

Sedimentation

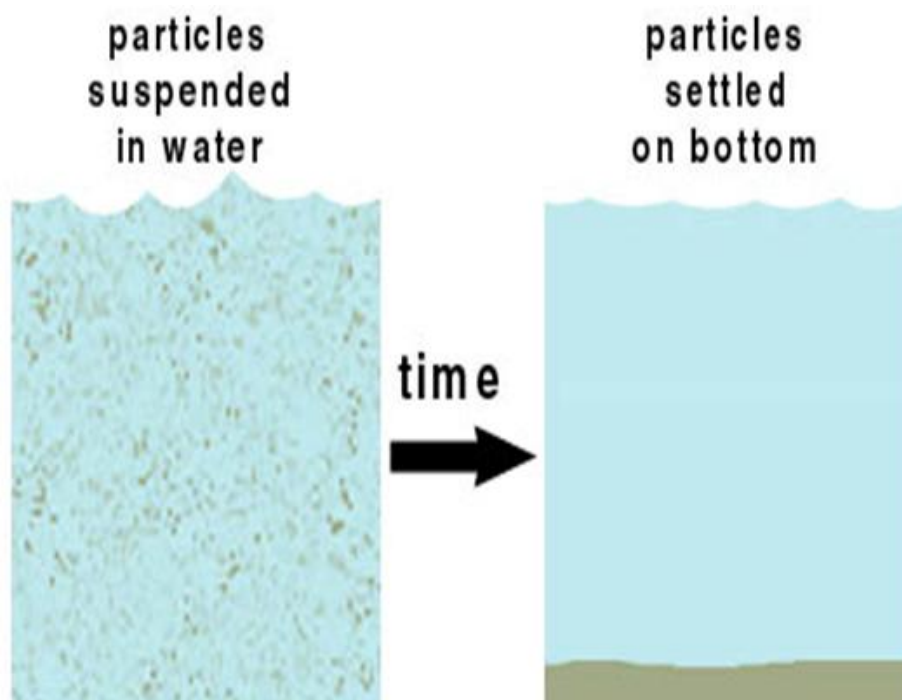
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- 8-Terminal velocity of particle for sedimentation
- 9-Terminal velocity of particle for hindered settling.

Introduction

Sedimentation describes the motion of molecules in solutions or particles in suspensions in response to an external force such as gravity, centrifugal force or electric force.

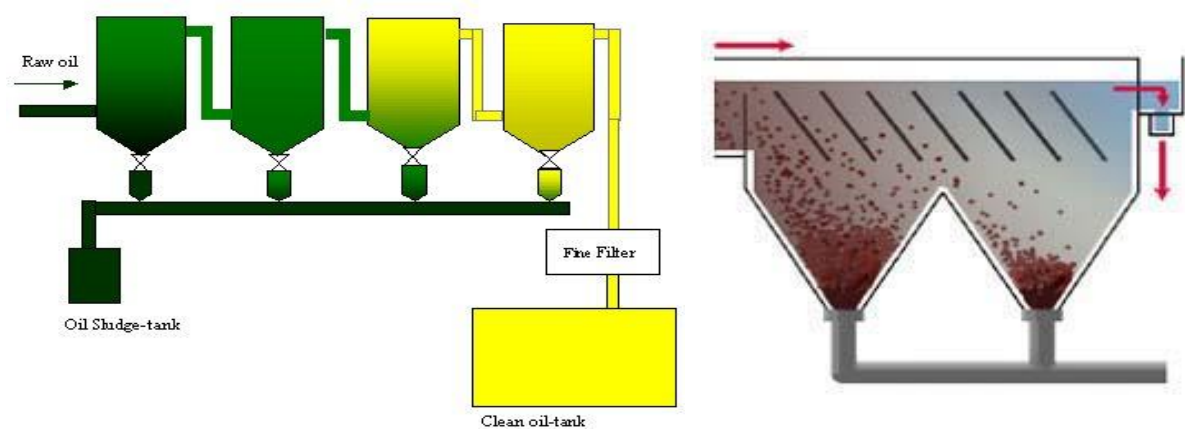
The separation of a dilute slurry or suspension by gravity settling into a clear fluid and a slurry of higher solids content is called sedimentation.



Objective & Application

is to remove the particles from the fluid stream so that the fluid is free of particle contaminants.

Applications of sedimentation: include removal of solids from liquid sewage wastes, settling of crystals from the mother liquor, separation of liquid-liquid mixture from a solvent-extraction stage in settler, water treatment, separation of flocculated particles, lime-soda softening iron and manganese removal, wastewater treatment, solids/sludge/residuals.



Theory for sedimentation

Whenever a particle is moving through a fluid, a number of forces will be acting on the particle.

First, a density difference is needed between the particle and the fluid.

If the densities of the fluid and particle are equal, the buoyant force on the particle will counter balance the external force and the particle will not move relative to the fluid.

There are **three forces** acting on the body:

- **Gravity Force**
- **Buoyant Force**
- **Drag Force**

Mechanics of particle motion in fluids

To describe, two properties need:

1- Drag coefficient

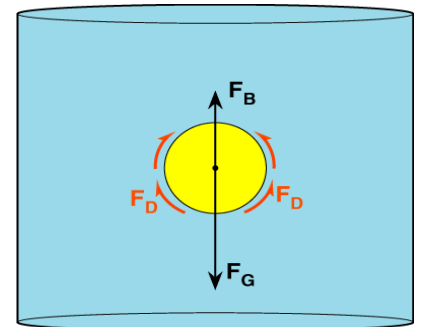
2-Terminal velocity

Drag Coefficient

For particle movement in fluids, drag force is a resistance to its motion.

Drag coefficient is a coefficient related to drag force.

Overall resistance of fluids act to particle can be described in term of drag force using drag coefficient



Comparing with fluid flow in pipe principle, drag coefficient is similar to friction coefficient or friction factor (f).

$$f = \frac{\text{shear stress}}{\left(\frac{\text{kinetic energy}}{\text{unit volume of flow}} \right)}$$

$$f = \frac{(F / A)}{\left(\frac{\frac{1}{2}mv^2}{V} \right)}$$

$$F = \frac{1}{2} f \rho A v^2$$

For drag coefficient:

$$C_D = \frac{\text{drag force per area}}{\left(\frac{\text{kinetic energy}}{\text{unit volume of flow}} \right)}$$

$$C_D = \frac{(F / A)}{\left(\frac{\frac{1}{2}mv^2}{V} \right)}$$

$$F = \frac{1}{2} C_D \rho A v^2$$

Frictional drag coefficient

1- For flat plate with a laminar boundary layer:

$$C_D = \frac{1.328}{N_R^{0.5}}$$

2- For flat plate with a turbulent boundary layer:

$$C_D = \frac{0.455}{(\log N_R)^{2.58}}$$

$$N_R = \frac{Dv\rho}{\mu}$$

D = length of plate = diameter of sphere

Frictional drag coefficient:

For *flat plate* with a transition region:

$$C_D = \frac{0.455}{(\log N_R)^{2.58}} - \frac{1700}{N_R}$$

$$N_R = \frac{Dv\rho}{\mu}$$

D = length of plate = diameter of sphere

Note: If a plate or circular disk is placed normal to the flow, the total drag will contain negligible frictional drag and does not change with Reynolds number .

Sphere object

At very low Reynolds number (< 0.2), Stoke law is applicable. The inertia forces may be neglected and those of viscosity alone considered.

$$C_D = \frac{24}{N_R}$$

Terminal or Settling Velocity

Settling velocity (v_t): The terminal velocity at which a particles falls through a fluid.

When a particle is dropped into a column of fluid it immediately accelerates to some velocity and continues falling through the fluid at that velocity (often termed the (*terminal settling velocity*).

The speed of the terminal settling velocity of a particle depends on properties of both the fluid and the particle:

Properties of the particle include:

The size if the particle (d).

The density of the material making up the particle (r_p).

The shape of the particle.

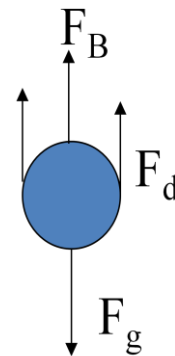
Particle Settling Velocity

Put particle in a still fluid... what happens?

