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<u>Course Outline</u>				
1	Introduction of unit operation			
2	Particle Size Reduction and Enlargement, Emulsification			
3	Motion of Particles in a Fluid			
4	Flow of fluid through granular beds and packed columns			
5	Sedimentation			
6	Agitation and mixing			
7	Fluidization			
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9	Membrane Separation Processes			
10	Centrifugal Separations			
11	Draying			
12	Crystallization			

Course Outline

Text book: Chemical Engineering – volume 2, 5th E, Particle Technology and Separation Processes.

Reference book:

1- Chemical Engineering Design volume 6. , 4th ED., Coulson and Richardson's.

2- Unit operations of chemical engineering 5th ED., Peter

Harriott.

Size Reduction And Emulsification

Materials are rarely found in the size range required, and it is often necessary either to decrease or to increase the particle size. In the materials processing industry, size reduction or comminution is usually carried out in order to increase the surface area because, in most reactions involving solid particles the rate of reactions is directly proportional to the area of contact with a second phase.

Mechanism of size reduction

Whilst the mechanism of the process of size reduction is extremely complex, in recent years a number of attempts have been made at a more detailed analysis of the problem. If a single lump of material is subjected to a sudden impact, it will generally break so as to yield a few relatively large particles and a number of fine particles, with relatively few particles of intermediate size. If the energy in the blow is increased, the larger particles will be of a rather smaller size and more numerous and, whereas the number of fine particles will be appreciably increased, their size will not be much altered. It therefore appears that the size of the fine particles is closely connected with the internal structure of the material, and the size of the larger particles is more closely connected with the process by which the size reduction is effected.

The energy required to effect size reduction is related to the internal structure of the material. The method of application of the force to the

particles may affect the breakage pattern. PRASHER suggests that four basic patterns may be identified

)a) Impact —particle concussion by a single rigid force.

)b) Compression—particle disintegration by two rigid forces.

)c) Shear —produced by a fluid or by particle–particle interaction.

)d) Attrition —arising from particles scraping against one another or against a rigid surface.

Energy for size reduction

Although it is impossible to estimate accurately the amount of energy required in order to effect a size reduction of a given material, a number of empirical laws have been proposed. The two earliest laws are due to KICK) and VON RITTINGER, and a third law due to BOND has also been proposed. These three laws may all be derived from the basic differential equation:

$$\frac{\mathrm{d}E}{\mathrm{d}L} = -CL^p$$

which states that the energy dE required to effect a small change dL in the size of unit mass of material is a simple power function of the size.

Writing $C = K_R f_c$, where f_c is the crushing strength of the material, and p=-2 then Rittinger's law is obtained as:

$$E = K_R f_c \left(\frac{1}{L_2} - \frac{1}{L_1} \right)$$

If p = -1, then:

$$E = C \ln \frac{L_1}{L_2}$$

and, writing $C = K_K f_c$:

$$E = K_K f_c \ln \frac{L_1}{L_2} \tag{2.4}$$

which is known as *Kick's law*. This supposes that the energy required is directly related to the reduction ratio L_1/L_2 which means that the energy required to crush a given amount of material from a 50 mm to a 25 mm size is the same as that required to reduce the size from 12 mm to 6 mm. In equations 2.3 and 2.4, K_R and K_K are known respectively as Rittinger's constant and Kick's constant. It may be noted that neither of these constants is dimensionless.

Bond has suggested a law intermediate between Rittinger's and Kick's laws, by putting p = -3/2 in equation 2.1. Thus:

$$E = 2C \left(\frac{1}{L_2^{1/2}} - \frac{1}{L_1^{1/2}} \right)$$

= $2C \sqrt{\left(\frac{1}{L_2}\right) \left(1 - \frac{1}{q^{1/2}}\right)}$ (2.5)

where:

the reduction ratio. Writing $C = 5E_i$, then:

$$E = E_i \sqrt{\left(\frac{100}{L_2}\right) \left(1 - \frac{1}{q^{1/2}}\right)}$$
(2.6)

Bond terms E_i the work index, and expresses it as the amount of energy required to reduce unit mass of material from an infinite particle size to a size L_2 of 100 µm, that is $q = \infty$. The size of material is taken as the size of the square hole through which 80 per cent of the material will pass. Expressions for the work index are given in the original papers^(8,9) for various types of materials and various forms of size reduction equipment.

Example

A material is crushed in a Blake jaw crusher such that the average size of particle is reduced from 50 mm to 10 mm with the consumption of energy of 13.0 kW/(kg/s). What would be the consumption of energy needed to crush the same material of average size 75 mm to an average size of 25 mm:

- a) assuming Rittinger's law applies?
- b) assuming Kick's law applies?

Which of these results would be regarded as being more reliable and why?

Solution

a) Rittinger's law.

This is given by:	$E = K_R f_c[(1/L_2) - (1/L_1)]$
Thus:	13.0 $K_R f_c[(1/10) - (1/50)]$
and:	$K_R f_c = (13.0 \times 50/4) = 162.5 \text{ kW/(kg mm)}$

(equation 2.3)

Thus the energy required to crush 75 mm material to 25 mm is:

$$E = 162.5[(1/25) - (1/75)] = 4.33 \text{ kJ/kg}$$

b) Kick's law.

This is given by:	$E = K_K f_c \ln(L_1/L_2)$	(equation 2.4)
Thus:	$13.0 = K_K f_c \ln(50/10)$	
and:	$K_K f_c = (13.0/1.609) = 8/08 \text{ kW/(kg/s)}$	

Thus the energy required to crush 75 mm material to 25 mm is given by:

 $E = 8.08 \ln(75/25) = 8.88 \text{ kJ/kg}$

The size range involved by be considered as that for coarse crushing and, because Kick's law more closely relates the energy required to effect elastic deformation before fracture occurs, this would be taken as given the more reliable result.

The principal types of size-reduction machines are as follows:

- A. Crushers (coarse and fine)
 - 1. Jaw crushers
 - 2. Gyratory crushers
 - 3. Crushing rolls
- B. Grinders (intermediate and fine)
 - 1. Hammer mills; impactors
 - 2. Rolling-compression mills
 - a. Bowl mills
 - b. Roller mills
 - 3. Attrition mills
 - 4. Tumbling mills
 - a. Rod mills
 - b. Ball mills; pebble mills
 - c. Tube mills; compartment mills
- C. Ultrafine grinders
 - 1. Hammer mills with internal classification
 - 2. Fluid-energy mills
 - 3. Agitated mills
- D. Cutting machines
 - 1. Knife cutters; dicers; slitters

The Grinding:

In the grinding process, materials are reduced in size by fracturing them. In the process, the material is stressed by the action of mechanical moving parts in the grinding machine and initially the stress is absorbed internally by the material as strain energy.

Grinding is achieved by mechanical stress followed by rupture and

the energy required depends upon:

1. the hardness of the material

2. the tendency of the material to crack (friability).

The force applied may be compression, impact, or shear, and both the magnitude of the force and the time of application affect the extent of grinding achieved.

Surface Area Formed by Grinding

When a uniform particle is crushed, after the first crushing the size of the particles produced will vary a great deal from relatively coarse to fine and even to dust. As the grinding continues, the coarser particles will be further reduced but there will be less change in the size of the fine particles. Careful analysis has shown that there tends to be a certain size that increases in its relative proportions in the mixture and which soon becomes the predominant size fraction.

The new surface produced was directly proportional to the energy input. For a given energy input the new surface produced was independent of :

(a) The velocity of impact,

(b) The mass and arrangement of the sample,

(c) The initial particle size, and

(d) The moisture content of the sample .

For example, wheat after first crushing gives a wide range of particle sizes in the coarse flour, but after further grinding the predominant fraction soon becomes Most reactions are related to the surface area available, so the surface area can have a considerable bearing on the properties of the material. The surface area per unit mass is called the specific surface. To calculate this in a known mass of material it is necessary to know:

•the particle-size distribution

•the shape factor of the particles.

The particle size gives one dimension that can be called the Typical Dimension, D_p of a particle. This has now to be related to the surface area. We can write, arbitrarily:

$$V_p = pD_p^3$$

$$A_p = 6qD_p^2$$

Where

 V_p is the volume of the particle. A_p is the area of the particle surface D_p is the typical dimension of the particle q is factor of the particle geometries. And , The ratio of surface area to volume is: $A_p/V_p = (6q/p)/D_p = 6\lambda/D_p$ $A_p = 6\lambda V_p / D_p$ $\lambda = q/p$ is a shape factor. It has been found experimentally

for example

	V_p	A _p	Specific Surface V _P / A _p	λ
Cube	D_p^{3}	$6D_p^2$	6/D _p	1
Sphere	$\pi D_{p}^{3}/6$	πD_p^2	6/D _p	1

A number of particles(N) is :

$$N = \frac{m}{m_p} = \frac{m}{\rho_p V_p}$$

Where :

m is a total mass of particles

m_p is a mass of particle

 ρ_p is a density of particles

So total area of the mass of particles (At) is:

$$A_{t} = NA_{p} = \left[\frac{m}{\rho_{p}V_{p}}\right] \left[\frac{6\lambda V_{p}}{D_{p}}\right]$$
$$A_{t} = \frac{6\lambda m}{\rho D_{p}}$$

Example :

In an analysis of ground salt using Tyler sieves, it was found that 38% of the total salt passed through a 7-mesh sieve and was caught on a 9-mesh sieve. For one of the finer fractions, 5% passed an 80-mesh sieve but was retained on a 115-mesh sieve. Estimate the surface areas of these two fractions in a 5 kg sample of the salt, if the density of salt is 1050 kg/m3 and the shape factor is 1.75. Aperture of Tyler sieves: 7 mesh = 2.83 mm 9 mesh = 2.00 mm 80 mesh = 0.177 mm

$$\begin{split} &115 \text{ mesh} = 0.125 \text{ mm} \ .\\ &Solution:\\ &Mean aperture 7 and 9 \text{ mesh} = 2.41 \text{ mm} = 2.4 \text{ x } 10^{-3}\text{m}\\ &Mean aperture 80 and 115 \text{ mesh} = 0.151 \text{ mm} = 0.151 \text{ x } 10^{-3}\text{m}\\ &A_t = (6 \text{ x } 1.75 \text{ x} 0.38 \text{ x } 5)/(1050 \text{ x } 2.41 \text{ x} 10^{-3})\\ &A_t = 7.88 \text{ m}^2\\ &A_t = (6 \text{ x } 1.75 \text{ x } 0.05 \text{ x } 5)/(1050 \text{ x } 0.151 \text{ x} 10^{-3})\\ &A_t = 16.6 \text{ m}^2. \end{split}$$

Emulsification

Emulsions are stable suspensions of one liquid in another, the liquids being immiscible. Stability of the emulsion is obtained by dispersion of very fine droplets of one liquid, called the disperse phase, through the other liquid, which is called the continuous phase. The emulsion is stable when it can persist without change, for long periods of time, without the droplets of the disperse phase coalescing with each other, or rising or settling. The stability of an emulsion is controlled by interfacial surface forces, droplets, • size the of disperse phase the continuous properties of phase and viscous • density difference between the two phases.

The dispersed particles in the emulsion have a very large surface area, which is created in the process of emulsification. Surface effects depend upon the properties of the materials of the two phases, but very often a third component is added which is absorbed at the interface and which helps to prevent the droplets from coalescing. These added materials are called **emulsifying agents** and examples are phosphates and glycerol monostearate. Stokes' Law gives a qualitative indication of the physical factors that influence the stability of an emulsion. This is because the relative flow of the particles under gravitational forces may break the emulsion, so stability is enhanced by small settling velocities. From eqn.

 $v = D^2 g(\rho_{\rm c} - \rho_{\rm d})/18\mu_{\rm c}$

where: v: settling velocity D:drops diameter g : specific gravity ρ_c , ρ_d :densities for continuous and dispersed face μ_c : viscosity of continuous face.

Motion of Particles in a Fluid

when a viscous fluid flows over a surface the fluid is retarded in the boundary layer which is formed near the surface and that the boundary layer increases in thickness with increase in distance from the leading edge.



Figure 3.2. Flow of fluid over a surface against a pressure gradient

The flow is characterised by the Reynolds number $(Re=ud\rho/\mu)$ in which ρ is the density of the fluid, μ is the viscosity of the fluid, d is the diameter of the sphere, and u is the velocity of the fluid relative to the particle.

Drag coefficients

The most satisfactory way of representing the relation between drag force and velocity involves the use of two dimensionless groups, similar to those used for correlating information on the pressure drop for flow of fluids in pipes.

The first group is the particle Reynolds number Re= $Re'(=ud\rho/\mu)$

The second is the group $R'/\rho u^2$, in which R^{\setminus} is the force per unit projected area of particle in a plane perpendicular to the direction of motion. For a sphere, the projected area is that of a circle of the same diameter as the sphere.

Thus:
$$R' = \frac{F}{(\pi d^2/4)}$$
 and
$$\frac{R'}{\rho u^2} = \frac{4F}{\pi d^2 \rho u^2}$$

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 $R'/\rho u^2$ is a form of *drag coefficient*, often denoted by the symbol C'_D . Frequently, a drag coefficient C_D is defined as the ratio of R' to $\frac{1}{2}\rho u^2$.

Thus:

$$C_D = 2C'_D = \frac{2R'}{\rho u^2}$$
(3.4)

It is seen that C'_D is analogous to the friction factor $\phi (= R/\rho u^2)$ for pipe flow, and C_D is analogous to the Fanning friction factor f.

When the force F is given by Stokes' law (equation 3.1), then:

$$\frac{R'}{\rho u^2} = 12 \frac{\mu}{u d \rho} = 12 R e'^{-1}$$

The relation between $R'/\rho u^2$ and Re' is conveniently given in graphical form by means of a logarithmic plot as shown in Figure 3.4. The graph may be divided into four regions as shown. The four regions are now considered in turn.

