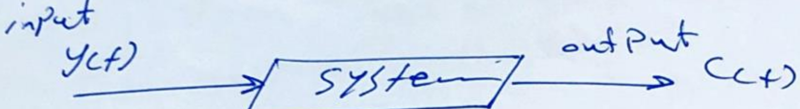
review



## Time - Domain Analysis of control systems:

**Time response:** It is evaluate the state & output response of the system with respect to time $(t)$




behavior of systems with respect of time.

- To study the system must be applied ~~by~~ to input ~~sy~~ signal (referece signal). (input), the Performance of the system is evaluated by study the response in time domain.

- The Time response of control system is usually divided into two Parts :- ①- Transent response $(y_t(t))$
② steady state response $(y_{ss}(t))$

In general The Time response $(y(t))$ can be written

$$\boxed{y(t) = y_T(t) + y_{ss}(t)}$$

In control systems, transient response $(y_T(t))$ is defind as the Part of ~~system~~ the time response that goes to zero when the time become very Large (goes to $\infty$)

$$\lim_{t \to \infty} y_T(t) = 0$$

The steady state response ($y_{ss}(t)$):- is the Part of total response that remains after the Transient has died out. (it can still vary in a fixed Pattern, such as sine wave or ramp function that increases with time.

- The transient response is very important in the control system which must be observed because ~~of~~ It is significant Part of dynamic behavior of the system and the deviation between output response and the input, before reach to the steady state. therefore must be control of the system during transient response and make it very short to transfer to the steady state region.

- The steady state response of control system is also very important, because it indicates where the system output ends up when the time is become Large. ~~the~~ (when compared with Input that indication of final accuracy of the system. In general if the steady state response ($y_{ss}(t)$) of the output not agree with the input the system has stead-state erro $E_{ss}(t)$

$$\boxed{e_{ss}(t) = y_{input} - y_{output}} \quad \boxed{r_{in}(t) - y_{out}(t)}$$

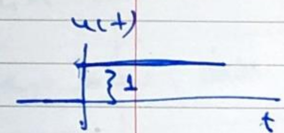
# Transiant & steady state response analysis

input → System → output

The Response of the System is the behavier of the output ~~system~~ system, depend on the input signal

usually test the ~~s~~ system response by known input ~~ref~~ signal

1- unit step $u(t) = 1 = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$

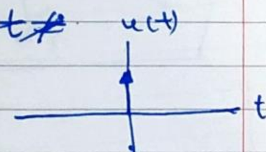



$u(s) = \frac{1}{s}$ from table of laplace transform.

2)- unit impulse $u(t) = \delta(t)$

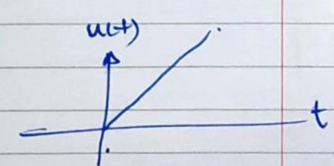
$u(s) = 1$

$u(t) = \begin{cases} 0 & t \neq 0 \\ \delta(t) & t = 0 \end{cases}$




3)- unit Ramp $u(t) = t$

$u(s) = \frac{1}{s^2}$




* input is known we must find output signal $Y(s)$ or $C(s)$ as time $y(t)$, $c(t)$ from T.F

$T.F = \frac{output}{input} \Rightarrow y(t) = T.F * u(t)$

$\boxed{C(s) = G(s)\, U(s)}$

Ex:- Find the steadystate response of the system (1st order) system

$$T.F = \frac{1}{TS+1} \quad , \quad U(S) = \text{unit step}$$

$$u(t) = 1 \Rightarrow U(S) = \frac{1}{S}$$

Sol:- $T.F = \frac{1/T}{S + 1/T}$ General form

$u(t) = 1$

* we convert the system to the S-domain
* easier than solving by time-domain

$$\text{time} \xrightarrow{\text{Laplace-T}} \text{S-domain} \xrightarrow{\mathcal{L}^{-1}} \text{time}$$

النظام بيتعامل مع time فقط

$$C(S) = U(S)\, T.F \Rightarrow \boxed{C(S) = \frac{1/T}{S + 1/T} * \frac{1}{S}}$$

$$C(S) = \frac{1}{(TS+1)S} \Rightarrow c(t) = \mathcal{L}^{-1} C(S)$$

$$\mathcal{L}^{-1} \frac{1}{S(TS+1)} = \frac{B}{S} + \frac{A}{TS+1}$$

~~B = Lim A ...~~

$$\frac{1}{\cancel{S(TS+1)}} = \frac{AS + B(TS+1)}{\cancel{S(TS+1)}}$$

$$1 = AS + BTS + B \qquad B = 1$$

$$(A + BT)S = 0 \Rightarrow A + T = 0$$

$$A = -T$$

③

sub in eq.:

$$\mathcal{L}^{-1}\frac{1}{s(Ts+1)} = \frac{1}{s} + \frac{-T}{Ts+1}$$

$$= \frac{1}{s} + \frac{-1}{s+\frac{1}{T}} \Rightarrow$$

$$\boxed{c(t) = 1 - e^{-\frac{1}{T}t}}$$ general eq. of output signal of system in steady state & transient.