Speed at which particle settles depends on:

Particle properties: $D, \rho_p, shape$

Fluid properties: ρ_f, μ, Re



STOKES Settling Velocity

Assumes spherical particle (**diameter = d_p**) laminar settling

F_G depends on the volume and density (ρ_p) of the particle and is given by:

$$F_G = \frac{\pi}{6} d_p^3 \times \rho_p g = \frac{\pi}{6} \rho_p g d_p^3$$

F_B is equal to the weight of fluid that is displaced by the particle:

$$F_B = \frac{\pi}{6} d_p^3 \times \rho_f g = \frac{\pi}{6} \rho_f g d_p^3$$

Where ρ_f is the density of the fluid.

F_D is known experimentally to vary with the size of the particle, the viscosity of the fluid and the speed at which the particle is traveling through the fluid.

Viscosity is a measure of the fluid's "resistance" to deformation as the particle passes through it.

$$F_D = \frac{1}{2} C_D \rho_f A_p v^2 = 3\pi d_p \mu v \qquad C_D = \frac{24}{N_R}$$

Where μ (the lower case Greek letter μ) is the fluid's dynamic viscosity and v is the velocity of the particle; $3\pi d_p$ is proportional to the area of the particle's surface over which viscous resistance acts.

From basic equation, $F = mg =$ resultant force:

$$F = ma = m \frac{dv}{dt} = F_G - F_B - F_D$$

With $v =$ terminal velocity or v_t :

$$F_G - F_B - F_D = m \frac{dv}{dt} = 0$$

In the case of $0.0001 < N_R < 0.2$, terminal velocity can be determined by using $C_D = 24/N_R$:

$$v_t = \sqrt{\frac{4 d_p (\rho_p - \rho_f) g}{3 C_D \rho_f}} = \frac{d_p^2 (\rho_p - \rho_f) g}{18 \mu}$$

In the case of $0.2 < N_R < 500$, terminal velocity can be determined by using C_D as:

$$C_D = \frac{24}{N_R} \left[1 + 0.15 N_R^{0.687} \right]$$

In the case of $500 < N_R < 200,000$, terminal velocity can be determined by using C_D as:

$$C_D = 0.44$$

Table 9.1 Comparative summary of equations of motion of spheres, disks and circular cylinders (Adopted from Lapple, 1956)

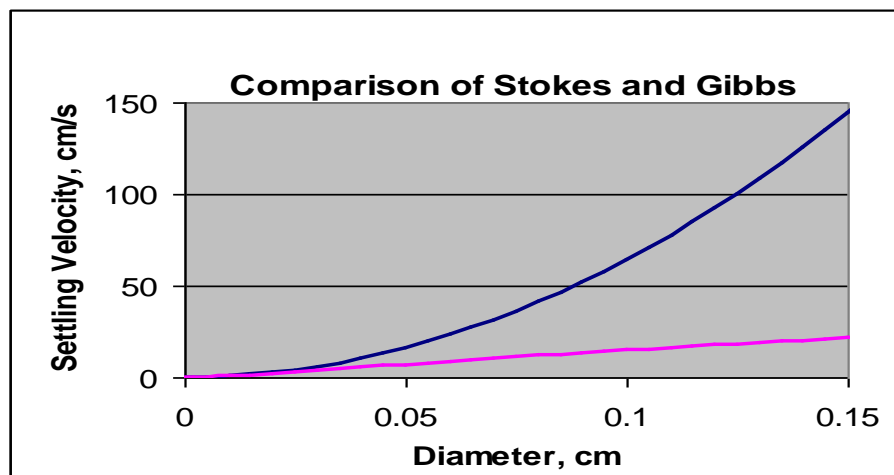
	Sphere (any direction ¹)	Thin disk (normal to face ¹)	Thin disk (parallel to face ¹)	Infinite circular cylinder (normal to axis ¹)
Reynolds No. equ.	$d_p V \rho_f / \eta$	$d_p V \rho_f / \eta$	$2LV \rho_f / \eta$	$d_p V \rho_f / \eta$
Frontal area A_p	$(\pi/4) d_p^2$	$(\pi/4) d_p^2$	$(d_p) L$	$(d_p) L$
Mass m_p	$\rho_p (\pi/6) d_p^3$	$\rho_p (\pi/4) d_p^2 L$	$\rho_p (\pi/4) d_p^2 L$	$\rho_p (\pi/4) d_p^2 L$
Drag relationships				
streamline flow				
$N_R < 0.2, F_D =$	$3\pi \eta V d_p$	$8\eta V d_p$	$(16/3) \eta V d_p$	$(4\pi/K) \eta V L$
$C_D N_R =$	24	$64/\pi$	$64/3$	$8\pi/K$
turbulent flow				
C_D (average)	0.44	1.12	—	1.2
N_R (range)	$1 \times 10^3 - 2 \times 10^5$	>1000	—	$1 \times 10^2 - 2 \times 10^5$
Terminal velocity				
V_t^2	$\frac{4g d_p (\rho_p - \rho_f)}{3C_{Df}}$	$\frac{2gL(\rho_p - \rho_f)}{C_{Df}}$	$\frac{gd_p \pi(\rho_p - \rho_f)}{2C_{Df}}$	$\frac{gd_p \pi(\rho_p - \rho_f)}{2C_{Df}}$

¹ Direction of flow or motion

L = Thickness of disk, length of rod or cylinder, length of flat plate along direction of flow or motion

$K = 2.002 \ln N_R$

Laminar (Stokes) vs. Turbulent (Gibbs) settling:



Stoke's Law has several limitations

i) It applies well only to perfect spheres.

The drag force ($3\pi d m v_t$) is derived experimentally only for spheres.

Non-spherical particles will experience a different distribution of viscous drag.

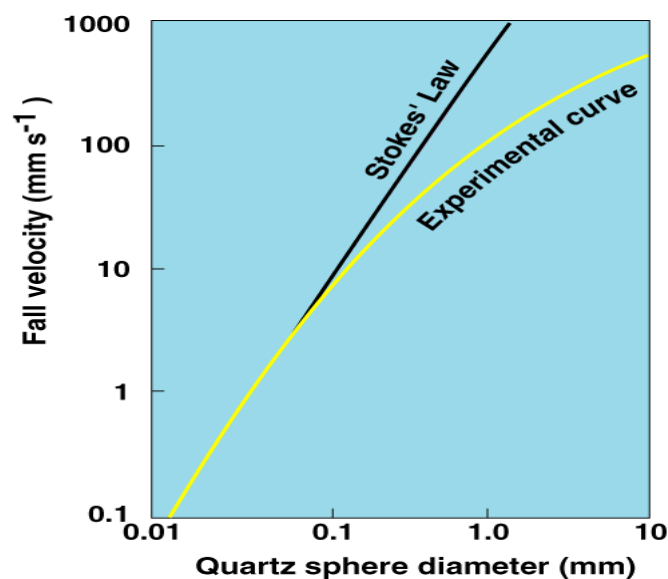
ii) It applies only to still water.

Settling through turbulent waters will alter the rate at which a particle settles; upward-directed turbulence will decrease v_t whereas downward-directed turbulence will increase v_t .

iii) It applies to particles **0.1 mm** or finer.

Coarser particles, with larger settling velocities, experience different forms of drag forces.

Stoke's Law overestimates the settling velocity of quartz density particles larger than **0.1 mm**.

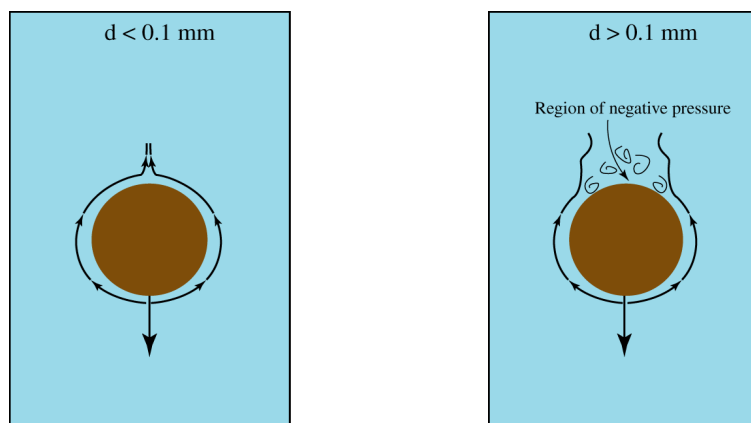


a) When settling velocity is low ($d < 0.1 \text{ mm}$) flow around the particle as it falls smoothly follows the form of the sphere.

