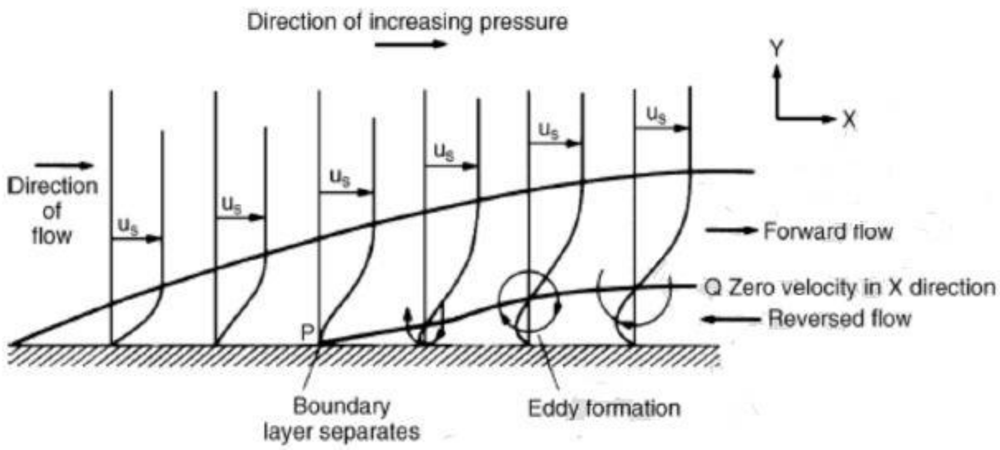
Motion of Particles in a Fluid

when a viscous fluid flows over a surface, the fluid is retarded in the boundary layer which is formed near the surface and that the boundary layer increases in thickness with increase in distance from the leading edge.



The flow is characterised by the Reynolds number (*Re*= *udρ/μ)* in which *ρ* is the density of the fluid, *μ* is the viscosity of the fluid, *d* is the diameter of the sphere, and *u* is the velocity of the fluid relative to the particle.

Drag coefficients

The most satisfactory way of representing the relation between drag force and velocity involves the use of two dimensionless groups, similar to those used for correlating information on the pressure drop for flow of fluids in pipes.

The first group is the particle Reynolds number *Re*= *Re*'= $\left(\frac{Ud\rho}{\mu}\right)$

The second is the group in which $\frac{R'}{\rho U^2}$ is the force per unit projected area of particle in a plane perpendicular to the direction of motion. For a sphere, the projected area is that of a circle of the same diameter as the sphere.

Thus: $\boldsymbol{R'} = \frac{\boldsymbol{F}}{(\boldsymbol{\pi d^2/4})}$

and $\frac{\boldsymbol{R'}}{\boldsymbol{\rho u^2}} = \frac{\boldsymbol{4F}}{\boldsymbol{\pi d^2 \rho u^2}}$

$\frac{\boldsymbol{R'}}{\boldsymbol{\rho u^2}}$ is a form of *drag coefficient* , often denoted by symbol $\boldsymbol{C'_D}$. Frequently, a drag coefficient C_D is defined as the ratio of R' to $\frac{1}{2}\boldsymbol{\rho u^2}$.

Thus : $C_D = 2C'_D = \frac{2R'}{\rho u^2}$

It is seen that C'_D is analogous to the friction factor $\emptyset(= R/\rho u^2)$ for pipe flow, and C_D is analogous to the fanning friction factor f.
When the force F is given by Stokes' law then:

$$\frac{R'}{\rho u^2} = 12\frac{\mu}{\rho u^2} = 12Re'^{-1} \qquad \ldots\ldots (4)$$

The relation between $\frac{R'}{\rho u^2}$ and Re' is conveniently given in graphical form by means of a logarithmic plot as shown in figure 3.4.The graph may be divided into four regions as shown. The four regions are now considered in turn.

