}{2+3} = \frac{2}{3}$$

Example:: Determine the current in the 4 Ω resistor for the circuit shown, using loop current method. All resistor value in ohms.



Solution::

Loop 1::

$$24 - 4I_1 - (I_1 - I_2) - 3(I_1 - I_3) = 0$$

$$(-8I_1 + I_2 + 3I_3 = -24) * -1$$

$$8I_1 - I_2 - 3I_3 = 24 \quad \text{—— (1)}$$

Loop 2::

$$12 - 2I_2 - (I_2 - I_1) - 12(I_2 - I_3) = 0$$

$$(I_1 - 15I_2 + 12I_3 = -12) * -1$$

$$-I_1 + 15I_2 - 12I_3 = 12 \quad \text{—— (2)}$$

Loop 3::

$$-10 - 12(I_3 - I_2) - 3(I_3 - I_1) - 2I_3 = 0$$

$$(+3I_1 + 12I_2 - 17I_3 = 10) \cdot -1$$

$$-3I_1 - 12I_2 + 17I_3 = -10 \quad \text{—— (3)}$$

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$$8I_1 - I_2 - 3I_3 = 24$$

$$-I_1 + 15I_2 - 12I_3 = 12$$

$$-3I_1 - 12I_2 + \cancel{15}\,17\,I_3 = -10$$

$$I_1 = \frac{\begin{vmatrix} 24 & -1 & -3 \\ 12 & 15 & -12 \\ -10 & -12 & 17 \end{vmatrix}}{\begin{vmatrix} 8 & \cancel{15} & -3 \\ -1 & 15 & -12 \\ -3 & -12 & \cancel{15}\,17 \end{vmatrix}} = 4.111\ A$$

$I_2 = 2.7\ A$

$I_3 = 2.05\ A$

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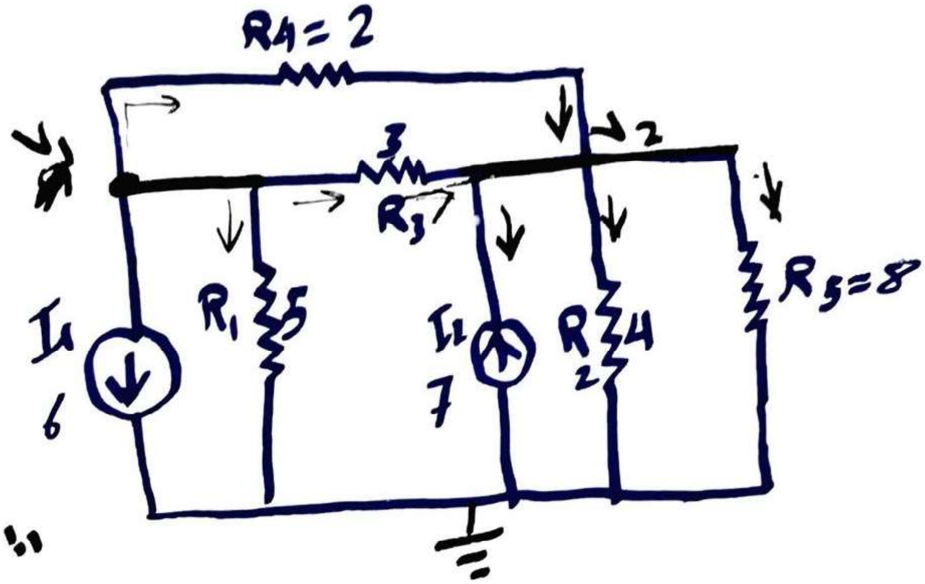
Xi: ⓐ Write the nodal equation for the circuit shown and solve for the nodal voltage.

ⓑ Determine the magnitude and polarity of the voltage across each resistor.



Solution:

①

$$-6 = \frac{v_1}{5} + \frac{v_1 - v_2}{2} + \frac{v_1 - v_2}{3}$$

$$-6 = v_1\left(\frac{1}{5} + \frac{1}{2} + \frac{1}{3}\right) - v_2\left(\frac{1}{2} + \frac{1}{3}\right)$$

$$-6 = v_1 \frac{31}{30} - v_2 \frac{5}{6} \quad — ①$$

②

$$\frac{v_1 - v_2}{2} + \frac{v_1 - v_2}{3} = -7 + \frac{v_2}{4} + \frac{v_2}{8}$$

$$v_1\left(\frac{1}{2} + \frac{1}{3}\right) - v_2\left(\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{8}\right) = -7$$

$$v_1 \frac{5}{6} - v_2 \frac{29}{24} = -7 \quad — ②$$

$$v_1 \frac{31}{36} - v_2 \frac{5}{6} = -6$$

$$v_1 \frac{5}{6} - v_2 \frac{29}{24} = -7$$

$$v_1 = \frac{\begin{vmatrix} -6 & -\frac{5}{6} \\ -7 & -29/24 \end{vmatrix}}{\begin{vmatrix} \frac{31}{36} & -\frac{5}{6} \\ \frac{5}{6} & -\frac{29}{24} \end{vmatrix}} = -2.556V$$

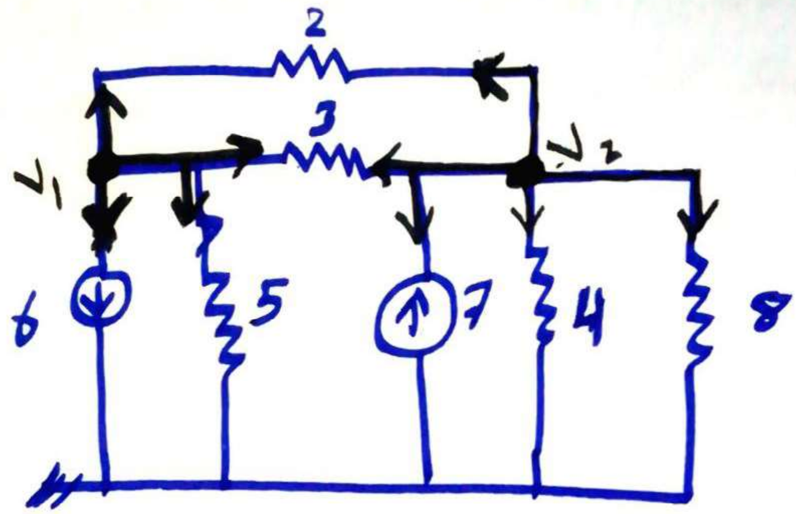
$$v_2 = \frac{\begin{vmatrix} 31/30 & -6 \\ 5/6 & -7 \end{vmatrix}}{\begin{vmatrix} 31/36 & -5/6 \\ 5/6 & -29/24 \end{vmatrix}} = \underline{\underline{4.03\,V}}$$

$$v_{R_1} = v_1 = -2.556V$$

$$v_{R_2} = v_{R_5} = 4.03V$$

$$v_{R_3} = v_{R4} = v_1 - v_2 = (-2.556 - 4.03)$$
$$= +6.586V$$

another Solution



v_1

$$6 + \frac{v_1}{5} + \frac{v_1 - v_2}{2} + \frac{v_1 - v_2}{3} = 0$$

$$v_1\left(\frac{1}{5} + \frac{1}{2} + \frac{1}{3}\right) - v_2\left(\frac{1}{2} + \frac{1}{3}\right) = -6$$

$$\frac{31}{30} v_1 - \frac{5}{6} v_2 = -6 \quad \text{— } 1$$

---

v_2

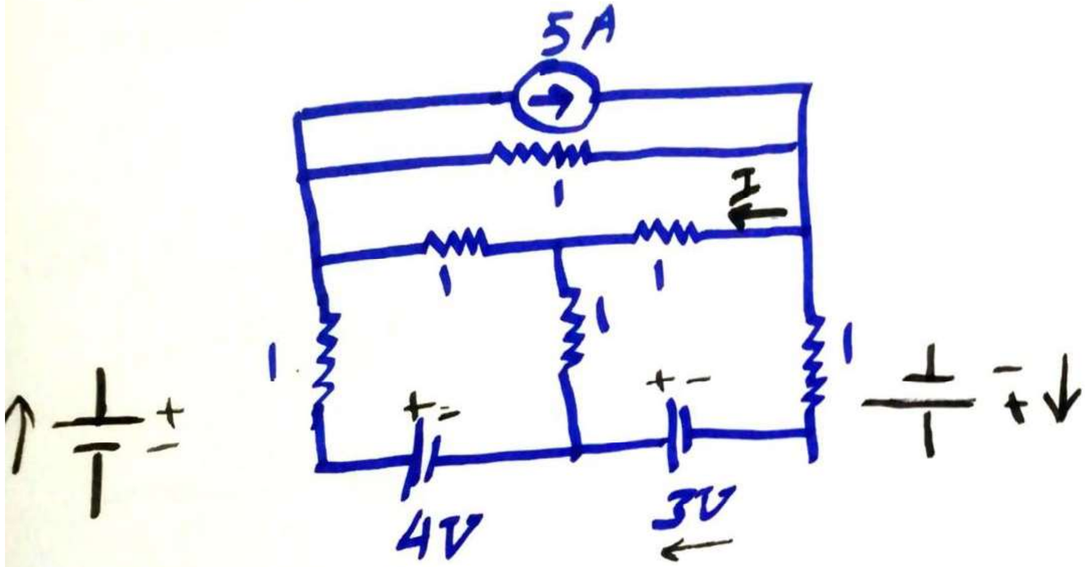
$$-7 + \frac{v_2}{4} + \frac{v_2}{8} + \frac{v_2 - v_1}{2} + \frac{v_2 - v_1}{3} = 0$$

$$-v_1\left(\frac{1}{2} + \frac{1}{3}\right) + v_2\left(\frac{1}{4} + \frac{1}{8} + \frac{1}{2} + \frac{1}{3}\right) = 7$$

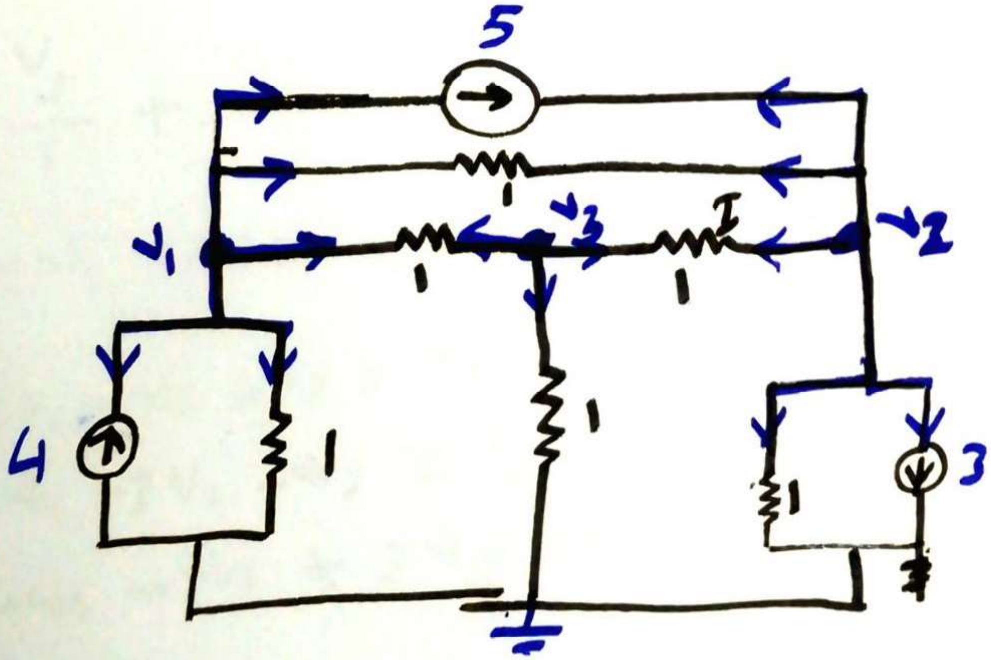
$$\left(-\frac{5}{6} v_1 + \frac{29}{24} v_2 = 7\right) * -1$$

$$\frac{5}{6} v_1 - \frac{29}{24} v_2 = -7 \quad \text{— } 2$$

Example: # Using the nodal voltage method, find the current I in the circuit shown. All resistor are in ohms.



$$I_1 = \frac{V}{R} = \frac{4}{1} = 4\,A \qquad I_2 = \frac{V}{R} = \frac{3}{1} = 3\,A$$



$$\frac{v_2 - v_3}{R} =$$

$$\frac{v_1}{1} - 4 + \frac{v_1 - v_2}{1} + \frac{v_1 - v_3}{1} + 5 = 0$$

$$v_1(1 + 1 + 1) - v_2 - v_3 = -1 \quad \text{—} (1)$$

$$3v_1 - v_2 - v_3 = -1$$

(2)

$$\frac{v_2 - v_3}{1} + \frac{v_2 - v_1}{1} - 5 + \frac{v_2}{1} + 3 = 0$$

$$-v_1 + v_2(1 + 1 + 1) - v_3 = 2$$

$$-v_1 + 3v_2 - v_3 = 2 \quad \text{—} (2)$$

(3)

$$\frac{v_3}{1} + \frac{v_3 - v_1}{1} + \frac{v_3 - v_2}{1} = 0$$

$$-v_1 - v_2 + 3v_3 = 0 \quad \text{—} (3)$$

$$3v_1 - v_2 - v_3 = -1$$

$$-v_1 + 3v_2 - v_3 = 2$$

$$-v_1 - v_2 + 3v_3 = 0$$

$\cancel{v_3 =}$

$$v_1 = 0$$

$$v_2 = \frac{\begin{vmatrix} 3 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & 0 & 3 \end{vmatrix}}{\begin{vmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{vmatrix}} = \frac{3}{4}\,V$$

$$v_3 = \frac{1}{4}\,V$$

$$v_{1\Omega} = v_2 - v_3 = \frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2}\,V$$

$$I = \frac{v_{1\Omega}}{R} = \frac{\frac{1}{2}}{1} = \frac{1}{2}\,A = 0.5A$$

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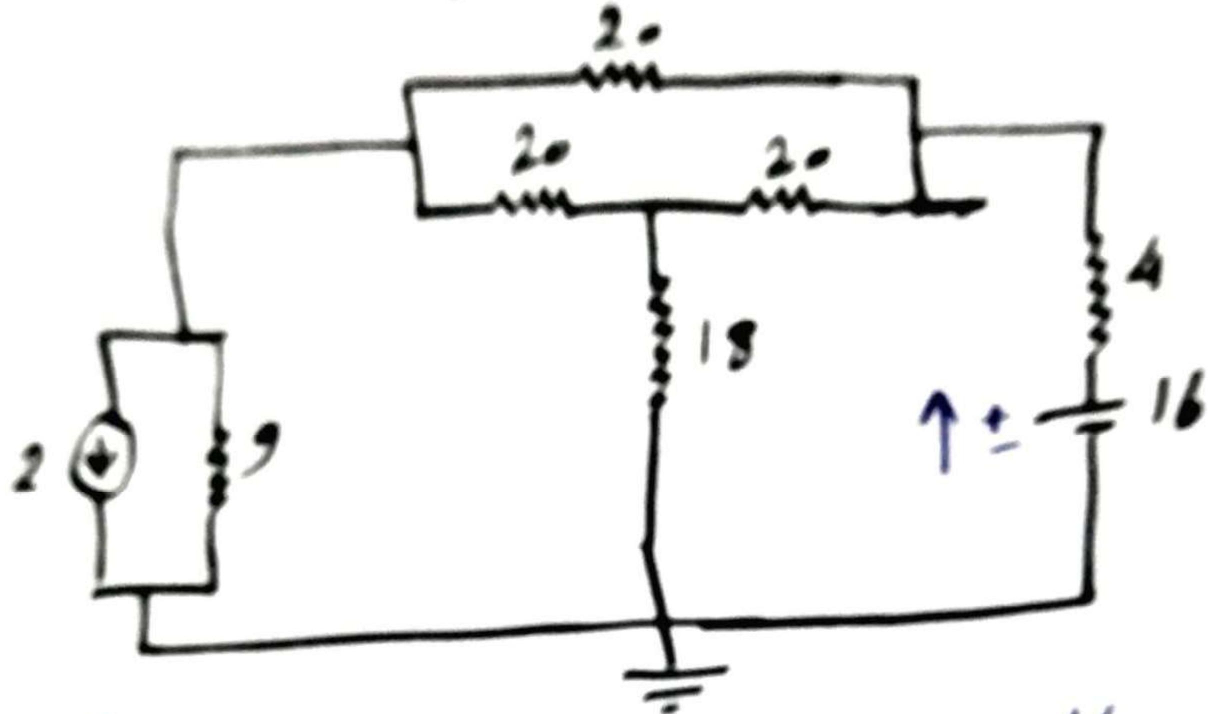
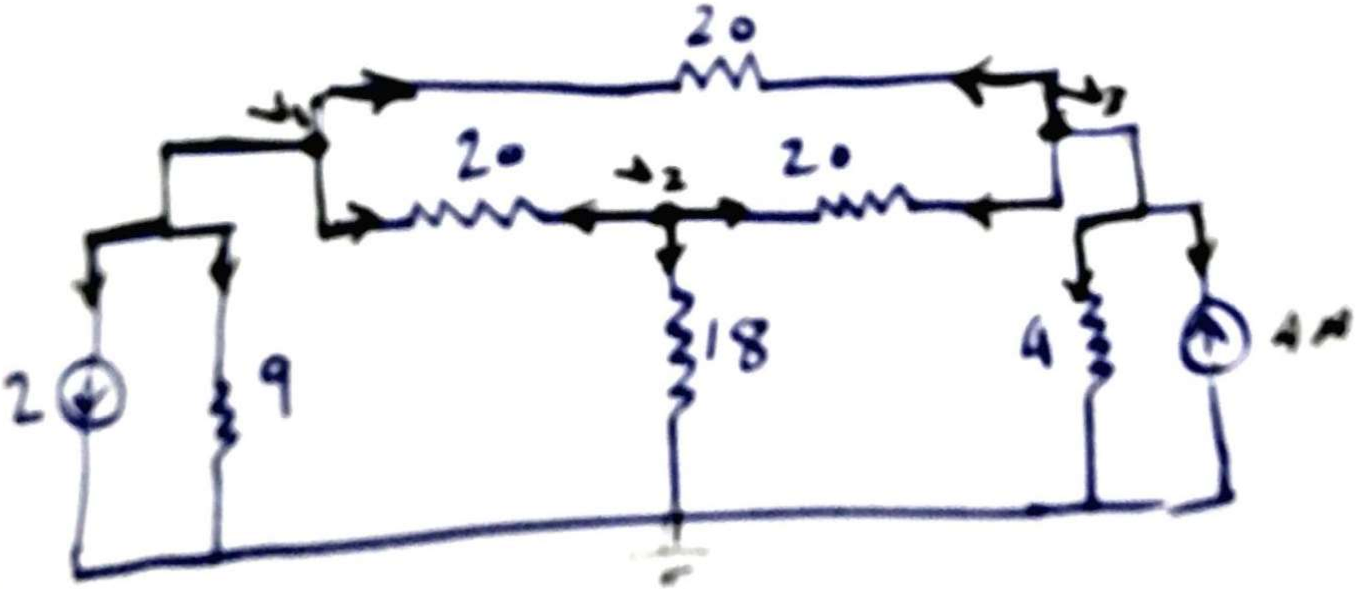
Example: For the network shown in Fig below write the equations and solve for the nodal voltage.