Drag forces (F_D) are only due to the **viscosity of the fluid**.

When settling velocity is high ($d > 0.1 \text{ mm}$) flow separates from the sphere and a wake of eddies develops in its lee.

Pressure forces acting on the sphere vary.

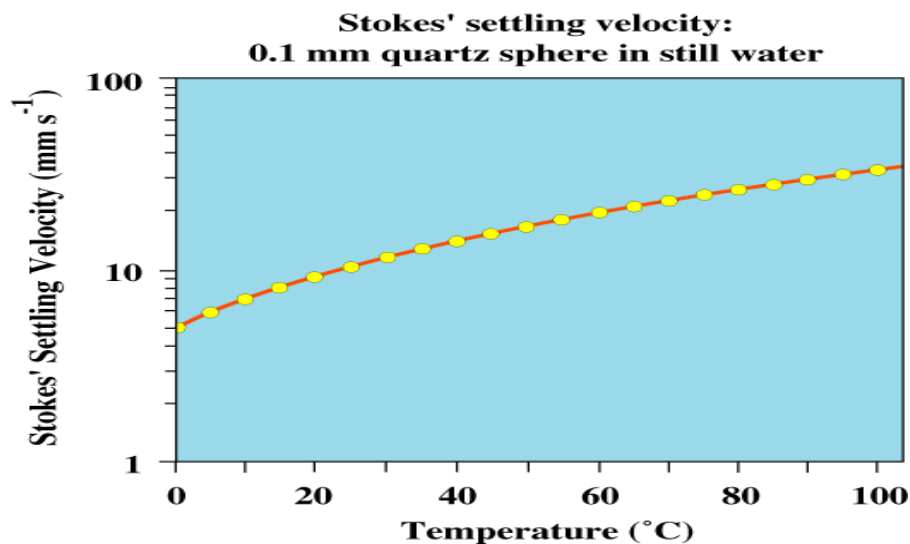


Negative pressure in the lee retards the passage of the particle, adding a new resisting force.

Stoke's Law neglects resistance due to pressure.

iv) Settling velocity is temperature dependant because fluid viscosity and density vary with temperature.

Temp. °C	μ Ns/m ²	ρ Kg/m ³	v_t mm/s
0	$1.792 \cdot 10^{-3}$	999.9	5
100	$2.84 \cdot 10^{-4}$	958.4	30



Grain size is sometimes described as a linear dimension based on Stoke's Law:

Stoke's Diameter (d_s): the diameter of a sphere with a Stoke's settling velocity equal to that of the particle.

$$v_t = \frac{(\rho_f - \rho_p)gd_s^2}{18\mu}$$

Set $d_s = d_p$ and solve for d_p .

$$d_p = \sqrt{\frac{18\mu v_t}{(\rho_f - \rho_p)g}}$$

Settling velocity of dust particles

Example:

Calculate the settling velocity of dust particles of **60 μm** diameter in air at **21°C** and **100 kPa** pressure. Assume that the particles are spherical and density = **1280 kg m^{-3}** , and that the viscosity of air = **1.8 x 10⁻⁵ N s m^{-2}** and density of air = **1.2 kg m^{-3}** .

Solution:

For **60 μm** particle:

$$v_t = \frac{gD_p^2(\rho_p - \rho)}{18\mu}$$

$$\begin{aligned} v &= \frac{(60 \times 10^{-6})^2 \times 9.81 \times (1280 - 1.2)}{(18 \times 1.8 \times 10^{-5})} \\ &= 0.14 \text{ m s}^{-1} \end{aligned}$$

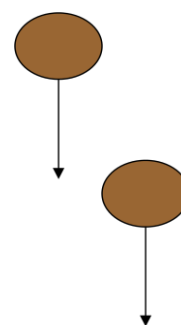
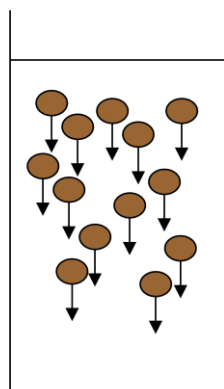
Checking the Reynolds number for the **60 μm** particles,

$$\begin{aligned} \text{Re} &= (vD_p/\mu) \\ &= (60 \times 10^{-6} \times 0.14 \times 1.2) / (1.8 \times 10^{-5}) \\ &= 0.56 \end{aligned}$$

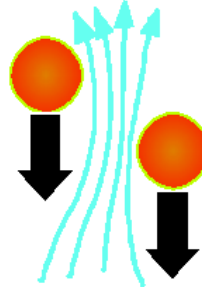
HINDERED SETTLING

If the settling is carried out with high concentrations of solids to liquid so that the particles are so close together that collision between the particles is practically continuous and the relative fall of particles involves repeated pushing apart of the lighter by the heavier particles it is called hindered settling.

particles interfere with each other



particle interactions change settling velocity



discrete particles:

higher solids concentration reduces velocity

Flocculating particles:

experiments only



Hindered Settling

ε = void fraction

ψ_p = empirical correlation fraction

$$= \frac{1}{10^{1.82(1-\varepsilon)}}$$

For turbulent flow ($Re > 10^4$) $v_s = \sqrt{\frac{10 * g (\rho_p - \rho_w) d}{3 \rho_w}}$

Settling velocity of spherical discrete particle under turbulent flow

Example:

Find the terminal settling velocity of a spherical discrete particle with diameter (0.5 mm) and specific gravity of 2.65 settling through water at 20 °C ?
 $\rho_w = 998 \text{ kg/m}^3$, $\mu = 1.002 \times 10^{-3}$

Solution:

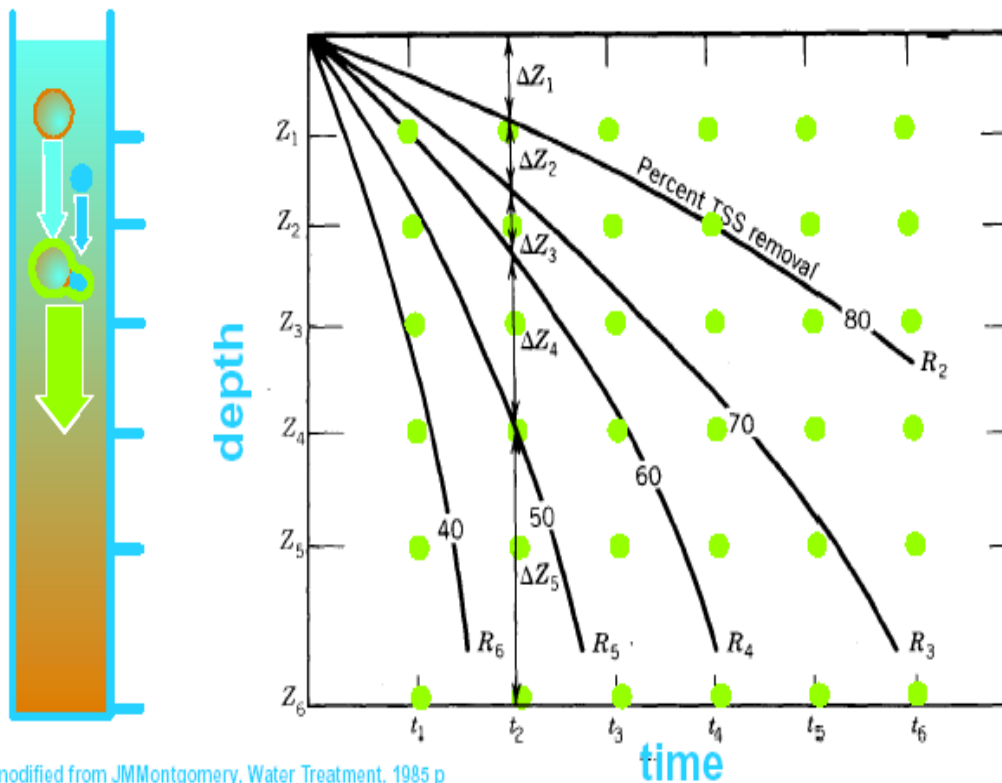
$Sp.gr = \rho_p / \rho_w$

$2.65 = \rho_p / 998$ ----- $\rho_p = 2644.7 \text{ kg/m}^3$ $v_s = \sqrt{\frac{10 * g (\rho_p - \rho_w) d}{3 \rho_w}}$