Region (a) (10⁻⁴ < Re' < 0.2)

In this region, the relationship between $\frac{R'}{\rho u^2}$ and Re' is a straight line of slope -1 represented by equation 3.5:

$$\frac{R'}{\rho u^2} = 12Re'^{-1} \qquad (\text{equation 3.5})$$



Figure 3.4. $R'/\rho u^2$ versus Re' for spherical particles

Thus:

Region (b) (0.2 < Re' < 500-1000)

In this region, the slope of the curve changes progressively from -1 to 0 as Re' increases. Several workers have suggested approximate equations for flow in this intermediate region. DALLAVELLE⁽⁶⁾ proposed that $R'/\rho u^2$ may be regarded as being composed of two component parts, one due to Stokes' law and the other, a constant, due to additional non-viscous effects.

$$\frac{R'}{\rho u^2} = 12Re'^{-1} + 0.22 \tag{3.8}$$

SCHILLER and NAUMANN⁽⁷⁾ gave the following simple equation which gives a reasonable approximation for values of Re' up to about 1000:

$$\frac{R'}{\rho u^2} = 12Re'^{-1}(1+0.15Re'^{0.687})$$
(3.9)

Region (c) $(500-1000 < \text{Re}' < \text{ca } 2 \times 10^5)$

In this region, *Newton's law* is applicable and the value of $R'/\rho u^2$ is approximately constant giving:

$$\frac{R'}{\rho u^2} = 0.22$$
 (3.10)

Region (d) (Re' > ca 2×10^5)

When Re' exceeds about 2×10^5 , the flow in the boundary layer changes from streamline to turbulent and the separation takes place nearer to the rear of the sphere. The drag force is decreased considerably and:

$$\frac{R'}{\rho u^2} = 0.05$$
 (3.11)

A comprehensive review of the various equations proposed to relate drag coefficient to particle Reynolds number has been carried out by CLIFT, GRACE and WEBER⁽⁸⁾. One of the earliest equations applicable over a wide range of values of Re' is that due to WADELL⁽⁹⁾ which may be written as:

$$\frac{R'}{\rho u^2} = \left(0.445 + \frac{3.39}{\sqrt{Re'}}\right)^2 \tag{3.12}$$

Subsequently, KHAN and RICHARDSON⁽¹⁰⁾ have examined the experimental data and suggest that a very good correlation between $R'/\rho u^2$ and Re', for values of Re' up to 10⁵, is given by:

$$\frac{R'}{\rho u^2} = [1.84Re'^{-0.31} + 0.293Re'^{0.06}]^{3.45}$$
(3.13)

For values of Re' < 2, correction factors for Stokes' law have been calculated from equations 3.9, 3.12 and these 3.13 and are these included in Table 3.1.

Total force on a particle

The force on a spherical particle may be expressed using equations 3.5, 3.9, 3.10 and 3.11 for each of the regions a, b, c and d as follows.

In region (a):
$$R' = 12\rho u^2 \left(\frac{\mu}{ud\rho}\right) = \frac{12u\mu}{d}$$
(3.14)

The projected area of the particle is $\pi d^2/4$. Thus the total force on the particle is given by:

$$F = \frac{12u\mu}{d} \frac{1}{4}\pi d^2 = 3\pi\mu du \tag{3.15}$$

This is the expression originally obtained by STOKES⁽¹⁾ already given as equation 3.1. In region (b), from equation 3.9:

$$R' = \frac{12u\mu}{d} (1 + 0.15Re^{0.687}) \tag{3.16}$$

and therefore:

$$F = 3\pi\mu du (1 + 0.15Re^{0.687}) \tag{3.17}$$

In region (c):

$$R' = 0.22\rho u^2$$
(3.18)
$$E = 0.22\rho u^{21} \pi d^2 = 0.055\pi d^2 c u^2$$
(3.19)

and:

$$F = 0.22\rho u \frac{1}{4}\pi u = 0.055\pi u \rho u$$

This relation is often known as Newton's law.

In region (d):

$$R' = 0.05\rho u^2 \tag{3.20}$$

(3.19)

$$F = 0.0125\pi d^2 \rho u^2 \tag{3.21}$$

Alternatively using equation 3.13, which is applicable over the first three regions (a), (b) and (c) gives:

$$F = \frac{\pi}{4} d^2 \rho u^2 (1.84 R e'^{-0.31} + 0.293 R e'^{0.06})^{3.45}$$
(3.22)

Terminal falling velocities

If a spherical particle is allowed to settle in a fluid under gravity, its velocity will increase until the accelerating force is exactly balanced by the resistance force. Although this state is approached exponentially, the effective acceleration period is generally of short duration for very small particles.

Again if the terminal falling velocity corresponds to a value of *Re* Greater than about 500, the drag on the particle is given by equation 3.19. Under terminal falling conditions, velocities are rarely high enough for Re to approach 105, with the small particles generally used in industry.

The accelerating force due to gravity is given by:

$$=(\frac{1}{6}\pi d^{3})(\rho_{s}-\rho)g$$
 (3.23)

where ρ_s is the density of the solid.

The terminal falling velocity u_0 corresponding to region (a) is given by:

$$(\frac{1}{6}\pi d^{3})(\rho_{s} - \rho)g = 3\pi \mu du_{0}$$
$$u_{0} = \frac{d^{2}g}{18\mu}(\rho_{s} - \rho)$$
(3.24)

and:

The terminal falling velocity corresponding to region (c) is given by:

$$(\frac{1}{6}\pi d^{3})(\rho_{s} - \rho)g = 0.055\pi d^{2}\rho u_{0}^{2}$$

$$u_{0}^{2} = 3dg \frac{(\rho_{s} - \rho)}{\rho}$$
(3.25)

or:

The problem is most effectively solved by the generation of a new dimensionless group which is independent of the particle velocity. The resistance force per unit projected area of the particle under terminal falling conditions R'_0 is given by:

$$R_{0}^{\prime} \frac{1}{4} \pi d^{2} = \frac{1}{6} \pi d^{3} (\rho_{s} - \rho) g$$

$$R_{0}^{\prime} = \frac{2}{3} d(\rho_{s} - \rho) g \qquad (3.33)$$

Thus:

or:

 $\frac{R'_0}{\rho u_0^2} = \frac{2dg}{3\rho u_0^2} (\rho_s - \rho)$ (3.34)

The dimensionless group $(R'_0/\rho u_0^2)Re'_0^2$ does not involve u_0 since:

$$\frac{R'_0}{\rho u_0^2} \frac{u_0^2 d^2 \rho^2}{\mu^2} = \frac{2dg(\rho_s - \rho)}{3\rho u_0^2} \frac{u_0^2 d^2 \rho^2}{\mu^2}$$
$$= \frac{2d^3(\rho_s - \rho)\rho g}{3\mu^2}$$
(3.35)

The group $\frac{d^3\rho(\rho_s-\rho)g}{\mu^2}$ is known as the Galileo number Ga or sometimes the Archimedes number Ar.

Thus:

$$\frac{R_0'}{\rho u_0^2} R e_0'^2 = \frac{2}{3} G a \tag{3.36}$$

Using equations 3.5, 3.9 and 3.10 to express $R'/\rho u^2$ in terms of Re' over the appropriate range of Re', then:

$$Ga = 18Re'_0$$
 (Ga < 3.6) (3.37)

$$Ga = 18Re'_0 + 2.7Re'^{1.687}_0(3.6 < Ga < ca. 10^5)$$
(3.38)

$$Ga = \frac{1}{3}Re_0^{\prime 2} \qquad (Ga > ca. \ 10^5)$$
(3.39)

 $(R'_0/\rho u_0^2)Re'_0^2$ can be evaluated if the properties of the fluid and the particle are known.

In Table 3.4, values of log Re' are given as a function of log $\{(R'/\rho u^2)Re'^2\}$ and the data taken from tables given by HEYWOOD⁽¹¹⁾, are represented in graphical form in Figure 3.6. In order to determine the terminal falling velocity of a particle, $(R'_0/\rho u_0^2)Re'_0^2$ is evaluated and the corresponding value of Re'_0 , and hence of the terminal velocity, is found either from Table 3.4 or from Figure 3.6.

$\log\{(R'/\rho u^2)Re'^2\}$ 0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	2.018 1.007 1.967 0.817 1.562 2.236 2.841	2.117 1.105 0.008 0.895 1.632 2.300 2.899	2.216 1.203 0.148 0.972 1.702 2.363 2.956	2.315 1.301 0.236 1.048 1.771 2.425 3.013	2.414 1.398 0.324 1.124 1.839 2.487 3.070	2.513 1.495 0.410 1.199 1.907 2.548 3.127	3.620 2.612 1.591 0.495 1.273 1.974 2.608 3.183	3.720 2.711 1.686 0.577 1.346 2.040 2.667 3.239	3.819 2.810 1.781 0.659 1.419 2.106 2.725 3.295

Table 3.4. Values of log Re' as a function of $\log\{(R'/\rho u^2)Re'^2\}$ for spherical particles



Figure 3.6. $(R'/\rho u^2)Re'^2$ and $(R'/\rho u^2)Re'^{-1}$ versus Re' for spherical particles

Example

What is the terminal velocity of a spherical steel particle, 0.40 mm in diameter, settling in an oil of density 820 kg/m³ and viscosity 10 mN s/m²? The density of steel is 7870 kg/m³.

Solution

For a sphere:

$$\frac{R'_0}{\rho u_0^2} R e'_0^2 = \frac{2d^3(\rho_s - \rho)\rho g}{3\mu^2}$$
(equation 3.35)
$$= \frac{2 \times 0.0004^3 \times 820(7870 - 820)9.81}{3(10 \times 10^{-3})^2}$$
$$= 24.2$$
$$\log_{10} 24.2 = 1.384$$
From Table 3.4:
$$\log_{10} R e'_0 = 0.222$$
Thus:
$$R e'_0 = 1.667$$
and:
$$u_0 = \frac{1.667 \times 10 \times 10^{-3}}{820 \times 0.0004}$$
$$= 0.051 \text{ m/s or } 51 \text{ mm/s}$$

MOTION OF PARTICLES IN A CENTRIFUGAL FIELD

For a spherical particle in a fluid, the equation of motion for the Stokes' law region is:

$$\frac{\pi}{6}d^3(\rho_s - \rho)r\omega^2 - 3\pi\mu d\frac{dr}{dt} = \frac{\pi}{6}d^3\rho_s\frac{d^2r}{dt^2}$$
(3.108)

As the particle moves outwards, the accelerating force increases and therefore it never acquires an equilibrium velocity in the fluid.

If the inertial terms on the right-hand side of equation 3.108 are neglected, then:

$$\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{d^2(\rho_s - \rho)r\omega^2}{18\mu}$$

$$= \frac{d^2(\rho_s - \rho)g}{18\mu}\frac{r\omega^2}{g}$$

$$= u_0 \left(\frac{r\omega^2}{g}\right)$$
(3.109)
(3.110)

Thus, the instantaneous velocity (dr/dt) is equal to the terminal velocity u_0 in the gravitational field, increased by a factor of $r\omega^2/g$.

Thus the time taken for a particle to move to a radius r from an initial radius r_1 is given by:

$$t = \frac{18\mu}{d^2\omega^2(\rho_s - \rho)} \ln \frac{r}{r_1}$$
(3.120)

For a suspension fed to a centrifuge, the time taken for a particle initially situated in the liquid surface $(r_1 = r_0)$ to reach the wall of the bowl (r = R) is given by:

$$t = \frac{18\mu}{d^2\omega^2(\rho_s - \rho)} \ln \frac{R}{r_0}$$
(3.121)

If h is the thickness of the liquid layer at the walls then:

Then:

$$h = R - r_0$$

$$\ln \frac{R}{r_0} = \ln \frac{R}{R - h} = -\ln\left(1 - \frac{h}{R}\right)$$

$$= \frac{h}{R} + \frac{1}{2}\left(\frac{h}{R}\right)^2 + \cdots$$

If h is small compared with R, then:

$$\ln \frac{R}{r_0} \approx \frac{h}{R}$$

Equation 3.120 then becomes:

$$t = \frac{18\mu h}{d^2\omega^2(\rho_s - \rho)R}$$

(3.122)

For the Newton's law region, the equation of motion is:

$$\frac{\pi}{6}d^{3}(\rho_{s}-\rho)r\omega^{2}-0.22\frac{\pi}{4}d^{2}\rho\left(\frac{\mathrm{d}r}{\mathrm{d}t}\right)^{2}=\frac{\pi}{6}d^{3}\rho_{s}\frac{\mathrm{d}^{2}r}{\mathrm{d}t^{2}}$$

This equation can only be solved numerically. If the acceleration term may be neglected, then:

μ.

$$\left(\frac{\mathrm{d}r}{\mathrm{d}t}\right)^2 = 3d\omega^2 \left(\frac{\rho_s - \rho}{\rho}\right) r$$
$$r^{-1/2} \frac{\mathrm{d}r}{\mathrm{d}t} = \left\{3d\omega^2 \left(\frac{\rho_s - \rho}{\rho}\right)\right\}^{1/2}$$
(3.123)

Thus:

Integration gives:

$$2(r^{1/2} - r_1^{1/2}) = \left\{ 3d\omega^2 \frac{\rho_s - \rho}{\rho} \right\}^{1/2} t$$
$$t = \left[\frac{\rho}{3d\omega^2(\rho_s - \rho)} \right]^{1/2} 2(r^{1/2} - r_1^{1/2})$$
(3.124)

or:

Flow of Fluids through Granular Beds and Packed Columns

The flow of fluids through beds composed of stationary granular particles is a frequent occurrence in the chemical industry and therefore expressions are needed to predict pressure drop across beds due to the resistance caused by the presence of the particles.

Darcy's law and permeability

The first experimental work on the subject was carried out by DARCYin 1830 in Dijon when he examined the rate of flow of water from the local fountains through beds of sand of various thicknesses. It was shown that the average velocity, as measured over the whole area of the bed, was directly proportional to the driving pressure and inversely proportional to the thickness of the bed. This relation, often termed Darcy's law, has subsequently been confirmed by a number of workers and can be written as follows:

$$u_c = K \frac{(-\Delta P)}{l} \tag{4.1}$$

where $-\Delta P$ is the pressure drop across the bed,

- *l* is the thickness of the bed,
- u_c is the average velocity of flow of the fluid, defined as (1/A)(dV/dt),
- A is the total cross sectional area of the bed,
- V is the volume of fluid flowing in time t, and
- K is a constant depending on the physical properties of the bed and fluid.