***Region* (a)** ($10^{-4} < Re' < 0.2$)

In this region , the relationship between $\frac{R'}{\rho u^2}$ and Re' is a straight line of slope -1 represented by equation (4):

$$\frac{R'}{\rho u^2} = 12Re'^{-1} \qquad \ldots..(4)$$

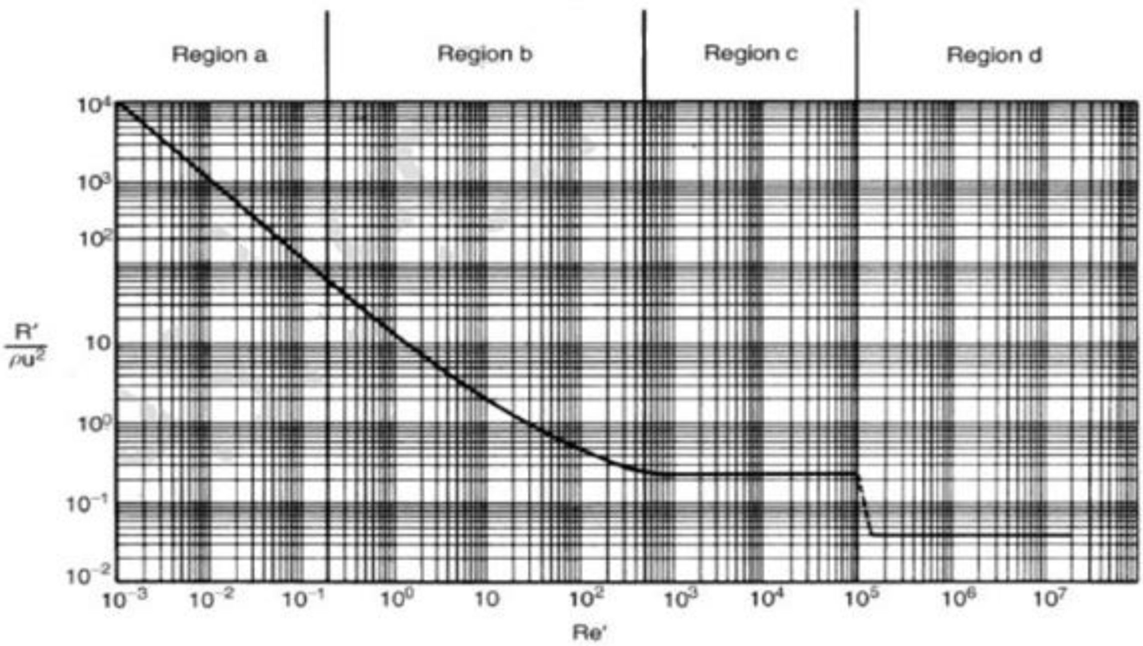


Figure 2. $\frac{R'}{\rho u^2}$ versus Re' for spherical particles

Region (b) ($0.2 < Re' <$ 500-1000)

In this region , the slope of the curve changes progressively from -1 to 0 as Re' increases.

Thus: $$\frac{R'}{\rho u^2} = 12Re'^{-1} + 0.22 \quad .....(5)$$

SCHILLER and NAUMANN gave the following simple equation which gives a reasonable approximation for values of *Re*' up to about 1000:

$$\frac{R'}{\rho u^2} = 12Re'^{-1}(1 + 0.15Re^{0.687}) \quad ......(6)$$

Region (c) *(500 –1000 < **Re'** < ca 2 × 10*^{5}*)*

In this region, *Newton's law* is applicable and the value of $\frac{R'}{\rho u^2}$ approximately constant giving:

$$\frac{R'}{\rho u^2} = 0.22 \quad ......(7)$$

***Region (d) (Re'** > **ca 2** × 10*^{5}*)*

When *Re'* exceeds about 2 × 10^5, the flow in the boundary layer changes from streamline to turbulent and the separation takes place nearer to the rear of the sphere. The drag force is decreased considerably and:

$$\frac{R'}{\rho u^2} = 0.05 \quad ....(8)$$

A comprehensive review of the various equations proposed to relate drag coefficient to particle Reynolds number has been carried out by CLIFT, GRACE and WEBER . One of the earliest equations applicable over a wide range of values of *Re'* is that due to WADELL*(*9*)* which may be written as:

$$\frac{R'}{\rho u^2} = \left(0.445 + \frac{3.39}{\sqrt{Re'}}\right)^2 \quad ......(9)$$

Subsequently, KHAN and RICHARDSON have examined the experimental data and suggest that a very good correlation between $R/\rho u^2$ and *Re'*, for values of *Re'* up to 10^5, is given by:

$$\frac{R'}{\rho u^2} = (1.84Re'^{-0.31} + 0.293Re'^{0.06})^{3.45} \quad ....(10)$$

Total force on a particle

The force on a spherical particle may be expressed using equations 4, 6, 7 and 8 for each of the regions *a*, *b*, *c* and *d* as follows.