$v_s = \sqrt{\frac{10 * 9.81 (2644.7 - 998) * 0.5 * 10^{-3}}{3 * 998}}$

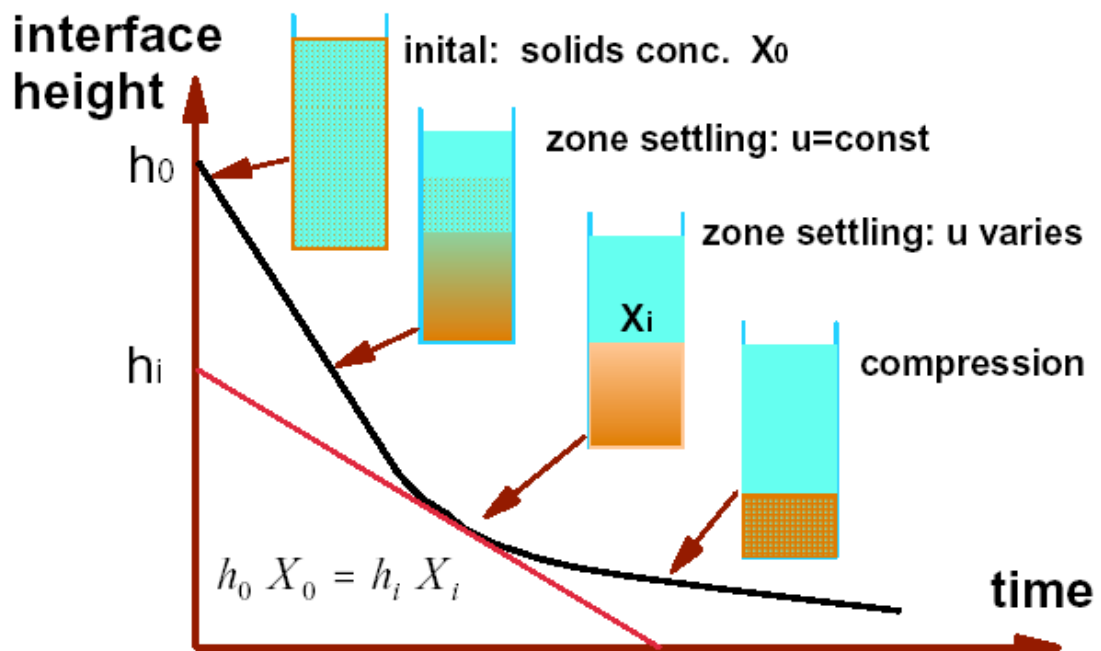
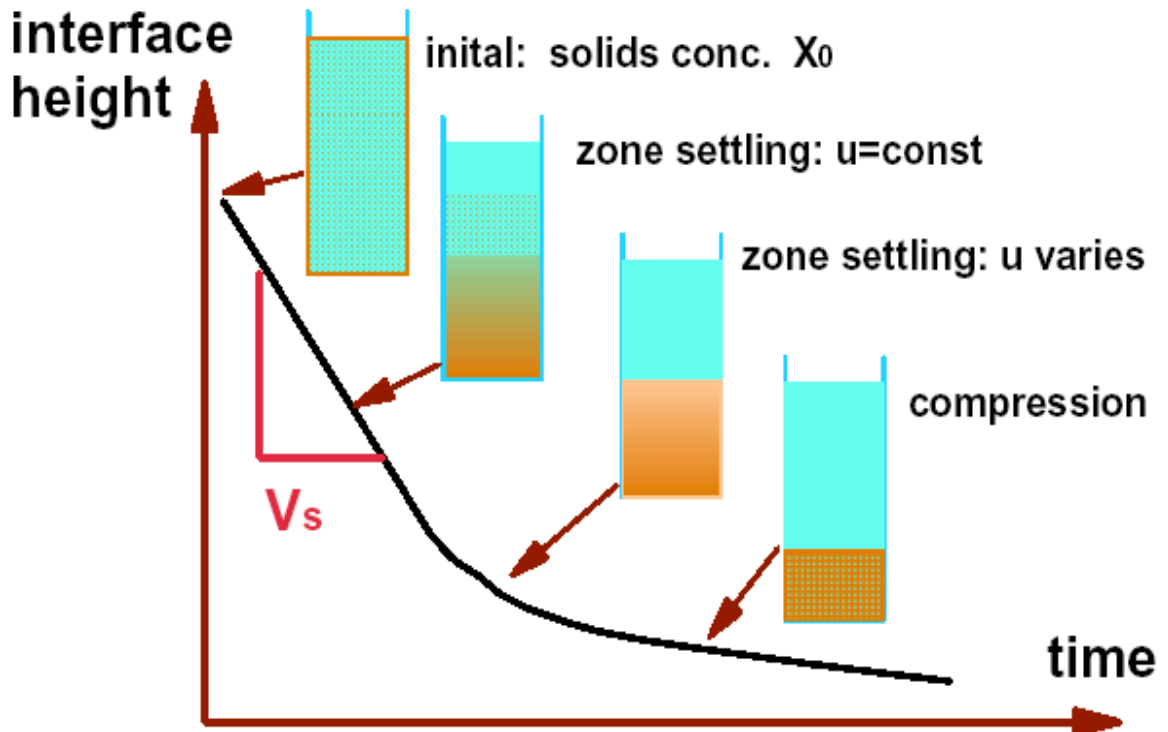
$v_s = 0.1642 \text{ m/sec}$

Flocculating Particles (Type II)



modified from JMMontgomery, Water Treatment, 1985 p

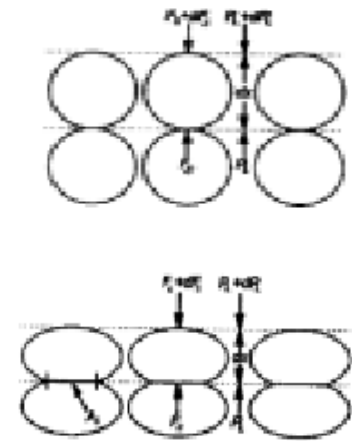
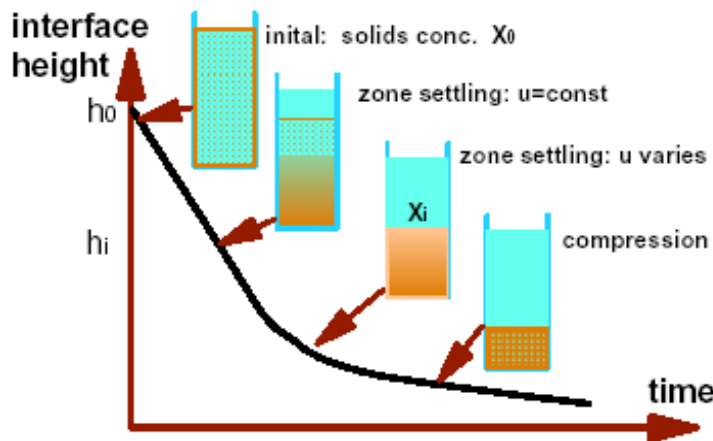
Zone Settling & Compression:



$$C_o h_o = C_c h_c = C_u h_u$$

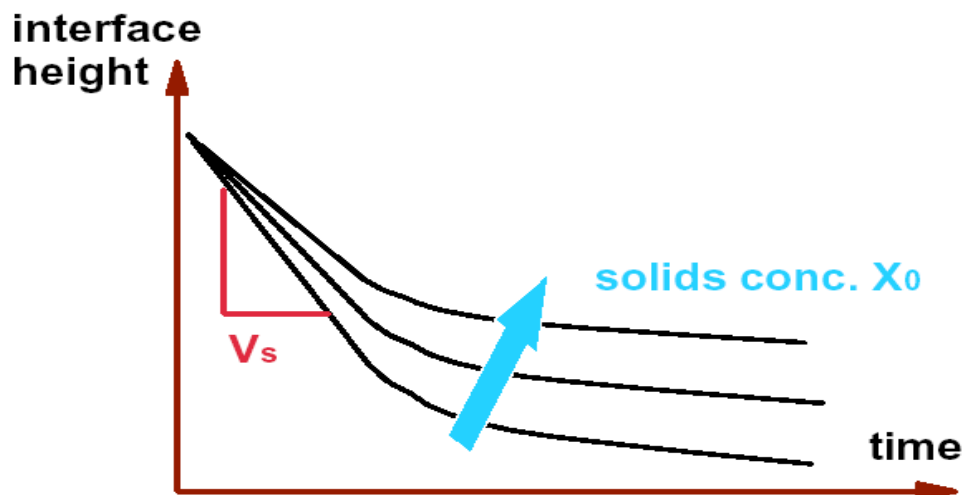
$$C_u = \frac{C_o h_o}{h_u}$$

Compression - Compaction:



$$\frac{dh}{dt} = -k(h - h_\infty)$$

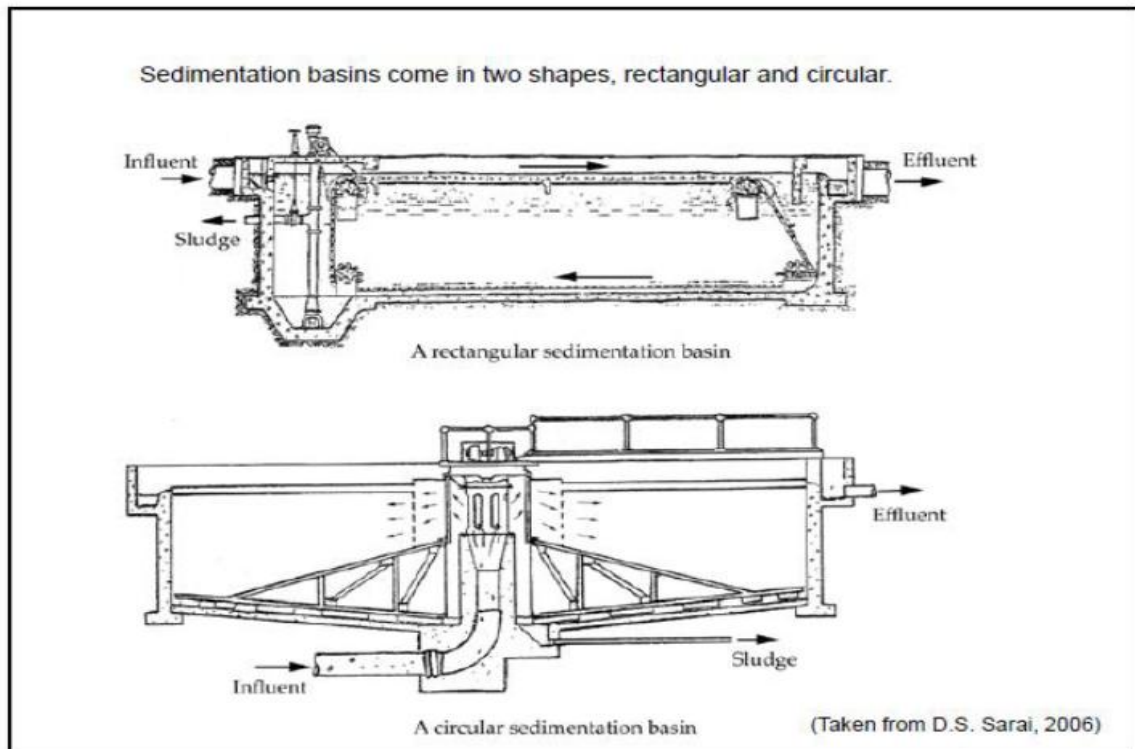
Zone Settling:



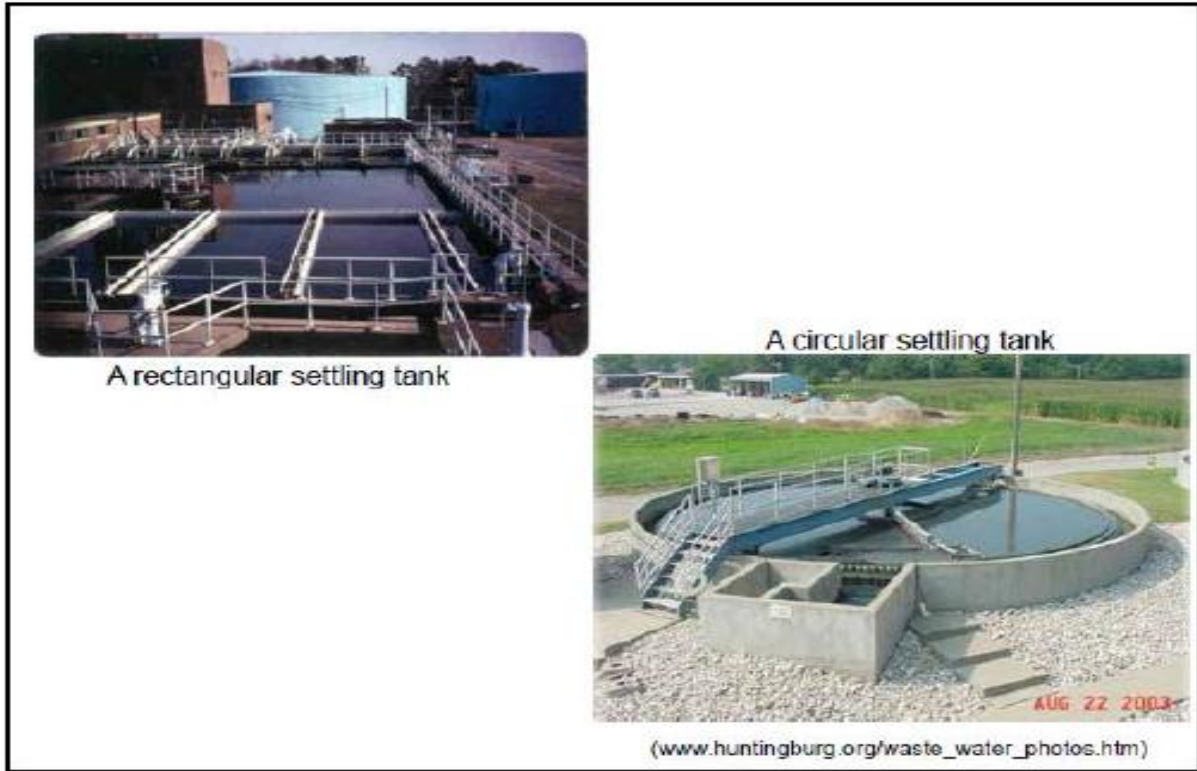
$$V_s = \frac{h_o - h_u}{t_u - t_o} = \frac{h_o - h_i}{t_i}$$

Sedimentation basins (settling tank) :

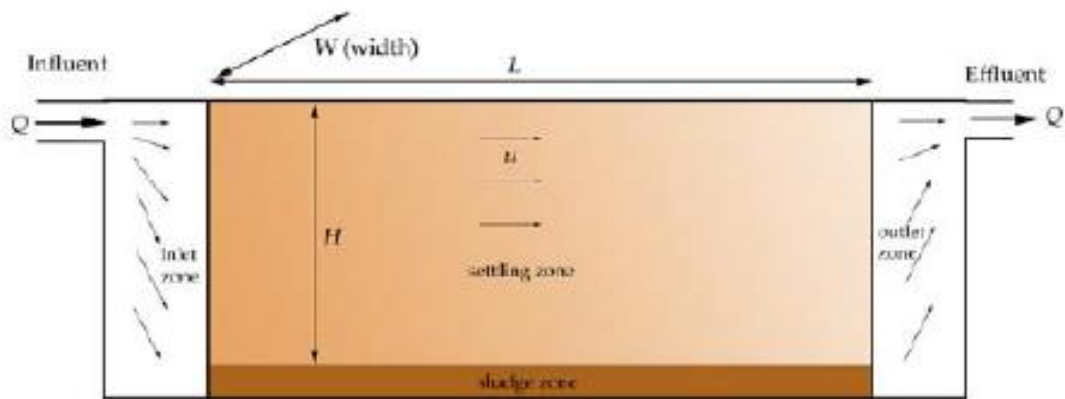
Sedimentation basins, also called settling tanks or *clarifiers*, are large tanks in which water is made to flow very slowly in order to promote the sedimentation of particles .



