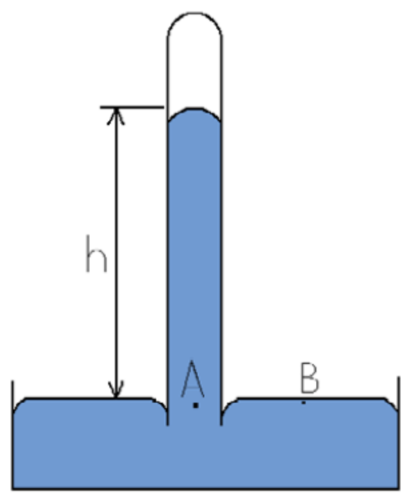
Lecture Four

Measurement of Pressure

4.1 Manometer

Manometer is a straight tube sealed at one end filled with mercury and the open end immersed in a container of mercury as shown in Figure. Mercury drains out of the tube creating a vacuum until the pressure at point A equals the pressure at point B which is just below the free mercury surface in the container. Pressure at points on the same plane within a fluid remains equal. Therefore, Pressure at point A, P_A, is equal to Pressure at point B, P_B. Pressure at point B is as same as the atmospheric pressure.



$$P_A = P_B$$

Pressure at point A is given by:

$$P_A = h\rho \boldsymbol{g}$$

Therefore, the atmospheric pressure:

$$P_{atm} = P_B = h\rho \boldsymbol{g}$$

Since $\rho \boldsymbol{g}$ is a constant, height of the mercury column could be used as a measure of the pressure. This is how mercury millimetre (Hg mm) became a unit of pressure.

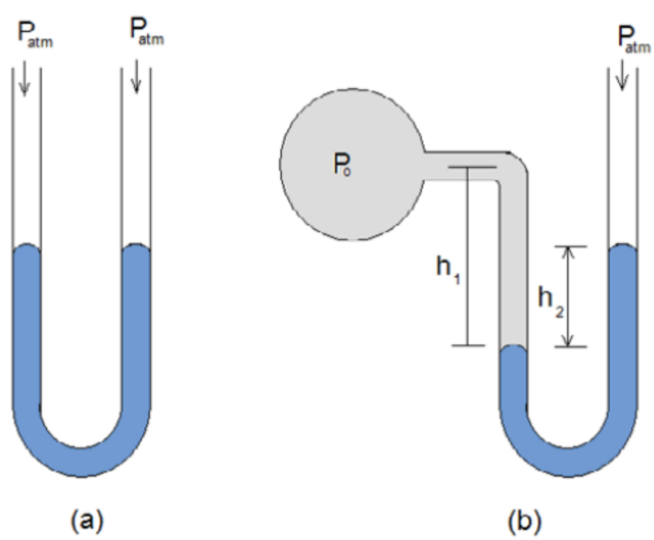
Manometer of this type is called the "barometer" and is used to measure atmospheric pressure.

Exercise

Considering the fact that the 1 atm = 101325 Pa, calculate the corresponding height of the mercury column in a barometer. Density of mercury is 13590kg/m³.

4.2 U-tube Manometer

Barometer has limited applications. It is widely used to measure atmospheric pressure. However, the principle could be used to measure pressure relative to the atmospheric pressure or pressure difference between two points. For this purpose, a U-tube partially filled with mercury is used. When the both ends of the U-tube are open to the atmosphere, the mercury column balances giving the same height in both arms of the U tube. When one end is open to the atmosphere and the other end to a vessel with a pressure different from the atmosphere, the mercury column moves to a new equilibrium position giving a height difference in the U-tube as shown in Figure.



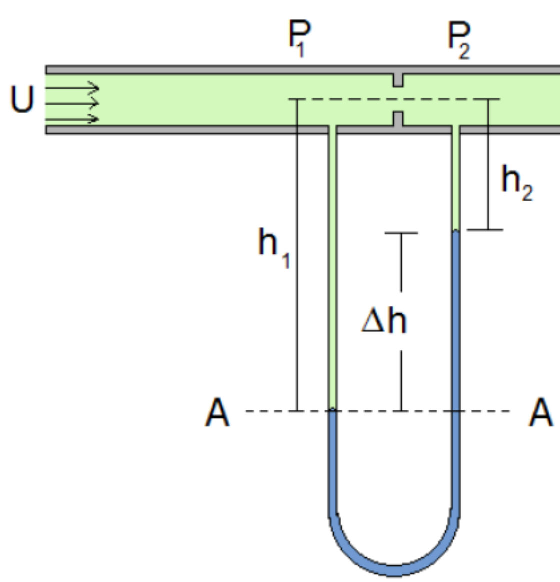
Suppose the fluid in the bulb has a density ρ_1 and the density of mercury to be ρ_m. Furthermore, assume the pressure inside the bulb to be P_0. Once the mercury column attains equilibrium, a simple force balance at a point just inside the static mercury meniscus will give:

$$P_0 + h_1\rho_1\boldsymbol{g} = P_{atm} + h_2\rho_M\boldsymbol{g}$$

Rearranging terms gives:

$$P_0 - P_{atm} = h_2\rho_M\boldsymbol{g} - h_1\rho_1\boldsymbol{g}$$

Manometers could be used to measure the pressure difference between two points. Consider an arrangement as shown in Figure below. A U-tube partially filled with a heavier liquid, mercury in most cases, connected to a pipe across a restriction in the pipe. Density of the fluid in the pipe is ρ_L and the density of the heavy liquid is ρ_m. Pressure at two tapings to which the manometer arms are connected are P_1 and P_2 (P1>P2).