* ~~in transient~~ plote the Behaivor of the system

| c(t) | t |
|---|---|
| 0 | 0 |
| 0.6 | 1 |
| | 2 |

* in transent we look on the $T_r$ & $T_s$ (rising time and settling time) ($T_r$ + $T_s$) must be small to increas the Performance of the system

$$T_r = \frac{2.2}{\text{pole}} = \frac{2.2}{a} = 2.2T \qquad G(s) = \frac{k}{s+a}$$

$$a = \frac{1}{T} \text{ priod Time.}$$

$$T_s = \frac{4}{a} = 4T \text{ in } 2\%$$

$$\frac{3}{a} = 3T \text{ in } 5\%$$

in Transent Priod must be small & in the steady start must be reduce the error $e$.

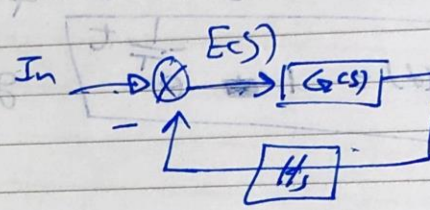
5

Find output in steady start $C_{ss}(t)$

$$C_{ss}(t) = \lim_{t \to \infty} C(t) \Rightarrow C_{ss}(s) = \lim_{s \to 0} s\,C(s)$$

$$C_{ss}(\infty) = 1 - e^{-\infty} = \underline{\underline{1}}$$

$$E(s) = R(s) - G(s)\,\underbrace{H(s)}_{1}$$




$$\therefore E(s) = U(s) - G(s)$$

$$e(t) = u(t) - c(t)$$

$$= 1 - \left(1 - e^{-\frac{1}{T}(t)}\right) = -e^{-\frac{1}{T}t}$$

$$e_{ss}(t) = \lim_{t \to \infty} e^{-\infty} = \underline{\underline{0}}$$

* T.F have all the c/c of the system

$S = j\omega = j2\pi f$

$T.F = \dfrac{S \mp Z \rightarrow \text{zero } (o)}{S \pm P \rightarrow \text{pols } (X)} = \dfrac{n(s)}{d(s)}$

$G(s) = \dfrac{S+5}{S^2+1} \Rightarrow$

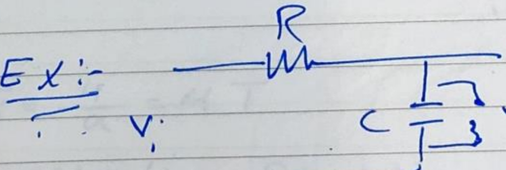
$n(s) = S+5 \Rightarrow \#\text{ zero} = 1$
$S = -5$ zero value

$d(s) = S^2+1 \Rightarrow S = \pm i \quad \#\text{ Pole} = 2$
pole value.

# of pole = degree of the $d(s)$ أكبر اس لل $\underline{\underline{s}}$
# of zero in system = degree of $(n(s))$

* usually we make $d(s)$ degree > degree of $n(s)$

Ex:-




$X_R = R$
$X_C = \dfrac{1}{CS}$

$G(s) = \dfrac{V_o(s)}{V_i(s)} \Rightarrow \dfrac{X_C}{X_C + X_R} = \dfrac{\frac{1}{CS}}{\frac{1}{CS}+R} = \boxed{\dfrac{1}{1+SRC}}$

1st order system.

$T.F = \dfrac{K}{S+a} = \dfrac{1/RC}{S+1/RC} = \dfrac{K}{S+a}$

$\boxed{a = \dfrac{1}{RC} \quad , \quad K = 1/RC}$

$$U(s) \longrightarrow \boxed{\frac{K}{s+a}} \longrightarrow C(s)$$

$u(t) = 1$

$u(s) = \frac{1}{s}$

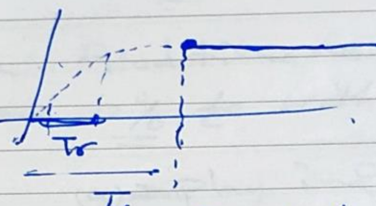
$C(s) = G(s) * U(s)$

$C(s) = \frac{K}{s(s+a)}$

$C(t) = \mathcal{L}^{-1} C(s)$

$\boxed{C(t) = 1 - e^{-at}}$ general eq. of out put system in stead start & transit

by Plot.