$V \longrightarrow I$ R Series

$I \longrightarrow V$ R Parallel

$$I = \frac{V}{R} = \frac{16}{4} = 4A$$



(1)

$$2 + \frac{v_1}{9} + \frac{v_1 - v_3}{20} + \frac{v_1 - v_2}{20} = 0$$

$$v_1\left(\frac{1}{9} + \frac{1}{20} + \frac{1}{20}\right) - \frac{1}{20}v_2 - \frac{1}{20}v_3 = 0 - 2$$

$$\frac{19}{90}v_1 - \frac{1}{20}v_2 - \frac{1}{20}v_3 = -2 \quad \text{—— (1)}$$

(2)

$$\frac{v_2 - v_1}{20} + \frac{v_2}{18} + \frac{v_2 - v_3}{20} = 0$$

$$-\frac{v_1}{20} + v_2\left(\frac{1}{20} + \frac{1}{18} + \frac{1}{20}\right) - \frac{v_3}{20} = 0$$

$$-\frac{v_1}{20} + \frac{7}{45}v_2 - \frac{v_3}{20} = 0 \quad \text{—— (2)}$$

(3)

$$\frac{v_3 - v_1}{20} + \frac{v_3 - v_2}{20} + \frac{v_3}{4} - 4 = 0$$

$$-\frac{v_1}{20} - \frac{v_2}{20} + v_3\left(\frac{1}{20} + \frac{1}{20} + \frac{1}{4}\right) = 4$$

$$-\frac{v_1}{20} - \frac{v_2}{20} + \frac{7}{20}v_3 = 4 \quad \text{—— (3)}$$

$$\frac{19}{80}v_1 - \frac{1}{20}v_2 - \frac{1}{20}v_3 = -2$$

$$-\frac{v_1}{20} + \frac{7}{45}v_2 - \frac{v_3}{20} = 0$$

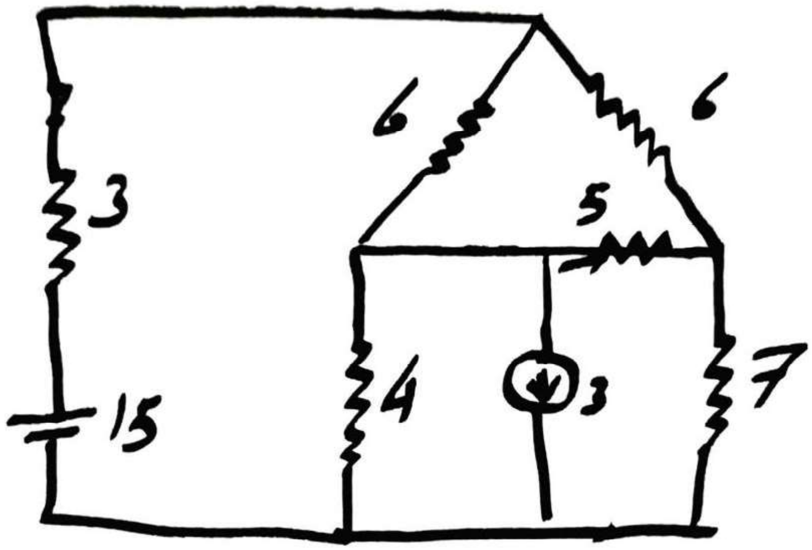
$$-\frac{v_1}{20} - \frac{v_2}{20} + \frac{7}{20}v_3 = 4$$

$v_1 = -6.64\,v$

$v_2 = 1.29\,v$

$v_3 = 10.66\,v$

Example 1: For the circuit shown, write the nodal equation and solve for the nodal voltage



+1

1

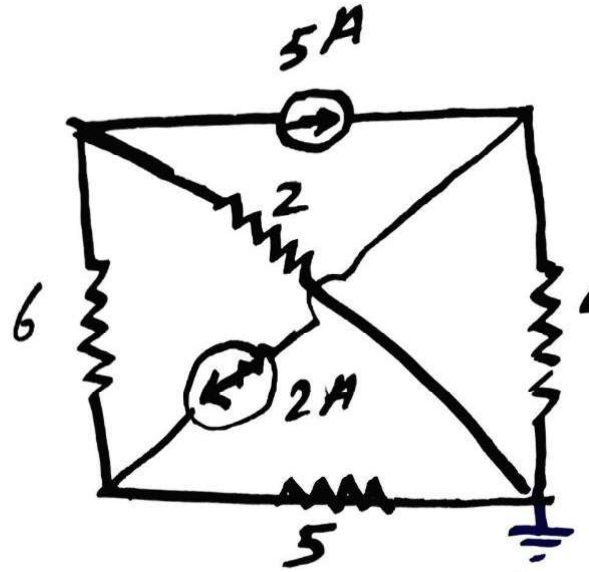
2

3

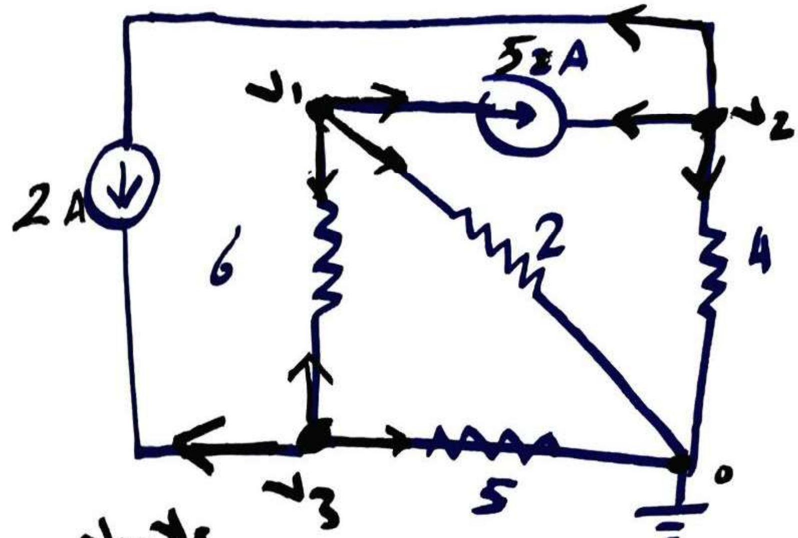
4

5

Example: For the network write the nodal equation and solve for the nodal voltage.



Solution:



①

$$5\cancel{2} + \frac{v_1}{2} + \frac{v_1 - v_3}{6} \cancel{*} = 0 \qquad v_1\left(\frac{1}{2} + \frac{1}{6}\right) - \frac{v_3}{6} = -\cancel{2}5$$

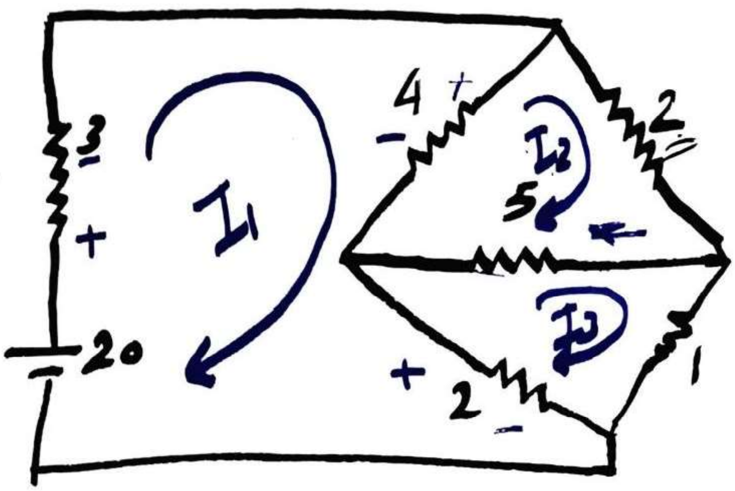
②

$$2 + (-5) + \frac{v_2}{4} = 0 \qquad \frac{v_2}{4} = 3 \Rightarrow v_2 = 12$$

③

$$\frac{v_3 - v_1}{6} + (-2) + \frac{v_3}{5} = 0 \qquad \frac{-v_1}{6} + v_3\left(\frac{1}{6} + \frac{1}{5}\right) = 2$$

Example:. For the bridge network shown, using the Loop current method find the current in R_5.



Solution:

Loop 1:

$$20 - 3I_1 - 4(I_1 - I_2) - 2(I_1 - I_3) = 0$$

$$(-3-4-2)I_1 + 4I_2 + 2I_3 = -20$$

$$9I_1 - 4I_2 - 2I_3 = 20 \quad \text{— ①}$$

Loop 2:

$$-4(I_2 - I_1) - 2(I_2) - 5(I_2 - I_3) = 0$$

$$(+4I_1 - 11I_2 + 5I_3 = 0) \text{ — ② } \times -1$$

$$-4I_1 + 11I_2 - 5I_3 = 0 \quad \text{— ②}$$

Loop 3

$$-2(I_3 - I_1) - 5(I_3 - I_2) - I_3 = 0$$

$$-2I_1 - 5I_2 + 8I_3 = 0 \quad \text{— ③}$$

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$9I_1 - 4I_2 - 2I_3 = 20$

$-4I_1 + 11I_2 - 5I_3 = 0$

$-2I_1 - 5I_2 + 8I_3 = 0$

$I_1 = 4$

$$I_2 = \frac{\begin{vmatrix} 9 & 20 & -2 \\ -4 & 0 & -5 \\ -2 & 0 & 8 \end{vmatrix}}{\begin{vmatrix} 9 & -4 & -2 \\ -4 & 11 & -5 \\ -2 & -5 & 8 \end{vmatrix}} = \frac{8}{3} = 2.66\ A$$

$$I_3 = \frac{8}{3} = 2.66\ A$$

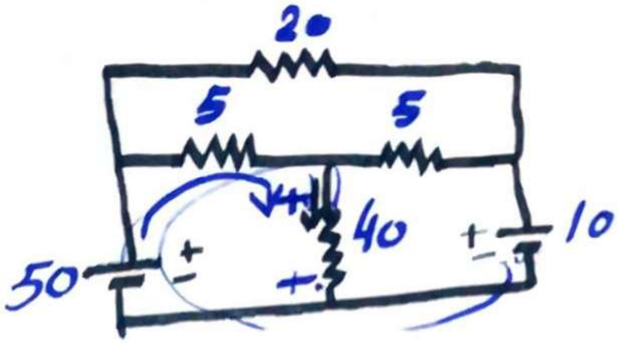
$$I_5 = I_2 - I_3 = \frac{8}{3} - \frac{8}{3} = 0\ A$$

Nodal = ??

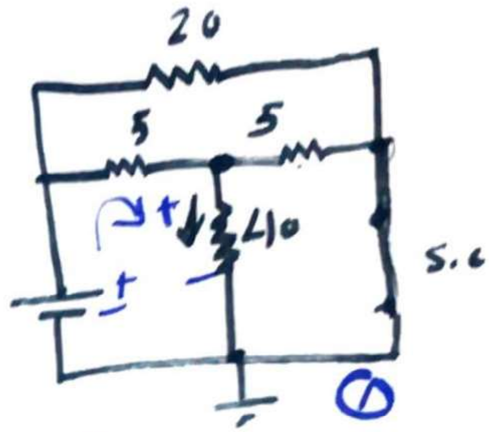
+1	1
+1	2
+1	3

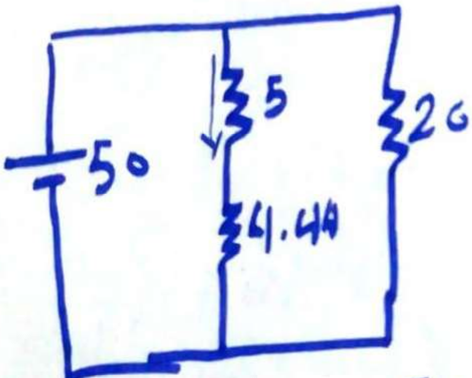
Example. Use the superposition theorem.

Find the current in the 40 Ω

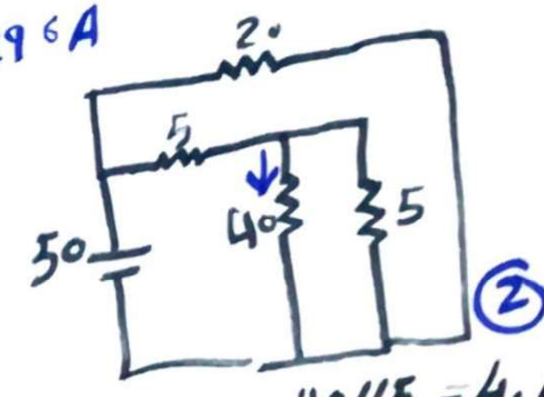


Solution:

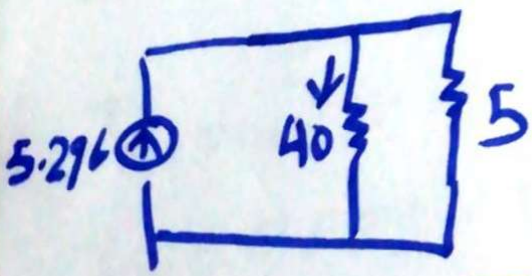


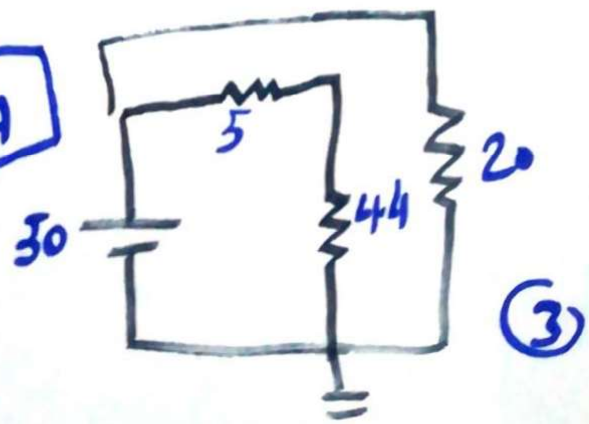


$$I = \frac{V}{R} = \frac{50}{5+4.44} = 5.296\,A$$



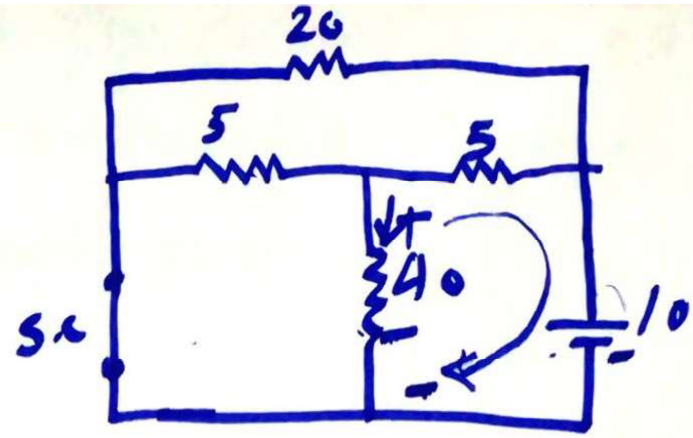
$40 // 5 = 4.44$



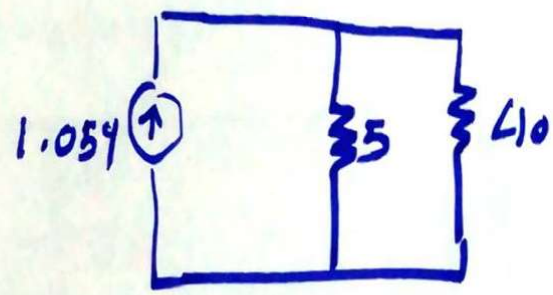


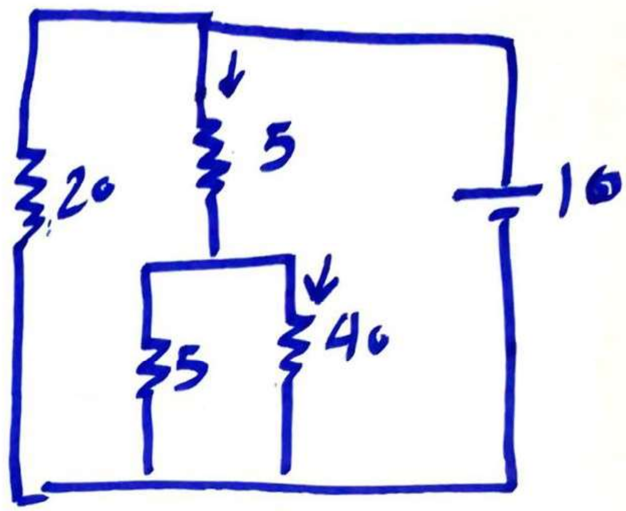
$$I_{40} = \frac{5.296 * 5}{5+40} \boxed{= 0.589A}$$

$$I = \frac{V}{R_T} = \frac{10}{5+4.44}$$



$$I = 1.059A$$





$$\bar{\bar{I}}_{40} = 1.059 * \frac{5}{5+40}$$

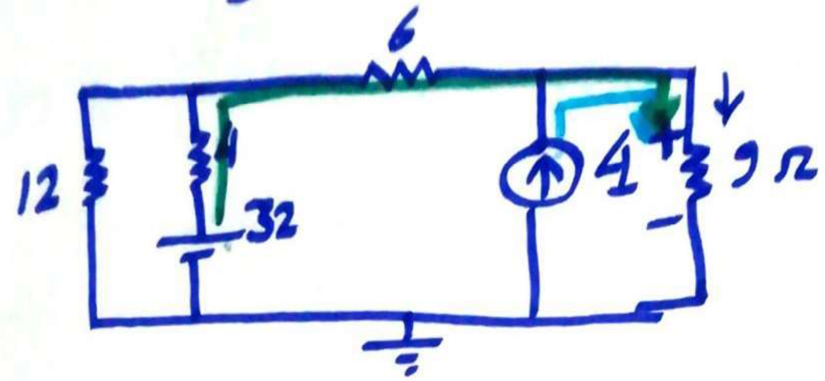
$$5 // 40 = 4.44$$

$$\boxed{\bar{\bar{I}}_{40} = 0.118\ A}$$

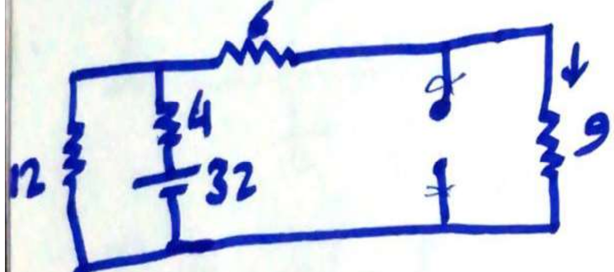
$$I_{40} = \bar{I}_4 + \bar{\bar{I}}_4 = \bar{I}_4 - (-\bar{\bar{I}}_4) = \bar{I}_4 + \bar{\bar{I}}_4$$

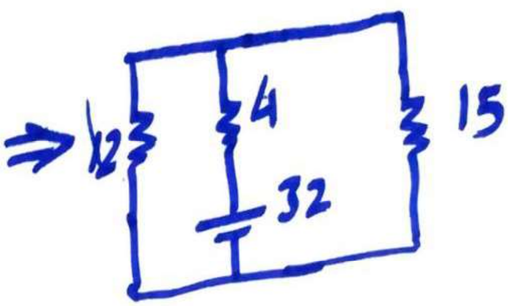
$$I_{40} = 0.589 + 0.118 = 0.707A$$

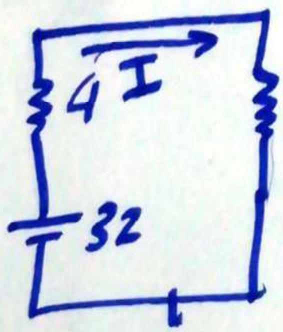
Example :: Find the current in the 9Ω using supperposition theorem.



Solution ::



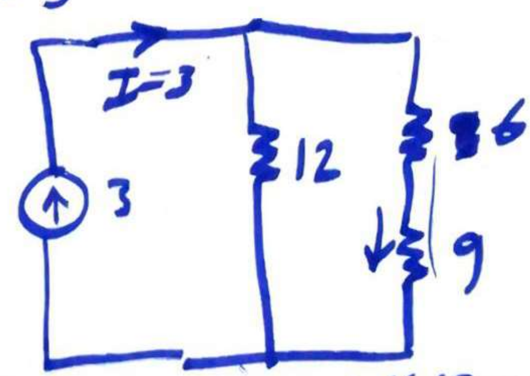




$15 // 12 = \frac{20}{3}$

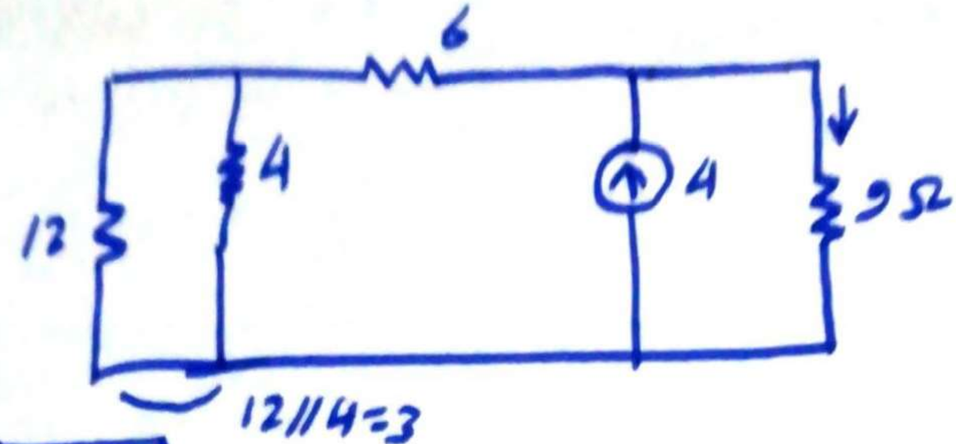
$I = \frac{V}{R_T} = \frac{32}{4+\frac{20}{3}} = 3\,A$

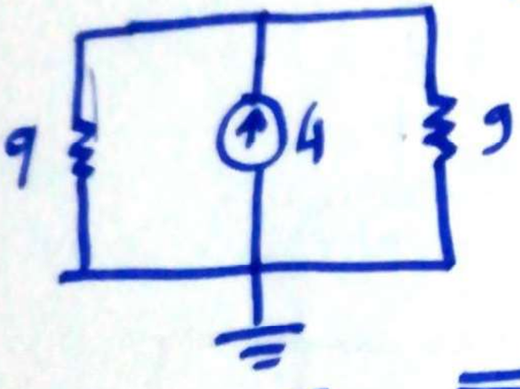
$I_9 = \frac{3 \times 12}{12+(6+9)} = \boxed{\frac{4}{3}A}$



$(9+6) // 12 = \frac{20}{3}$

(2)

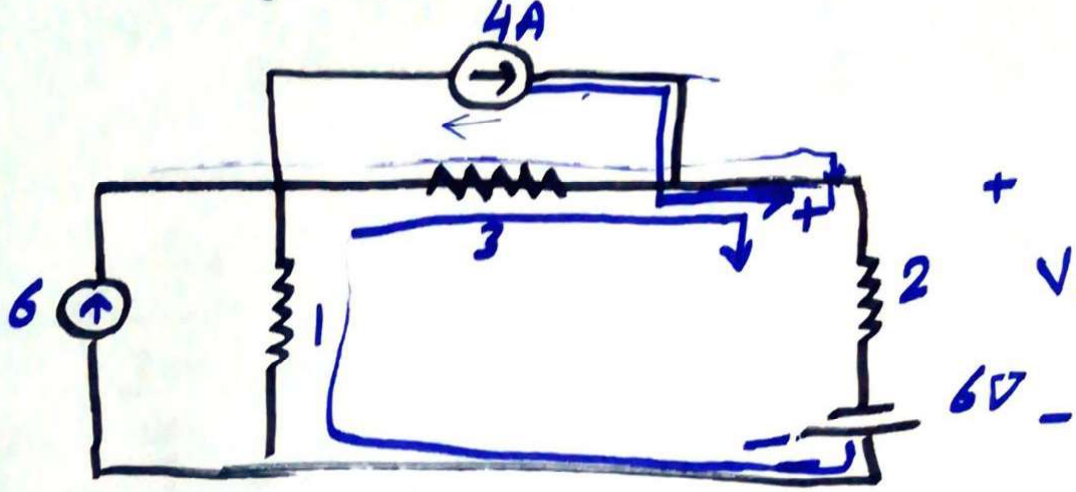


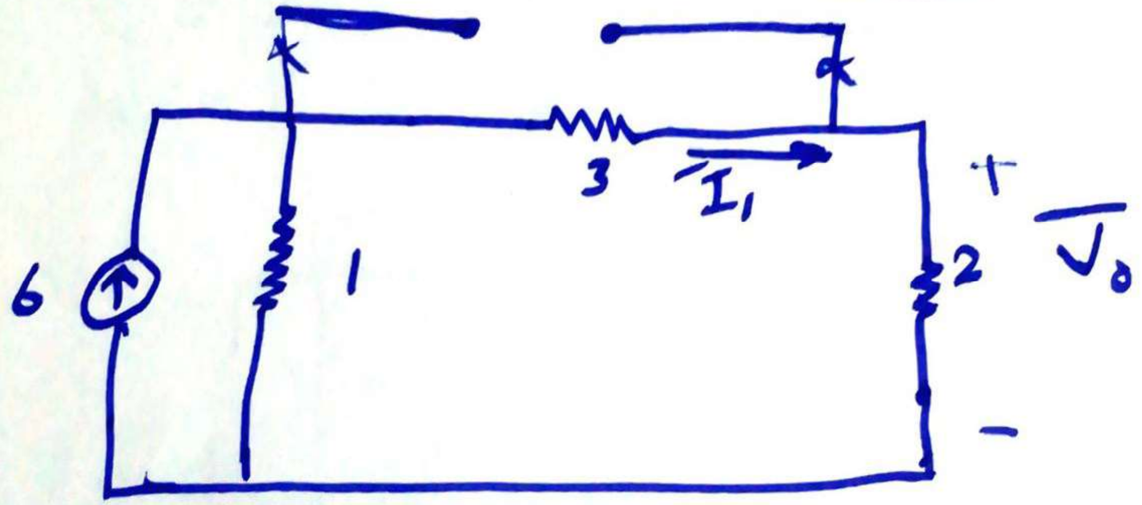


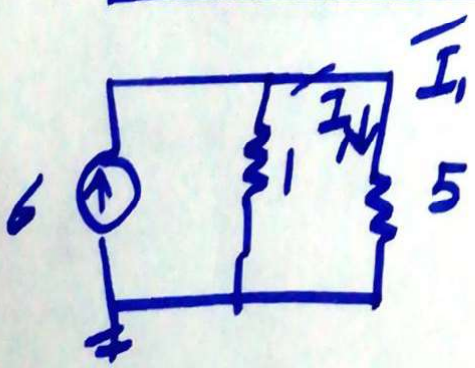
$$\bar{\bar{I}}_9 = \frac{4 * 9}{9+9} = 2A$$

$$I_{9\Omega} = +\bar{I}_9 + \bar{\bar{I}}_9 = \frac{4}{3} + 2 = \frac{10}{3} A$$

Example:: find the Value of the output Voltage V_o using superposition.



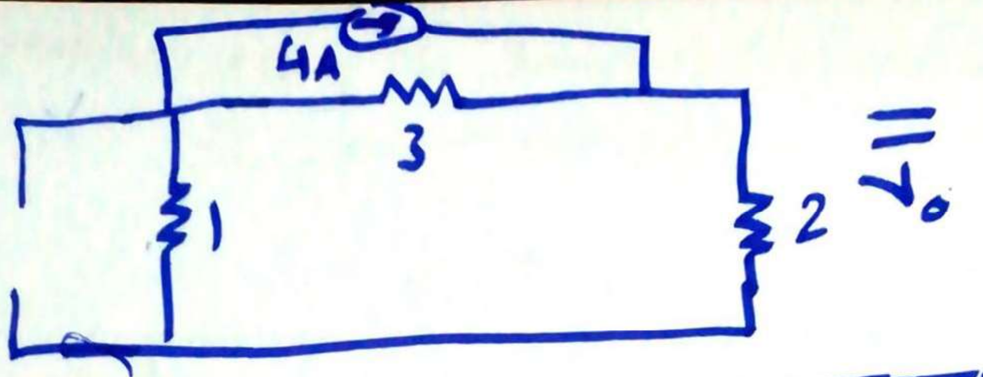




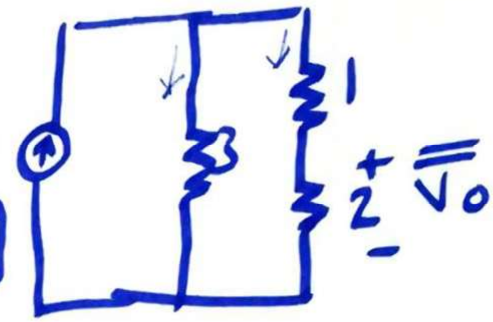
$$\bar{I}_1 = \frac{6*1}{1+5}$$

$$\bar{I}_1 = \frac{6*1}{1+2+3} = 1A$$

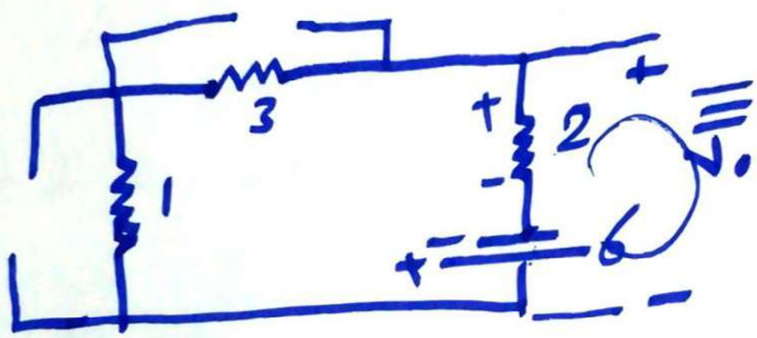
$$\bar{V}_o = \bar{I}_1 * 2 = 1*2 = \boxed{2V}$$



$$\overline{\overline{I}}_1 = \frac{4 * 3}{3+1+2} = 2A$$



$$\overline{\overline{V}}_o = \overline{\overline{I}}_1 * 2 = 2 * 2 = \boxed{4V}$$



$$V_{2\Omega} = \frac{6 * 2}{1+3+2} = 2V$$

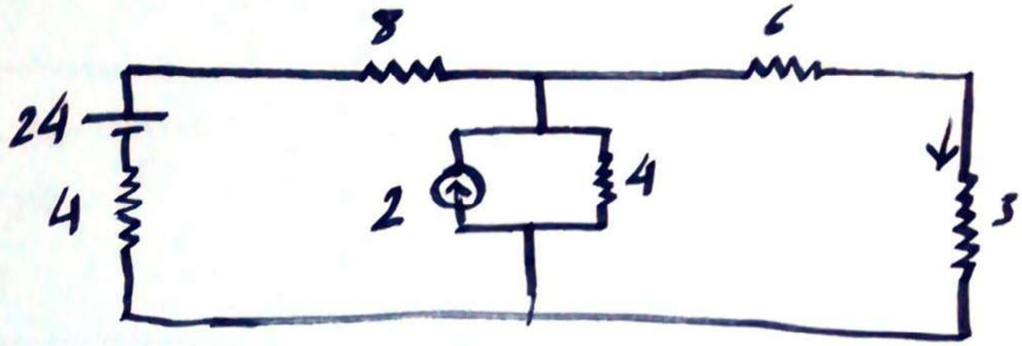
$$\overline{\overline{\overline{V}}}_o = 2 - 6 = \boxed{-4}$$

