Lecture Three

Fluid Statics

3.1 Introduction

Statics is the area of fluid mechanics that studies fluids at rest. It also extends to fluids in motion when there is no relative motion between adjacent fluid particles (e.g. rigid body motion). In engineering context, fluid statics provide an essential body of knowledge to design liquid storage tanks considering forces acting on viewing glasses, sluice gates, required wall thicknesses etc.

Imagine a small fluid element surrounded by the rest of the fluid. The boundary of this fluid particle experience shear stress due to intermolecular forces. The fluid element experiences the gravitational force irrespective to its motion or the position. The force acting on the fluid element due to the gravity is given by the product ***mg*** where m and **g** are the mass of the element and the acceleration due to gravity respectively. Such forces appear in fluids due to external fields such as gravity or electromagnetic fields are called the body forces. Body forces acts on the whole volume of the fluid particle. Therefore, for fluids at rest it is important to understand the role of the pressure.

3.1 Pressure

In weather reports, you might have heard of "high pressure" or "low pressure" regions that make clouds to move. This refers to the force air mass above the ground applying on a unit area on the ground.

Pressure is defined as the total force applied normal (perpendicular) to a unit surface area.

$$Pressure = \frac{Total\ static\ force\ exerted\ normal\ to\ the\ area}{Area\ on\ which\ the\ force\ is\ applied}$$

Consider a force F applied on an area A as shown in Figure 2.1.

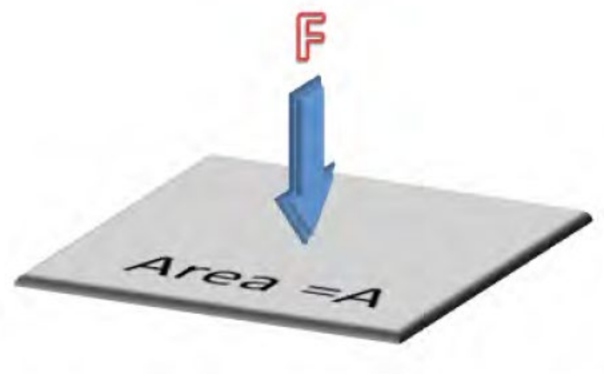


Figure 2.1. Definition of pressure. Force F acting on an area of A m^2.

$$\text{Pressure} = \frac{F}{A}$$

Pressure has the units N/m^2. N/m^2 is called a Pascal (P) in honour of *Blaise Pascal*, a French mathematician and a physicist whose work on static fluids lead to understand the concept of pressure. The other most widely used unit is mercury millimetres (Hg mm).

3.3 Variation of Pressure with Depth

Consider a static fluid at equilibrium. Pressure at any arbitrary point is indicated by P.