The linear relation between the rate of flow and the pressure difference leads one to suppose that the flow was streamline, because the Reynolds number for the flow through the pore spaces in a granular material is low, since both the velocity of the fluid and the width of the channels are normally small. The resistance to flow then arises mainly from viscous drag. Equation 4.1 can then be expressed as:

$$u_c = \frac{K(-\Delta P)}{l} = B \frac{(-\Delta P)}{\mu l}$$
(4.2)

where μ is the viscosity of the fluid and *B* is termed the permeability coefficient for the bed, and depends only on the properties of the bed.

Specific surface and voidage

The general structure of a bed of particles can often be characterised by the specific surface area of the bed S_B and the fractional voidage of the bed e.

 S_B is the surface area presented to the fluid per unit volume of bed when the particles are packed in a bed. Its units are (length)⁻¹.

e is the fraction of the volume of the bed not occupied by solid material and is termed the fractional voidage, or porosity. It is dimensionless.

Thus the fractional volume of the bed occupied by solid material is (1 - e).

S is the specific surface area of the particles and is the surface area of a particle divided by its volume. Its units are again $(length)^{-1}$. For a sphere, for example:

$$S = \frac{\pi d^2}{\pi (d^3/6)} = \frac{6}{d}$$

It can be seen that S and S_B are not equal due to the voidage which is present when the particles are packed into a bed. If point contact occurs between particles so that only a very small fraction of surface area is lost by overlapping, then:

$$S_B = S(1-e) \tag{4.4}$$

(4.3

Streamline flow—Carman–Kozeny equation

$$u = \frac{d_t^2}{32\mu} \frac{(-\Delta P)}{l_t} \tag{4.5}$$

where: μ is the viscosity of the fluid,

- u is the mean velocity of the fluid,
- d_t is the diameter of the tube, and
- l_t is the length of the tube.

If the free space in the bed is assumed to consist of a series of tortuous channels, equation 4.5 may be rewritten for flow through a bed as:

$$u_1 = \frac{d_m'^2}{K'\mu} \frac{(-\Delta P)}{l'}$$
(4.6)

where: d'_m is some equivalent diameter of the pore channels,

K' is a dimensionless constant whose value depends on the structure of the bed,

- l' is the length of channel, and
- u_1 is the average velocity through the pore channels.

In a cube of side X, the volume of free space is eX^3 so that the mean cross-sectional area for flow is the free volume divided by the height, or eX^2 . The volume flowrate through this cube is u_cX^2 , so that the average linear velocity through the pores, u_1 , is given by:

$$u_1 = \frac{u_c X^2}{e X^2} = \frac{u_c}{e}$$
(4.7)

Although equation 4.7 is reasonably true for random packings, it does not apply to all regular packings. Thus with a bed of spheres arranged in cubic packing, e = 0.476, but the fractional free area varies continuously, from 0.215 in a plane across the diameters to 1.0 between successive layers.

For equation 4.6 to be generally useful, an expression is needed for d'_m , the equivalent diameter of the pore space. KOZENY^(5,6) proposed that d'_m may be taken as:

$$d'_{m} = \frac{e}{S_{B}} = \frac{e}{S(1 - e)}$$

$$\frac{e}{S_{B}} = \frac{\text{volume of voids filled with fluid}}{\text{wetted surface area of the bed}}$$

$$= \frac{\text{cross-sectional area normal to flow}}{\text{wetted surface area of the bed}}$$
(4.8)

where:

wetted perimeter

The hydraulic mean diameter for such a flow passage has been shown in Volume 1, Chapter 3 to be:

$$4\left(\frac{\text{cross-sectional area}}{\text{wetted perimeter}}\right)$$

It is then seen that:

$$\frac{e}{S_B} = \frac{1}{4}$$
 (hydraulic mean diameter)

Then taking $u_1 = u_c/e$ and $l' \propto l$, equation 4.6 becomes:

$$u_{c} = \frac{1}{K''} \frac{e^{3}}{S_{B}^{2}} \frac{1}{\mu} \frac{(-\Delta P)}{l}$$
$$= \frac{1}{K''} \frac{e^{3}}{S^{2}(1-e)^{2}} \frac{1}{\mu} \frac{(-\Delta P)}{l}$$
(4.9)

K'' is generally known as Kozeny's constant and a commonly accepted value for K'' is 5. As will be shown later, however, K'' is dependent on porosity, particle shape, and other factors. Comparison with equation 4.2 shows that *B* the permeability coefficient is given by:

$$B = \frac{1}{K''} \frac{e^3}{S^2 (1-e)^2}$$
(4.10)

Inserting a value of 5 for K'' in equation 4.9:

$$u_c = \frac{1}{5} \frac{e^3}{(1-e)^2} \frac{-\Delta P}{S^2 \mu l}$$
(4.11)

(equation 4.3)

For spheres: S = 6/d and:

$$u_c = \frac{1}{180} \frac{e^3}{(1-e)^2} \frac{-\Delta P d^2}{\mu l}$$
(4.12)

$$= 0.0055 \frac{e^3}{(1-e)^2} \frac{-\Delta P d^2}{\mu l}$$
(4.12*a*)

For non-spherical particles, the Sauter mean diameter d_s should be used in place of d. This is given in Chapter 1, equation 1.15.

Streamline and turbulent flow

The modified Reynolds number Re_1 is obtained by taking the same velocity and characteristic linear dimension d'_m as were used in deriving equation 4.9. Thus:

$$Re_{1} = \frac{u_{c}}{e} \frac{e}{S(1-e)} \frac{\rho}{\mu}$$

$$= \frac{u_{c}\rho}{S(1-e)\mu}$$
(4.13)

The friction factor, which is plotted against the modified Reynolds number, is $R_1/\rho u_1^2$, where R_1 is the component of the drag force per unit area of particle surface in the direction of motion. R_1 can be related to the properties of the bed and pressure gradient as follows. Considering the forces acting on the fluid in a bed of unit cross-sectional area and thickness l, the volume of particles in the bed is l(1 - e) and therefore the total surface is Sl(1 - e). Thus the resistance force is $R_1Sl(1 - e)$. This force on the fluid must be equal to that produced by a pressure difference of ΔP across the bed. Then, since the free cross-section of fluid is equal to e:

$$-\Delta P)e = R_1 Sl(1-e)$$

$$R_1 = \frac{e}{S(1-e)} \frac{(-\Delta P)}{l}$$
(4.14)

Thus

$$\frac{R_1}{\rho u_1^2} = \frac{e^3}{S(1-e)} \frac{(-\Delta P)}{l} \frac{1}{\rho u_c^2}$$
(4.15)

Carman found that when $R_1/\rho u_1^2$ was plotted against Re_1 using logarithmic coordinates, his data for the flow through randomly packed beds of solid particles could be correlated approximately by a single curve (curve A, Figure 4.1), whose general equation is:

$$\frac{R_1}{\rho u_1^2} = 5Re_1^{-1} + 0.4Re_1^{-0.1} \tag{4.16}$$

From equation 4.16 it can be seen that for values of Re_1 less than about 2, the second term is small and, approximately:

$$\frac{R_1}{\rho u_1^2} = 5Re_1^{-1} \tag{4.18}$$

SAWISTOWSKI⁽⁹⁾ compared the results obtained for flow of fluids through beds of hollow packings (discussed later) and has noted that equation 4.16 gives a consistently low result for these materials. He proposed:

$$\frac{R_1}{\rho u_1^2} = 5Re_1^{-1} + Re_1^{-0.1} \tag{4.19}$$

For flow through ring packings which as described later are often used in industrial packed columns, Ergun⁽¹⁰⁾ obtained a good semi-empirical correlation for pressure drop as follows:

$$\frac{-\Delta P}{l} = 150 \frac{(1-e)^2}{e^3} \frac{\mu u_c}{d^2} + 1.75 \frac{(1-e)}{e^3} \frac{\rho u_c^2}{d}$$
(4.20)

Writing d = 6/S (from equation 4.3):

$$\frac{-\Delta P}{Sl\rho u_c^2} \frac{e^3}{1-e} = 4.17 \frac{\mu S(1-e)}{\rho u_c} + 0.29$$
$$\frac{R_1}{\rho u_1^2} = 4.17 R e_1^{-1} + 0.29 \tag{4.21}$$

or:

Dependence of K___ on bed structure

CARMAN⁽¹⁾ has shown that:

$$K'' = \left(\frac{l'}{l}\right)^2 \times K_0 \tag{4.22}$$

where (l'/l) is the tortuosity and is a measure of the fluid path length through the bed compared with the actual depth of the bed,

 K_0 is a factor which depends on the shape of the cross-section of a channel through which fluid is passing.

K_0 is equal to 2.0, and for streamline flow through a rectangle where the ratio of the lengths of the sides is 10 : 1, $K_0 = 2.65$.

Wall effect. In a packed bed, the particles will not pack as closely in the region near the wall as in the centre of the bed, so that the actual resistance to flow in a bed of small diameter is less than it would be in an infinite container for the same flowrate per unit area of bed cross-section. A correction factor f_w for this effect has been determined experimentally by Coulson⁽¹⁵⁾. This takes the form:

$$f_w = \left(1 + \frac{1}{2}\frac{S_c}{S}\right)^2 \tag{4.23}$$

where S_c is the surface of the container per unit volume of bed.

Equation 4.9 then becomes:

$$u_c = \frac{1}{K''} \frac{e^3}{S^2 (1-e)^2} \frac{1}{\mu} \frac{(-\Delta P)}{l} f_w$$
(4.24)

Equations 4.9 and 4.16, which involve e/S_B as a measure of the effective pore diameter, are developed from a relatively sound theoretical basis and are recommended for beds of small particles when they are nearly spherical in shape. The correction factor for wall effects, given by equation 4.23, should be included where appropriate. With larger particles which will frequently be far from spherical in shape, the correlations are not so reliable. As shown in Figure 4.1, deviations can occur for rings at higher values of Re₁. Efforts to correct for nonsphericity, though frequently useful, are not universally effective, and in such cases it will often be more rewarding to use correlations, such as equation 4.19, which are based on experimental data for large packings. The values of K'' shown on Figure 4.2 apply to equation 4.24.



Figure 4.2. Variation of Kozeny's constant K" with voidage for various shapes

Fluidization

It is the operation that can change a system a solid particles into fluidlike suspension in gas or liquid. This method of contacting of this two phase mixture have some unusual characteristics that are widely used in many fields of chemical industry. Simplified diagram showing the idea of fluidization is presented in Figure below Gas is delivered from the bottom of the reactor, goes through a gas distributor to provide inform distribution through whole profile of bed and flows through packed bed of solids.



Figure 1: Schematic diagram of fluidization

- At low gas velocities the drag force is to small to lift the bed, which remains fixed. Increasing gas velocity causes solids to move upward and create fluid bed. Depending on the velocity of gas we can distinguish different modes of fluidization (Fig..2) from bubbling fluidization, through turbulent and fast fluidization modes up to pneumatic transport of solids.



FIGURE 1.2: Fluidization type depending on gas velocity

- Another important issue concerning the fluidization proces is pressure drop through a fixed bed. Fig. 1.3 presents changes in pressure drop with changing gas velocity.



- The velocity at which the pressure is stabilized is called minimum fludization velocity.

If the particles are spherical the bed can be described by means of their diameter distribution, but in real application most particles are nonspherical which yields a question about the way to decribe this kind of beds. However, the most wildly used is the one called sphericity (φs) defined as the ratio of the surface of sphere to the surface of particle with the same volume. For spherical particles φs = 1 and for other shapes $0 \le \varphi s \le 1$.

Pressure drop

Pressure drop through fixed bed of solids of uniform size (dp) of the length L is given by Ergun correlation:

$$\frac{\Delta P}{L} = 150 \frac{(1-\epsilon_m)^2}{\epsilon_m^3} \frac{\mu u_0}{(\phi_s d_p)^2} + 1.75 \frac{1-\epsilon_m}{\epsilon_m^3} \frac{\rho_g u_o^2}{\phi_s dp}$$

where μ is gas viscosity, d_p is solid diameter, ρ_g is gas density, u_0 is superficial gas velocity, and ϵ_m is the fractional voidage, which usually can be found experimentally for each specific system.

If flow conditions within the bed are streamline, the relation between fluid velocity u_c , pressure drop $(-\Delta P)$ and voidage *e* is given, for a fixed bed of spherical particles of diameter *d*, by the Carman-Kozeny equation (4.12*a*) which takes the form:

$$u_c = 0.0055 \left(\frac{e^3}{(1-e)^2}\right) \left(\frac{-\Delta P d^2}{\mu l}\right)$$
(6.2)

For a fluidised bed, the buoyant weight of the particles is counterbalanced by the frictional drag. Substituting for $-\Delta P$ from equation 6.1 into equation 6.2 gives:

$$u_{c} = 0.0055 \left(\frac{e^{3}}{1-e}\right) \left(\frac{d^{2}(\rho_{s}-\rho)g}{\mu}\right)$$
(6.3)

Minimum fluidizing velocity

At the beginning of this section one has to revise definitions of two dimensionless numbers:

Reynolds:

$$Re = \frac{d_p u_{mf} \rho_g}{\mu}$$

and Archimedes :

$$Ar = \frac{d_p^3 \rho_g (\rho_s - \rho_g) g}{\mu^2}$$

- Under fluidization conditions, pressure–drop equals effective weight of solid, as intraparticle forces disappear and solids float in the bed exhibiting 'liquid–like 'behavior. For a fluidized bed of length of L and bed-porosity of ϵ .

$$\Delta P_{bed}A_t = A_t L_{mf} (1 - \epsilon_{mf}) [(\rho_s - \rho_g)g]$$

Rearranging and combining with Ergun equation gives a quadratic in u_{mf} which can be presented in dimensionless form of the equation

$$\frac{1.75}{\epsilon_{mf}^3 \phi_s} R e_{p,mf}^2 + \frac{150(1 - \epsilon_{mf})}{\epsilon_{mf}^3 \phi_s^2} R e_{p,mf} = Ar$$

For fine particles expression proposed by Wen and Yu can be used to obtain reynolds number in minimum fludization conditions:

$$Re_{p,mf} = (33.7^2 + 0.0494Ar)^{1/2} - 33.7$$

For laminar flow and spherical particles:

$$u_{mf} = 0.0055 \left(\frac{e_{mf}^3}{1 - e_{mf}}\right) \frac{d^2(\rho_s - \rho)g}{\mu}$$

For small particles $Re_{rp,mf} < 20$

$$\Rightarrow U_{mf} = \frac{d_{p}^{2} (\rho_{p} - \rho_{f})g}{150 \ \mu_{f}} \left(\frac{\epsilon_{mf}^{3} \ \varphi_{s}^{2}}{1 - \epsilon_{mf}}\right)$$
For large particles (Re_{s,pmf} > 1000)
$$\Rightarrow U_{mf}^{2} = \frac{d_{p} (\rho_{p} - \rho_{f})g}{1.75 \ \rho_{f}} \ \epsilon_{mf}^{3} \ \varphi_{s}$$

To avoid or reduce carryover of particles form the fluidized bed, keep the gas velocity between U_{mf} and U_t . Recall

Terminal velocity, $u_t = \frac{gd_p^2(\rho_p - \rho_f)}{18\mu_f}$ for low Reynolds number and, $U_t = 1.75 \frac{\sqrt{gd_P(\rho_P - \rho_f)}}{\rho_P}$ for high Reynolds number

With the expressions for U_{mf} and U_t known for small (viscous-flow) and large (inertial flow) particles or Reynolds number, one can take the ratio of U_t and U_{mf} :

For small Re,P:
$$\frac{U_{t}}{U_{mf}} = \frac{150 (1 - \epsilon_{mf})}{18 \epsilon_{mf}^{2} \phi_{s}^{2}} = 8.33 \frac{(1 - \epsilon_{mf})}{\phi_{s}^{2} \epsilon_{mf}^{2}}$$

For spherical particles, $\phi_{\rm s}=1$ and assuming $\varepsilon_{mf}=0.45, U_t=50~U_{mf}$

Therefore, a bed that fluidizes at 1cm/s could preferably be operated with velocities < 50 cm/s, with few particles carried out or entrained with the exit gas.