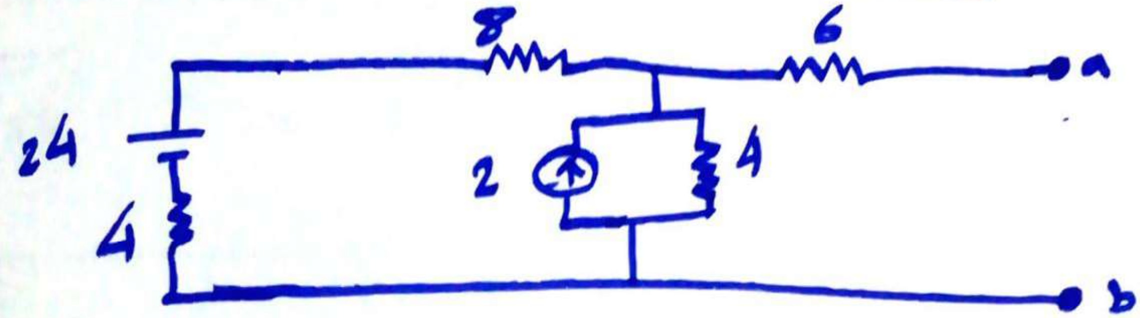
$\boxed{\overline{V}_o = 2V}$ $\boxed{\overline{\overline{V}}_o = 4V}$ $\boxed{\overline{\overline{\overline{V}}}_o = -4}$

$$V_o = \overline{V}_o + \overline{\overline{V}}_o + \overline{\overline{\overline{V}}}_o$$

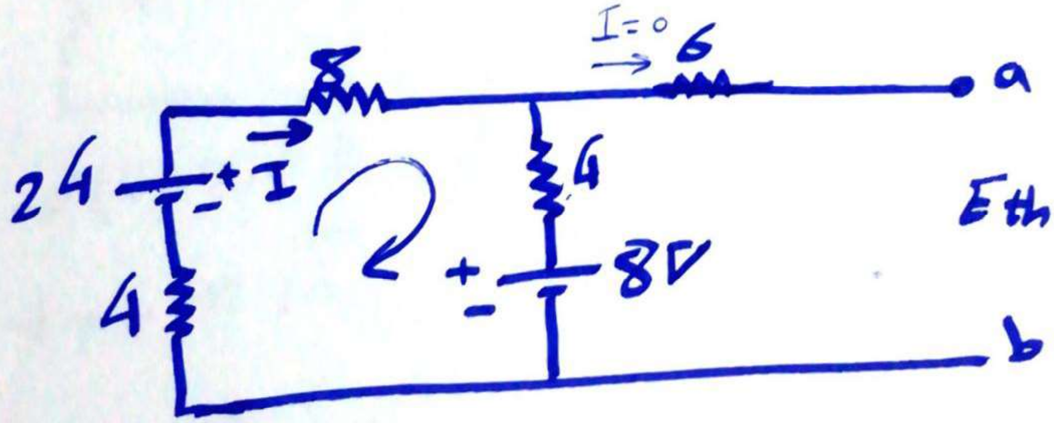
$$V_o = 2V + 4V + (-4) = 2V$$

Example:- Using the Thevenin's theorem to find the current in the 3Ω.





$E = I R = 2 * 4 = 8 V$

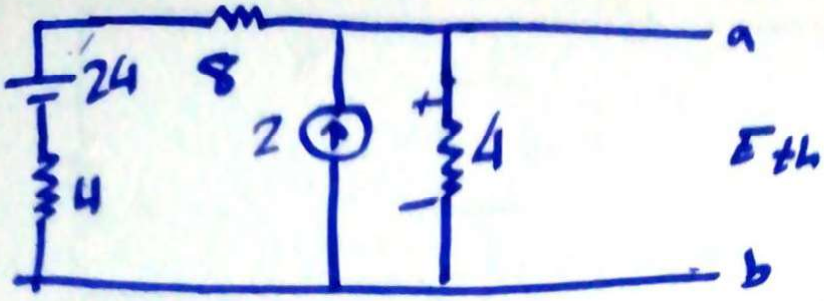


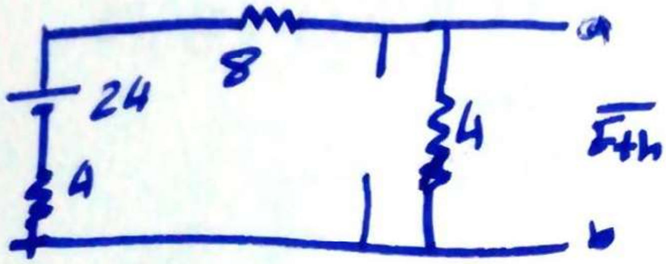
E_{th}

$$I = \frac{24-8}{8+4+4} = 1 A$$

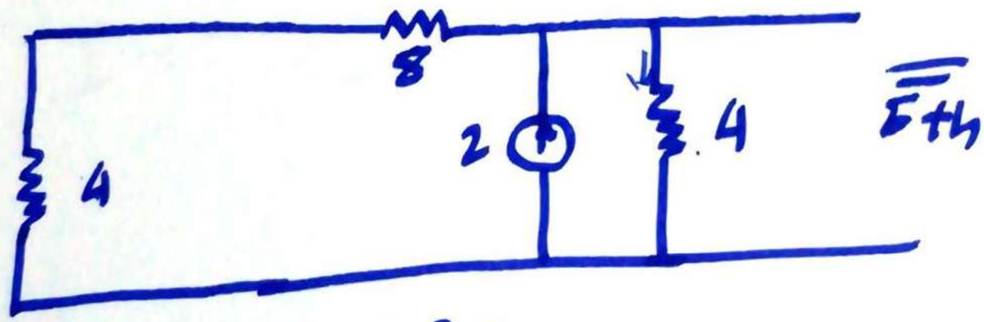
$$V_{4\Omega} = 1 * 4 = 4V$$

$$E_{th} = 8 + 4 = 12 V$$





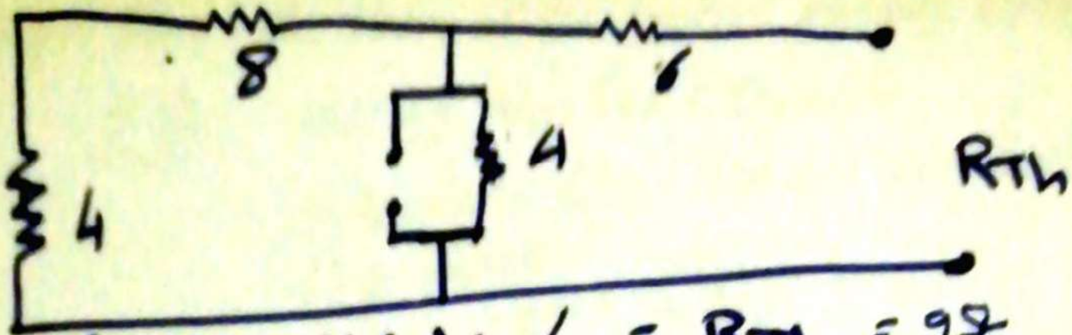
$$\overline{E}_{th} = V_{4\Omega} = \frac{24 * 4}{8+4+4} = 6\,V$$



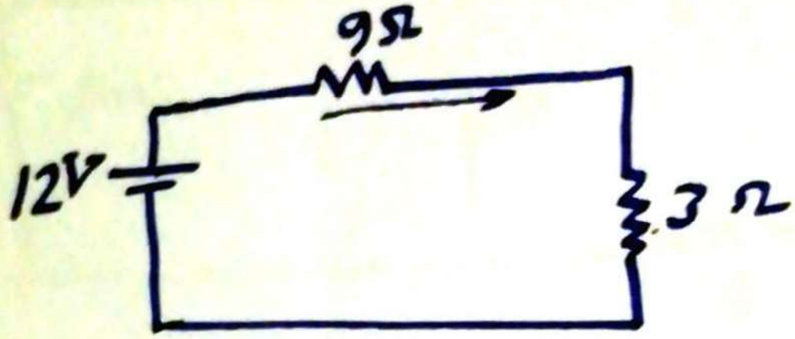
$$R_T = (4+8) // 4 = 3\,\Omega$$

$$V_T = \overline{\overline{E}}_{Th} = I\,R_T = 2 * 3 = 6\,V$$

$$E_{Th} = +\overline{E}_{Th} + \overline{\overline{E}}_{Th} = 6 + 6 = 12\,V$$

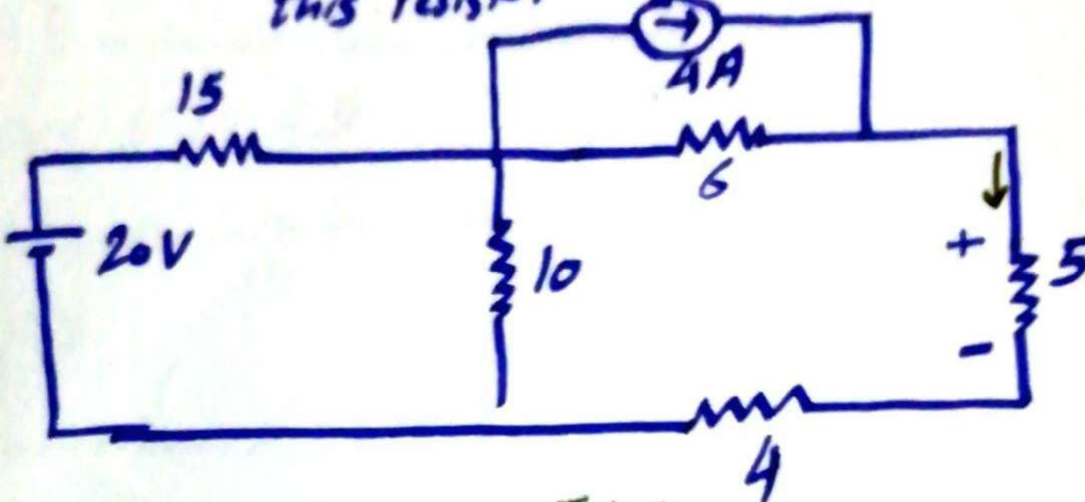


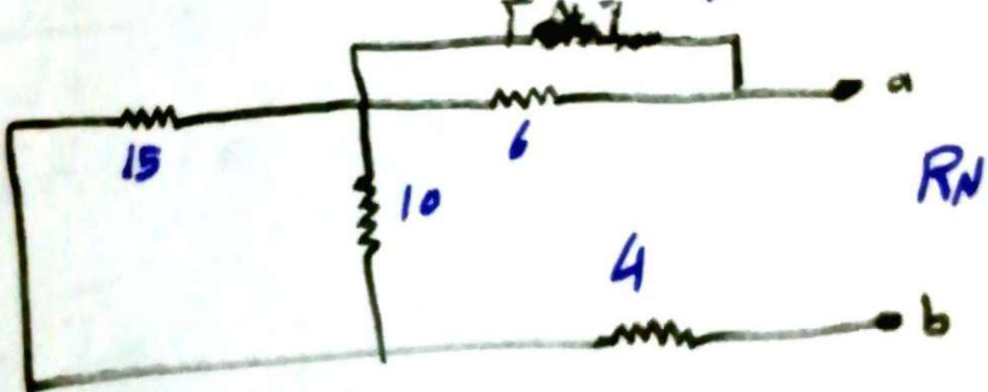
$$((8+4) \parallel 4) + 6 = R_{Th} = 9\Omega$$



$$I = \frac{E_{Th}}{R_T} = \frac{12}{9+3} = \frac{12}{12} = 1\ A$$

Example: Find the value of the current passing through 5Ω using Norton's theorem. Calculate the power absorbed by this resistor