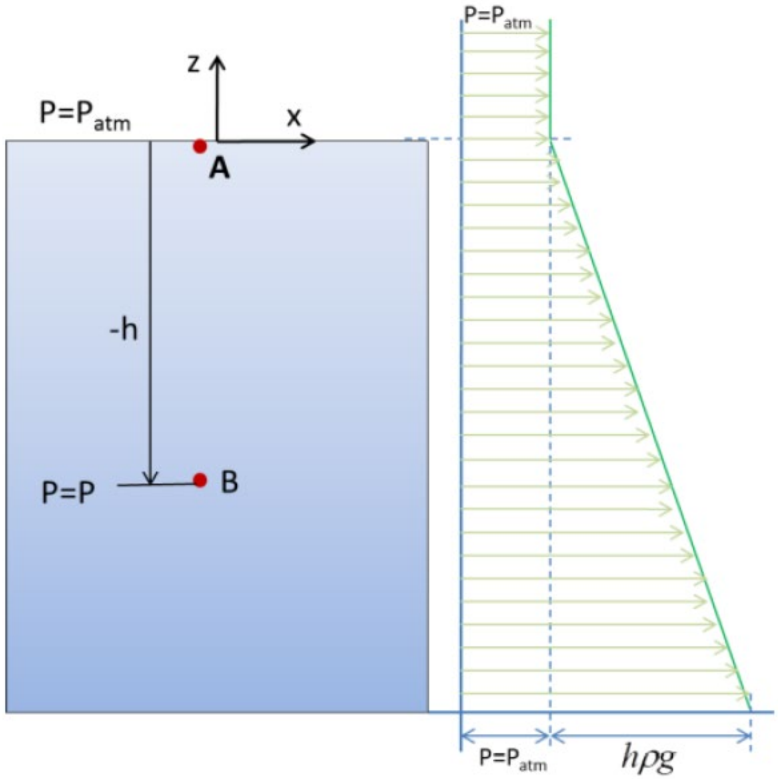


Figure 2.4. Pressure variation in the direction of gravity

$$\frac{dP}{dz} = -\rho \boldsymbol{g}$$

This equation gives an important relationship between the pressure and the height of the fluid. Integrating it from 0 to a depth of h gives:

$$\int_{P_{atm}}^{P} dp = \int_{0}^{-h} -\rho \boldsymbol{g} dz$$

$$\int_{P_{atm}}^{P} dp = \int_{0}^{-h} -\rho \boldsymbol{g} dz \quad \ldots \quad \text{Equation 3.1}$$

$P - Patm = \rho gh$

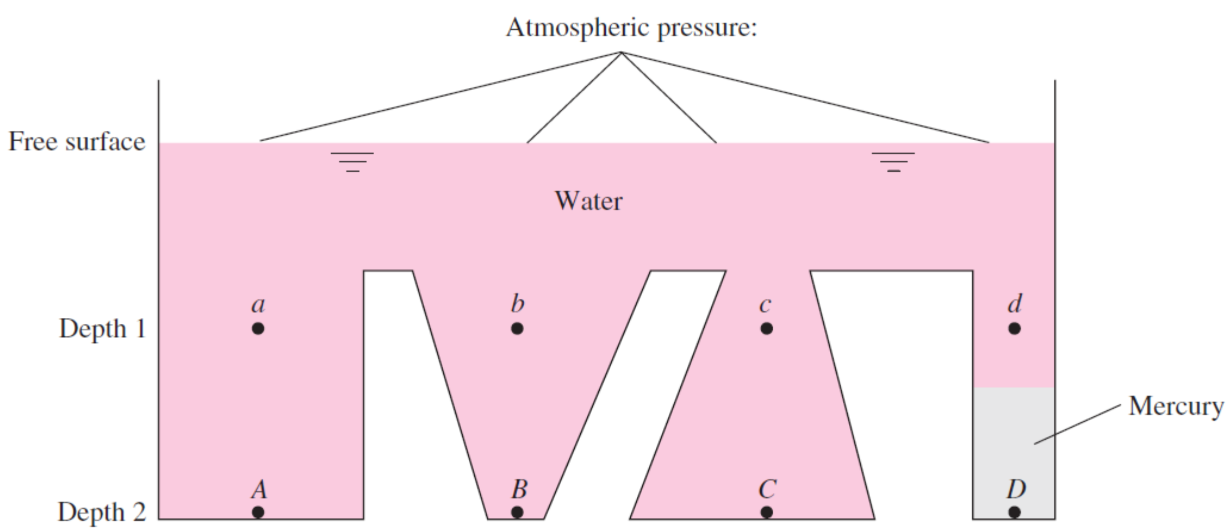
$P = \rho gh + Patm$ … Equation 3.2

Equation 3.1 applies when

1. Fluid is static
2. Gravity is the only body force
3. z axis is vertical and upward

Equation 3.1 suggests that the pressure at any point of a fluid at rest is given by the sum of ρ gh and the pressure above the body of fluid. For example, if it is a tank full of water open to atmosphere, then the pressure at the bottom of the tank is given by ρ gh + atmospheric pressure.

Example1:



Points a, b, c, and d are at equal depths in water and therefore have identical pressures. Points A, B, and C are also at equal depths in water and have identical pressures higher than a, b, c, and d. Point D has a different pressure from A, B, and C because it is not connected to them by a water path.