For large Re, p:
$$\frac{U_t}{U_{mf}} = \frac{\frac{2.32}{1/2}}{\frac{\epsilon_{mf}}{\epsilon_{mf}}}$$

Or, $u_t = 7.7 \; u_{mf}$ for $\varepsilon_{mf} = 0.45$,

Therefore, operating safety margin in a bed of coarse particles is smaller and there is a disadvantage for the use of coarse particles in a fluidized bed.

However, make a note that the operating particle size is also decided by the other factors such as grinding cost, pressure-drop, heat and mass-transfer aspects.

Example 6.1

A bed consists of uniform spherical particles of diameter 3 mm and density 4200 kg/m³. What will be the minimum fluidising velocity in a liquid of viscosity 3 mNs/m² and density 1100 kg/m³?

Solution

By definition:

Galileo number,
$$Ga = d^3 \rho (\rho_s - \rho) g / \mu^2$$

= $((3 \times 10^{-3})^3 \times 1100 \times (4200 - 1100) \times 9.81) / (3 \times 10^{-3})^2$
= 1.003×10^5

Assuming a value of 0.4 for e_{mf} , equation 6.14 gives:

$$Re'_{mf} = 25.7\{\sqrt{(1 + (5.53 \times 10^{-5})(1.003 \times 10^{5}))} - 1\} = 40$$

and:

$$u_{mf} = (40 \times 3 \times 10^{-3})/(3 \times 10^{-3} \times 1100) = 0.0364$$
 m/s or 36.4 mm/s

Example

Oil, of density 900 kg/m³ and viscosity 3 mNs/m², is passed vertically upwards through a bed of catalyst consisting of approximately spherical particles of diameter 0.1 mm and density 2600 kg/m³. At approximately what mass rate of flow per unit area of bed will (a) fluidisation, and (b) transport of particles occur?

Solution

(a) Equations 4.9 and 6.1 may be used to determine the fluidising velocity, u_{mf} .

$$u = (1/K'')(e^3/(S^2(1-e)^2)(1/\mu)(-\Delta P/l)$$
 (equation 4.9)

$$-\Delta P = (1 - e)(\rho_s - \rho)lg$$

(equation 6.1)

where *S* = surface area/volume, which, for a sphere, = $\pi d^2/(\pi d^3/6) = 6/d$.

Substituting K'' = 5, S = 6/d and $-\Delta P/l$ from equation 6.1 into equation 4.9 gives:

$$u_{mf} = 0.0055(e^3/(1-e))(d^2(\rho_s - \rho)g)/\mu$$

Hence :

$$G'_{mf} = \rho u = (0.0055e^3/(1-e))(d^2(\rho_s - \rho)g)/\mu$$

In this problem, $\rho_s = 2600 \text{ kg/m}^3$, $\rho = 900 \text{ kg/m}^3$, $\mu = 3.0 \times 10^{-3} \text{ Ns/m}^2$ and $d = 0.1 \text{ mm} = 1.0 \times 10^{-4} \text{ m}$.

As no value of the voidage is available, e will be estimated by considering eight closely packed spheres of diameter d in a cube of side 2d. Thus:

volume of spheres = $8(\pi/6)d^3$

volume of the enclosure = $(2d)^3 = 8d^3$

and hence: voidage, $e = [8d^3 - 8(\pi/6)d^3]/8d^3 = 0.478$, say, 0.48.

Thus :

$$G'_{mf} = 0.0055(0.48)^3 (10^{-4})^2 ((900 \times 1700) \times 9.81) / ((1 - 0.48) \times 3 \times 10^{-5})$$
$$= 0.059 \text{ kg/m}^2\text{s}$$

(b) Transport of the particles will occur when the fluid velocity is equal to the terminal falling velocity of the particle.

Using Stokes' law :
$$u_0 = d^2 g(\rho_s - \rho)/18\mu$$
 (equation 3.24)
= $((10^{-4})^2 \times 9.81 \times 1700)/(18 \times 3 \times 10^{-3})$
= 0.0031 m/s

The Reynolds number = $((10^{-4} \times 0.0031 \times 900)/(3 \times 10^{-3}) = 0.093$ and hence Stokes' law applies.

The required mass flow = $(0.0031 \times 900) = 2.78 \text{ kg/m}^2\text{s}$

An alternative approach is to make use of Figure 3.6 and equation 3.35,

$$(R/\rho u^2)Re^2 = 2d^3\rho g(\rho_s - \rho)/3\mu^2$$

= (2 × (10⁻⁴)³ × (900 × 9.81) × 1700)/(3(3 × 10⁻³)²) = 1.11

From Figure 3.6, Re = 0.09

Hence: $u_0 = Re(\mu/\rho d) = (0.09 \times 3 \times 10^{-3})/(900 \times 10^{-4}) = 0.003 \text{ m/s}$ and: $G' = (0.003 \times 900) = 2.7 \text{ kg/m}^2 \text{s}$

Filtration

- Removal of solids from fluid (gas or liquid) by a filtering medium on which solid particles are deposited.
- For filtration, external force is applied to a (gas or liquid + solid) mixture to make it flow through the medium.

- Filtration, when applied to gas cleaning, usually refers to the removal of fine particles like dust from air or flue gas. In such case, a polymeric fiber or cloth is wrapped over a pretreated metallic cylinder, capable of capturing micron size particles, including soot and fly-ash.

- Very large size ceramic based filters for high temperatur applications are also commercially available.

- The liquid-solid filtration is often called "cake-filtration", because the separation of solids from the slurry by the filtering medium is effective during the initial stages of filtration. Later, the 'cakes' or deposits collected over the medium act as the filter. Therefore, cake thickness increases during filtration and the resistance (hydraulic) offered by the cake-material is larger than that by the filtering medium.

There are two types of operation:

a. Constant-pressure

b. Constant filtering rate

In the 1st case, filtering rate varies with time, whereas in the 2nd case, pressure–drop increases with time.

For ideal cake filtration, cake should be stable and large porosity. There are two common types of filters:

a. The plate and frame press

b. Rotary-drum filter

Thus, the main factors to be considered when selecting equipment and operating conditions are:

(a) The properties of the fluid, particularly its viscosity, density and corrosive properties.

(b) The nature of the solid—its particle size and shape, size distribution, and packing characteristics.

(c) The concentration of solids in suspension.

(d) The quantity of material to be handled, and its value.

(e) Whether the valuable product is the solid, the fluid, or both.

(f) Whether it is necessary to wash the filtered solids.

(g) Whether very slight contamination caused by contact of the suspension or filtrate with the various components of the equipment is detrimental to the product.

(h) Whether the feed liquor may be heated.

(i) Whether any form of pretreatment might be helpful.



Principle of filtration

The most important factors on which the rate of filtration depends will be:
- (a) The drop in pressure from the feed to the far side of the filter medium.
- (b) The area of the filtering surface.
- (c) The viscosity of the filtrate.
- (d) The resistance of the filter cake.
- (e) The resistance of the filter medium and initial layers of cake.

FILTRATION THEORY

Because the particles forming the cake are small and the flow through the bed is slow, streamline conditions are almost invariably obtained, and, at any instant, the flowrate of the filtrate may be represented by the following form of equation:

$$u_c = \frac{1}{A} \frac{dV}{dt} = \frac{1}{5} \frac{e^3}{(1-e)^2} \frac{-\Delta P}{S^2 \mu l}$$
(7.1)

where V is the volume of filtrate which has passed in time t, A is the total cross-sectional area of the filter cake, uc is the superficial velocity of the filtrate, l is the cake thickness, S is the specific surface of the particles, e is the voidage, μ is the viscosity of the filtrate, and P is the applied pressure difference.



Mechanisms of filtration: (a) cake filter; (b) clarifying filter; (c) crossflow filter.

The initial stages in the formation of the cake are therefore of special importance for the following reasons:

(a) For any filtration pressure, the rate of flow is greatest at the beginning of the process since the resistance is then a minimum.

(b) High initial rates of filtration may result in plugging of the pores of the filter cloth and cause a very high resistance to flow.

(c) The orientation of the particle in the initial layers may appreciably influence the structure of the whole filter cake.

For incompressible cakes e in equation 7.1 may be taken as constant and the quantity $e^3/[5(1 - e)^2S^2]$ is then a property of the particles forming the cake and should be constant for a given material.

Thus:

$$\frac{1}{A} \frac{dV}{dt} = \frac{-\Delta P}{\mathbf{r}\mu l}$$
(7.2)
where:

$$\mathbf{r} = \frac{5(1-e)^2 S^2}{e^3}$$
(7.3)

For incompressible cakes, \mathbf{r} is taken as constant, although it depends on rate of deposition, the nature of the particles, and on the forces between the particles. \mathbf{r} has the dimensions of \mathbf{L}^{-2} and the units \mathbf{m}^{-2} in the SI system.

If v is the volume of cake deposited by unit volume of filtrate then:

$$v = \frac{lA}{V}$$
 or $l = \frac{vV}{A}$ (7.6)

Substituting for *l* in equation 7.2:

$$\frac{1}{A}\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{(-\Delta P)}{\mathbf{r}\mu}\frac{A}{vV}$$
$$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{A^2(-\Delta P)}{\mathbf{r}\mu vV}$$
(7.8)

or:

Equation 7.8 may be regarded as the basic relation between -P, V, and t. Two important types of operation are: (i) where the pressure

difference is maintained constant and (ii) where the rate of filtration is maintained constant.

For a filtration at constant rate

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{V}{t} = \text{constant}$$

$$\frac{V}{t} = \frac{A^2(-\Delta P)}{\mathbf{r}\mu Vv} \tag{7.9}$$

$$\frac{t}{V} = \frac{\mathbf{r}\mu v}{A^2(-\Delta P)}V \tag{7.10}$$

or:

so that:

and $-\Delta P$ is directly proportional to V.

For a filtration at constant pressure difference

$$\frac{V^2}{2} = \frac{A^2(-\Delta P)t}{\mathbf{r}\mu v} \tag{7.11}$$

(7.12)

or:

Thus for a constant pressure filtration, there is a linear relation between V_2 and t or between t/V and V.

 $\frac{t}{V} = \frac{\mathbf{r}\mu v}{2A^2(-\Delta P)}V$

Filtration at constant pressure is more frequently adopted in practice, although the pressure difference is normally gradually built up to its ultimate value.

If this takes a time t_1 during which a volume V_1 of filtrate passes, then integration of equation 7.12 gives:

$$\frac{1}{2}(V^2 - V_1^2) = \frac{A^2(-\Delta P)}{\mathbf{r}\mu v}(t - t_1)$$
Or
(7.13)

$$\frac{t - t_1}{V - V_1} = \frac{\mathbf{r}\mu v}{2A^2(-\Delta P)}(V - V_1) + \frac{\mathbf{r}\mu v V_1}{A^2(-\Delta P)}$$
(7.14)

Thus, there where is a linear relation between V 2 and t and between $(t - t_1)/(V - V_1)$ and $(V - V_1)$, where $(t - t_1)$ represents the time of the constant pressure filtration and $(V - V_1)$ the corresponding volume of filtrate obtained.

Flow of filtrate through the cloth and cake combined

If the filter cloth and the initial layers of cake are together equivalent to a thickness *L* of cake as deposited at a later stage in the process, and if $-\Delta P$ is the pressure drop across the cake and cloth combined, then:

$$\frac{1}{A}\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{(-\Delta P)}{\mathbf{r}\mu(l+L)}$$
(7.15)

which may be compared with equation 7.2.

Thus:

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{A(-\Delta P)}{\mathbf{r}\mu\left(\frac{Vv}{A} + L\right)} = \frac{A^2(-\Delta P)}{\mathbf{r}\mu v\left(V + \frac{LA}{v}\right)}$$
(7.16)

This equation may be integrated between the limits t = 0, V = 0 and $t = t_1$, $V = V_1$ for constant rate filtration, and $t = t_1$, $V = V_1$ and t = t, V = V for a subsequent constant pressure filtration.