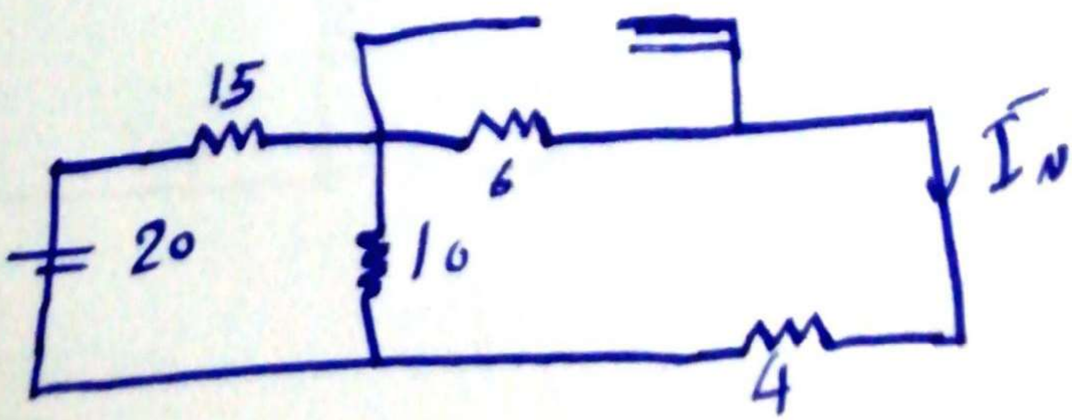


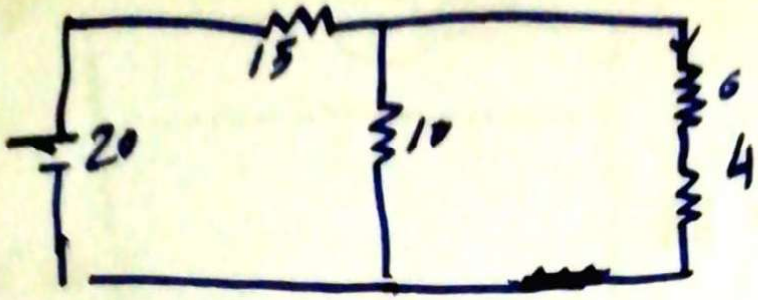


$(15 // 10) = 6$

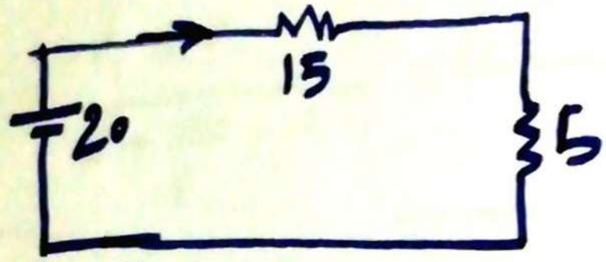
$R_N = (15 // 10) + 4 + 6 = 16\Omega$

I_N





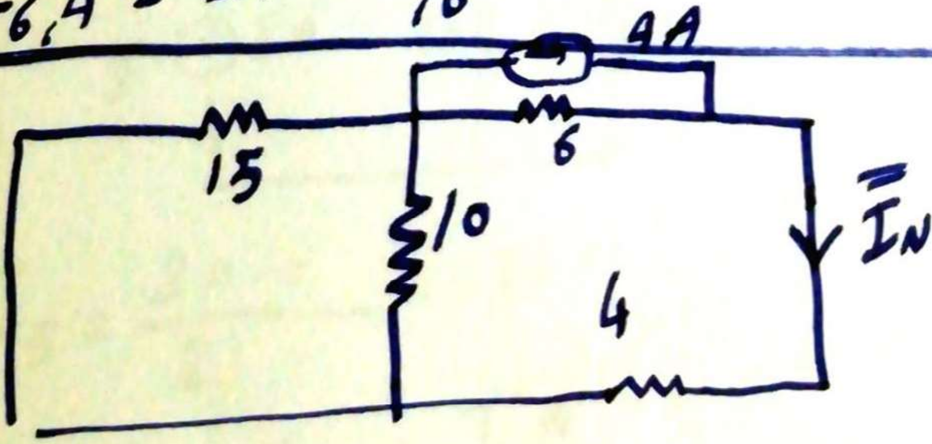
$$(6+4) \parallel 10 = 5$$

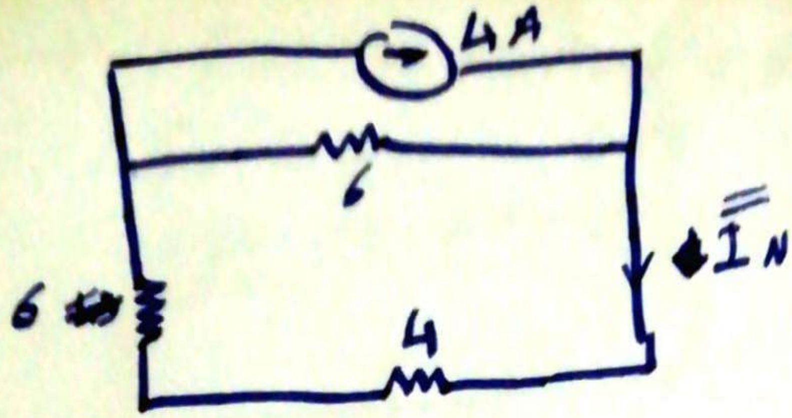


$$I = \frac{20}{20} = 1$$

$$V_{5\Omega} = IR = 1 * 5 = 5\,V$$

$$I_{6,4} = \bar{I}_N = \frac{5}{10} = \frac{1}{2} = \boxed{0.5\,A}$$

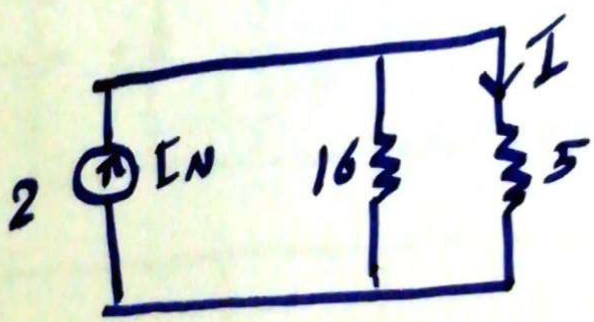




$$\bar{\bar{I}}_N = \frac{4\times 6}{6+6+4} = \frac{3}{2} = 1.5\,A$$

$$\boxed{\bar{I}_N = 0.5\,A} \qquad \boxed{\bar{\bar{I}}_N = 1.5\,A}$$

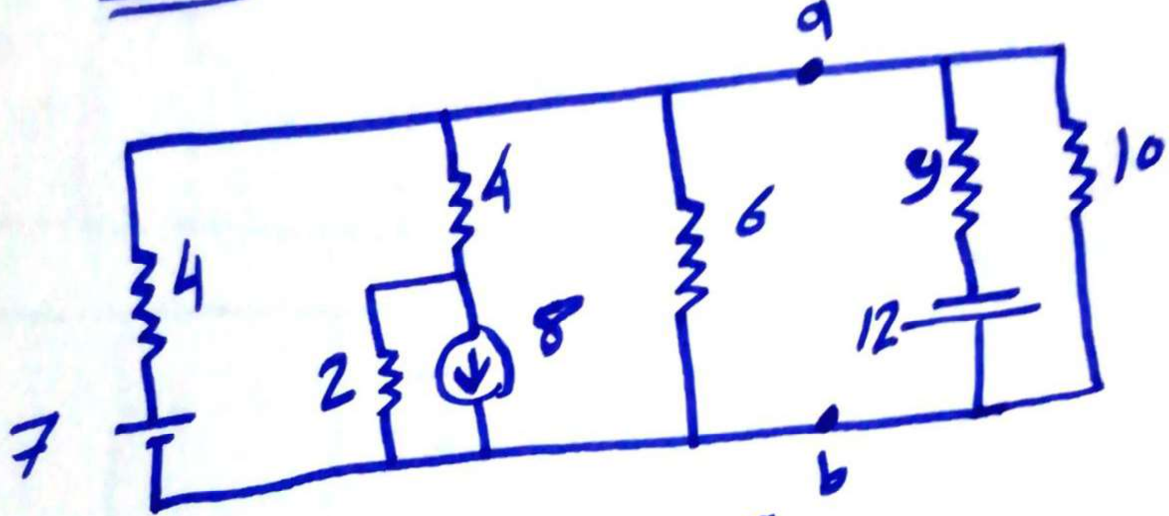
$$I_N = \bar{I}_N + \bar{\bar{I}}_N = 0.5 + 1.5 = 2\,A$$

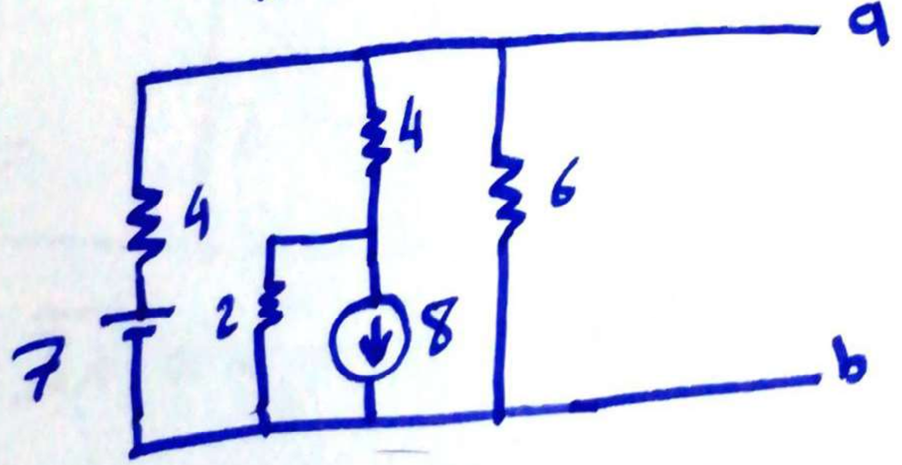


$$I_5 = \frac{2 * 16}{21} = 1.52\,A$$

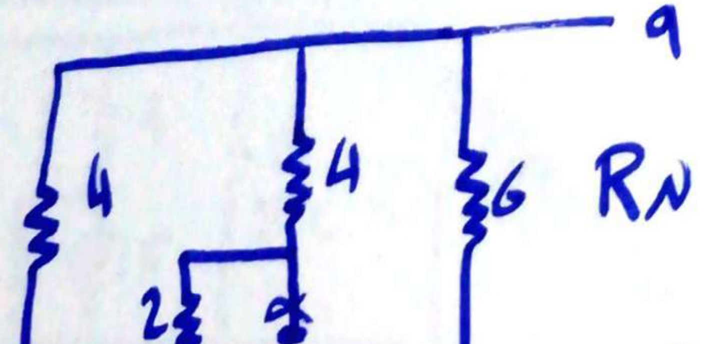
$$P = V * I = I^2 * R = (1.52)^2 * 5 = 11.6$$

Example: Find the Norton equivalent circuit for the portion of the network to the <u>Left</u> of (a-b) in the circuit shown.

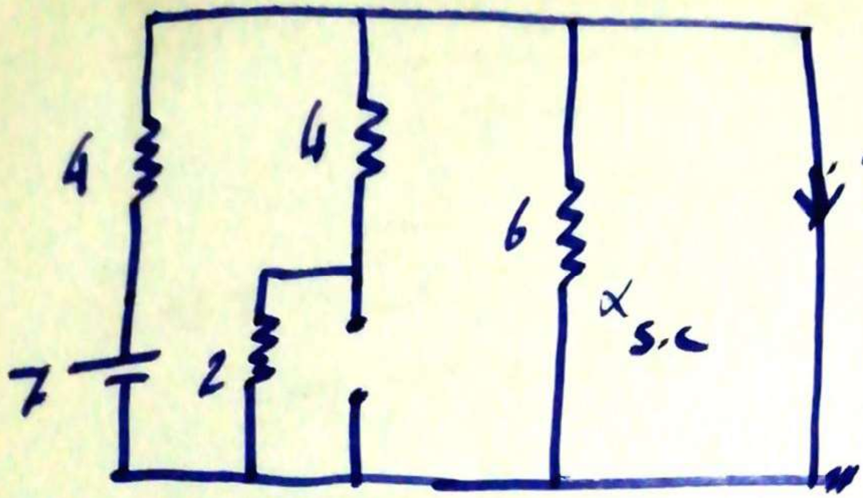


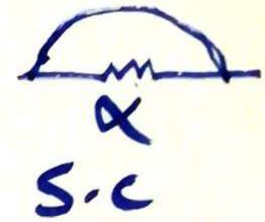


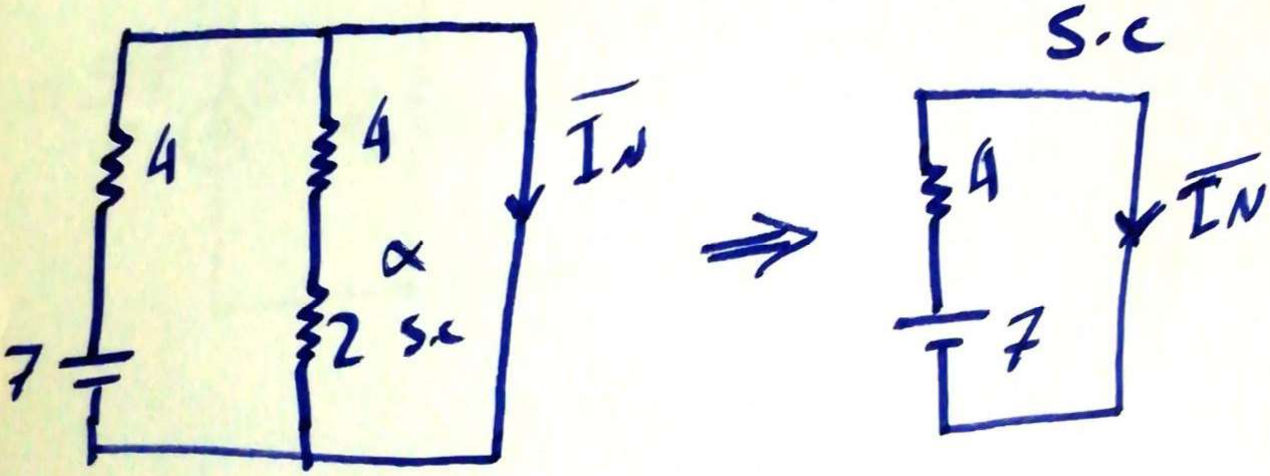
R_N



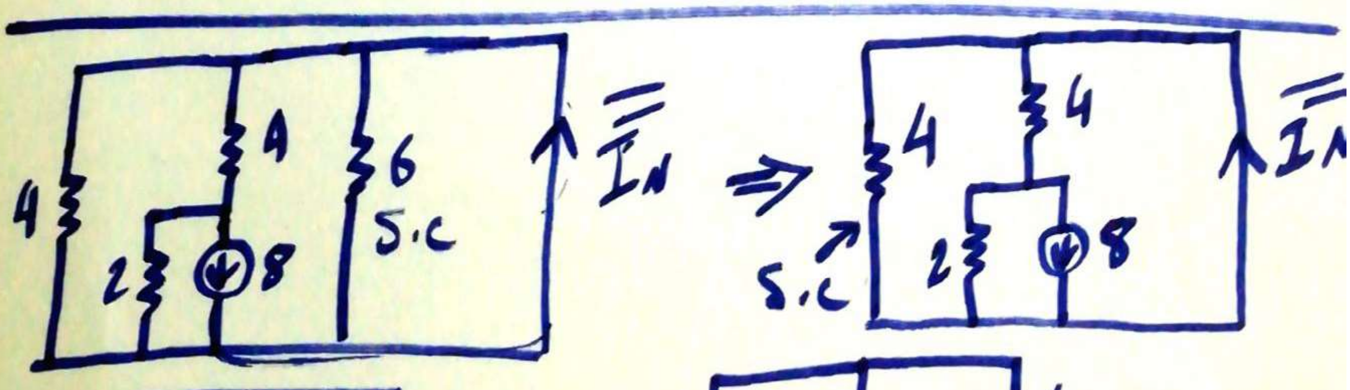
$$R_N = 4 \,//\, (4+2) \,//\, 6 = 1.714\,\Omega$$

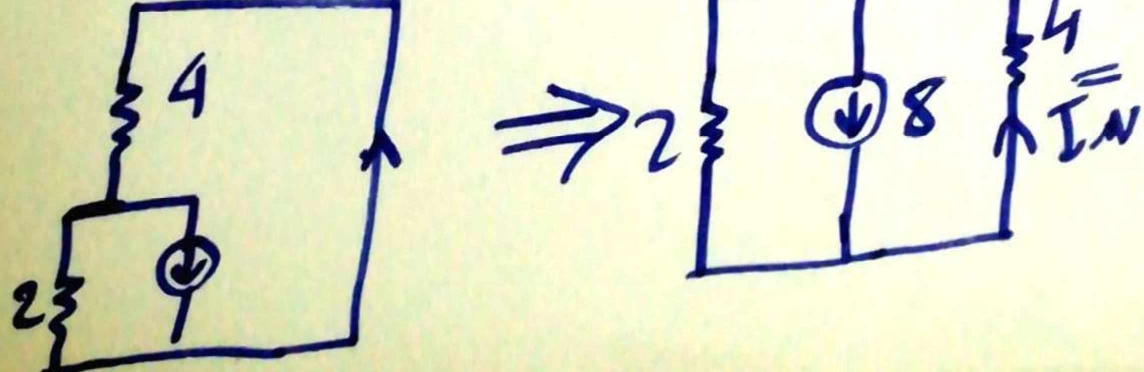






$$\overline{I_N} = \frac{7}{4} = 1.75A$$

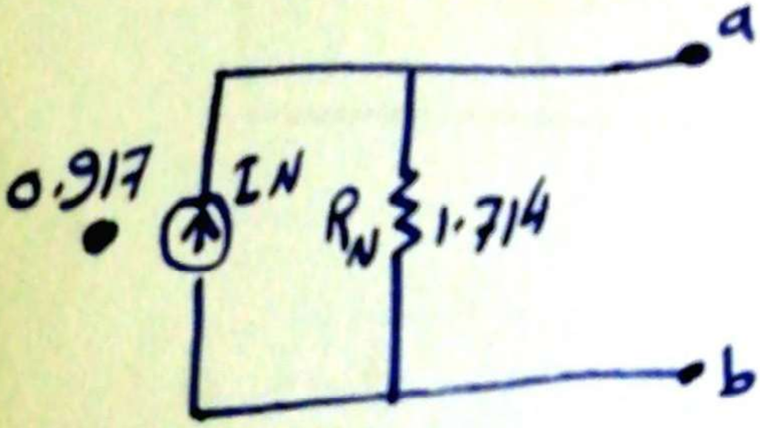




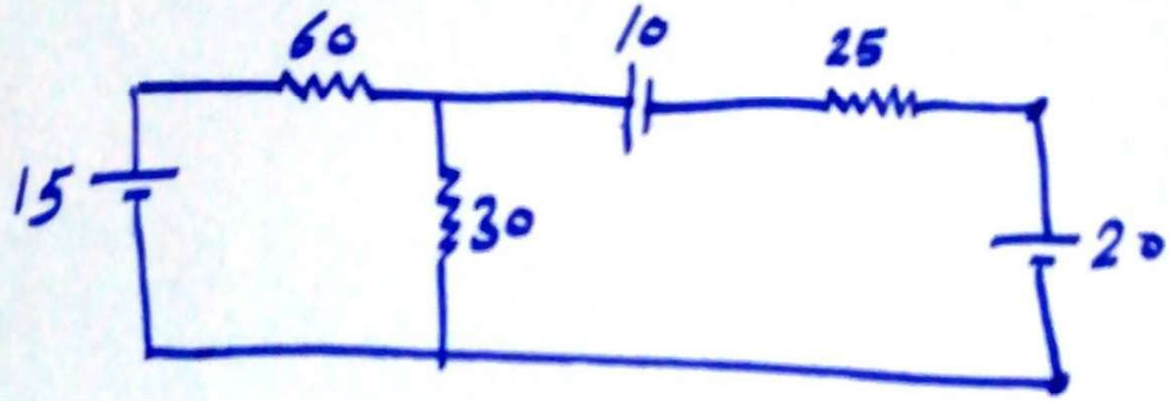
$$\bar{I}_N = \frac{8 * 2}{2+4} = 2.667\,A$$

$$I_N = -\bar{I}_N + \bar{\bar{I}}_N$$

$$= -1.75 + 2.667 = 0.917\,A$$



Example: For the circuit, find the current through the 20V voltage source using Thevenin's theorem.