In *region* (*a*): $$R' = 12\rho u^2\left(\frac{\mu}{ud\rho}\right) = \frac{12u\mu}{d} \quad ....(11)$$

The projected area of the particle is $\pi d^2/4$. Thus the total force on the particle is given by:

$$F = \frac{12u\mu}{d}\frac{1}{4}\pi d^2 = 3\pi\mu du \quad \text{.........} (12)$$

This is the expression originally obtained by STOKES already given as equation (1). In ***region*** (***b***), from equation (6):

$$R' = \frac{12u\mu}{d}(1 + 0.15Re^{0.687}) \quad \text{.......}(13)$$

and therefore: $F = 3\pi\mu du(1 + 0.15Re^{0.687})$ ......(14)

In ***region*** (***c***): $R' = 0.22\rho u^2$ ....(15)

and: $F = 0.22\rho u^2 \frac{1}{4}\pi d^2 = 0.055\pi d^2\rho u^2$ ....(15)

This relation is often known as Newton's law.

In ***region*** (***d***):

$$R' = 0.05\rho u^2 \quad \text{.....}(16)$$

$$F = 0.0125\pi d^2\rho u^2 \quad \text{.......} (17)$$

Alternatively using equation (10), which is applicable over the first three regions (*a*), (*b*) and (*c*) gives:

$$F = \frac{\pi}{4}d^2\rho u^2(1.84Re'^{-0.31} + 0.293Re'^{0.06})^{3.45}$$

Terminal falling velocities

If a spherical particle is allowed to settle in a fluid under gravity, its velocity will increase until the accelerating force is exactly balanced by the resistance force. Although this state is approached exponentially, the effective acceleration period is generally of short duration for very small particles.

Again if the terminal falling velocity corresponds to a value of ***Re*** Greater than about ***500***, the drag on the particle is given by equation (15). Under terminal falling conditions, velocities are rarely high enough for ***Re*** to approach 10^5, with the small particles generally used in industry.

Equation for one-dimensional motion of particle through fluid

Expression for acceleration of a particle settling in a fluid:

$$m\frac{du}{dt} = F_e - F_b - F_D$$

Where , $F_e = ma_e$

Acceleration in external field = a_e = g in gravity settling, and $\omega^2 r$ in centrifugal field

F_b = Buoyant force = $\frac{m}{\rho_P}\rho$

FD = Drag force, $F_D = \frac{C_D \rho u^2 A_p}{2}$

C_D = Drag coefficient, A_P = Projected area

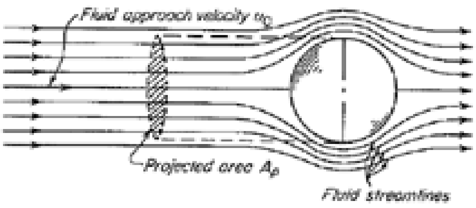
- F_D always *increases with velocity* , and soon acceleration becomes 0.
- Terminal velocity is the *constant velocity* the particle attains when acceleration becomes **0**.

Drag coefficient & terminal velocity

$$C_D = \frac{F_D/A_P}{\rho u_0^2/2}$$

$$m\frac{du}{dt} = F_e - F_b - F_D = mae - \frac{m}{\rho_P}\rho a_e - \frac{C_D \rho u^2 A_p}{2}$$

$$\frac{du}{dt} = a_e - \frac{1}{\rho_p}\rho a_e - \frac{C_D \rho u^2 A_p}{2m}$$



$$a_e\left(1 - \frac{\rho}{\rho_p}\right) = \frac{C_D \rho u^2 A_p}{2m}$$

$$u^2 = \frac{2mg(\rho_p - \rho)}{C_D \rho \rho_p A_P} = \frac{2\left(\frac{\pi}{6} D_p^3 \rho_p\right) g(\rho_p - \rho)}{C_D \rho \rho_p \left(\frac{\pi D_p^2}{4}\right)} =$$