For the period of *constant rate filtration*:

or: $\frac{V_1}{t_1} = \frac{A^2(-\Delta P)}{\mathbf{r}\mu v \left(V_1 + \frac{LA}{v}\right)}$ or: $\frac{t_1}{V_1} = \frac{\mathbf{r}\mu v}{A^2(-\Delta P)}V_1 + \frac{\mathbf{r}\mu L}{A(-\Delta P)}$ or: $V_1^2 + \frac{LA}{v}V_1 = \frac{A^2(-\Delta P)}{\mathbf{r}\mu v}t_1$ (7.17)

For a subsequent constant pressure filtration:

$$\frac{1}{2}(V^2 - V_1^2) + \frac{LA}{v}(V - V_1) = \frac{A^2(-\Delta P)}{\mathbf{r}\mu v}(t - t_1)$$
(7.18)

(7.19)

$$(V - V_1 + 2V_1)(V - V_1) + \frac{2LA}{v}(V - V_1) = \frac{2A^2(-\Delta P)}{\mathbf{r}\mu v}(t - t_1)$$

or:

$\frac{t-t_1}{V-V_1} = \frac{\mathbf{r}\mu v}{2A^2(-\Delta P)}(V-V_1) + \frac{\mathbf{r}\mu vV_1}{A^2(-\Delta P)} + \frac{\mathbf{r}\mu L}{A(-\Delta P)}$

Washing of the filter cake

Washing may be regarded as taking place in two stages. First, filtrate is displaced from the filter cake by wash liquid during the period of displacement washing and in this way up to 90 per cent of the filtrate may be removed. During the second stage, diffusional

washing, solvent diffuses into the wash liquid from the less accessible voids and the following relation applies:

$$\left(\frac{\text{volume of wash liquid passed}}{\text{cake thickness}}\right) = (\text{constant}) \times \log\left(\frac{\text{initial concentration of solute}}{\text{concentration at particular time}}\right)$$

Membrane Separation Processes

Whilst effective product separation is crucial to economic operation in the process industries, certain types of materials are inherently difficult and expensive to separate. Important examples include:

(a) Finely dispersed solids, especially those which are compressible, and a density close to that of the liquid phase, have high viscosity, or are gelatinous.

(b) Low molecular weight, non-volatile organics or pharmaceuticals and dissolved salts.

(c) Biological materials which are very sensitive to their physical and chemical environment.

The processing of these categories of materials has become increasingly important in recent years, especially with the growth of the newer biotechnological industries and with the increasingly sophisticated nature of processing in the food industries. When difficulties arise in the processing of materials of biological origin, it is worth asking, how does nature solve the problem? The solution which nature has developed is likely to be both highly effective and energy efficient, though it may be slow in process terms. Nature separates biologically active materials by means of membranes. a membrane may be defined as "an interphase separating two phases and selectively controlling the transport of materials between those phases". A membrane is an interphase rather than an interface because it occupies a finite, though normally small, element of space. Human beings are all surrounded by a membrane, the skin, and membranes control the separation of materials at all levels of life, down to the outer layers of bacteria and subcellular components.

As discussed by LONSDALE(2), since the 1960s a new technology using synthetic membranes for process separations has been rapidly developed by materials scientists, physical chemists and chemical engineers. Such membrane separations have been widely applied to a range of conventionally difficult separations. They potentially offer the advantages of ambient temperature operation, relatively low capital and running costs, and modular construction. In this chapter, the nature and scope of membrane separation processes are outlined, and then those processes most frequently used industrially are described more fully.

CLASSIFICATION OF MEMBRANE PROCESSES

Industrial membrane processes may be classified according to the size range of materials which they are to separate and the driving force used in separation.

Name of process	Driving force	Separation size range	Examples of materials separated
Microfiltration	Pressure gradient	10-0.1 μm	Small particles, large colloids, microbial cells
Ultrafiltration	Pressure gradient	<0.1 µm-5 nm	Emulsions, colloids, macromolecules, proteins
Nanofiltration	Pressure gradient	~1 nm	Dissolved salts, organics
Reverse osmosis (hyperfiltration)	Pressure gradient	<1_nm	Dissolved salts, small organics
Electrodialysis	Electric field gradient	<5 nm	Dissolved salts
Dialysis	Concentration gradient	<5 nm	Treatment of renal failure

Table 8.1. Classification of membrane separation processes for liquid systems

THE NATURE OF SYNTHETIC MEMBRANES

Membranes used for the pressure-driven separation processes, microfiltration, ultrafiltration and reverse osmosis, as well as those used for dialysis, are most commonly made of polymeric materials. Initially most such membranes were cellulosic in nature. These are now being replaced by polyamide, polysulphone, polycarbonate and a number of other advanced polymers. These synthetic polymers have improved chemical stability and better resistance to microbial degradation. Membranes have most commonly been produced by a form of phase inversion known as immersion precipitation. This process has four main steps:

(a) the polymer is dissolved in a solvent to 10–30 per cent by mass,

(b) the resulting solution is cast on a suitable support as a film of thickness, approximately $100 \ \mu m$,

(c) the film is quenched by immersion in a non-solvent bath, typically

water or an aqueous solution,

(d) the resulting membrane is annealed by heating. The third step gives a polymer-rich phase forming the membrane, and a polymer-depleted phase forming the pores.

- A parameter often quoted in manufacturer's literature is the nominal molecular weight cut-off (MWCO) of a membrane. This is based on studies of how solute molecules are rejected by membranes. {The nominal molecular weight cut-off is normally defined as the molecular weight of a solute for which R = 0.95. Values of MWCO typically lie in the range 2000–100,000 kg/kmol with values of the order of 10,000 being most common}. A solute will pass through a membrane if it is sufficiently small to pass through a pore, if it does not significantly interact with the membrane and if it does not interact with other, larger solutes. It is possible to define a solute rejection coefficient R by:

 $R = 1 - (C_p/C_f)$

where C_f is the concentration of solute in the feed stream and C_p is the concentration of solute in the permeate.



Figure 8.3. Dependence of rejection coefficient on molecular weight for ultrafiltration membranes

GENERAL MEMBRANE EQUATION

The general membrane equation is an attempt to state the factors which may be important in determining the membrane permeation rate for pressure driven processes. This takes the form:

$$J = \frac{|\Delta P| - |\Delta \Pi|}{(R_m + R_c)\mu}$$
(8.2)

where J is the membrane flux^{*}, expressed as volumetric rate per unit area, $|\Delta P|$ is the pressure difference applied across the membrane, the transmembrane pressure, $\Delta \Pi$ is the difference in osmotic pressure across the membrane, R_m is the resistance of the membrane, and R_c is the resistance of layers deposited on the membrane, the filter cake and gel foulants. If the membrane is only exposed to pure solvent, say water, then equation 8.2 reduces to $J = |\Delta P|/R_m\mu$. For microfiltration and ultrafiltration membranes where solvent flow is most often essentially laminar through an arrangement of tortuous channels, this is analogous to the Carman–Kozeny equation discussed in Chapter 4.

CROSS-FLOW MICROFILTRATION

The concept of *cross-flow* microfiltration, described by BERTERA, STEVEN and $METCALFE^{(4)}$, is shown in Figure 8.4 which represents a cross-section through a rectangular or tubular membrane module. The particle-containing fluid to be filtered is pumped at a velocity in the range 1–8 m/s parallel to the face of the membrane and with a pressure difference of 0.1–0.5 MN/m² (MPa) across the membrane. The liquid permeates through the membrane and the feed emerges in a more concentrated form at the exit of the module.



Figure 8.4. The concept of cross-flow filtration⁽⁴⁾

All of the membrane processes listed in Table 8.1 are operated with such a cross-flow of the process feed. The advantages of cross-flow filtration over conventional filtration are:

- (a) A higher overall liquid removal rate is achieved by prevention of the formation of an extensive filter cake.
- (b) The process feed remains in the form of a mobile slurry suitable for further processing.
- (c) The solids content of the product slurry may be varied over a wide range.
- (d) It may be possible to fractionate particles of different sizes.

A flow diagram of a simple cross-flow system⁽⁴⁾ is shown in Figure 8.5. This is the system likely to be used for batch processing or development rigs and is, in essence, a basic pump recirculation loop. The process feed is concentrated by pumping it from the tank and across the membrane in the module at an appropriate velocity. The partially concentrated *retentate* is recycled into the tank for further processing while the *permeate* is stored or discarded as required. In cross-flow filtration applications, product washing is frequently necessary and is achieved by a process known as *diafiltration* in which wash water is added to the tank at a rate equal to the permeation rate.

In practice, the membrane permeation rate falls with time due to membrane fouling; that is blocking of the membrane surface and pores by the particulate materials, as shown in Figure 8.6. The rate of fouling depends on the nature of the materials being processed, the nature of the membrane, the cross-flow velocity and the applied pressure. For example, increasing the cross-flow velocity results in a decreased rate of fouling. Backflushing



Figure 8.6. The time-dependence of membrane permeation rate during cross-flow filtration: (a) Low cross-flow velocity, (b) Increased cross-flow velocity, (c) Backflushing at the bottom of each "saw-tooth"the membrane using permeate is often used to control fouling as shown in Figure 8.6c.

Ideally, cross-flow microfiltration would be the pressure-driven removal of the process liquid through a porous medium without the deposition of particulate material. The flux decrease occurring during cross-flow microfiltration shows that this is not the case. If the decrease is due to particle deposition resulting from incomplete removal by the cross-flow liquid, then a description analogous to that of generalised cake filtration theory, discussed in Chapter 7, should apply. Equation 8.2 may then be written as:

$$J = \frac{|\Delta P|}{(R_m + R_c)\mu}$$
(8.3)

where R_c now represents the resistance of the cake, which if all filtered particles remain in the cake, may be written as:

$$R_c = \frac{\mathbf{r}VC_b}{A_m} = \frac{\mathbf{r}V_s}{A_m} \tag{8.4}$$

where **r** is the specific resistance of the deposit, V the total volume filtered, V_s the volume of *particles* deposited, C_b the bulk concentration of particles in the feed (particle volume/feed volume) and A_m the membrane area. The specific resistance may theoretically be related to the particle properties for spherical particles by the Carman relationship, discussed in Chapter 4, as:

$$\mathbf{r} = 180 \left(\frac{1-e}{e^3}\right) \left(\frac{1}{d_s^2}\right) \tag{8.5}$$

where e is the void volume of the cake and d_s the mean particle diameter.

Combining equations 8.3 and 8.4 gives:

$$J = \frac{1}{A_m} \frac{\mathrm{d}V}{\mathrm{d}t} = \frac{|\Delta P|}{(R_m + \mathbf{r}VC_b/A_m)\mu}$$
(8.6)

Solution of equation 8.6 for V at constant pressure gives:

$$\frac{t}{V} = \frac{R_m \mu}{|\Delta P|A_m} + \frac{C_b \mathbf{r} \mu V}{2|\Delta P|A_m^2}$$
(8.7)

yielding a straight line on plotting t/V against V.

ULTRAFILTRATION

Ultrafiltration is one of the most widely used of the pressure-driven membrane separation processes. The solutes retained or rejected by ultrafiltration membranes are those with molecular weights of 10³ or greater, depending mostly on the MWCO of the membrane chosen. The process liquid, dissolved salts and low molecular weight organic molecules (500–1000 kg/kmol) generally pass through the membrane. The pressure difference applied across the membrane is usually in the range 0.1–0.7 MN/m² and membrane permeation rates are typically 0.01–0.2 m³/m² h. In industry, ultrafiltration is always operated in the cross-flow mode.

The separation of process liquid and solute that takes place at the membrane during ultrafiltration gives rise to an increase in solute concentration close to the membrane surface, as shown in Figure 8.7. This is termed concentration polarisation and takes place within the boundary film generated by the applied cross-flow. With a greater concentration at the membrane, there will be a tendency for solute to diffuse back into the bulk feed according to Fick's Law, discussed in Volume 1, Chapter 10. At steady state, the rate of back-diffusion will be equal to the rate of removal of solute at the membrane, minus the rate of solute leakage through the membrane:



Figure 8.7. Concentration polarisation at a membrane surface

Here solute concentrations C and C_p in the permeate are expressed as mass fractions, D is the diffusion coefficient of the solute and y is the distance from the membrane. Rearranging and integrating from $C = C_f$ when y = l the thickness of the film, to C = C_w , the concentration of solute at the membrane wall, when y = 0, gives:

$$-\int_{C_w}^{C_f} \frac{\mathrm{d}C}{C - C_p} = \frac{J}{D} \int_0^l \mathrm{d}y \tag{8.12}$$

or:

$$\frac{C_w - C_p}{C_f - C_p} = \exp\left(\frac{Jl}{D}\right) \tag{8.13}$$

If it is further assumed that the membrane completely rejects the solute, that is, R = 1and $C_p = 0$, then:

$$\frac{C_w}{C_f} = \exp\left(\frac{Jl}{D}\right) \tag{8.14}$$

where the ratio C_w/C_f is known as the polarisation modulus. It may be noted that it has been assumed that l is independent of J and that \underline{D} is constant over the whole range of C at the interface. The film thickness is usually incorporated in an overall mass transfer coefficient h_D , where $h_D = D/l$, giving:

$$J = h_D \ln\left(\frac{C_w}{C_f}\right) \tag{8.15}$$

The mass transfer coefficient is usually obtained from correlations for flow in nonporous ducts. One case is that of laminar flow in channels of circular cross-section where the parabolic velocity profile is assumed to be developed at the channel entrance. Here the solution of Lévêque⁽⁷⁾, discussed by BLATT et al.⁽⁸⁾, is most widely used. This takes the form:

$$Sh = 1.62 \left(Re \, Sc \, \frac{d_m}{L} \right)^{1/3} \tag{8.16}$$

where Sh is the Sherwood number $(h_D d_m/D), d_m$ is the hydraulic diameter, L is the channel length, Re is the Reynolds number $(ud_m\rho/\mu)$, Sc the Schmidt number $(\mu/\rho D)$, with u being the cross-flow velocity, ρ the fluid density and μ the fluid viscosity. This gives:

$$h_D = 1.62 \left(\frac{u D^2}{d_m L}\right)^{1/3}$$
(8.17)

or for tubular systems:

$$h_D = 0.81 \left(\frac{\dot{\gamma}}{L} D^2\right)^{1/3} \tag{8.18}$$

where $\dot{\gamma}$, the shear rate at the membrane surface equals $8u/d_m$, as shown in Volume 1, Chapter 3.