The **overflow rate** (also known as the **surface loading** or the **surface overflow rate**) is equal to the settling velocity of the smallest particle which the basin will remove. Surface loading is calculated by dividing the flow by the surface area of the tank. Overflow rate should usually be less than 1,000 gal/day-ft.² The weir loading is another important factor in sedimentation basin efficiency. **Weir loading**, Is another factor in sedimentation efficiency weir loading also known as **weir overflow rate**, is the number of gallons of water passing over a foot of weir per day. The standard weir overflow rate is **10,000 to 14,000 gpd/ft** and should be less than **20,000 gpd/ft**. Longer weirs allow more water to flow out of the sedimentation basin without exceeding the recommended water velocity.



Consider what goes on in a rectangular settling basin.



Key parameters are:

- H = depth of settling zone
- L = length of settling zone
- W = width of settling zone
- V = volume of settling zone
- Q = volumetric flowrate
- u = flow speed
- θ = transit time = hydraulic retention time

Relations are:

$$u = \frac{Q}{HW}$$

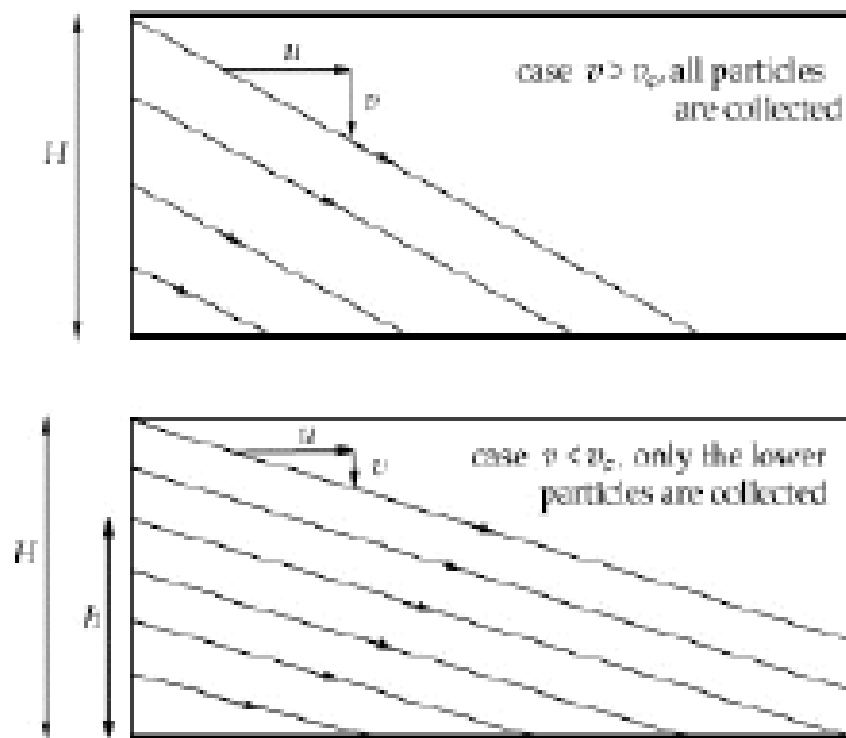
$$\theta = \frac{L}{u} = \frac{HLW}{Q} = \frac{V}{Q}$$

If a particle settles with vertical speed v , its vertical fall over the length of the tank is

$$h = v \theta = v \frac{L}{u}$$

This length h is either longer than the settling depth H or it is not.

- If $h \geq H$, then the particle hits the bottom before the end of the tank and is collected.
- If $h < H$, then the particle may or may not hit the bottom, depending on the level at which it starts, If it starts close to the bottom, it will settle on the bottom, but if it starts too high it will not fall down enough and will escape with the outflow.



$$1- V = \frac{h \cdot Q}{H \cdot A}$$

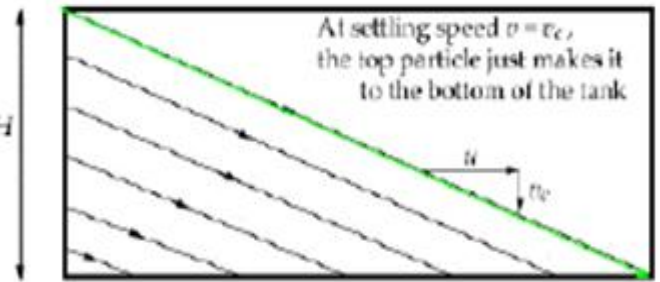
It is easy to show that, if $h < H$, the particles in the lowest h portion of the tank are collected and that those starting within the top $H - h$ portion do not get collected.

This leads us to define a critical settling speed, namely the settling speed of the particles that get barely all collected.

$$h = H \quad \text{for} \quad v_c = \frac{H}{\theta} = \frac{Hu}{L}$$

in terms of the volumetric flowrate $h=H$

$$v_c = \frac{H}{L} \frac{Q}{WH} = \frac{Q}{WL} = \frac{Q}{A}$$



This critical speed is called the *overflow rate*

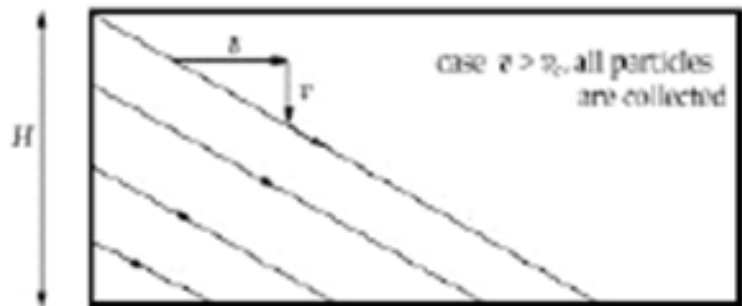
Note: How in this definition, Q is not divided by the cross-sectional area WH but by the horizontal area of the tank, $WL = A$

Collecting efficiency:

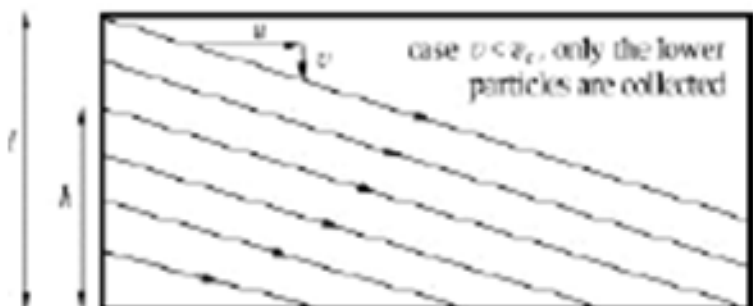
For particles settling with speed v faster than v_c , the collection efficiency is **100%**

For particles settling with speed v slower than v_c , the collection efficiency is $\frac{h}{H}$.

$$\eta = 100\%$$



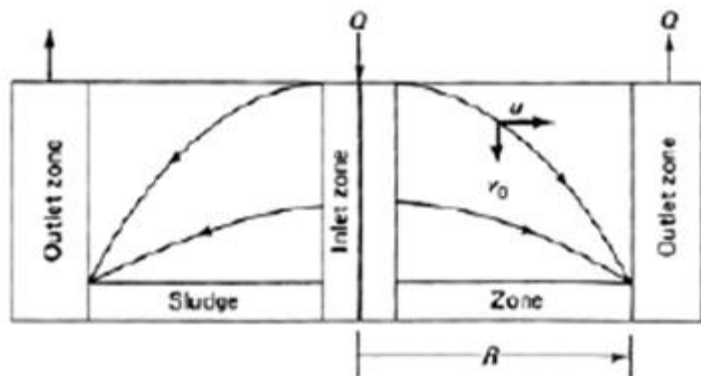
$$\eta = \frac{h}{H} = \frac{vL}{uH} = \frac{v}{v_c} (< 1)$$



And, how does it work in a **circular** sedimentation tank ?