Tr:- rising Time:- the Priod to rising the out Put signal form (%10 to 90%)

$Tr = \frac{2.2}{a} \Rightarrow a = \frac{1}{RC}$ , $RC = T$ Priod time

$\boxed{Tr = 2.2T}$

Tr must be very small to increase the Performance of the system.

$Ts = \frac{4}{a} = 4T$

setling time Priod time to transfer the system from 0% to within 2% of steady state.

* Tr & Ts must be small

9

$T_r$ = rising time — time Priod to rising the signal output from (10% to 90%)

$$T_r = \frac{2.2}{a} \Rightarrow a = \frac{1}{RC} \Rightarrow RC = T \text{ (Priod time)}$$

$$= \frac{1}{T}$$

$$\boxed{T_r = 2.2\,T} \Rightarrow \text{nutral time (tranfent time)}$$

يجب أن تكون هذه الفترة صغيرة ($T_r$) لذلك يجب أن أقلل T

$T = RC$ أقلل ← $R$ لذلك [illegible] النظام

- $T_s$ stling time — Priod time to transfer the system from $0\%$ to within $2\%$ of ~~stay~~ state

$$T_s = \frac{4}{a} = 4T \quad (a) \Rightarrow \text{play important role}$$

$T_r$, $T_s$ must be small By reduce Resistance value

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$$y(t) = y_{ss}\left(1 - e^{-\frac{t}{RC}}\right)$$

in stabity only look on the Pole

في الـ stability نعتبر فقط الـ Pole ~~[illegible]~~

$$G(s) = \frac{K}{s+a} \qquad s + a = 0$$

يعني الـ $s+a=0$

حيث اذا كانت $s+a=0$

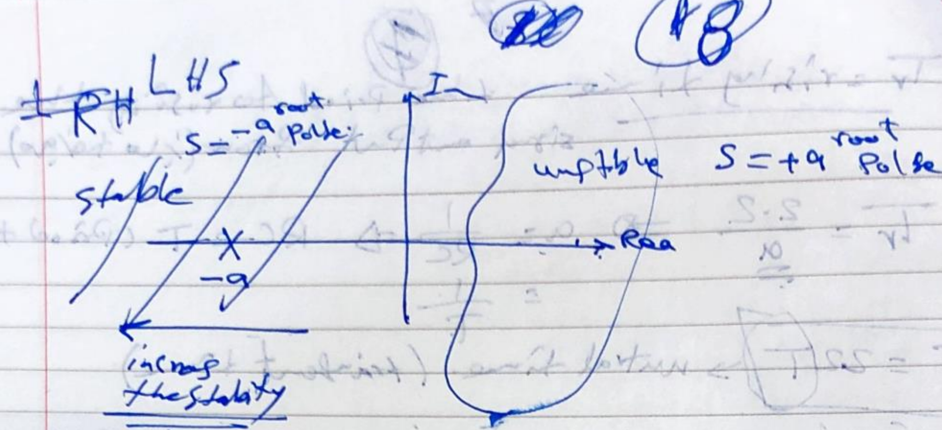
هذا يعني $G(s) = \frac{k}{0} = \infty$

unstable.

لذلك يجب أن ابتعد عن الـ zero

if $a > 0 \Rightarrow s =$ nagitive

48




هنا لك تعارض بين Performance of system ($T_r$, $T_s$) حيث انا نحتاج (a) كبيرة

$T_r = \frac{2.2}{a}$ = حيث انا نريد صغيرة لكي يكون اداء النظام افضل

اما في الـ stability حيث ان تكون a ~~كبيرة~~ نوعاً ما صغيرة لكي يكون النظام stable .

لا يمكن ان نحصل على اداء عالي و استقراريه عالية في النظام الواحد

لان الاداء يريد الـ Pole كبير اما الـ stability تريد a صغيرة جداً

$T_r = \frac{2.2}{a}$

$\underline{\underline{\mathcal{9}}}$

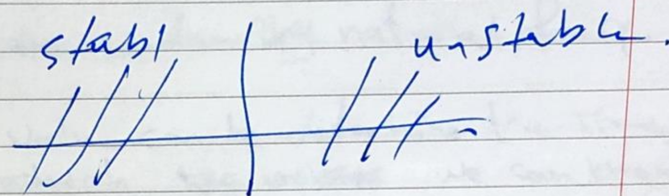
* In steady state only look to the Polle.