For the case of turbulent flow the DITTUS-BOELTER⁽⁹⁾ correlation given in Volume 1, Chapters 9 and 10, is used:

$$Sh = 0.023 Re^{0.8} Sc^{0.33}$$
(8.19)

which for tubular systems gives:

$$h_D = 0.023 \frac{u^{0.8} D^{0.67}}{d_m^{0.2}} \left(\frac{\rho}{\mu}\right)^{0.47} \tag{8.20}$$

and for thin rectangular flow channels, with channel height b:

$$h_D = 0.02 \frac{u^{0.8} D^{0.67}}{b^{0.2}} \left(\frac{\rho}{\mu}\right)^{0.47} \tag{8.21}$$

For both laminar and turbulent flow it is clear that the mass transfer coefficient and hence the membrane permeation rate may be increased, where these equations are valid, by increasing the cross-flow velocity or decreasing the channel height. The effects are greatest for turbulent flow. For laminar flow the mass transfer coefficient is decreased if the channel length is increased. This is due to the boundary layer increasing along the membrane module. The mass transfer coefficient is, therefore, averaged along the membrane length.

This boundary-layer theory applies to mass-transfer controlled systems where the membrane permeation rate is independent of pressure, for there is no pressure term in the model. In such cases it has been proposed that, as the concentration at the membrane increases, the solute eventually precipitates on the membrane surface. This layer of precipitated solute is known as the *gel-layer*, and the theory has thus become known as the *gel-polarisation* model proposed by MICHAELS⁽¹⁰⁾. Under such conditions C_w in equation 8.15 becomes replaced by a constant C_G the concentration of solute in the gel-layer, and:

$$J = h_D \ln\left(\frac{C_G}{C_f}\right) \tag{8.22}$$

Example 8.1

Obtain expressions for the optimum concentration for minimum process time in the diafiltration of a solution of protein content S in an initial volume V_0 .

(a) If the gel-polarisation model applies.

It may be assumed that the extent of diafiltration is given by:

$$V_d = \frac{\text{Volume of liquid permeated}}{\text{Initial feed volume}} = \frac{V_p}{V_0}$$

Solution

(a) Assuming the gel-polarisation model applies

The membrane permeation rate, $J = h_D \ln(C_G/C_f)$

(equation 8.22)

where C_G and C_f are the gel and the bulk concentrations respectively.

In this case: $C_f = S/V_0$

and the volume V_d liquid permeated, $V_p = V_d S / C_f$

The process time per unit area, $t = V_p/J$

$$= V_d S / (C_f h_D \ln(C_G / C_f))$$

Assuming C_f and h_D are constant, then:

$$dt/dC_f = -V_d S/[h_D C_f^2 \ln(C_G/C_f)] + V_d S/\{h_D C_f^2 [\ln(C_G/C_f)]^2\}$$

If, at the optimum concentration C_f^* and $dt/dC_f = 0$, then:

 $1 = \ln(C_G/C_f^*)$

and:

REVERSE OSMOSIS

A classical demonstration of osmosis is to stretch a parchment membrane over the mouth of a tube, fill the tube with a sugar solution, and then hold it in a beaker of water. The level of solution in the tube rises gradually until it reaches a steady level. The static head developed would be equivalent to the osmotic pressure of the solution if the parchment were a perfect semipermeable membrane, such a membrane having the property of allowing the solvent to pass through but preventing the solute from passing through.

The pure solvent has a higher chemical potential than the solvent in the solution and so diffuses through until the difference is cancelled out by the pressure head. If an additional pressure is applied to the liquid column on the solution side of the membrane then it is possible to force water back through the membrane. This pressure-driven transport of water from a solution through a membrane is known as *reverse osmosis*. It may be noted that it is not quite the reverse of osmosis because, for all real membranes, there is always a certain transport of the solute along its chemical potential gradient, and this is not reversed. The phenomenon of reverse osmosis has been extensively developed as an industrial process for the concentration of low molecular weight solutes and especially for the desalination, or more generally demineralisation, of water.

Many models have been developed to explain the semi-permeability of reverse osmosis membranes and to rationalise the observed behaviour of separation equipment. These have included the postulation of preferential adsorption of the solute at the solution–membrane interface, hydrogen bonding of water in the membrane structure, and the exclusion of ions by the membrane due to dielectric effects. They are all useful in explaining aspects of membrane behaviour, although the most common approach has been to make use of the theories of the thermodynamics of irreversible processes proposed by SPIEGLER and KEDEM⁽²⁰⁾. This gives a phenomenological description of the relative motion of solution components within the membrane, and does not allow for a microscopic explanation of the flow and rejection properties of the membrane. In the case of reverse osmosis however, the thermodynamic approach is combined with a macroscopic *solution–diffusion* description of membrane transport as discussed by SOLTANIEH and GILL⁽²¹⁾. This implies that the membrane is non-porous and that solvent and solutes can only be transported across the membrane by first dissolving in, and subsequently diffusing through, the membrane.

For any change to occur a chemical potential gradient must exist. For a membrane system, such as the one under consideration, HAASE⁽²²⁾ and BELFORT⁽²³⁾ have derived the following simplified equation for constant temperature:

$$d\mu_i = v_i dP + \left(\frac{\partial \mu_i}{\partial C_i}\right)_T dC_i + z_i \mathbf{F} d\phi$$
(8.30)

where μ_i is the chemical potential of component *i*, v_i is the partial molar volume of component *i*, z_i is the valence of component *i*, ϕ is the electrical potential and **F** is Faraday's constant. This equation may be applied to any membrane process. For ultrafiltration, only the pressure forces are usually considered. For electrodialysis, the electrical and concentration forces are more important, whereas, in the present case of reverse osmosis both pressure and concentration forces need to be considered. Integrating across the thickness of the membrane for a two-component system with subscript 1 used to designate the solvent (water) and subscript 2 used to designate the solute, for the solvent:

$$\Delta \mu_{1} = \int \left(\frac{\partial \mu_{1}}{\partial C_{1}}\right)_{P,T} dC_{1} + \int v_{1} dP$$
$$= \int \left(\frac{\partial \mu_{1}}{\partial C_{2}}\right)_{P,T} dC_{2} + \int v_{1} dP \qquad (8.31)$$

When $\Delta \mu_1$ becomes small, only the osmotic pressure difference $\Delta \Pi$ remains. Thus, for constant v_1 :

$$\Delta \mu_1 = v_1(|\Delta P| - |\Delta \Pi|) \tag{8.32}$$

For the solute:

$$\Delta \mu_2 = \int \left(\frac{\partial \mu_2}{\partial C_2}\right)_{P,T} \mathrm{d}C_2 + \int v_2 \,\mathrm{d}P \tag{8.33}$$

and for dilute solutions, $\mu_2 = \mu_2^0 + \mathbf{R}T \ln C_2$, and for constant v_2 :

$$\Delta \mu_2 = \mathbf{R} T \Delta \ln \mathbf{C}_2 + v_2 |\Delta P| \tag{8.34}$$

where the second term on the right-hand side is negligible compared with the first term. In the present case:

$$\Delta \mu_2 = \mathbf{R}T \Delta \ln C_2 \approx \left(\frac{\mathbf{R}T}{C_2}\right) \Delta C_2 \text{ (for low values of } C_2)$$
(8.35)

Incorporating the model of diffusion across the membrane, and writing Fick's law in the generalised form⁽²⁴⁾, (using $\mu = \mu^0 + \mathbf{R}T \ln C$)*:

$$J = -\frac{DC}{\mathbf{R}T}\frac{\mathrm{d}\mu}{\mathrm{d}y} \tag{8.36}$$

where *y* is distance in the direction of transfer.

It is found for the solvent that:

$$J_1 = K_1(|\Delta P| - |\Delta \Pi|) \tag{8.37}$$

where the permeability coefficient is described in terms of a diffusion coefficient, water concentration, partial molar volume of water, absolute temperature and effective membrane thickness. For the solute it is found that:

$$J_2 = K_2 |\Delta C_2| \tag{8.38}$$

where K_2 is described in terms of a diffusion coefficient, distribution coefficient and effective membrane thickness. It is clear from these equations that solvent (water) flow only occurs if $|\Delta P| > |\Delta \Pi|$, though solute flow is independent of $|\Delta P|$. Thus, increasing the operating pressure increases the effective separation. This explains why reverse osmosis plants operate at relatively high pressure. For example, the osmotic pressure of brackish water containing 1.5–12 kg/m³ salts is 0.1–0.7 MN/m² (MPa) and the osmotic pressure of sea water containing 30–50 kg/m³ salts is 2.3–3.7 MN/m² (MPa). In practice, desalination plants operate at 3–8 MN/m² (MPa).

MEMBRANE MODULES AND PLANT CONFIGURATION

Membrane equipment for industrial scale operation of microfiltration, ultrafiltration and reverse osmosis is supplied in the form of modules. The area of membrane contained in these basic modules is in the range $1-20 \text{ m}^2$. The modules may be connected together in series or in parallel to form a plant of the required performance. The four most common types of membrane modules are tubular, flat sheet, spiral wound and hollow fibre, as shown in Figures 8.9–8.12.



Figure 8.9. Tubular module



Figure 8.10. Schematic diagram of flat-sheet module



Figure 8.11. Schematic diagram of spiral-wound module



Figure 8.12. (a) Hollow-fibre module and, (b), a single fibre

Membrane modules can be configured in various ways to produce a plant of the required separation capability. A simple batch recirculation system has already been described in Section 8.5. Such an arrangement is most suitable for small-scale batch operation, but larger scale plants will operate either as *feed and bleed* or *continuous single-pass* operations, as shown in Figure 8.13.



Figure 8.13. Schematic flow-diagrams of (a) Single-stage "feed and bleed", (b) Multiple-stage "feed and bleed", (c) Continuous single-pass membrane plants

Example 8.2

As part of a downstream processing sequence, 10 m³ of a process fluid containing 20 kg m⁻³ of an enzyme is to be concentrated to 200 kg/m³ by means of ultrafiltration. Tests have shown that the enzyme is completely retained by a 10,000 MWCO surface-modified polysulphone membrane with a filtration flux given by:

$$J = 0.04 \ln(250/C_f)$$

where J is the flux in m/h and C_b is the enzyme concentration in kg m⁻³. Four hours is available for carrying out the process.

(a) Calculate the area of membrane needed to carry out the concentration as a simple batch process, (b) Use the following approximation for estimating the average flux during a simple batch process:

$$J_{av} = J_f + 0.27(J_i - J_f)$$

where J_{av} is the average flux, J_i is the initial flux and J_f is the final flux. Is this approximation suitable for design purposes in the present case?

Solution

(a) From the gel-polarisation model:

$$J = \frac{1}{A} \frac{\mathrm{d}V}{\mathrm{d}t} = h_D \ln\left(\frac{C_G}{C_f}\right)$$
$$C_f = C_0 \left(\frac{V_0}{V}\right)$$

Also:

where C_0 and V_0 are the initial concentration and volume, respectively and C_f and V are the values at subsequent times.

Combining these equations gives:

$$\frac{\mathrm{d}V}{\mathrm{d}t} = A \left(h_D \ln\left(\frac{C_G}{C_0}\right) - h_D \ln\left(\frac{V_0}{V}\right) \right)$$
$$\int_{V_0}^{V_t} \frac{\mathrm{d}V}{\left(J_0 - h_D \ln\left(\frac{V_0}{V}\right)\right)} = A \int_0^t \mathrm{d}t$$

MEMBRANE SEPARATION PROCESSES

V	$\left(J_0 - h_D \ln(V_0/V)\right)^{-1}$			
10	9.90			
5	13.64			
3	18.92			
2	27.30			
1	112.40			

The data are plotted in Figure 8.14.



Figure 8.14. Graphical integration for Example 8.2

The area under the curve = 184.4Operation for four hours gives 46.1 m^2 membrane area

(b) $J_0 = 0.04 \ln(250/20) = 0.101 \text{ m/h}$ $J_f = 0.04 \ln(250/200) = 0.008 \text{ m/h}$ $J_{av} = 0.008 + 0.27(0.101 - 0.008) = 0.033 \text{ m/h}$

For the removal of 9 m³ filtrate in 4 hours: Area = $(9/4)/0.033 = 68.2 \text{ m}^2$ membrane The approximation is not suitable for design purposes.

Example 8.3

An ultrafiltration plant is required to treat 50 m³/day of a protein-containing waste stream. The waste contains 0.5 kg/m³ of protein which has to be concentrated to 20 kg/m³ so as to allow recycling to the main process stream. The tubular membranes to be used are available as 30 m² modules. Pilot plant studies show that the flux J through these membranes is given by:

$$J = 0.02 \ln \left(\frac{30}{C_f}\right) \qquad \text{m/h}$$

where C_f is the concentration of protein in kg/m³. Due to fouling, the flux never exceeds 0.04 m/h.

Estimate the minimum number of membrane modules required for the operation of this process (a) as a single *feed and bleed* stage, and (b) as two *feed and bleed* stages in series. Operation for 20 h/day may be assumed.

Solution

(a) with a single *feed and bleed* stage, the arrangement is shown in Figure 8.15:



Figure 8.15. Single 'feed and bleed' stage

It is assumed that Q_0 is the volumetric flowrate of feed, Q_2 the volumetric flowrate of concentrate, C_0 the solute concentration in the feed, C_2 the solute concentration in the concentrate, F the volumetric flowrate of membrane permeate, and A the required membrane area. It is also assumed that there is no loss of solute through the membrane.

The concentration (C_l) at which the flux becomes fouling-limited is:

$$0.04 = 0.02 \ln \left(\frac{30}{C_l}\right)$$
$$C_l \approx 4 \text{ kg/m}^3$$

or:

That is, below this concentration the membrane flux is 0.04 m/h.

This does not pose a constraint for the single stage as the concentration of solute C_2 will be that of the final concentrate, 20 kg/m³.

 $Q_0 C_0 = Q_2 C_2$

 $Q_0 = F + Q_2$

Conservation of solute gives:

A fluid balance gives:

(i)

(ii)

Combining these equations and substituting known values:

 $2.438 = A \, 0.02 \ln \left(\frac{30}{20}\right)$ $\underline{A} = 301 \, \mathrm{m}^2$

and:

Thus, <u>10 modules</u> will almost meet the specification for the single-stage process.