H.W

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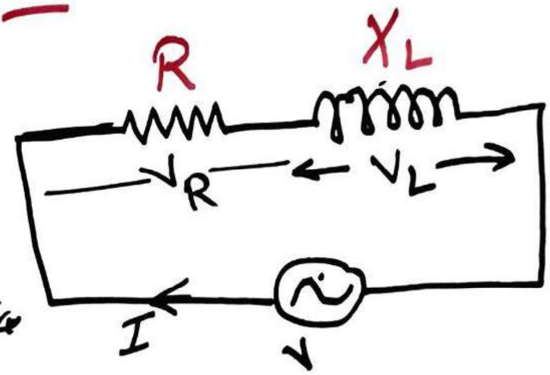
Series and Parallel AC circuits

1- Series circuits

1.1 Ac through R and L

V = the rms value of voltage

$V_{rms} = \frac{V}{\sqrt{2}}$

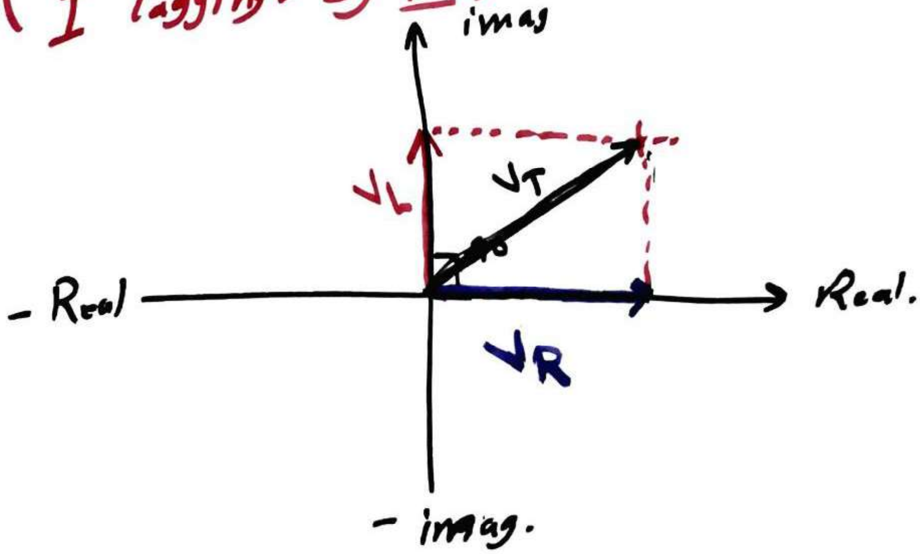


I = The rms value of the current

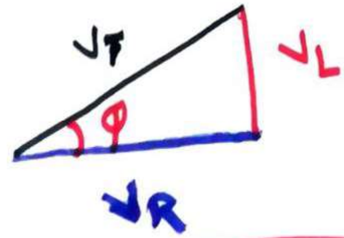
$I_{rms} = \frac{I}{\sqrt{2}}$

$V_R = IR$ (V inphase I) $\varphi = 0$

$V_L = I X_L$ ← (V leading I by 90°. I lagging V by 90°)



$$V_T^2 = V_R^2 + V_L^2$$



$$\boxed{V = \sqrt{V_R^2 + V_L^2}}$$

$$V = \sqrt{(IR)^2 + (IX_L)^2}$$

$$V = \sqrt{I^2(R^2 + X_L^2)}$$

$$\boxed{V = I\sqrt{R^2 + X_L^2}} \;**$$

$$\boxed{I = \frac{V}{\sqrt{R^2 + X_L^2}}}$$

$$\boxed{Z = \sqrt{R^2 + X_L^2}}$$

$$I = \frac{V}{Z}$$

$$V = IZ$$

φ Phase

$$\tan\varphi = \frac{\text{مقابل}}{\text{مجاور}} = \frac{V_L}{V_R} = \frac{\cancel{I}X_L}{\cancel{I}R} = \frac{X_L}{R}$$

$$\frac{\varphi}{\tan^{-1}} = \frac{X_L}{R} \Rightarrow \boxed{\varphi = \tan^{-1}\frac{X_L}{R}}$$

$X = R + yi$

$\bar{X}\cancel{X} = \sqrt{R^2 + y^2}$

$\phi = \tan^{-1}\frac{y}{R}$

$X = \bar{X}\angle\phi$

$R\angle 0$ $\quad$ $X_L\angle 90$

$$\boxed{\begin{aligned} Z &= R\angle 0 + X_L\angle 90 \\ \bar{Z} &= R + jX_L \end{aligned}}$$

$R\cos\phi = R$

$R\sin\phi = 0i$

$R + \cancel{j0}$

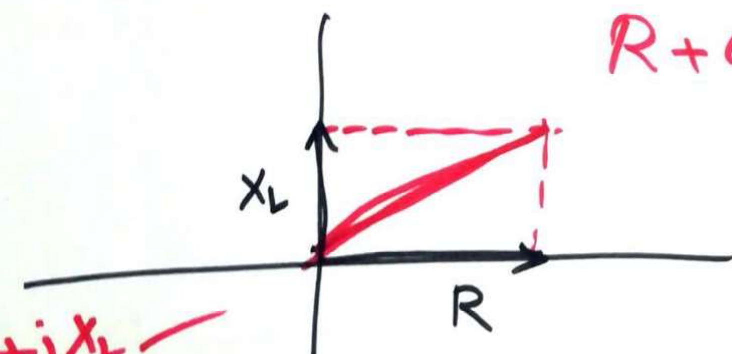
$X_L \cos 90 = 0$

$jX_L \sin 90 = iX_L$

$0 + iX_L$

Impedance diagram

$R + 0$

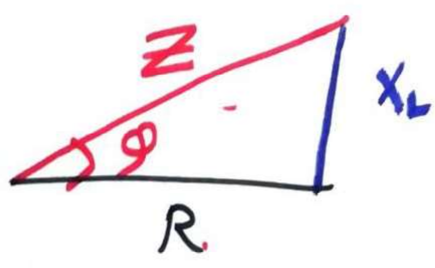


$\bar{Z} = R + jX_L$

$\bar{Z} = \sqrt{R^2 + X_L^2}$

$\phi = \tan^{-1}\frac{X_L}{R}$

$\bar{Z} = Z\angle\phi$



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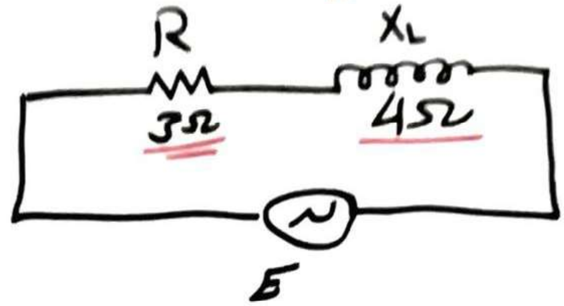
Example :- for the circuit shown. determine the total $\bar{Z}_T$ impedance and draw the impedance diagram



Solution:-

$4\angle 90$

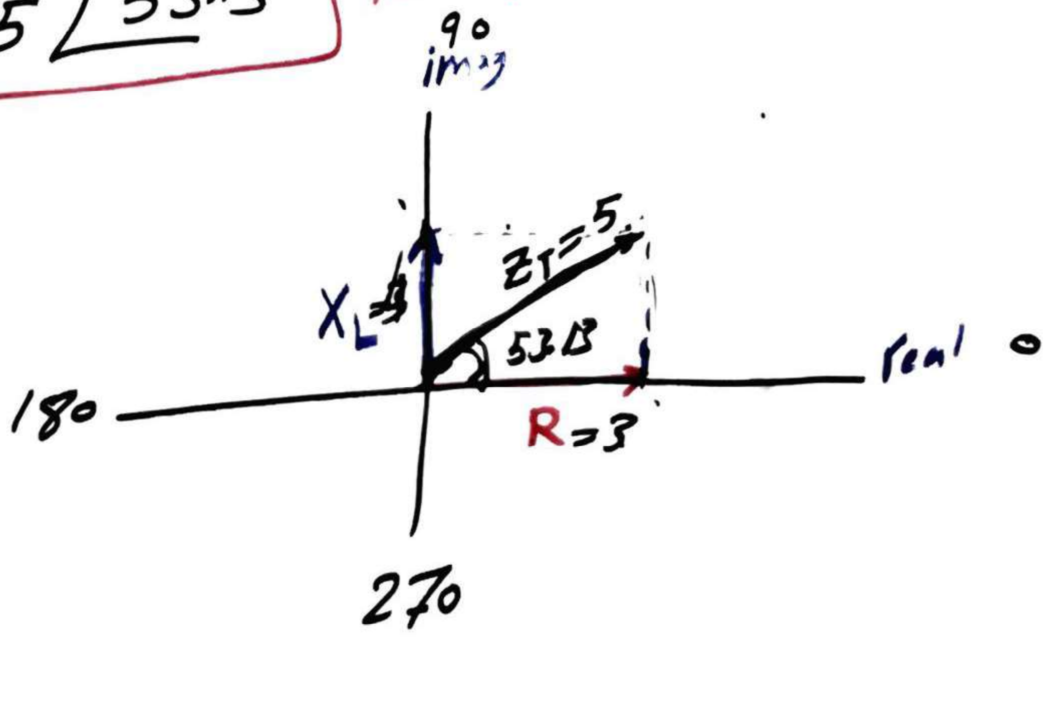
$\bar{Z}_T = R + jX_L$

$\boxed{Z_T = 3 + j4}$ Rectangular form

$Z = \sqrt{3^2 + 4^2} = 5$

$\phi = \tan^{-1}\left(\frac{4}{5}\right) = 53.13^\circ$

$\boxed{\bar{Z}_T = 5\angle 53.13}$ ~~Polar form~~ Polar form



Power factor (P_F)

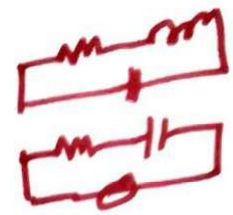
I leading V -||-

I lagging V

* $P_f = \cos\varphi$
* $P_F = \dfrac{R}{Z_T}$

lagging

leading -||-



* The Apparent Power (S)

$$S = VI = (IZ)I = I^2 Z \quad \boxed{(VA)}$$

$$S = \sqrt{Q^2 + P^2} = \sqrt{(I^2R)^2 + (I^2X_L)^2}$$

* The Active Power (P)

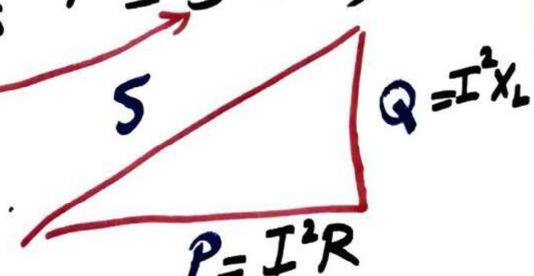
$$P = I^2R = VI\cos\varphi = S\cos\varphi$$

$$\boxed{W}$$

* The Reactive Power (Q)

$$Q = I^2X_L = VI\,\cancel{\cos}\sin\varphi = S\sin\varphi$$

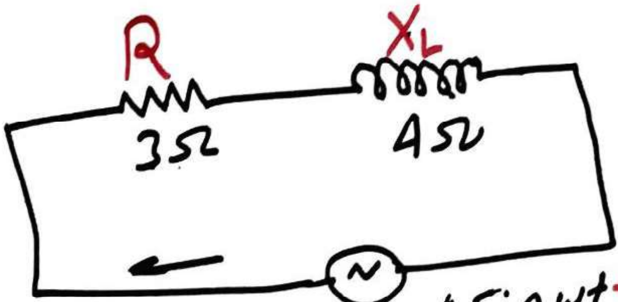
$$\boxed{VAR}$$



* The quality factor of the Coil

$$Q_{factor} = \frac{1}{P_f} = \frac{1}{\cos\phi} = \frac{Z_T}{R}$$

Example:- For the circuit shown, draw the phasor diagram of the voltage across each element and applied voltage and determine
- The power factor
- The active power and reactive pow
- The apparent power



$v = 141.4 \sin \omega t + 0)$

$\times \sin(\omega t + \varphi)$

Solution

$$V = \frac{141.4}{\sqrt{2}} = 100\angle 0 \rightleftharpoons 100 + j0$$

$$\overline{Z}_T = \overline{Z}_1 + \overline{Z}_2$$

$$Z_T = 3 + j4 \rightleftharpoons Z_T = 5\angle 53.13^\circ$$

$$I = \frac{V}{Z_T} = \frac{100\angle 0}{5\angle 53.13} = \frac{100}{3+j4}$$

$$I = \frac{100}{5}\angle 0 - 53.13 = 20\angle -53.13$$

$$V_R = IR \Rightarrow 20\angle -53.13 * 3\angle 0$$

$$\boxed{V_R = 60\angle -53.13}$$

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$$V_L = I \cdot X_L = 20\angle{-53.13} * 4\angle{90}$$

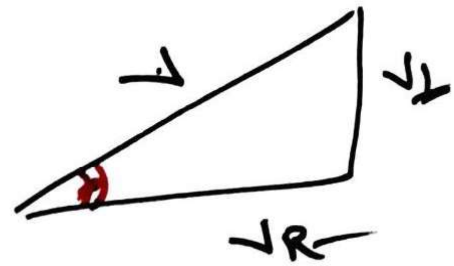
$$= \cancel{80\angle}$$

$$= (20 * 4)\angle{-53.13+90}$$

$$\boxed{V_L = 80\angle{36.87}}$$

$$\cancel{V_T = V_L + V_R}$$

$$V_T = V_R + V_L$$



$$V_T = 60\angle{-53.13} + 80\angle{36.87}$$

$$V_T = 36 - j48 + 64 + j48$$

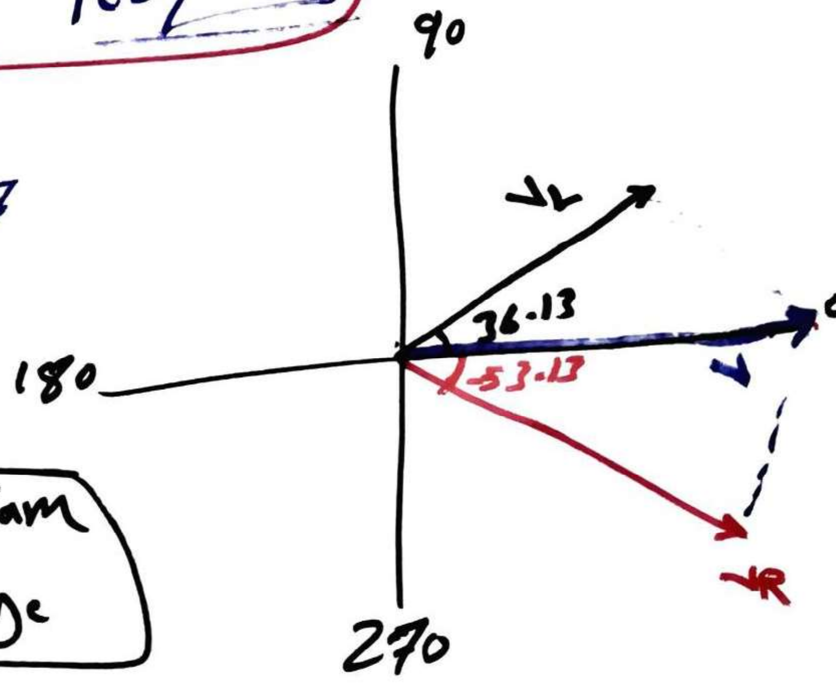
$$V_T = 100 + 0$$

$$\boxed{V_T = 100 \qquad 100\angle 0}$$

$$V_R = 60\angle{-53.13}$$

$$V_L = 80\angle{36.87}$$

$$V_T = 100\angle 0$$



Phasor diagram
Voltage

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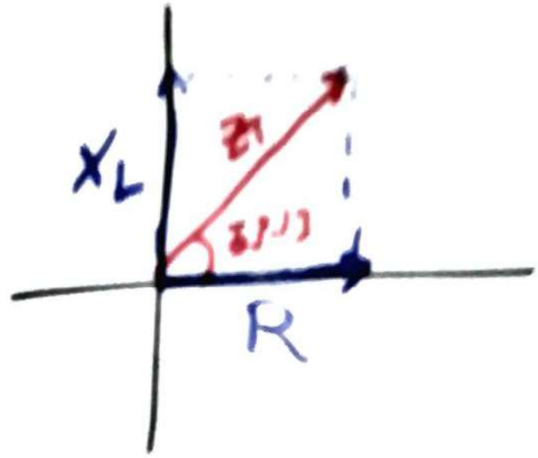
$R = 3\angle 0$

$X_L = 4\angle 90$

$Z_T = 5\angle 53.13$



Power factor

$\varphi = -53.13$

$\boxed{P_f = \cos(-53.13) = 0.6 \text{ logging}}$

$P = I^2R = IV\cos\varphi$

$I = 20\angle -53.13$ $\quad V = 100\angle 0$

$\varphi = -53.13$

$R = 3\angle 0$

$P = (20)^2 * 3 = 1200\ W$

$P = IV\cos\varphi = 20 * 100 * \cos 53.13 = 1200\ W$

$X_L = 4\angle 90$

$\cancel{IV = I^2 Z}$

$Q = I^2 X_L = IV\sin\varphi$

$Q = (20)^2 * 4 = 1600\ VAR$

$Q = IV\sin\varphi = 20 * 100 \sin 53.1$

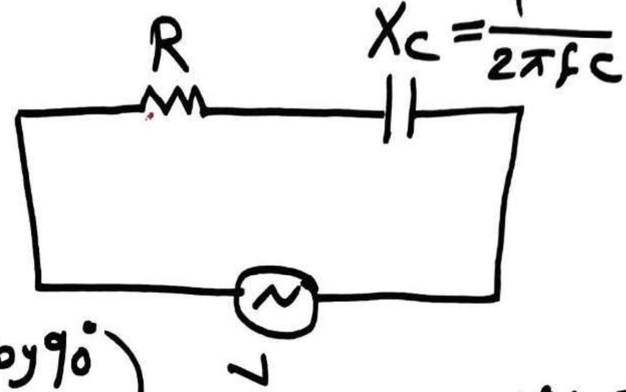
$= 1600\ VAR$

$1.6\ KVAR$

$$S = \sqrt{P^2 + Q^2} = 1968 \text{ VA}$$

$$S = IV = 20 * 100 = 2000 \text{ VA}$$

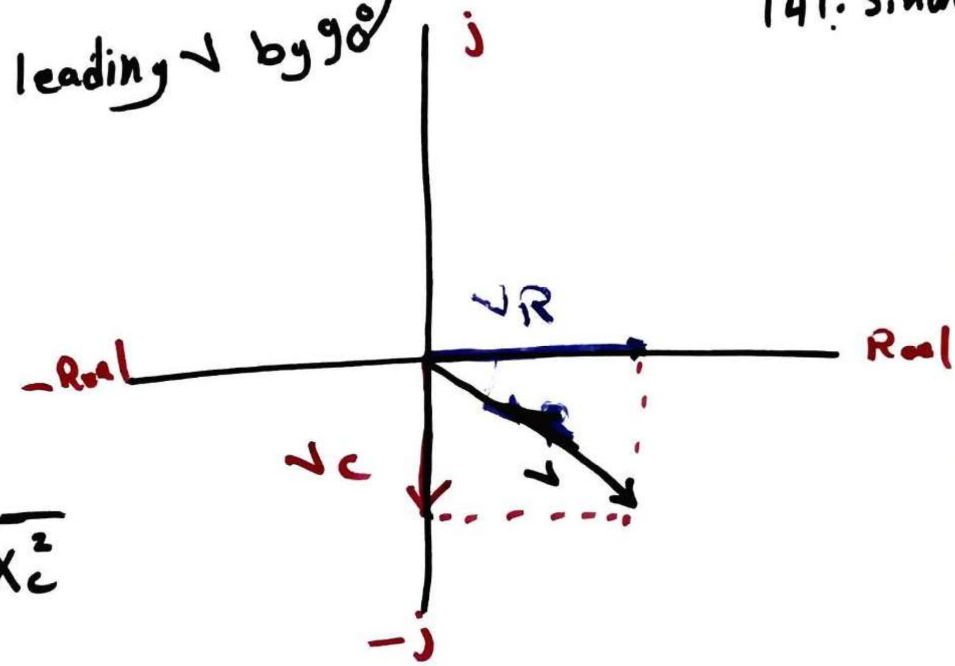
AC Through R and C



$V_R = IR$ V inphase I

$V_C = I X_C$ (V lagging I by 90°; I leading V by 90°)

