Lecture Two

2.1 Newton's Law of Viscosity

As already pointed out, different liquids flow at different rates given all other conditions remains same. This means there is some property that affects the way fluids flow. This property is called *viscosity*.

Viscosity of a fluid originates from the nature of molecular interactions. Liquids, unlike gasses, when liquids flow under applied force, molecules are in motion and continuously dislocating from its molecular arrangement with respect to other molecules. To dislocate a molecule, a certain amount of energy is required. Viscosity is the energy that needs to dislocate a mole of a fluid.

Viscosity characterizes the flow of fluids. Newton, studying the flow realised that the applied shear force and the amount of deformation relate to one another. For example consider a rectangular fluid packet as shown in figure 1.3. A shear force F is applied to the upper surface at time t=0. During a small period of δt, upper surface moves a small distance δx deforming the rectangle to its new position shown in (b).

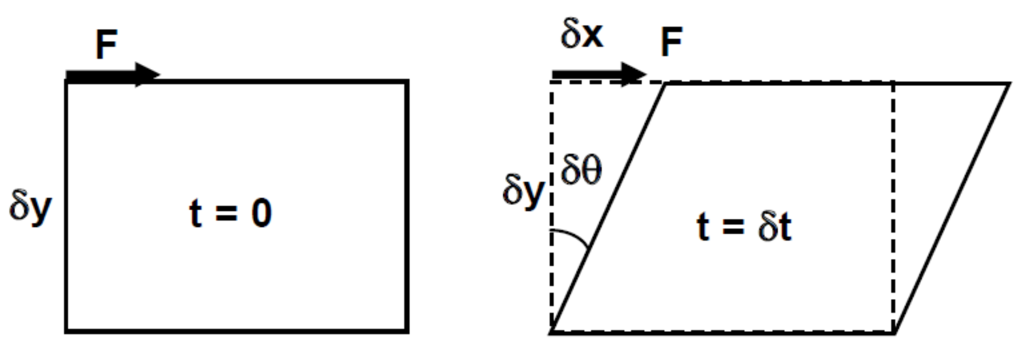


Figure 1.3. Deformation of a rectangular fluid element under applied shear stress

As long as the force F is applied, the fluid element will continue to deform. The rate of deformation is given by the rate at which the angle δθ changes. The rate of deformation is proportional to the shear stress applied. Shear stress is normally designated by the Greek letter τ (tau).

$$\tau = \frac{F}{A}$$

The rate of deformation is given by the rate at which the angle δθ increases with applied shear force τ. Considering the proportionality

$$\tau \propto \frac{\delta\theta}{\delta t}$$

The angle δθ is given by

$$tan\delta\theta = \frac{\delta x}{\delta y}$$

For small angles tan δθ ≈ δθ

Therefore,

$$\tau \propto \frac{\delta x}{\delta y\ \delta t}$$

Since $\lim_{\delta t \to 0} \frac{\delta x}{\delta t} \to du$, where *du* is the velocity induced by the applied force.

$$\tau \propto \frac{du}{dy}$$

Newton postulated that proportionality constant is the viscosity. This gives

$$\tau = \mu \frac{du}{dy}$$

This equation achieves dimensional homogeneity only if μ has units Pa.s (Pascal seconds). However, it is common practice

to give the viscosity in *Poise* (P) or *centipoises* (cP), a unit named after French physicist Jean Marie Poiseuille.

1P = 1 g / cm.s

The term $\frac{du}{dy}$ is called the *velocity gradient*.

Above equation shows that the shear stress is linearly proportional to the velocity gradient. Fluids that show this linear relationship is called *Newtonian fluids*. Water, air, and crude oil are some examples of Newtonian fluids. However, there are fluids that do not show the linear relationship. They are called non-Newtonian fluids. Polymer melts, xanthan gum and resins are some examples for non-Newtonian fluids. In non-Newtonian fluids the viscosity often depends on the shear rate and also the duration of shearing. We will discuss non Newtonian fluids later in the lecture series.