The space in the tube above the heavy fluid in the manometer is filled with the same fluid that flows in the pipe. This type of setting, when the pipe contains water and the heavy fluid is the mercury, is called "water over mercury" manometer. Heavy liquid attains equilibrium forming a height difference Δh. The line A-A marks the level of the heavy liquid (mercury). Pressure at this level in both arms should be equal.

Therefore, the pressure difference is given by:

$$P_1 + h_1\rho_L\boldsymbol{g} = P_2 + h_2\rho_L\boldsymbol{g} + \Delta h\rho_M\boldsymbol{g}$$

$$P_1 - P_2 = \Delta h\rho_L\boldsymbol{g}\left(\frac{\rho_M}{\rho_L} - 1\right)$$

4.3 Inclined-tube manometer

Manometers shown in Figures above are easy to use with pressure differences that would give considerable height differences. To measure small pressure differences the sensitivity of the manometer has to be increased. This is achieved by inclining one arm of the U-tube as shown in Figure 2.9.

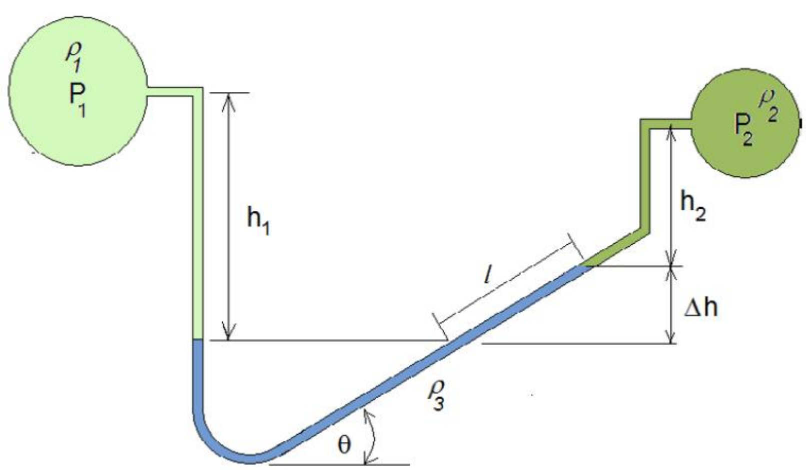


Figure 2.9 Inclined tube manometer

Once again by equating the pressure at equal levels, one can write:

$$P_1 + h_1\rho_1\boldsymbol{g} = P_2 + h_2\rho_2\boldsymbol{g} + lsin\theta\rho_3\boldsymbol{g}$$

Rearranging terms gives:

$$P_1 - P_2 = -(h_1\rho_1 - h_2\rho_2)\boldsymbol{g} + lsin\theta\rho_3\boldsymbol{g}$$

If the pipes contain the same fluid (i.e. $\rho_1 = \rho_2$) then above equation reduces to:

$$P_1 - P_2 = lsin\theta\rho_3\boldsymbol{g} - \Delta h\rho_1\boldsymbol{g}$$

Further simplification gives:

$$P_1 - P_2 = lsin\theta\boldsymbol{g}(\rho_3 - \rho_1)$$

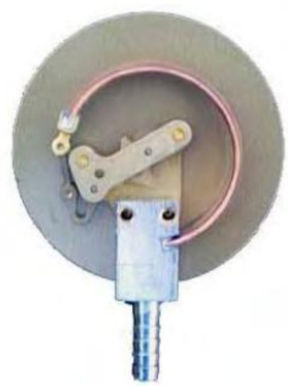
This result is obtained using the relationship:

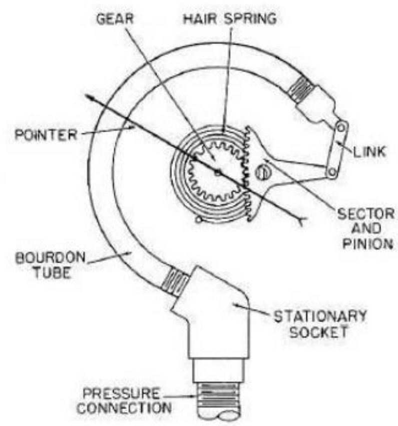
$$lsin\theta = \Delta h$$

By selecting an appropriate inclination angle θ , one can increase l to be a measurable length.

4.4 Bourdon gauge

Manometers are somewhat difficult to use. As a result more compact, liquid free measuring techniques are invented. Of these, the Bourdon gauge is a widely used measuring technique. Bourdon gauge measures the pressure relative to the atmospheric pressure. It contains a coiled tube connected to an indicator. The metal tube (made of copper in most cases), when expand under higher pressure, moves the indicator on a dial. The dial is calibrated so that the pressure could be read directly. The mechanism is shown in Figure.





Transducers that generate an electrical signal proportional to the applied pressure are widely used in chemical industries where process control is carried out remotely. These transducers use electric properties such as capacitance or piezoelectric voltage induction to infer pressure. These transducers, though expensive compared to traditional gauges, provide high accuracy and has small footprint on the system.