$$\frac{4D_p g(\rho_p - \rho)}{3C_D \rho}$$

$$u = \sqrt{\frac{4D_p g(\rho_p - \rho)}{3C_D \rho}}$$

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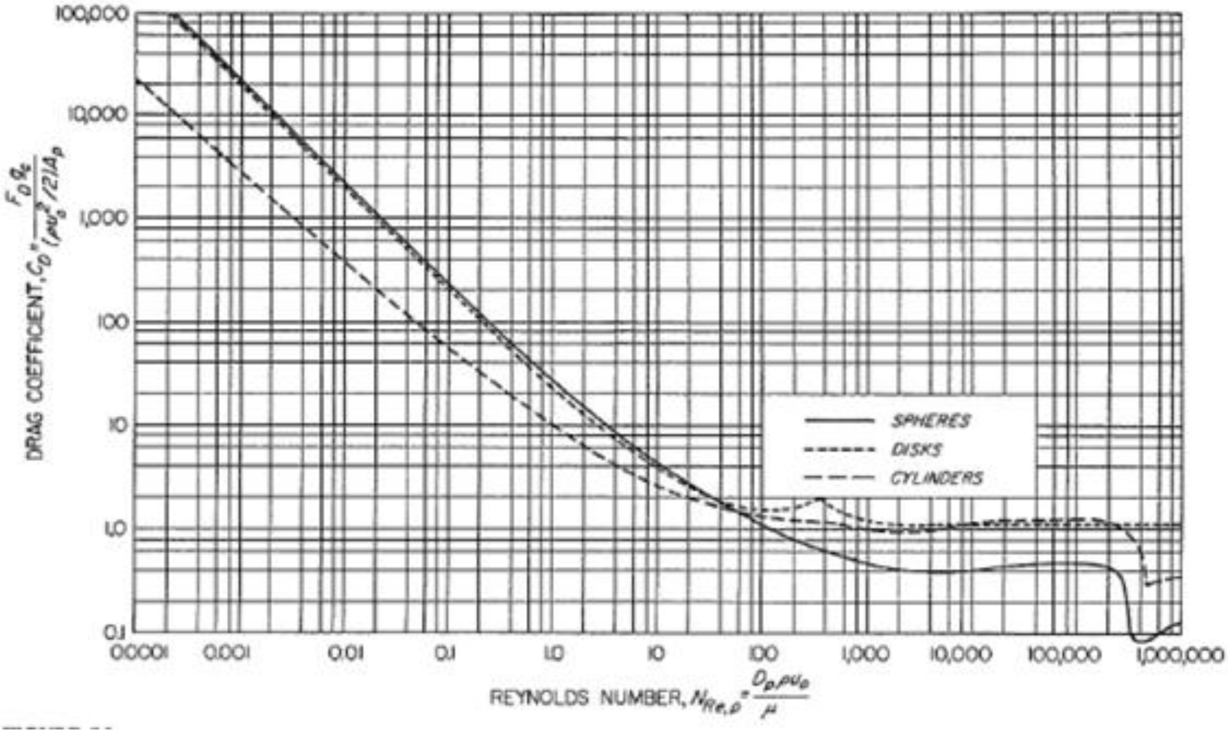

Figure (4): Drag coefficients for spheres, disks, and cylinders.

Trial and error method for determination of terminal velocity

$$u = \sqrt{\frac{4D_p g(\rho_p - \rho)}{3C_D \rho}} \quad .....(1)$$

- Assume a value of u.
- Calculate Reynolds number of particle, $N_{Re.p}$.
- $N_{Re.p} = \frac{D_p u \rho}{\mu}$
  - ❖ ***Dp*** = *Diameter of particle*, ***u*** = *velocity of particle*, ***ρ*** = *density of fluid*, ***μ*** = *viscosity of fluid.*
- Determine C_D from chart of C_D *vs* $N_{Re.p}$.
- Calculate u from eqution (1).
- Compare calculated u with assumed **u** , if error is not within limit restart from step 1 for second trial.