The viscosity μ is called the absolute or dynamic viscosity. There is another related measure of viscosity called kinematic viscosity often designated by the Greek letter ν (nu).

$$\nu = \frac{\mu}{\rho}$$

Kinematic viscosity has the units m^2/s. Units of kinematic viscosity in cgs system, cm^2/s is called Stokes. It is so named in honour of the Irish mathematician and physicist George Gabriel Stokes. Kinematic viscosity could be understood as the area a fluid can cover during a unit period of time under the influence of gravity (during a second).

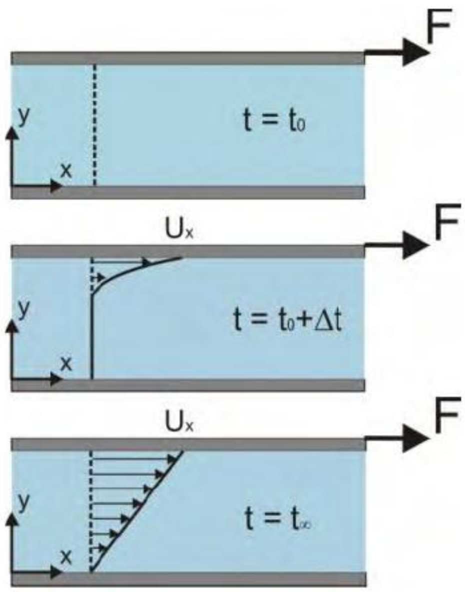


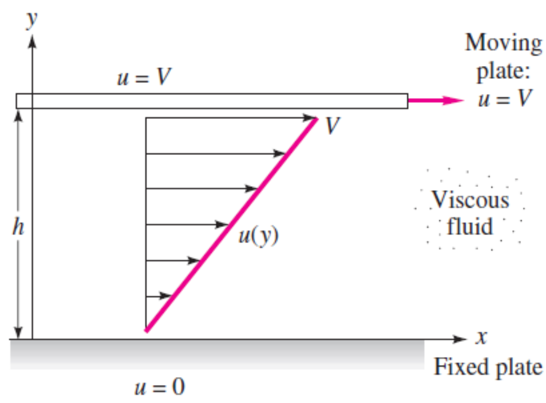
Figure 1.4. Development of velocity over time for a suddenly accelerated plate

As mentioned before, viscosity affects the fluid flow by setting a velocity gradient proportional to the shear stress applied on the fluid. For example consider a fluid trapped between two plates. If one plate, say the top one, is pushed forward at the constant velocity U while holding the bottom plate stationary all the time, the fluid start to move slowly. With time the velocity will penetrate down to the bottom plate generating a velocity profile along the depth as shown in Figure 1.4. The fluid elements at the top plate are being dragged at the same velocity as the plate. Fluid elements slightly below is dragged along by the first layer. All subsequent layers are dragged by the one above. On the other hand you can view that as the layer below slowing down the one above by offering some friction; hence the energy dissipation.

Viscosity is measured using a wide range of viscometers that measures the time taken to flow a known amount of the liquid or measuring the shear rate indirectly measuring the torque of a shaft rotating in the liquid. Ostwalt, Cannon-Fenske and Saybolt viscometers measures the flow time and cone and plate type viscometers use the torque measurements.

Example1:

Suppose that the fluid being sheared in below figure is crude oil at 20 C. Compute the shear stress in the oil if V = 3 m/s and h = 2 cm. Given that the oil viscosity μ= 0.29 kg/(m-s).



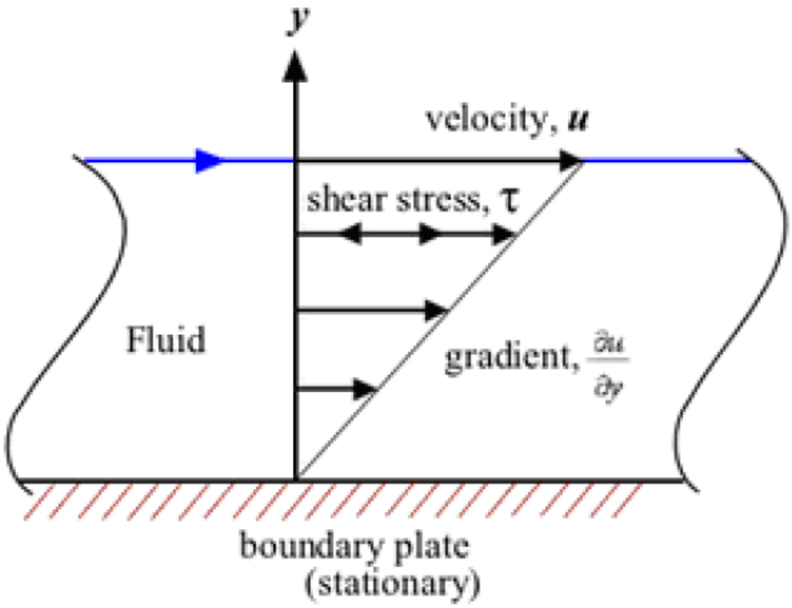
Solution:

$$\tau = \mu \frac{du}{dy}$$

$$\tau = \mu \frac{V}{h} = \left(0.29 \frac{\text{kg}}{\text{m} \cdot \text{s}}\right) \frac{(3\ \text{m/s})}{(0.02\ \text{m})} = 43.5 \frac{\text{kg} \cdot \text{m/s}^2}{\text{m}^2} = 43.5 \frac{\text{N}}{\text{m}^2} \approx 44\ \text{Pa}$$

Example 2:

In a fluid the velocity measured at a distance of 75mm from the boundary is 1.125m/s. The fluid has absolute viscosity 0.048 Pa s and relative density 0.913. What is the velocity gradient and shear stress at the boundary assuming a linear velocity distribution? Determine its kinematic viscosity.



Solution:

$$Gradient = \frac{dV}{dy} = \frac{1.125}{0.075} = 15\ s^{-1}$$

$$\tau = \mu \frac{dV}{dy} = 0.048 x 15 = 0.720\ Pa.s$$

$$\upsilon = \frac{\mu}{\rho} = \frac{0.048}{913} = 5.257 x 10^{-5}\ m^2/s$$

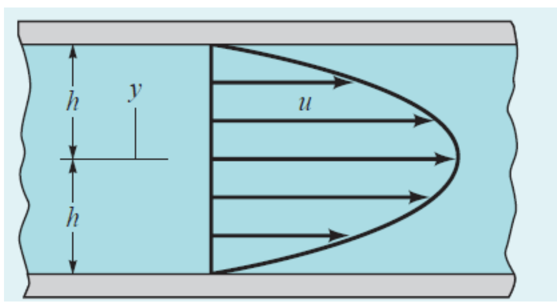
Example 3:

The velocity distribution for the flow of a Newtonian fluid between two wide, parallel plates is given by the equation:

$$u = \frac{3V}{2}\left[1 - \left(\frac{y}{h}\right)^2\right]$$

where V is the mean velocity V =2 ft /s. The fluid has a viscosity of 0.04 lb . s / ft^2, and h = 0.2 in.

Determine: (a) the shearing stress acting on the bottom wall, and (b) the shearing stress acting on a plane parallel to the walls and passing through the centerline (midplane).



Solution:

$$\tau = \mu \frac{du}{dy}$$

Thus, if the velocity distribution $u = u(y)$ is known, the shearing stress can be determined at all points by evaluating the velocity gradient, du/dy . For the distribution given:

$$\frac{du}{dy} = -\frac{3Vy}{h^2}$$

(a) Along the bottom wall $y = -h$ so that:

$$\frac{du}{dy} = \frac{3V}{h}$$

and therefore the shearing stress is

$$\tau_{\substack{\text{bottom}\\ \text{wall}}} = \mu\left(\frac{3V}{h}\right) = \frac{(0.04 \text{ lb} \cdot \text{s/ft}^2)(3)(2 \text{ ft/s})}{(0.2 \text{ in.})(1 \text{ ft}/12 \text{ in.})}$$
$$= 14.4 \text{ lb/ft}^2 \text{ (in direction of flow)}$$

(b) Along the midplane where y=0, so that:

$$\frac{du}{dy} = 0$$

and thus the shearing stress is:

$$\tau_{\text{midplane}} = 0$$

COMMENT: we see that the velocity gradient (and therefore the shearing stress) varies linearly with *y* and in this particular example varies from 0 at the center of the channel to 14.4 lb/ft^2 at the walls. This is shown in Figure below. For the more general case the actual variation will, of course, depend on the nature of the velocity distribution.