141. sinωt

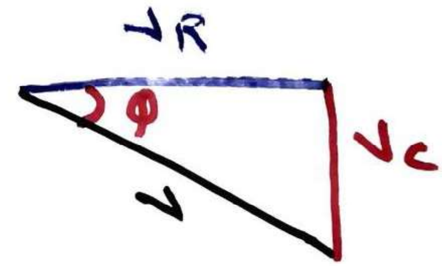


$$V = \sqrt{V_R^2 + V_C^2}$$

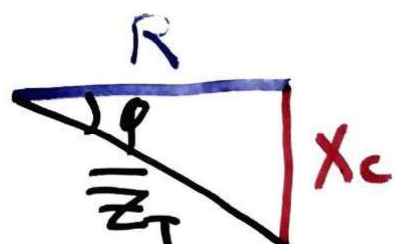
$$V = I\sqrt{R^2 + X_C^2}$$

$$V = I Z_T$$

$$Z_T = \sqrt{R^2 + X_C^2}$$



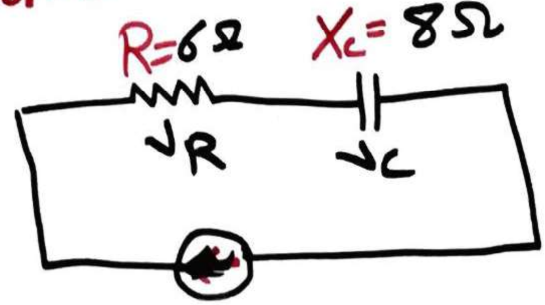
$$\tan\varphi = \frac{\text{مقابل}}{\text{مجاور}} = \frac{X_C}{R}$$



$$\boxed{\varphi = \tan^{-1}\frac{X_C}{R}}$$

$$P_F = \cos\varphi = \frac{V_R}{V} = \frac{R}{Z_T}$$ leading

Example :- For the circuit shown, draw the phasor diagram. voltage



$I = 5\angle 53.13$

Solution (1)

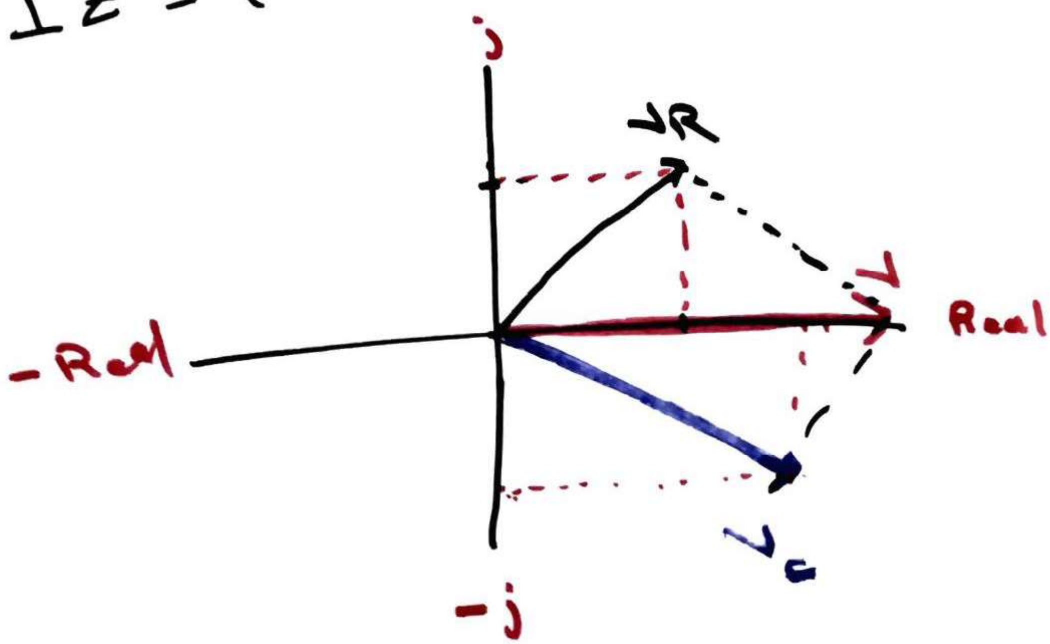
$$I = 5\angle 53.13 = 3 + 4i$$

$$Z_T = R - jX_c = 6 - j8$$

$$v_R = IR = (3+4i)(6) = \boxed{18+24i}$$

$$v_c = IX_c = (3+4i)(-8i) = \boxed{\begin{matrix}-24i+32 \\ = 32-24i\end{matrix}}$$

$$v = IZ = (3+4i)(6-i8) = \boxed{50}$$



Solution 2

$$I = 5\angle 53.13$$

$$Z_T = 6 - 8i = \sqrt{6^2+8^2}\,\angle \tan^{-1}\frac{-8}{6}$$

$$Z_T = 10\angle -53.13$$

$R\angle 0$

$X_C\angle -90$

$X_L\angle 90$

$$V_R = IR = 5\angle 53.13 * 6\angle 0$$

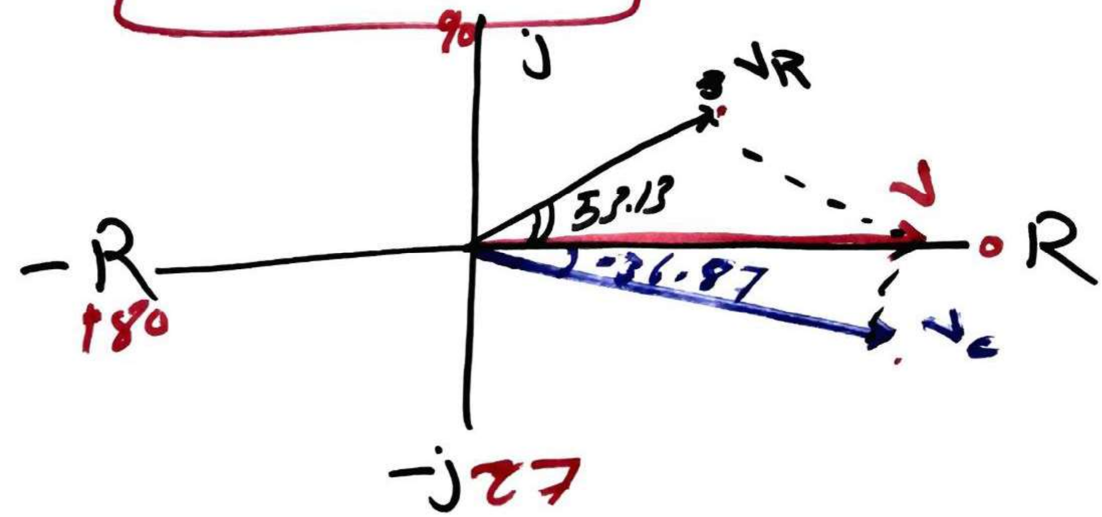
$$\boxed{V_R = 30\angle 53.13}$$

$$V_C = I X_C = 5\angle 53.13 * 8\angle -90$$

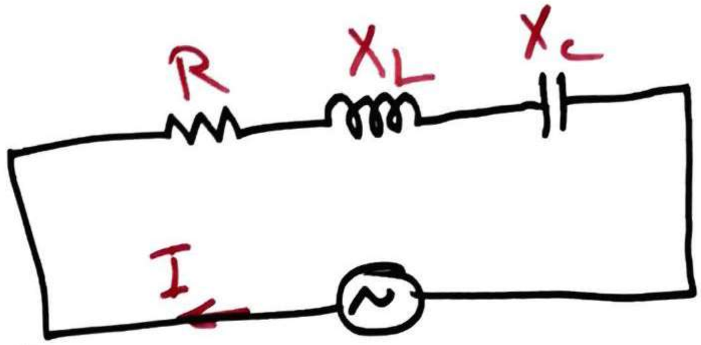
$$\boxed{V_C = 40\angle -36.87}$$

$$V = E = IZ = 5\angle 53.13 * 10\angle -53.13$$

$$\boxed{V = 50\angle 0}$$



3- AC through RLC series circuit



$$Z = R\angle 0 + X_L\angle 90 + X_C\angle{-90}$$

$$Z = R + jX_L - jX_C$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

$$\bar{Z} = Z\angle\phi$$

Example. For the circuit shown. determine

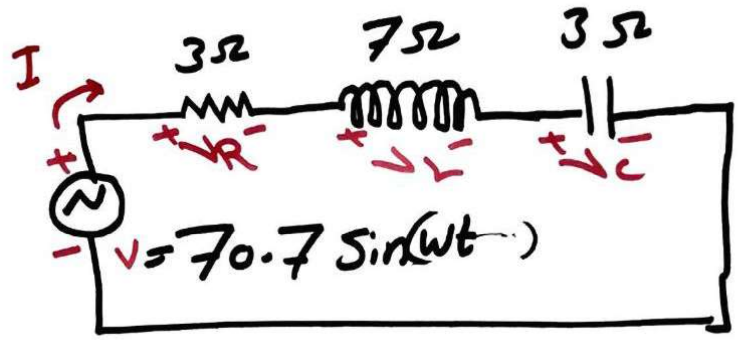
1- $\bar{Z}_T$, and draw the impedance diagram

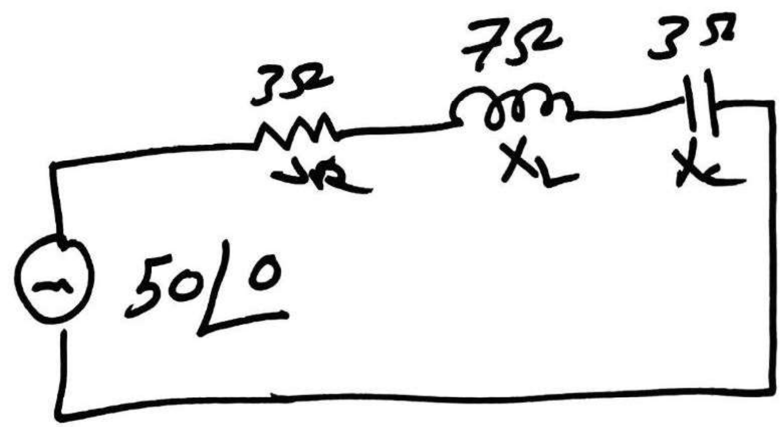
2- $\bar{I}$, $\bar{V}_R$, $\bar{V}_L$, $\bar{V}_C$ in the phasor domain and draw the phasor diagram.

3- i, v_R, v_L, v_C in time domain

4- the power factor of the circuit

5- the active power, reactive power and apparent power.