Terminal velocity in Stokes law range and Newton's law range

$$u_t = \sqrt{\frac{4D_p g(\rho_p - \rho)}{3C_D \rho}}$$

Stokes law range, particle Reynolds number less than 1.0.

$$C_D = \frac{24}{N_{Re.p}}$$ معادلة مهمة

- Where $N_{Re.p} = \frac{D_p u_0 \rho}{\mu}$

$$u_t = \frac{g D_p^2 (\rho_p - \rho)}{18\mu}$$

Newton's law range: **1000 <** $N_{Re.p}$ **< 200000** , C_D **= 0.44**

$$u_t = 1.75\sqrt{\frac{4 D_p g (\rho_p - \rho)}{\rho}}$$

$$u_t = \sqrt{\frac{4 D_p g (\rho_p - \rho)}{3 C_D \rho}}$$

$$u_t = \sqrt{\frac{4 D_p g (\rho_p - \rho)}{3\frac{24}{\frac{D_p u_0 \rho}{\mu}}\rho}} = \sqrt{\frac{g D_p^2 (\rho_p - \rho) u_t}{18\mu}}$$

$$u_t = \frac{g D_p^2 (\rho_p - \rho)}{18\mu}$$

Example (1):

What is the terminal velocity of a spherical steel particle, 0.40 *mm* in diameter, settling in an oil of density 820 kg/m^3 and viscosity 10 $mN\ s/m^2$? The density of steel is 7870 kg/m^3?

Solution:

For a sphere:

$$\frac{R_o'}{\rho u_o^2} Re_o'^2 = \frac{2d^3(\rho_s - \rho)\rho g}{3\mu^2}$$

$$= \frac{2 \times 0.0004^3 \times 820(7870 - 820)9.81}{3(10 \times 10^{-3})^2} = 24.2$$

$log_{10} 24.2 = 1.384$

From table (3) or figure (5)