(b) with two feed and bleed stages in series, the arrangement is shown in Figure 8.16:



Figure 8.16. Two 'feed and bleed' stages in series

In addition to the symbols previously defined, Q_1 will be taken as the volumetric flowrate of retenate at the intermediate point, C_1 the concentration of solute in the retentate at this point, F_1 and F_2 the volumetric flowrates of membrane permeate in the first and second stages respectively, and A_1 and A_2 the required membrane areas in these respective stages.

Conservation of solute gives:

$$Q_0 C_0 = Q_1 C_1 = Q_2 C_2 \tag{iii}$$

A fluid balance on stage 1 gives:

$$Q_0 = Q_1 + F_1 \tag{iv}$$

A fluid balance on stage 2 gives:

$$Q_1 = Q_2 + F_2 \tag{v}$$

Substituting given values in equations (iv) and (v) gives:

$$2.5 = \frac{1.25}{C_1} + 0.02 A_1 \ln\left(\frac{30}{C_1}\right)$$
(vi)

$$\frac{1.25}{C_1} = 0.0625 + 0.00811A_1 \tag{vii}$$

or:

The procedure is to use trial and error to estimate the value of C_1 that gives the optimum values of A_1 and A_2 . Thus:

If $C_1 = 5 \text{ kg/m}^3$, then, $A_1 = 63 \text{ m}^2$ and $A_2 = 23 \text{ m}^2$. That is, an arrangement of 3 modules -1 module is required.

If $C_1 = 4 \text{ kg/m}^3$, then $A_1 = 55 \text{ m}^2$ and $A_2 = 31 \text{ m}^2$. That is, an arrangement of 2 modules -1 module is almost sufficient.

If $C_1 = 4.5 \text{ kg/m}^3$, then $A_1 = 59 \text{ m}^2$ and $A_2 = 27 \text{ m}^2$. That is, an arrangement of 2 modules -1 module which meets the requirement.

This arrangement requires the minimum number of modules.

Centrifugal Separations

There is now a wide range of situations where centrifugal force is used in place of the gravitational force in order to effect separations. The resulting accelerations may be several thousand times that attributable to gravity.

Centrifugal fields can be generated in two distinctly different ways:

- (a) By introducing a fluid with a high tangential velocity into a cylindrical or conical vessel, as in the hydrocyclone and in the cyclone separator.
- (b) By the use of the centrifuge. In this case the fluid is introduced into some form of rotating bowl and is rapidly accelerated.

Using of centrifuge :

(a) For separating particles on the basis of their size or density.

(b) For separating immiscible liquids of different densities.

(c) For filtration of a suspension. In this case centrifugal force replaces the force of gravity or the force attributable to an applied pressure difference across the filter.

(d) For the drying of solids and, in particular, crystals.

- (e) For breaking down of emulsions and colloidal suspensions.
- (f) For the separation of gases.
- (g) For mass transfer processes.

For an element of liquid in a centrifuge bowl which is rotating at an angular velocity of ω , the centrifugal acceleration is $r\omega^2$, compared with the gravitational acceleration of g. The ratio $r\omega^2/g$ is one measure of the separating effect obtained in a centrifuge relative to that arising from the gravitational field.



The figure above shows an element of the free surface of the liquid in a bowl which is rotating at a radius r0 about a vertical axis at a very low speed; the centrifugal and gravitational fields will then be of the same order of magnitude. The centrifugal force per unit mass is $r_0\omega^2$ and the corresponding gravitational force is g. These two forces are perpendicular to one another and may be combined as shown to give the resultant force which must, at equilibrium, be at right angles to the free surface. Thus, the slope at this point is given by:

$$\frac{\mathrm{l}z_0}{\mathrm{l}r_0} = \frac{\mathrm{radial\ component\ of\ force}}{\mathrm{axial\ component\ of\ force}} = \frac{r_0\omega^2}{g} \tag{9.1}$$

where z_0 is the axial coordinate of the free surface of the liquid.

Equation 9.1 may be integrated to give:

$$z_0 = \frac{\omega^2}{2g}r_0^2 + \text{constant}$$

If z_a is the value of z_0 which corresponds to the position where the free surface is at the axis of rotation ($r_0 = 0$), then:

$$z_0 - z_a = \frac{\omega^2}{2g} r_0^2 \tag{9.2}$$

CENTRIFUGAL PRESSURE

the pressure gradient at a radius r:

$$\frac{\partial P}{\partial r} = \rho \omega^2 r$$

The centrifugal pressure gradient is a function of radius of rotation r, and increases towards the wall of the basket. Integration of equation 9.3 at a given height gives the pressure P exerted by the liquid on the walls of the bowl of radius R when the radius of the inner surface of the liquid is r_0 as:

$$P = \frac{1}{2}\rho\omega^2(R^2 - r_0^2)$$

SEPARATION OF IMMISCIBLE LIQUIDS OF DIFFERENT DENSITIES



Figure 9.3. Separation of two immiscible liquids in a centrifuge

of the outer weir r_w must be such that the pressure developed at the wall of the bowl of radius *R* by the heavy liquid alone as it flows over the weir is equal to that due to the two liquids within the bowl. Thus, applying equation 9.4 and denoting the densities of the light and heavy liquids by ρ_1 and ρ_2 respectively and the radius of the interface between the two liquids in the bowl as r_s :

$$\frac{1}{2}\rho_2\omega^2(R^2 - r_w^2) = \frac{1}{2}\rho_2\omega^2(R^2 - r_s^2) + \frac{1}{2}\rho_1\omega^2(r_s^2 - r_i^2)$$
$$\frac{r_s^2 - r_i^2}{r_s^2 - r_w^2} = \frac{\rho_2}{\rho_1}$$
(9.6)

or:

If Q_1 and Q_2 are the volumetric rates of feed of the light and heavy liquids respectively, on the assumption that there is no slip between the liquids in the bowl and that the same, then residence time is required for the two phases, then:

$$\frac{Q_1}{Q_2} = \frac{r_s^2 - r_i^2}{R^2 - r_s^2}$$
(9.7)

The retention time is given by:

$$t_R = \frac{V'}{Q_1 + Q_2} = \frac{V'}{Q}$$

where Q is the total feed rate of liquid, Q_1 and Q_2 refer to the light and heavy liquids respectively, and V' is the volumetric holdup of liquid in the bowl.

Approximately:

$$V' \approx \pi (R^2 - r_i^2) H \tag{9.9}$$

where H is the axial length (or clarifying length) of the bowl.

Thus:

$$t_R = \frac{Q}{\pi (R^2 - r_i^2)H}$$
(9.10)

SEDIMENTATION IN A CENTRIFUGAL FIELD

Because centrifuges are normally used for separating fine particles and droplets, it is necessary to consider only the Stokes' law region in calculating the drag between the particle and the liquid.

$$\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{d^2(\rho_s - \rho)r\omega^2}{18\mu}$$
$$= u_0 \frac{r\omega^2}{g}$$

At the walls of the bowl of radius r, dr/dt is given by:

$$\left(\frac{\mathrm{d}r}{\mathrm{d}t}\right)_{r=R} = \frac{d^2(\rho_s - \rho)R\omega^2}{18\mu} \tag{9.13}$$

The time taken to settle through a liquid layer of thickness h at the walls of the bowl is given by integration of equation 9.11 between the limits $r = r_0$ (the radius of the inner surface of the liquid), and r = R to give equation 3.121. This equation may be simplified where $R - r_0 (= h)$ is small compared with R, as in equation 3.122.

Then:
$$t_R = \frac{18\mu h}{d^2(\rho_s - \rho)R\omega^2}$$
(9.14)

 t_R is then the minimum retention time required for all particles of size greater than d to be deposited on the walls of the bowl. Thus, the maximum throughput Q at which all particles larger than d will be retained is given by substitution for t_R to give:

$$Q = \frac{d^2(\rho_s - \rho)R\omega^2 V'}{18\mu h}$$
$$Q = \frac{d^2(\rho_s - \rho)g}{18\mu} \frac{R\omega^2 V'}{hg}$$
$$\frac{d^2(\rho_s - \rho)g}{18\mu} = u_0$$

where u_0 is the terminal falling velocity of the particle in the gravitational field and hence:

$$Q = u_0 \frac{R\omega^2 V'}{hg}$$

Writing the capacity term as:

$$\Sigma = \frac{R\omega^2 V'}{hg}$$
$$= \frac{\pi R(R^2 - r_0^2) H\omega^2}{hg}$$
$$= \pi R(R + r_0) H \frac{\omega^2}{g}$$
$$Q = u_0 \Sigma$$

Then:

For cases where the thickness h of the liquid layer at the walls is comparable in order of magnitude with the radius R of the bowl, it is necessary to use equation 3.121 in place of equation 9.14 for the required residence time in the centrifuge or:

$$t_{R} = \frac{18\mu}{d^{2}(\rho_{s} - \rho)\omega^{2}} \ln \frac{R}{r_{0}}$$
(9.20)

(from equation 3.121)

$$Q = \frac{d^2(\rho_s - \rho)\omega^2 V'}{18\mu \ln(R/r_0)}$$
(9.21)

$$=\frac{d^{2}(\rho_{s}-\rho)g}{18\mu}\frac{\omega^{2}V'}{g\ln(R/r_{0})}$$
(9.22)

$$=u_0\Sigma \tag{9.23}$$

$$\Sigma = \frac{\omega^2 V'}{g \ln(R/r_0)}$$
$$= \frac{\pi (R^2 - r_i^2) H}{\ln(R/r_0)} \frac{\omega^2}{g}$$
(9.24)

In this case:

Example 9.1

In a test on a centrifuge all particles of a mineral of density 2800 kg/m3 and of size 5 μ m, equivalent spherical diameter, were separated from suspension in water fed at a volumetric throughput rate of 0.25 m3/s. Calculate the value of the capacity factor. What will be the corresponding size cut for a suspension of coal particles in oil fed at the rate of 0.04 m3/s? The density of coal is 1300 kg/m3 and the density of the oil is 850 kg/m3 and its viscosity is 0.01 Ns/m2.

It may be assumed that Stokes' law is applicable.

Solution

The terminal falling velocity of particles of diameter 5 μ m in water, of density $\rho = 1000 \text{ kg/m}^3$ and, of viscosity $\mu = 10^{-3} \text{ Ns/m}^2$, is given by:

$$u_0 = \frac{d^2(\rho_s - \rho)g}{18\mu} = \frac{25 \times 10^{-12} \times (2800 - 1000) \times 9.81}{18 \times 10^{-3}}$$
 (equation 3.24)
= 2.45 × 10⁻⁵ m/s

From the definition of Σ :

 $Q = u_0 \Sigma$ (equation 9.19) $\Sigma = \frac{0.25}{(2.45 \times 10^{-5})} = 1.02 \times 10^4 \text{ m}^2.$

and:

For the coal-in-oil mixture:

$$u_0 = \frac{Q}{\Sigma} = \frac{0.04}{(1.02 \times 10^4)} = 3.92 \times 10^{-6} \text{ m/s.}$$

From equation 3.24:

$$d^{2} = \frac{18\mu u_{0}}{(\rho_{s} - \rho)g}$$
$$= \frac{18 \times 10^{-2} \times 3.92 \times 10^{-6}}{(1300 - 850) \times 9.81}$$
$$d = 4.0 \times 10^{-6} \text{ m or } 4 \text{ }\mu\text{m}.$$

and:

Drying

The drying of materials is often the final operation in a manufacturing process, carried out immediately prior to packaging or dispatch. Drying refers to the final removal of water, or another solute, and the operation often follows evaporation, filtration, or crystallization.

drying is carried out for one or more of the following reasons:

(a) To reduce the cost of transport.

(b) To make a material more suitable for handling as, for example, with soap powders, dyestuffs and fertilizers.

(c) To provide definite properties, such as, for example, maintaining the free-flowing nature of salt.

(d) To remove moisture which may otherwise lead to corrosion. One example is the drying of gaseous fuels or benzene prior to chlorination.

GENERAL PRINCIPLES

Humidity *H*, mass of water per unit mass of dry air.

Since:

then:

 $\frac{\text{moles of water vapour}}{\text{moles of dry air}} = \frac{P_w}{(P - P_w)}$ $\mathcal{H} = \frac{18P_w}{29(P - P_w)}$

where P_w is the partial pressure of water vapour and P is the total pressure.

Humidity of saturated air \mathcal{H}_0 . This is the humidity of air when it is saturated with water vapour. The air then is in equilibrium with water at the given temperature and pressure.

Percentage humidity

$$= \frac{\text{Humidity of air}}{\text{Humidity of saturated air}} \times 100 = \frac{\mathcal{H}}{\mathcal{H}_0} \times 100$$

Percentage relative humidity, *R*

 $= \frac{\text{Partial pressure of water vapour in air}}{\text{Vapour pressure of water at the same temperature}} \times 100$
Humid volume, is the volume of unit mass of dry air and its associated vapour. Then, under ideal conditions, at atmospheric pressure:

humid volume =
$$\frac{22.4}{29} \left(\frac{T}{273}\right) + \frac{22.4\mathscr{H}}{18} \left(\frac{T}{273}\right) \text{ m}^3/\text{kg}$$

where T is in degrees K,

Saturated volume is the volume of unit mass of dry air, together with the water vapour required to saturate it.

Humid heat is the heat required to raise unit mass of dry air and associated vapour through 1 degree K at constant pressure or $1.00 + 1.88 \mathscr{H}$ kJ/kg K.

Dew point is the temperature at which condensation will first occur when air is cooled. Wet bulb temperature. If a stream of air is passed rapidly over a water surface, vaporisation occurs, provided the temperature of the water is above the dew point of the air. The temperature of the water falls and heat flows from the air to the water. If the surface is sufficiently small for the condition of the air to change inappreciably and if the velocity is in excess of about 5 m/s, the water reaches the wet bulb temperature θ_w at equilibrium.