$G(s) = \boxed{\dfrac{1}{s+a}}$ . $s + a = \underline{\underline{0}}$

حيث ان a عن ن
السيط = $\alpha \leq 0$
unstable.

to make the system stable in every time.
though all cases must the poles > 0

$S = \underline{\underline{-a}}$




* هناك مفارقة بين الاداء المقام اثناء ال transent و جعل النظام
stable اثناء ال steady stat من خلال موقع ال Polse حيث

$Tr = \dfrac{2.2}{a} \Rightarrow$ Tr is ~~smalt~~ small by make $a \gg$

stablity the Polle must be small
$\underline{s \; a \ll 0}$

② محاضرة من القسم

## 2nd order system :—

$$\boxed{T.F = \frac{W_n^2}{\underbrace{1}_{\text{must be} = 1} S^2 + 2 \zeta w_n S + W_n^2}}$$

2nd order general T.F

where

$\zeta$ = damping ratio (zeta)

$w_n$ = ~~under~~ undamping natural freq.

* From $\zeta$ (zeta) value can be determine the Time response, ~~which has values~~ we can know the system response from ($\zeta$) value below.

$\zeta = 0$ undamped time response

$0 < \zeta \leq 1$ underdamped time response

$\zeta = 1$ critical damped time response

$\zeta > 1$ over damped time response.

* The Time response of system in 2nd order system dependion the ($\zeta$) value.

Ex: $T.F = \frac{1}{S^2 + 2S + 1}$ system has this T.F what the response of the system?

Sol: general T.F of 2nd order system is

$$\frac{w_n^2}{S^2 + 2 \zeta w_n S + w_n^2}$$

$\omega_n^2 = 1 \Rightarrow \omega_n = 1$

$2 = 2\zeta\omega_n \Rightarrow \cancel{2 = 2\zeta} \quad 2 = 2\zeta * 1$

$\boxed{\zeta = 1}$ The time of response is (critical damped).

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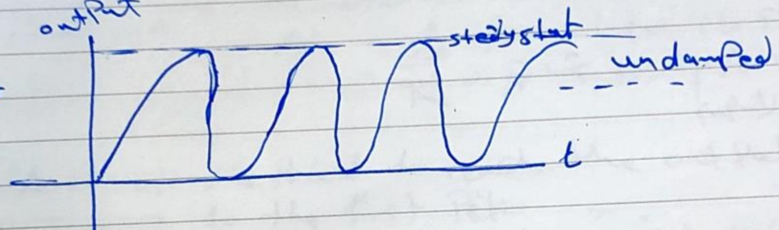
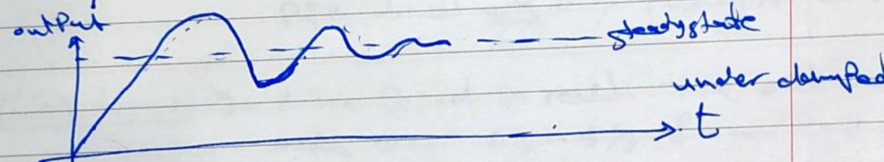
* The Time response depend on the ($\zeta$) value is