① $Z_T = R + jX_L - jX_C$

$Z_T = 3 + j7 - j3$ $(3\angle 0° + 7\angle 90° + 3\angle)$

$X_L - X_C = 7-4 = \underline{4i}$

$\underline{Z_T = 3 + j4}$

$Z_T = \sqrt{3^2+4^2}\angle \tan^{-1}\frac{4}{3}$

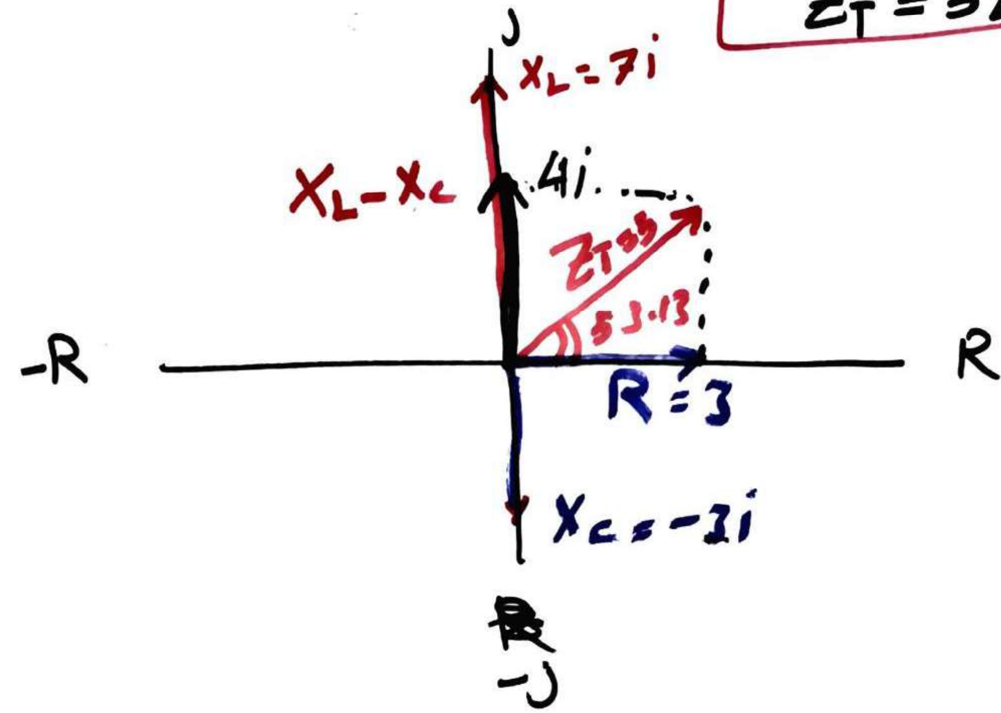
$\boxed{Z_T = 5\angle 53.13}$

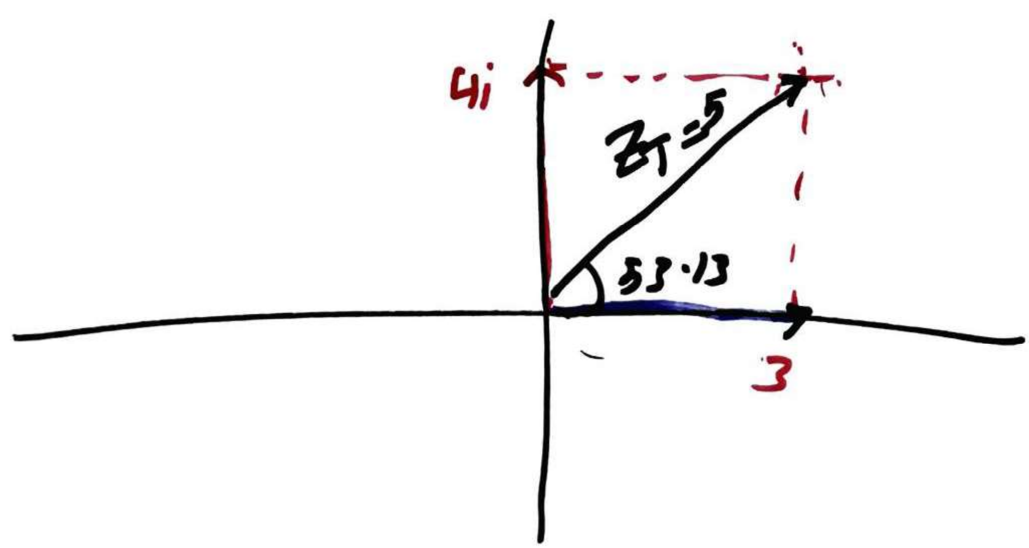
$Z = \sqrt{3^2+(7-3)^2}$

$Z = 5$

$\phi = \tan^{-1}\frac{7-3}{4} = 53.4$

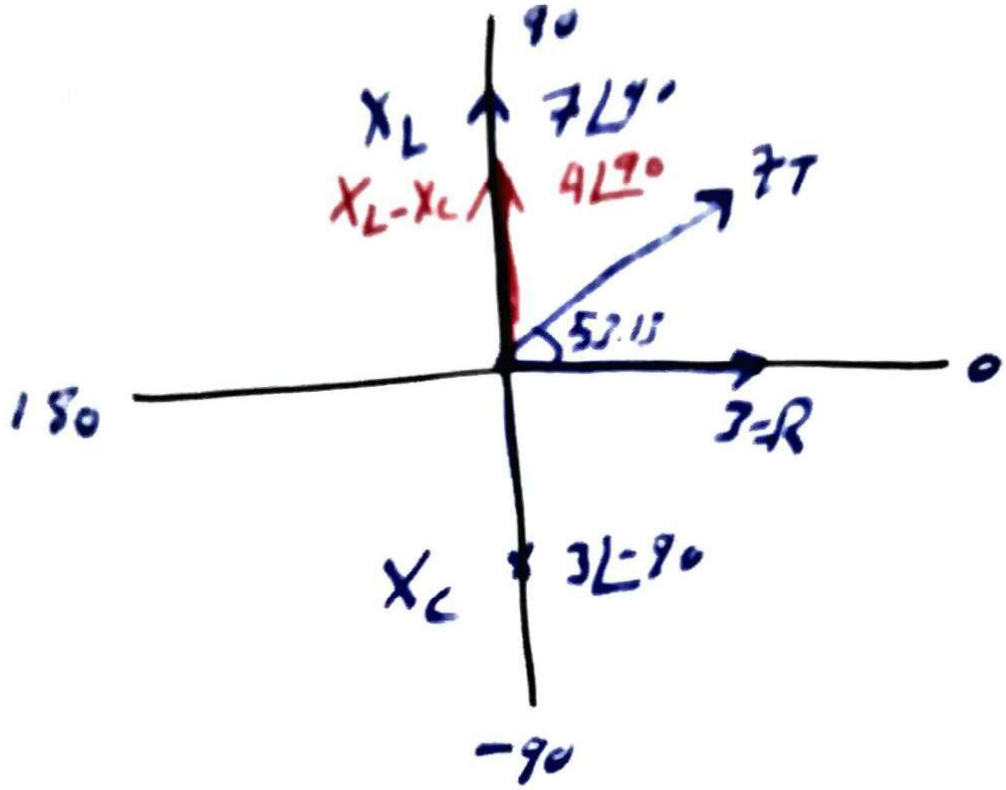
$Z_T = 5\angle 53.13$





$$Z_T = 3\angle 0 + 7\angle 90 + 3\angle -90$$

$$Z_T = 5\angle +53.13$$



② $Z_T = 5\angle 53.13$

$$E_{rms} = \frac{70.7}{\sqrt{2}} = \boxed{50\angle 0}$$

$$I = \frac{V}{Z} = \frac{50\angle 0}{5\angle 53.13} = \boxed{10\angle -53.13}$$

$$V_R = IR = 10\angle -53.13 * 3\angle 0$$

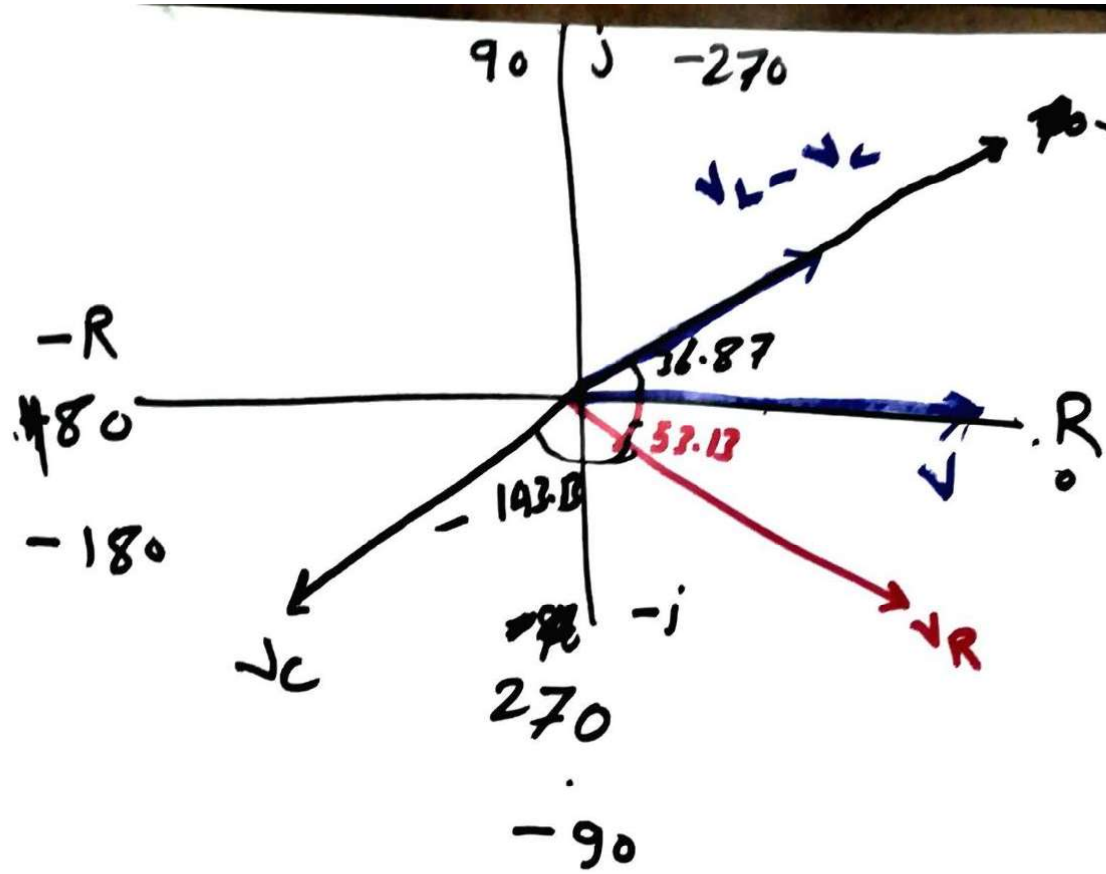
$$\boxed{V_R = 30\angle -53.13}$$

$$V_L = IX_L = 10\angle -53.13 * 7\angle 90$$

$$\boxed{V_L = 70\angle 36.87}$$

$$V_C = IX_C = 10\angle -53.13 * 3\angle -90$$

$$\boxed{V_C = 30\angle -143.13}$$



③ $I = 10\angle -53.13$

$$i = 10\sqrt{2}\,\sin(wt - 53.13)$$

$$\boxed{i = 14.14\sin(wt - 53.13)}$$

$$V_R = 30\angle -53.13$$

$$v_R = 30\sqrt{2}\,\sin(wt - 53.13$$

$$\boxed{v_R = 42.42\sin(wt - 53.13)}$$

$V_L = 70\angle 36.87$

$v_L = 70\sqrt{2}\,\sin(wt+36.87)$

$$\boxed{v_L = 98.98\,\sin(wt+36.87)}$$

$V_C = 30\angle -143.13$

$v_c = 30\sqrt{2}\,\sin(wt-143.13)$

$$\boxed{v_c = 42.42\,\sin(wt-143.13)}$$

(4) $P_F = \cos\varphi = \cos 53.13$

$= 0.6$ lagging

4) active power

$P = I^2 R$

$I = 10\angle -53.13$

$R = 3\angle 0$

$P = VI\cos\varphi$

$= 50 * 10 \cos 53.13 = 300$

$P = 100 * 3 = 300\,W$

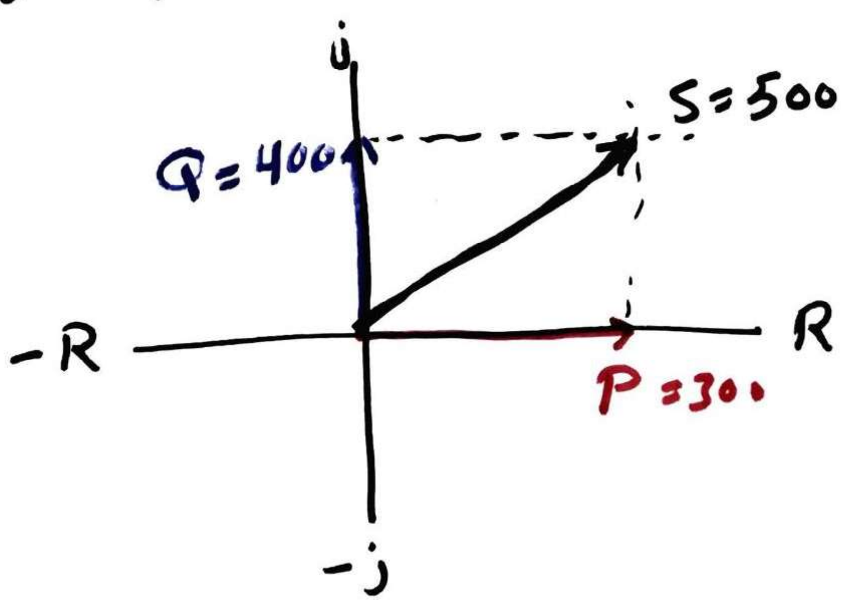
Reactive power

$Q = I^2 (X_L - X_C) = 10^2 * 4 = 400$ VAR

$Q = IV \sin\varphi = 50 * 10 \sin(53.13) = 400$

apparent power =

$S = IV = 10 * 50 = 500$ VA



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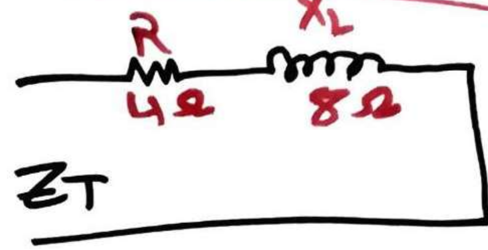
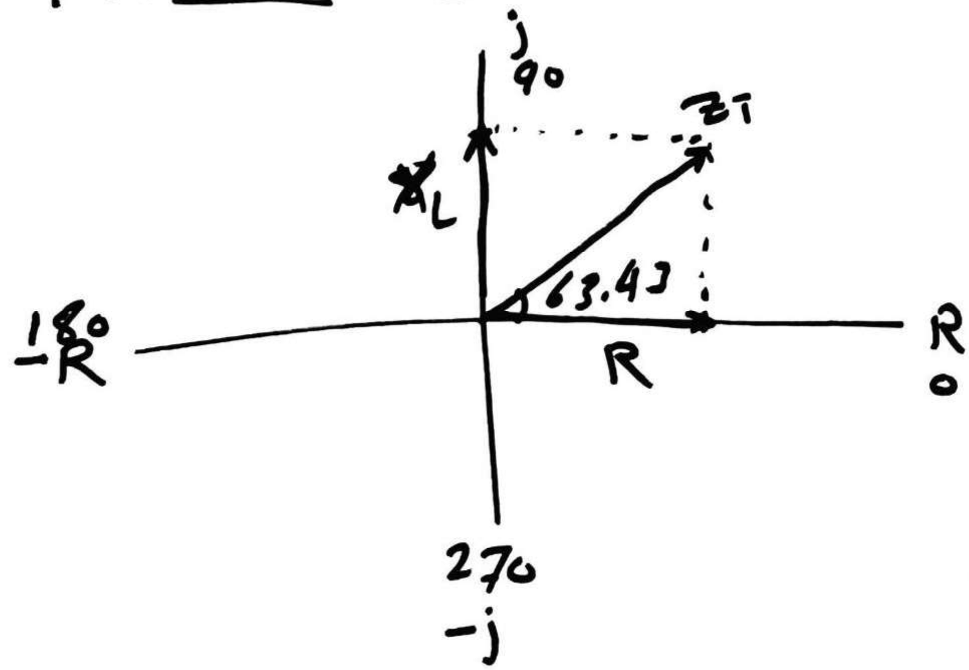
Example :- Draw the impedance diagram for the circuit shown and find the ~~total~~ total impedance.



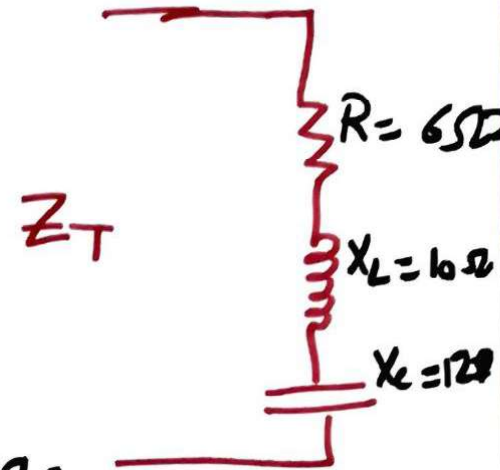
Solution

$$\bar{Z}_T = 4 + j8 = 4\angle 0 + 8\angle 90$$

$$\bar{Z}_T = 8.944\angle 63.43^\circ \ \Omega$$



Example :- Determine the input impedance to the series network shown.



Solution

$$Z_T = 6 + j10 - j12$$

$$= 6\angle 0 + 10\angle 90 + 12\angle -90^\circ$$

$$= 6 + j(10 - 12)$$

$Z_T = 6$ [redacted] $j2$

$$Z_T = 6.325\angle -18.43$$

P.F < lagging / leading ??

$$P.F = \cos\phi = \frac{R}{Z}$$

Example:- A 60 Hz sinusoidal voltage $v = 141 \sin \omega t$ a series R-L circuit. The value of the resistance and the inductance are 3 and 0.0106 H respectively.

ⓐ Compute the rms value of the current in the circuit and it's phase angle with respect to the voltage.

ⓑ Write the expression for the instantaneous current in the circuit.

ⓒ Find the average power.

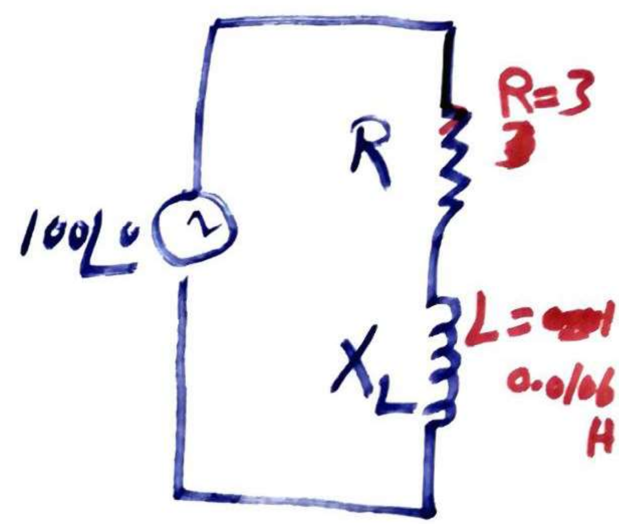
ⓓ Calculate the P.F of circuit.

Solution :-

$$\boxed{f = 60\ Hz}$$

$$v = 141 \sin \omega t \qquad V = \frac{141}{\sqrt{2}} = 100$$

$$\boxed{V = 100\angle 0}$$



(a) $I = ?$

$Z_T = R + jX_L$

$R = 3$
$L = 0.0106\,H$

$X_L = 2\pi f L = 2\pi * 60 * 0.0106 = 4\Omega$

$Z_T = 3 + j4$

$Z = 5\angle 53.13$

$I = \dfrac{V}{Z} = \dfrac{100\angle 0}{5\angle 53.13} = 20\angle -53.13$

I logging Voltage

(b) $i = 20 * \sqrt{2}\ \sin(\omega t + (-53.13))$
$i = 28.28\ \sin(\omega t - 53.13)$

(c) averag power = Active pow

$P = I^2 R = 20^2 * 3 = 1200\ W$

(d) $P.F = \cos\varphi = \cos(53.13) = 0.6$ logging

Example:- A two elements series circuit is connect across an A.C circuit having source ~~e=√2 (200) sin~~ $e = \sqrt{2}(200)\sin(wt+20)$. The current circuit is found to be $i = \sqrt{2}(10)\cos(\underset{314}{\overset{wt}{341t}} - 25)$

Determine the parameter of ~~e~~ the circuit.

Solution

* Two ~~elements~~ elements

$$e = \sqrt{2}(200)\sin(wt+20)$$
$$i = \sqrt{2}(10)\cos(341t-25)$$

$$E = \frac{(200)\cancel{\sqrt{2}}}{\cancel{\sqrt{2}}} = 200$$

$$\phi = 20$$

$$\boxed{E = 200\angle 20}$$

$$i = \sqrt{2}(10)\cos(341t - 25) = \sqrt{2}\,10\sin(\underset{314}{\overset{wt}{341t}}\underbrace{-25+90}_{+65^\circ})$$

$$I = \frac{10(\cancel{\sqrt{2}})}{\cancel{\sqrt{2}}} = 10 \qquad \phi = +65$$

$$\boxed{I = 10\angle 65}$$

$$Z_T = \frac{V}{I} = \frac{200\angle 20}{10\angle 65} = 20\angle -45$$

$$Z_T = 20\angle -45 = \underbrace{\overset{R}{14.14} - j\overset{X_c}{14.14}}$$

$R = 14.14$

$X_C = 14.14$

$\omega \cancel{t} = \cancel{341}\, 314\, t$

$\omega = \cancel{341}\, 314 = 2\pi f$

R ~~&~~ C

$$X_C = \frac{1}{2\pi f C}$$

$$14.14 = \frac{1}{\omega C}$$

$$14.14 = \frac{1}{\cancel{341} * C}$$

$$C = \frac{1}{\cancel{341}\ 314 * 14.14} = \cancel{2.03} \; 2.25 * 10^{-4}\ F$$

$R = 14.14\ \Omega$

$C\ 2.25 * 10^{-4}\ F$

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