The radial velocity u varies the radius r . decreasing so that the volumetric flow through the enlarging cross- section remains constant:

$$u = \frac{Q}{2\pi rH}$$



The slope of the settling curve follows the equation

$$\frac{dh}{dr} = \frac{v}{u} = \frac{2\pi rHv}{Q} \implies h = \frac{(\pi R_{outer}^2 - \pi R_{inner}^2)Hv}{Q} = \frac{AHv}{Q}$$

The collecting efficiency is

$$\eta = \frac{h}{H} = \frac{Av}{Q} = \frac{v}{(Q/A)} = \frac{v}{v_c} \text{ same as for the rectangular tank}$$

Typical design values for sedimentation basins

Parameter	Range	Typical values	Units
Rectangular Basin			
Length	15 – 90	25 – 40	m
Depth	3 – 5	3.5	m
width	3 - 24	6 - 10	m
Circular Basin			
Diameter	4 – 60	12 – 45	m
Depth	3 - 5	4.5	m
Water Treatment			
Overflow rate	35 - 110	40 - 80	m/day
Wastewater Treatment			
Overflow rate	10 - 60	16 - 40	m/day

In time t , vertical distance covered is v_t .

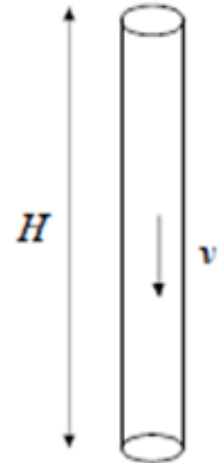
If $v_t < H$, then fraction (v_t / H) has been collected.

If $v_t > H$, then **100%** has settled.

There is a distribution of particles with various settling velocities.

Define: $m(v)$ as the probability distribution.

Put another way, $m(v) dv$ = mass fraction of particles with settling speed between v and $(v + dv)$.



Detention time :

$$t = \frac{\text{depth}}{\frac{\text{tank volume}}{\text{flow rate}}}$$

$$\text{Weir length: } Lw = \frac{Qc}{W.\text{loading}}$$

Where:

Lw = weir length (ft,m)

Qc = flow in one tank (gal/day , m^3 /sec)

في حالة اكثر من خزان = عدد الخزانات / معدل الجريان

$W.L$ = Weir loading (gal/day.ft , kg/sec.m)

Depth : the tank depth is calculated as defined:

$$d = \frac{V}{A}$$

Where : d = depth (ft , m) V = volume (ft^3 , m^3)

A = Surface area (ft^2 , m^2)

For all tank the length has been defines as follows:

$$L = 4W \quad W = \sqrt{\frac{V}{4d}}$$

$$Qc = \frac{Q}{n}$$

Where :

W = Width of tank ,

Qc = flow in one tank

Q = total flow

n = number of tank

$$A = \frac{Qc}{O.R}$$

A = Surface area

Qc = flow rate

$O.R$ = Over flow rate

To calculate tank surface area: $A = Qc / O.R$.

To calculate tank depth: $d = V / A$

To calculate width of a rectangular tank where length is four times the width:

$$W = \sqrt{\frac{V}{4d}}$$

Example:

Find dimension of rectangular basin having volume = 3 MLd , $t = 4 \text{ hr}$,
velocity = 10 cm/min

Solution:

Time = 4x60/1 hour = 240 min

Velocity = 10 cm / min

Length = 240 min x 0.1 = 24 m

Volume of water at 4 hour = $(3 \times 10^6 / 10^3) \times (4/24) = 500 \text{ m}^3$

Cross area = $V/L = 500/24 = 20.8 \text{ m}^3$

Assume working depth of 3.

Width = $20.8/3 = 7 \text{ m}$

Extra depth 1m of sludge + 0.5 free

Board = $3 + 1.5 = 4.5 \text{ m}$

Settling tank has size 4.5x24x7

$$\text{Volume per hour} = 3 \times 10^6 / 24 = 125000 \text{ m}^3$$

Surface loading (SOR , surface over flow rate):

$$\text{SOR} = (3 \times 10^6 / 24) \times (1 / 24 \times 7)$$

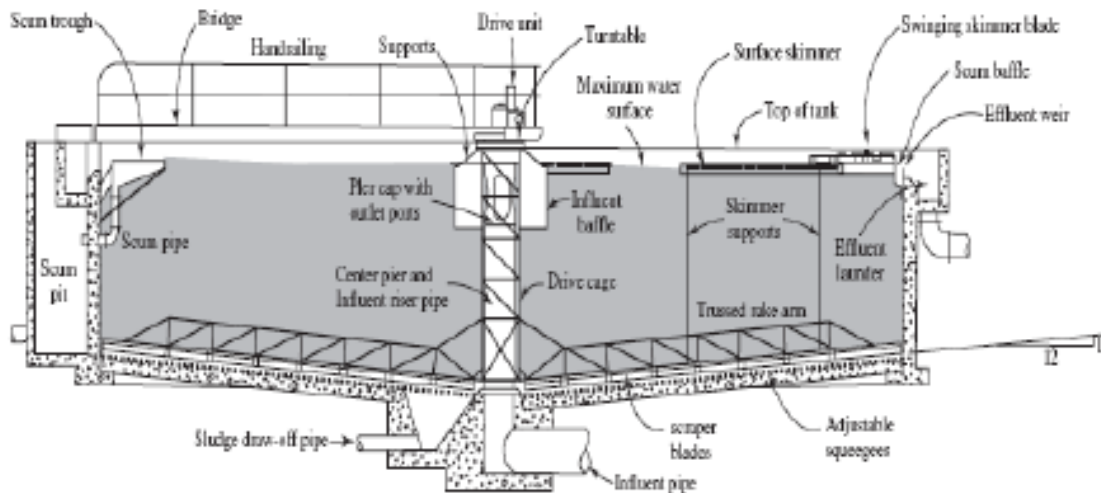
Advantages of circular basin :

1. No dead spaces
2. Low maintenance cost of equipment used for sludge collection and ease of design and construction.
3. The walls of circular tanks act as tension rings, which permit thinner walls than those for rectangular basins. As a result, the circular tanks have a lower capital cost per unit surface area than the rectangular tanks.

(تعمل جدران الخزانات الدائرية كحلقات توتر ، مما يسمح بجدران أرق من تلك الموجودة في الأحواض المستطيلة. نتيجة لذلك ، تتمتع الخزانات الدائرية بتكلفة رأسمالية أقل لكل وحدة مساحة من الخزانات المستطيلة.)

Disadvantages of circular basin :

1. low hydraulic detention efficiency. (كفاءة احتجاز هيدروليكي منخفضة)
2. high risk of short-circuiting. (ارتفاع خطر حدوث قصر في الدائرة الكهربائية)



Sedimentation tank components:

A- Influent Structure (هيكل مؤثر)

purpose:

(1) Dissipate energy of incoming flow by means of baffles.

(تبديد طاقة التدفق الوارد عن طريق الحواجز)

(2) Distribute the flow equally along the basin width.

(توزيع التدفق بالتساوي على طول عرض الحوض)

(3) Prevent short circuiting by disturbing the thermal or density stratification.

(منع قصر الدائرة عن طريق إزعاج الطبقات الحرارية أو الكثافة)

(4) Promote flocculation.(تعزيز التكتل)

(5) Keep low head loss.(الحفاظ على فقدان الرأس المنخفض)

Details of influent structure:

(1) Inlet channel: has a minimum velocity of 0.3m/s to prevent settling of

solids..(قناة المدخل: سرعة لا تقل عن 0.3 متر / ثانية لمنع ترسب المواد الصلبة)..

(2) Submerged ports or an overflow weir: Ports have velocities between 4.5 and 9 m/min at design average flow. The spacing between the ports is normally 1–2m with a maximum spacing of 3m.