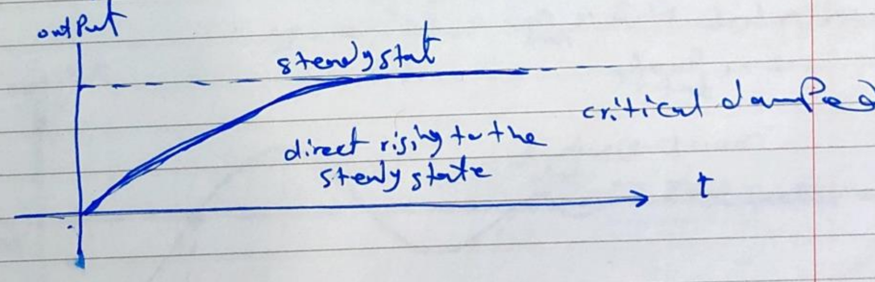
$\zeta = 0$



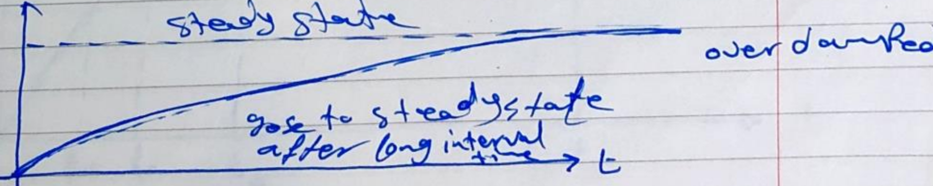

$0 < \zeta < 1$




$\zeta = 1$




$\zeta > 1$



# Time respponse 2nd order

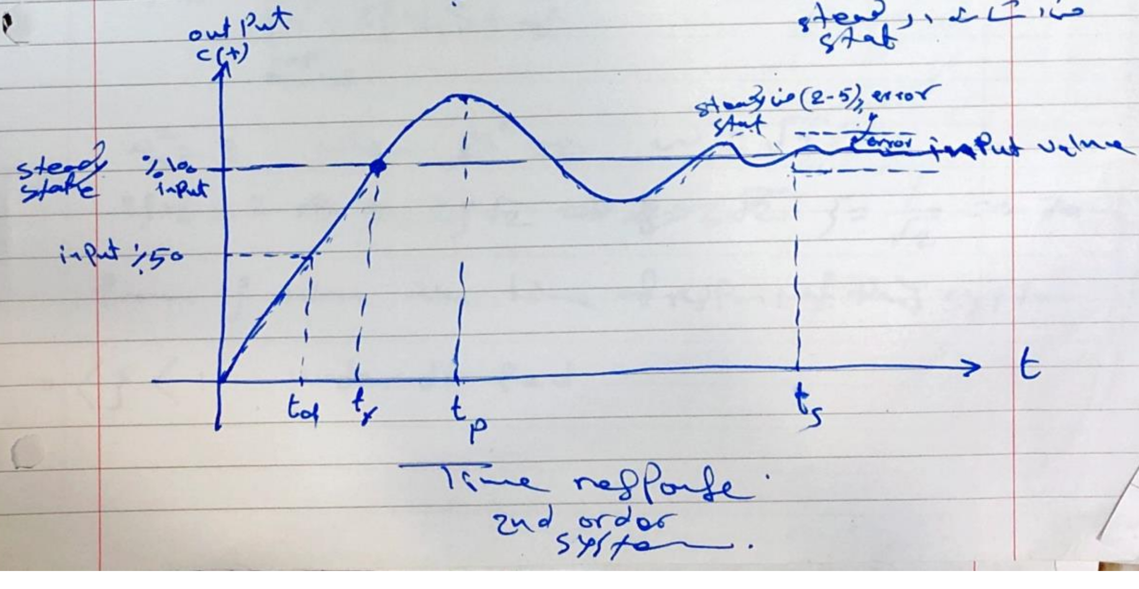
we must satisfied ($t_r$, $t_d$, $t_p$, $t_s$, $MP\%$).

**$t_r$ (timerising)** the Period time when the out Put respponse reach to 100% from input
(the priod take transent to start steadystate)
(2% — 90% from steadystate)
الوقت اللازم لوصول إشارة الـ output الى ١٠٠٪ من قيمة الـ input

**$t_d$ (delay time):** the Period time when the output reach to 50% from input (to steady state)
الوقت اللازم لوصول إشارة الـ output الى نصف قيمة الـ input

**$t_p$ (Pik time)** the Priod to reach the out Put signal to the first Pik.
الوقت اللازم لوصول إشارة الـ output الى اول Pik

**$t_s$ (setling time):** time Priod to reach the outPut make error 2% - 5% from input in steady state.
الوقت اللازم لوصول إشارة الـ output الى خطأ ٢٪ - ٥٪
ضمن منطقة الـ steady stat




Time respponse 2nd order system.

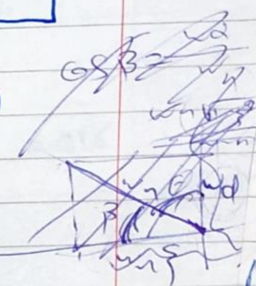
$\boxed{t_r = \frac{\pi - \beta}{w_d}}$ $\pi = 3.14$ , in radian $\boxed{\beta = \cos^{-1}\xi}$

$w_d$ = damped natural freq.