The rate of heat transfer from gas to liquid is given by:

$$Q = hA(\theta - \theta_w) \tag{16.1}$$

The mass rate of vaporisation is given by:

$$G_{v} = \frac{h_{D}AM_{w}}{\mathbf{R}T}(P_{w0} - P_{w})$$

$$= \frac{h_{D}AM_{A}}{\mathbf{R}T}[(P - P_{w})_{\text{mean}}(\mathcal{H}_{w} - \mathcal{H})]$$

$$= h_{D}A\rho_{A}(\mathcal{H}_{w} - \mathcal{H})$$
(16.2)

The rate of heat transfer required to effect vaporisation at this rate is given by:

$$G_v = h_D A \rho_A (\mathscr{H}_w - \mathscr{H}) \lambda \tag{16.3}$$

At equilibrium, the rates of heat transfer given by equations 16.1 and 16.3 must be equal, and hence:

$$\mathscr{H} - \mathscr{H}_w = -\frac{h}{h_D \rho_A \lambda} (\theta - \theta_w) \tag{16.4}$$

In this way, it is seen that the wet bulb temperature θ_w depends only on the temperature and humidity of the drying air.

In these equations:

h is the heat transfer coefficient,

 h_D is the mass transfer coefficient,

A is the surface area,

 θ is the temperature of the air stream,

 θ_w is the wet bulb temperature,

 P_{w0} is the vapour pressure of water at temperature θ_w ,

 M_A is the molecular weight of air,

 M_w is the molecular weight of water,

- **R** is the universal gas constant,
- T is the absolute temperature,
- \mathcal{H} is the humidity of the gas stream,
- \mathscr{H}_w is the humidity of saturated air at temperature θ_w ,
- ρ_A is the density of air at its mean partial pressure, and
- λ is the latent heat of vaporisation of unit mass of water.

Rate of drying

In drying, it is necessary to remove free moisture from the surface and also moisture from the interior of the material. If the change in moisture content for a material is determined as a function of time, a smooth curve is obtained from which the rate of drying at any given moisture content may be evaluated. The form of the drying rate curve varies with the structure and type of material, and two typical curves are shown in Figure 16.2. In curve 1, there are two well-defined zones: AB, where the rate of drying is constant and BC, where there is a steady fall in the rate of drying as the moisture content is reduced. The moisture content at the end of the constant rate period is represented by point B, and this is known as the critical moisture content. Curve 2 shows three stages, DE, EF and FC. The stage DE represents a constant rate period, and EF and FC are falling rate periods. In this case, the Section EF is a straight line, however, and only the portion FC is curved. Section EF is known as the first falling rate period and the final stage, shown as FC, as the second falling rate period. The drying of soap gives rise to a curve of type 1, and sand to a curve of type 2. A number of workers, including SHERWOOD⁽¹⁾ and NEWITT and co-workers(2-7), have contributed various theories on the rate of drying at these various stages.



Figure 16.2. Rate of drying of a granular material

The rate of drying in the constant rate period is given by:

$$W = \frac{\mathrm{d}w}{\mathrm{d}t} = \frac{hA\Delta T}{\lambda} = k_G A(P_s - P_w) \tag{16.9}$$

where:

W is the rate of loss of water,

- h is the heat transfer coefficient from air to the wet surface,
- ΔT is the temperature difference between the air and the surface,
 - λ is the latent heat of vaporisation per unit mass,
- k_G is the mass transfer coefficient for diffusion from the wet surface through the gas film,
- A is the area of interface for heat and mass transfer, and
- $(P_s P_w)$ is the difference between the vapour pressure of water at the surface and the partial pressure in the air.

Time for drying

If a material is dried by passing hot air over a surface which is initially wet, the rate of drying curve in its simplest form is represented by BCE, shown in Figure



The use of a rate of drying curve in estimating the time for drying

- where: w is the total moisture,
 - w_e is the equilibrium moisture content (point E),
 - $w w_e$ is the free moisture content, and
 - w_c is the critical moisture content (point C).

Constant-rate period

During the period of drying from the initial moisture content w_1 to the critical moisture content w_c , the rate of drying is constant, and the time of drying t_c is given by:

$$t_c = \frac{w_1 - w_c}{R_c A} \tag{16.10}$$

where: R_c is the rate of drying per unit area in the constant rate period, and

A is the area of exposed surface.

Falling-rate period

During this period the rate of drying is, approximately, directly proportional to the free moisture content $(w - w_e)$, or:

$$-\left(\frac{1}{A}\right)\frac{\mathrm{d}w}{\mathrm{d}t} = m(w - w_e) = mf \quad (\mathrm{say}) \tag{16.11}$$
$$-\frac{1}{mA}\int_{w_e}^{w}\frac{\mathrm{d}w}{(w - w_e)} = \int_{0}^{t_f} \mathrm{d}t$$
$$\frac{1}{mA}\ln\left[\frac{w_c - w_e}{w - w_e}\right] = t_f$$
$$t_f = \frac{1}{mA}\ln\left(\frac{f_c}{f}\right) \tag{16.12}$$

Thus:

or:

and:

Thus:

Total time of drying

The total time t of drying from w_1 to w is given by $t = (t_c + t_f)$.

The rate of drying R_c over the constant rate period is equal to the initial rate of drying in the falling rate period, so that $R_c = mf_c$.

$$t_c = \frac{(w_1 - w_c)}{mAf_c} \tag{16.13}$$

and the total drying time,
$$t = \frac{(w_1 - w_c)}{mAf_c} + \frac{1}{mA} \ln\left(\frac{f_c}{f}\right)$$
$$= \frac{1}{mA} \left[\frac{(f_1 - f_c)}{f_c} + \ln\left(\frac{f_c}{f}\right)\right]$$
(16.14)

Example 16.1

A wet solid is dried from 25 to 10 per cent moisture under constant drying conditions in 15 ks (4.17 h). If the critical and the equilibrium moisture contents are 15 and 5 per cent respectively, how long will it take to dry the solid from 30 to 8 per cent moisture under the same conditions?

 $w_1 = 0.25 \text{ kg/kg}, w = 0.10 \text{ kg/kg}, w_c = 0.15 \text{ kg/kg}$ and $w_e = 0.05 \text{ kg/kg}$

Solution

For the first drying operation:

Thus:

$$f_1 = (w_1 - w_e) = (0.25 - 0.05) = 0.20 \text{ kg/kg}$$

$$f_c = (w_c - w_e) = (0.15 - 0.05) = 0.10 \text{ kg/kg}$$

$$f = (w - w_e) = (0.10 - 0.05) = 0.05 \text{ kg/kg}$$

From equation 16.14, the total drying time is:

or:

$$t = (1/mA)[(f_1 - f_c)/f_c + \ln(f_c/f)]$$

$$15 = (1/mA)[(0.20 - 0.10)/0.10 + \ln(0.10/0.05)]$$
and:

$$mA = 0.0667(1.0 + 0.693) = 0.113 \text{ kg/s}$$

For the second drying operation:

$$w_1 = 0.30 \text{ kg/kg}, w = 0.08 \text{ kg/kg}, w_c = 0.15 \text{ kg/kg}$$
 and $w_e = 0.05 \text{ kg/kg}$

Thus:

$$f_c = (w_c - w_e) = (0.15 - 0.05) = 0.10 \text{ kg/kg}$$
$$f = (w - w_e) = (0.08 - 0.05) = 0.03 \text{ kg/kg}$$

 $f_1 = (w_1 - w_e) = (0.30 - 0.05) = 0.25 \text{ kg/kg}$

The total drying time is then

$$t = (1/0.113)[(0.25 - 0.10)/0.10 + \ln(0.10/0.03)]$$

= 8.856(1.5 + 1.204)
= 23.9 ks (6.65 h)

Example 16.2

Strips of material 10 mm thick are dried under constant drying conditions from 28 to 13 per cent moisture in 25 ks (7 h). If the equilibrium moisture content is 7 per cent, what is the time taken to dry 60 mm planks from 22 to 10 per cent moisture under the same conditions assuming no loss from the edges? All moistures are given on a wet basis.

The relation between E, the ratio of the average free moisture content at time t to the initial free moisture content, and the parameter J is given by:

E	1	0.64	0.49	0.38	0.295	0.22	0.14
J	0	0.1	0.2	0.3	0.5	0.6	0.7

It may be noted that $J = kt/l^2$, where k is a constant, t the time in ks and 2l the thickness of the sheet of material in millimetres.

Solution

For the 10 mm strips Initial free moisture content = (0.28 - 0.07) = 0.21 kg/kg. Final free moisture content = (0.13 - 0.07) = 0.06 kg/kg.

Thus:

when t = 25 ks, E = (0.06/0.21) = 0.286

and from Figure 16.4, a plot of the given data,

Thus:
and:
$$J = 0.52$$

 $0.52 = (k \times 25)/(10/2)^2$
 $k = 0.52$



Figure 16.4. Drying data for Example 6.2

For the 60 mm planks

Initial free moisture content = (0.22 - 0.07) = 0.15 kg/kg. Final free moisture content = (0.10 - 0.07) = 0.03 kg/kg.

$$E = (0.03/0.15) = 0.20$$

From Figure 16.4: $J = 0.63$
and hence: $t = Jl^2/k$
 $= 0.63(60/2)^2/0.52 = 1090$ ks (12.6 days)

Example 16.3

A granular material containing 40 per cent moisture is fed to a countercurrent rotary dryer at a temperature of 295 K and is withdrawn at 305 K, containing 5 per cent moisture. The air supplied, which contains 0.006 kg water vapour/kg dry air, enters at 385 K and leaves at 310 K. The dryer handles 0.125 kg/s wet stock.

Assuming that radiation losses amount to 20 kJ/kg dry air used, determine the mass flowrate of dry air supplied to the dryer and the humidity of the exit air.

The latent heat of water vapour at 295 K = 2449 kJ/kg, specific heat capacity of dried material = 0.88 kJ/kg K, the specific heat capacity of dry air = 1.00 kJ/kg K, and the specific heat capacity of water vapour = 2.01 kJ/kg K.

Solution

This example involves a heat balance over the system. 273 K will be chosen as the datum temperature, and it will be assumed that the flowrate of dry air = G kg/s.

Heat in:

(a) Air

G kg/s dry air enter with 0.006G kg/s water vapour and hence the heat content of this stream

 $= [(1.00G) + (0.006G \times 2.01)](385 - 273) = 113.35G \text{ kW}$

(b) Wet solid

0.125 kg/s enter containing 0.40 kg water/kg wet solid, assuming the moisture is expressed on a wet basis.

Thus: mass flowrate of water = $(0.125 \times 0.40) = 0.050$ kg/s

and:

Hence:

the heat content of this stream = $[(0.050 \times 4.18) + (0.075 \times 0.88)](295 - 273) = 6.05$ kW

mass flowrate of dry solid = (0.125 - 0.050) = 0.075 kg/s

Heat out:

(a) Air

Heat in exit air = $[(1.00 G) + (0.006 G \times 2.01)](310 - 273) = 37.45G kW$. Mass flowrate of dry solids = 0.075 kg/s containing 0.05 kg water/kg wet solids. Hence:

water in the dried solids leaving = $(0.05 \times 0.075)/(1 + 0.05) = 0.0036$ kg/s

and:

the water evaporated into gas steam =
$$(0.050 - 0.0036) = 0.0464$$
 kg/s.

Assuming evaporation takes place at 295 K, then:

heat in the water vapour =
$$0.0464[2.01(310 - 295) + 2449 + 4.18(295 - 273)]$$

= 119.3 kW

and:

the total heat in this stream = (119.30 + 37.45G) kW.

(b) Dried solids

The dried solids contain 0.0036 kg/s water and hence heat content of this stream is:

$$= [(0.075 \times 0.88) + (0.0036 \times 4.18)/(305 - 273)] = 2.59 \text{ kW}$$

(c) Losses

These amount to 20 kJ/kg dry air or 20m kW.

(

Heat balance

and:

$$113.35 G + 6.05) = (119.30 + 37.45 G + 2.59 + 20 G$$

G = 2.07 kg/s

Water in the outlet air stream = $(0.006 \times 2.07) + 0.0464 = 0.0588$ kg/s

and:

the humidity
$$\mathscr{H} = (0.0588/2.07) = 0.0284$$
 kg/kg dry air

Falling rate period, diffusion control

In the falling-rate period, the surface is no longer completely wetted and the rate of drying steadily falls. In the previous analysis, it has been assumed that the rate of drying per unit effective wetted area is a linear function of the water content, so that the rate of drying is given by:

$$\left(\frac{1}{A}\right)\frac{\mathrm{d}w}{\mathrm{d}t} = -m(w - w_e) \tag{equation 16.11}$$

16.5. DRYING EQUIPMENT

16.5.1. Classification and selection of dryers

Because of the very wide range of dryer designs available, classification is a virtually impossible task. PARKER⁽¹⁸⁾ takes into account, however, the means by which material is transferred through the dryer as a basis of his classification, with a view to presenting a guide to the selection of dryers. Probably the most thorough classification of dryer types has been made by KRÖLL⁽¹⁹⁾ who has presented a decimalised system based on the following factors:

- (a) Temperature and pressure in the dryer,
- (b) The method of heating,
- (c) The means by which moist material is transported through the dryer.
- (d) Any mechanical aids aimed at improving drying,
- (e) The method by which the air is circulated,
- (f) The way in which the moist material is supported,
- (g) The heating medium, and
- (h) The nature of the wet feed and the way it is introduced into the dryer.

Example 16.4

A 100 kg batch of granular solids containing 30 per cent moisture is to be dried in a tray drier to 15.5 per cent of moisture by passing a current of air at 350 K tangentially across its surface at a velocity of 1.8 m/s. If the constant rate of drying under these conditions is 0.0007 kg/s m² and the critical moisture content is 15 per cent, calculate the approximate drying time. Assume the drying surface to be 0.03 m²/kg dry mass.

Solution

In 100 kg feed, mass of water = $(100 \times 30/100) = 30$ kg

and:

mass of dry solids = (100 - 30) = 70 kg

For b kg water in the dried solids: 100b/(b+70) = 15.5

and the water in the product, b = 12.8 kg

Thus:

initial moisture content, $w_1 = (30/70) = 0.429$ kg/kg dry solids

final moisture content, $w_2 = (12.8/70) = 0.183$ kg/kg dry solids

and water to be removed = (30 - 12.8) = 17.2 kg

The surface area available for drying $= (0.03 \times 70) = 2.1 \text{ m}^2$ and hence the rate of drying during the constant period = $(0.0007 \times 2.1) = 0.00147$ kg/s.

As the final moisture content is above the critical value, all the drying is at this constant rate and the time of drying is:

$$t = (17.2/0.00147) = 11,700 \text{ s or } 11.7 \text{ ks} (3.25 \text{ h})$$