$log_{10} Re_o' = 0.222$

Thus: $Re_o' = 1.667$

and : $u_o = \frac{1.667 \times 10 \times 10^{-3}}{820 \times 0.0004}$

$= \mathbf{0.051}$ *m/s* or 51 *mm/s*

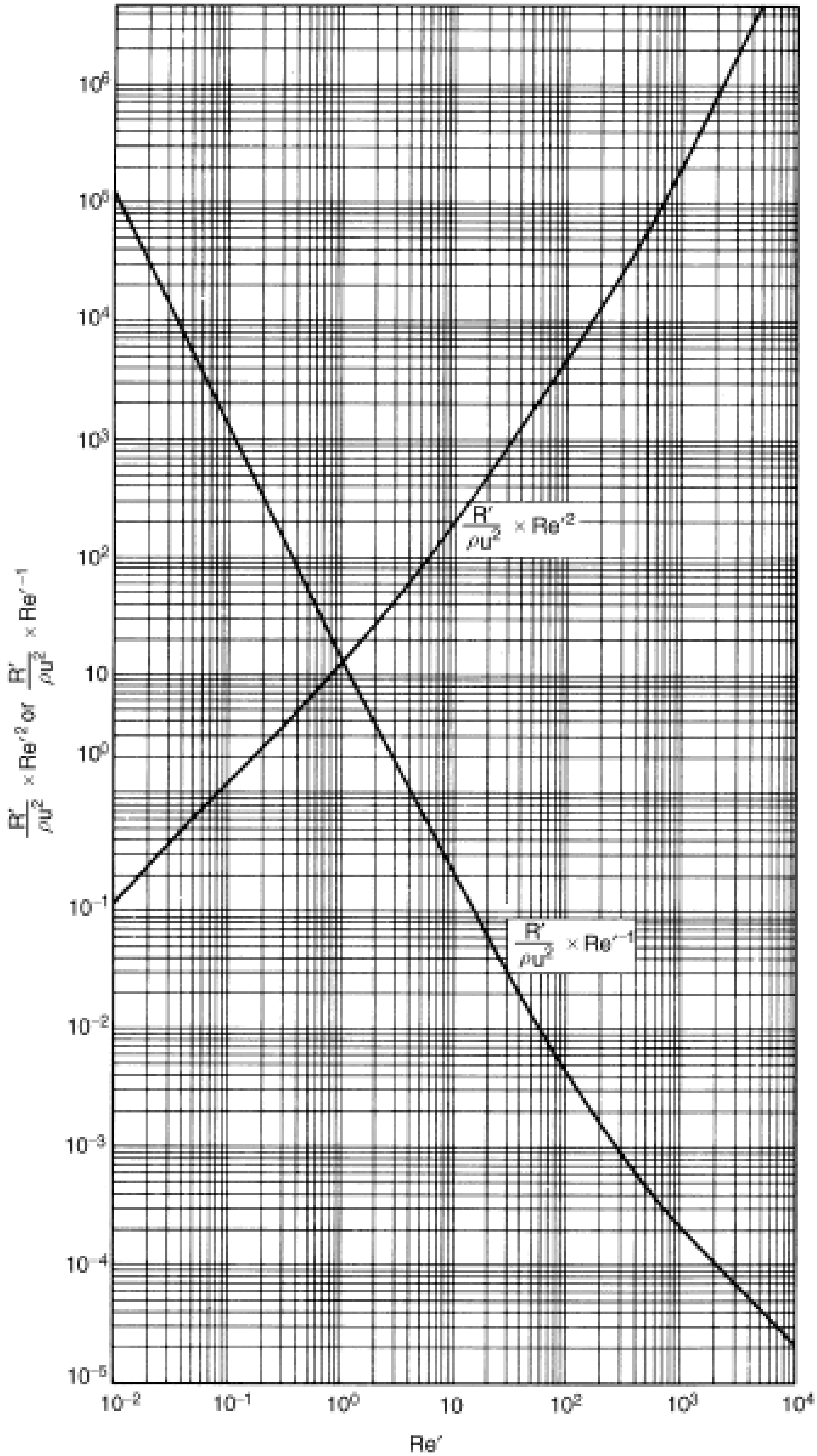

Figure (5).$(R'/\rho u^2)Re'^2$ $and (R'/\rho u^2)Re'^{-1}$ $versus\ Re'$ $for\ spherical\ particles$

Example(2):

A finely ground mixture of galena and limestone in the proportion of **1** to **4** by mass is subjected to elutriation by an upward-flowing stream of water flowing at a velocity of **5 *mm/s***. Assuming that the size distribution for each material is the same, and is as shown in the following table, estimate the percentage of galena in the material carried away and in the material left behind. The viscosity of water is **1 *mN s/m*²** and *Stokes' equation* may be used.

Diameter(μ *m*)	20	30	40	50	60	70	80	100
Undersize (per cent by mass)	15	28	48	54	64	72	78	88

The densities of galena and limestone are **7500** and **2700** kg/m^3, respectively.

Solution:

The first step is to determine the size of a particle which has a settling velocity equal to that of the upward flow of fluid, that is **5 *mm/s***.

Taking the largest particle, $d = (100 \text{ x } 10^{-6}) = 0.0001\ m$

and: $$Re' = \frac{(5\times10^{-3}\times0.0001\times1000)}{(1\times10^{-3})} = 0.5$$

thus, for the bulk of particles, the flow will be within region (a) in figure (2) and the velocity is given by Stokes's equation:

$$u_o = \frac{gD_p^2(\rho_s-\rho)}{18\mu}$$

For a particle of galena settling in water at 5 mm/s:

$$5\times10^{-3} = \frac{9.81\times D_p^2(7500-1000)}{18\times10^{-3}} = 3542500\ D_p^2$$

$$\therefore\ D_p^2 = \frac{5\times10^{-3}}{3542500} = 1.4114\times10^{-9}$$

$$and\ \ D_p = \sqrt{1.4114\times10^{-9}} = 3.76\times10^{-5}\ m\ \text{ or }\ 37.6\ \mu m$$

For a particle of limestone settling at 5 mm/s:

$$5\times10^{-3} = \frac{9.81\times D_p^2(2700-1000)}{18\times10^{-3}} = 926500\ D_p^2$$

$$\therefore\ D_p^2 = \frac{5\times10^{-3}}{926500} = 5.397\times10^{-9}$$

$$and\ \ D_p = \sqrt{5.397\times10^{-9}} = 7.346\times10^{-5}\ m\ \text{ or } 73.5\ \mu m$$

Thus particles of galena of less than **37.6 *μm*** and particles of limestone of less than **73.5 *μm*** will be removed in the water stream.

Interpolation of the data given shows that **43** per cent of the galena and **74** per cent of the limestone will be removed in this way.

In **100 *kg*** feed, there is **20 *kg*** galena and **80 *kg*** limestone.

Therefore galena removed = (20×0.43) = **8.6 *kg***, leaving **11.4 kg**, and limestone removed = (80×0.74) = **59.2 *kg***, leaving **20.8 kg**.

The *material removed*:

concentration of galena = $\frac{(8.6\times100)}{(8.6+59.2)} = \mathbf{12.7}$ per cent by mass

the *material remaining*:

concentration of galena = $\frac{(11.4\times100)}{(11.4+20.8)} = \mathbf{35.4}$ per cent by mass