$\boxed{w_d = w_n\sqrt{1-\xi^2}}$ $\boxed{t_d = \frac{1+0.7\xi}{w_n}}$

$\boxed{t_p = \frac{\pi}{w_d}}$ , $t_s =$ 2% $\frac{4}{\xi w_n}$ ; 5% $\frac{3}{\xi w_n}$

$\boxed{Mp\% = e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}} \times 100\%}$ $= e^{-\left(\frac{\xi}{w_d}\right)\pi w_n}$



Ex:- Time response ($t_r$, $t_p$, $t_s$, $M_p$)

$$\frac{Y(s)}{R(s)} = \frac{10}{s^2+2s+2} = T.F$$

Sol: From general form of 2nd order system T.F

$$T.F = \frac{w_n^2}{\underbrace{1}_{\text{must equal (1)}} s^2 + 2\xi w_n s + w_n^2}$$

$w_n^2 = 2$ when $(1) s^2 \Rightarrow w_n = \sqrt{2}$

$2\xi w_n = 2 \Rightarrow 2 = 2\xi\sqrt{2} \Rightarrow$ $\xi = \frac{1}{\sqrt{2}} = 0.70$

from $\xi$ value the time of respons of this system

$0 < \xi < 1$ under damped

radain

$t_r = \frac{\pi - \beta}{\omega_d}$ $\beta = \cos^{-1}\zeta = \cos^{-1}(0.707) = 0.7856$

$w_d = w_n\sqrt{1-\zeta^2} = \sqrt{2}\sqrt{1-(0.707)^2} = \boxed{0.7655}$

$t_r = \frac{3.14 - 0.7855}{0.7655} = \boxed{3.1 \text{ sec}}$

$t_p = \frac{\pi}{\omega_d} = \frac{3.14}{0.7655} = \boxed{4.1 \text{ Sec}}$

$t_s$ when $2\% = \frac{4}{\zeta\omega_n} = \frac{4}{0.707 * \sqrt{2}} = 4$ sec.

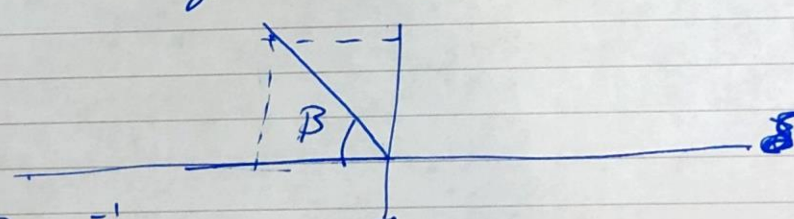
$5\% = \frac{3}{\zeta\omega_n} = 3$ sec.

$M_p = e^{-\pi\zeta/\sqrt{1-\zeta^2}} = e^{\frac{-\pi(0.707)}{\sqrt{1-0.707^2}}} \approx 0.016$

$\boxed{MP\% = 1.6\%}$

* $t_s = \frac{4}{\sigma}$ $\sigma = \zeta\omega_n$ secund [illegible] 2%

$t_s = \frac{4}{\sigma}$ $\sigma = \zeta\omega_n$ 5%



$\beta = \cos^{-1}$

$$\boxed{t_r = \frac{\pi - \beta}{w_d}}$$

$\pi = 3.14$, in radian , $\boxed{\beta = \cos^{-1} \xi}$

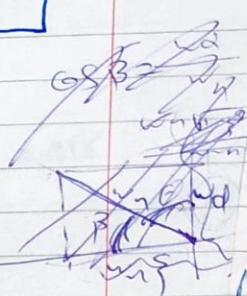
$w_d$ = damped natural freq.

$$\boxed{w_d = w_n \sqrt{1-\xi^2}}$$

$$\boxed{t_d = \frac{1+0.7\xi}{w_n}}$$

$$\boxed{t_p = \frac{\pi}{w_d}} \quad , \quad \boxed{t_s = \begin{cases} 2\% & \frac{4}{\xi w_n} \\ 5\% & \frac{3}{\xi w_n} \end{cases}}$$

$$\boxed{M_p\% = e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}} \times 100\%} = e^{-\left(\frac{\xi}{w_d}\right)\pi w_n}$$





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$w_n^2 = 2$ when $(1)s^2 \Rightarrow w_n = \sqrt{2}$

$2\xi w_n = 2 \Rightarrow 2 = 2\xi\sqrt{2} \Rightarrow \cancel{\xi = \xi\sqrt{2}}$ $\xi = \frac{1}{\sqrt{2}} = 0.70$

from $\xi$ value the time of respons of this system

$0 < \xi < 1$ under damped