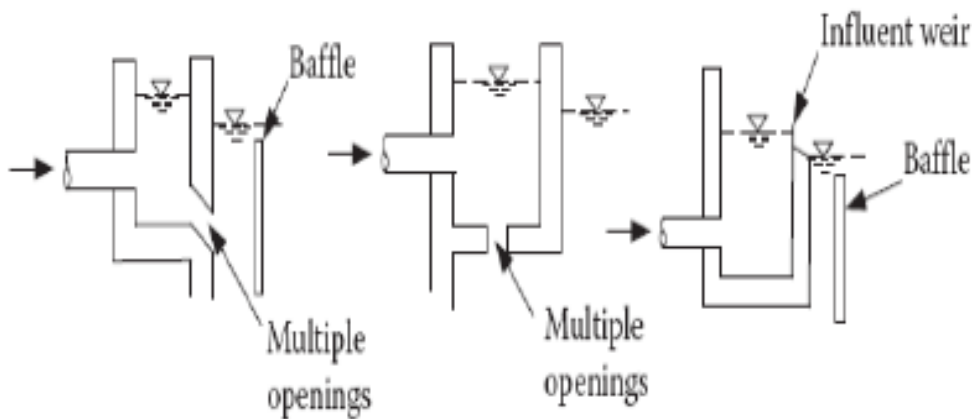
(المنافذ المغمورة أو السد الفائض تتمتع الموانئ بسرعات تتراوح بين 4.5 و 9 م / دقيقة بمتوسط تدفق التصميم. عادة ما تكون التباعد بين المنافذ 1 - 2 م مع أقصى تباعد 3 م).

(3) A perforated baffle (5% porosity) with ports size between (5-10cm), is typically installed 0.6–0.9m away from the inlet ports and the lower end is about 15–30 cm below the inlet ports. The top of the baffle is kept below the average water surface to allow scum to pass over the top.

(يتم تركيب حاجز مثقوب (5% مسامية) بمنافذ حجمها بين (5-10 سم) ، عادة على بعد 0.6 -

0.9 متر من منافذ الدخول ويكون الطرف السفلي حوالي 15-30 سم تحت منافذ الدخول. يتم

الاحتفاظ بالجزء العلوي من الحاجز تحت سطح الماء المتوسط للسماح بمرور حثالة فوق القمة).



B-Effluent structure

purposes:

(1) Provide uniform distribution of flow over a large area

توفير توزيع منتظم للتدفق على مساحة كبيرة

(2) Minimize lifting of the particles in to the effluent.

قلل من رفع الجسيمات إلى النفايات السائلة

(3) Reduce the escape of floating matter to the effluent.

الحد من تسرب المواد العائمة إلى النفايات السائلة

Details of Effluent structure

1.Overflow weir type : v-notch or straight

2.Baffle in front of an overflow weir: It stops the floating matter from escaping into the effluent (0.6 m submergence minimum)

مقاطع أمام حاجز الفائض: يمنع المادة العائمة من الهروب إلى النفايات السائلة (0.6 متر غمر كحد أدنى)

3.Effluent launder or channel. (غسيل أو قناة النفايات السائلة)

4.Outlet box.(صندوق تنفيذ)

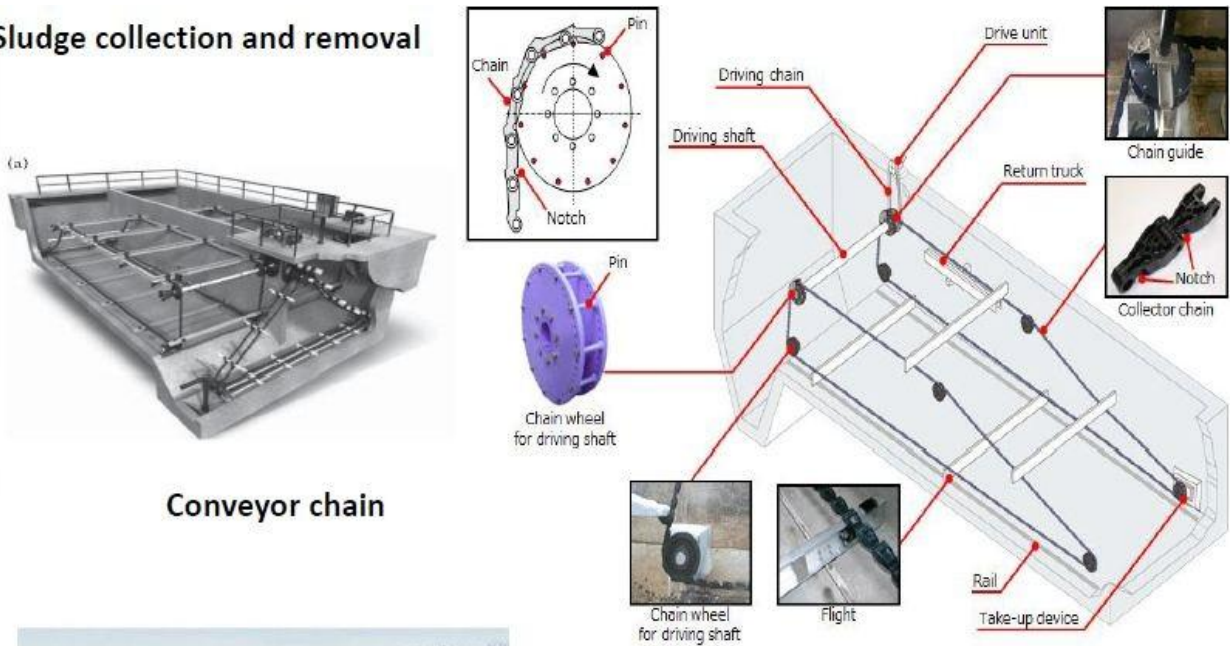
V-notch is preferred, Why?

•It provide larger head, satisfactory to prevent slime and sludge accumulation

إنه يوفر رأسًا أكبر ومرضيًا لمنع تراكم الوحل والحماة

•The capillary rise can be ignored when V-notch is use.

Sludge collection and removal



Conveyor chain



Bridge drive scraper

Bottom slope of sedimentation tank :

- Rectangular : 1-2%
- Circular: 4-10%

Example:

A sand particle has an average diameter of **1 mm** and a shape factor of **0.90** and a specific gravity of **2.1**, determine the terminal velocity of the particle settling in water at **20 °C** (kinematic viscosity of water = $1.003 \times 10^{-6} \text{ m}^2/\text{s}$ and specific gravity = 1). Drag coefficient can be computed using the following equation:

Solution:

$$\mu_f = 1.003 \times 10^{-6} \times 10^3 = 1.003 \times 10^{-3} \text{ kg/m.s}$$

Settling velocity using Stokes law is:

$$u_t = \frac{g(\rho_p - \rho_f)D_p^2}{18\mu_f} = \frac{9.81 \times (1 \times 10^{-3})^2 \times ((2.1 - 1) \times 1000)}{18 \times 1.003 \times 10^{-3}} = 0.597 \text{ m/sec}$$

$$Re = \phi \frac{\rho_f u_t D_p}{\mu_f} = 0.9 \frac{10^3 \times 0.597 \times (1 \times 10^{-3})}{1.003 \times 10^{-3}} = 536.32$$

Since $Re > 1$, therefore, Newton's law should be used for finding terminal velocity in transition zone. For initial assumption of settling velocity, *stokes law* is used. This initially assumed velocity is used to determine the *Reynolds number* which is further used to find settling velocity. This iterative procedure is repeated till initial assumed velocity is approximately equal to settling velocity calculated from *Newton's equation*.

Initial drag coefficient is calculated as:

$$C_D = \frac{24}{Re} + \frac{3}{\sqrt{Re}} + 0.34 = 0.5142$$

$$u_t = \sqrt{\frac{4(\rho_p - \rho_f)gD_p}{3\rho_f C_D}} = 0.1763$$

Now, iterative procedure is continued:

u_t (previous calculated)	Re	C_D	u_t	Difference
0.5977	536.3272	0.5143	0.1763	0.4214
0.1763	158.2037	0.7302	0.1480	0.0283
0.148	132.7684	0.7811	0.1431	0.0049
0.1431	128.3690	0.7917	0.1421	0.0010
0.1421	127.5052	0.7939	0.1419	0.0002
0.1419	127.3315	0.7943	0.1419	0.0000

Final settling velocity = 0.1419 m/s