Example 2:

Calculate the static pressure at the bottom of a **3000 m deep oil reservoir**, given:

- Depth (h) = 3000 m
- Density of crude oil (ρ) = 800 kg/m³
- Acceleration due to gravity (g) = 9.81 m/s²
- Atmospheric pressure (P_0) = 101,325 Pa (standard atmospheric pressure)

Solution

The static pressure (P) at a depth h in a fluid is given by:

$$P = P_0 + \rho g h$$

where:

- P_0: Atmospheric pressure (reference pressure).
- ρ: Density of the fluid.
- g: Acceleration due to gravity.
- h: Depth of the fluid column.

$$P = 101{,}325 \text{ Pa} + (800 \text{ kg/m}^3) \cdot (9.81 \text{ m/s}^2) \cdot (3000 \text{ m})$$

$$P = 101{,}325 \text{ Pa} + 23{,}544{,}000 \text{ Pa}$$

$$P = 23{,}645{,}325 \text{ Pa}$$

Example 3:

Calculate the static pressure at the bottom of a **10-meter-high water tank**. Assume the density of water is 1000 kg/m^3 and atmospheric pressure is 101,325 Pa.

Solution

$$P = P_0 + \rho g h$$

$$P = 101{,}325 + (1000 \cdot 9.81 \cdot 10)$$

$$P = 101{,}325 + 98{,}100 = 199{,}425 \text{Pa}$$

$$P = 199{,}425 / 100{,}000 = 1.99 \text{ bar}$$

Example 4:

A natural gas pipeline is **5 km long** and has a **vertical rise of 200 meters**. The density of natural gas is 0.8 kg/m^3. Calculate the static pressure difference between the bottom and top of the pipeline. Ignore friction losses.

Solution

$\Delta P = \rho g h$

$\Delta P = 0.8 \cdot 9.81 \cdot 200$

$\Delta P = 1{,}569.6 \text{Pa}$

$\Delta P = 1{,}569.6 / 100{,}000 = 0.0157 \text{bar}$

3.4 Measurement of Pressure

Pressure is a very important characteristic of fluids. As a result, there are many measuring techniques used to measure pressure. As shown in equation 3.2, height of a liquid column could be used to measure pressure.

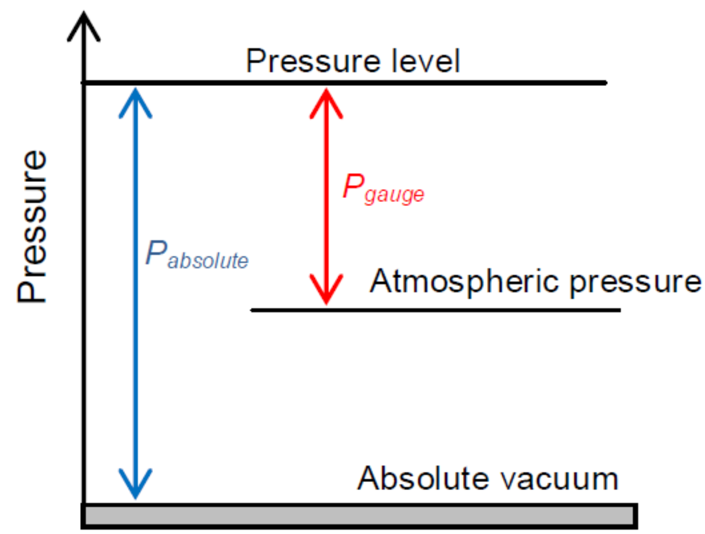


Figure 2.5. Relationship between gauge, absolute and atmospheric pressures.

It is essential to understand that the pressure is measured relative to the atmospheric pressure. In other words, some measuring techniques measure the pressure difference between the fluid and the atmosphere. This is called the gauge pressure.

For instance, P-Patm gives the gauge pressure. To obtain the absolute pressure one has to add the atmospheric pressure to the gauge pressure. Absolute pressure is measured relative to an absolute vacuum (zero pressure).

$$P_{gauge} = P_{absolute} - P_{atmosp\ here}$$

The absolute value of the atmospheric pressure is 101325 Pa. In most cases 1.01×10^5 Pa (101kPa) is used for simplicity.