# Flow of Fluids through Granular Beds and Packed Columns

The flow of fluids through beds composed of stationary granular particles is a frequent occurrence in the chemical industry and therefore expressions are needed to predict pressure drop across beds due to the resistance caused by the presence of the particles.

## Pressure drop in laminar flow

***Darcy*** (1856) mentioned the expression:

(Pressure gradient) α (Liquid velocity)

$$\left(\frac{-\Delta P}{H}\right) \propto U$$

$U$= Superficial fluid velocity through the bed

($-\Delta P$) = Frictional pressure drop across a bed depth H.

Superficial fluid velocity = fluid volumetric flow rate / cross- sectional area of bed $\left(\frac{Q}{A}\right)$

***Hagen-Poiseuille*** equation for laminar flow through a tube:

$$\left(\frac{-\Delta P}{H}\right) = \frac{32\mu U}{D^2}$$

Where ***D*** is the tube diameter and $\mu$ is the fluid viscosity

Packed bed of equivalent diameter ***De***, equivalent length ***He*** and carrying fluid with a velocity ***Ui***

$$\left(\frac{-\Delta P}{H_e}\right) = K_1 \frac{\mu U_i}{{D_e}^2}$$

$$U_i = \frac{U}{\varepsilon}$$

$\varepsilon$ is the voidage or void fraction

$$He = K_2 . H$$

$$De = \frac{4 \times flow\ area}{wetted\ perimeter\ \text{المحزم المبتل}}$$

Flow area = $\varepsilon A$ , where $A$ = cross-sectional area

***Wetted perimeter*** = $S_B A$, where $S_B$ = **particle surface area per unit volume of the bed Total particle surface area in the bed** = $S_B A H$

***For a pipe*** : wetted perimeter $= \frac{wetted\ \ surface}{length} = \frac{\pi DL}{L}$

***For the packed bed***: wetted perimeter $= \frac{S_B AH}{H} = S_B A$

$S_V$ =Surface area per unit volume of **particles** :

$$S_V(1-\varepsilon) = S_B$$

$$\left(\frac{surface\ of\ particles}{volume\ of\ particles}\right) \times \left(\frac{volume\ of\ particles}{volume\ of\ bed}\right) = \left(\frac{surface\ of\ particles}{volume\ of\ bed}\right)$$

$$D_e = \frac{4\varepsilon A}{S_B A} = \frac{4\varepsilon}{S_V(1-\varepsilon)}$$

$$\left(\frac{-\Delta P}{H_e}\right) = K_1 \frac{\mu U_i}{{D_e}^2}$$

$$U_i = \frac{U}{\varepsilon}\ \ , He\ = \ K_2 . H\ \ , D_e = \frac{4\varepsilon A}{S_B A} = \frac{4\varepsilon}{S_V(1-\varepsilon)}$$

$$\left(\frac{-\Delta P}{H_e}\right) = K_3 \frac{(1-\varepsilon)^2}{\varepsilon^3} \mu U S_V^2 \quad where\ K_3 = K_1 K_2$$

***Carman-Kozeny*** equation for laminar flow through randomly packed particles

$K_3$ depends on particle shape and surface properties (**~ 5**).

***For monosized sphere*** **:** ***S*** **= 6 /** ***x***

***Carman-Kozeny*** equation spherical particles:

$$\left(\frac{-\Delta P}{H_e}\right) = 180 \frac{\mu U (1-\varepsilon)^2}{x^2 \varepsilon^3}$$

# Turbulent flow

Randomly packed bed of monosized spheres

$$\left(\frac{-\Delta P}{H}\right) = 1.75 \frac{\rho_f U^2}{x} \frac{(1-\varepsilon)}{\varepsilon^3}$$ ***Burke-Pummer*** equation

## General Equation

Ergun (1952) proposed:

$$\left(\frac{-\Delta P}{H_e}\right) = 150\frac{\mu U(1-\varepsilon)^2}{x^2\varepsilon^3} + 1.75\frac{\rho_f U^2}{x}\frac{(1-\varepsilon)}{\varepsilon^3}$$

a- *Laminar flow* : pressure gradient

1-Increases linearly with superficial fluid velocity

2- Independent of fluid density.

b-*Turbulent flow*: pressure gradient

1- Increases with the square of superficial fluid velocity

2- Independent of fluid viscosity.

## Friction factor

$$Re^* = \frac{xU\rho_f}{\mu(1-\varepsilon)}$$

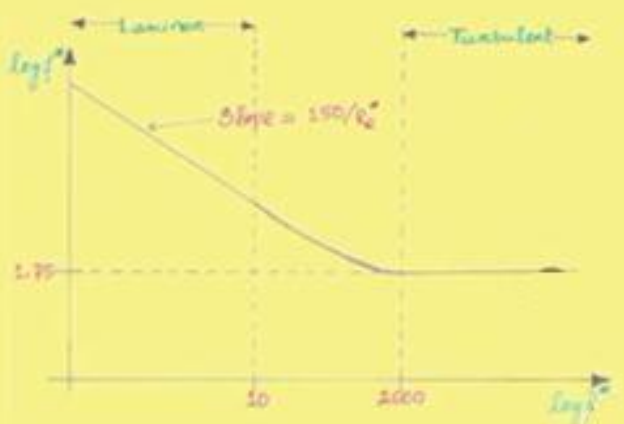
| Region | Value |
|---|---|
| Laminar | Re* < 10 |
| Transitional | 10 < Re* < 2000 |
| Turbulent | 2000 < Re* |

Friction factor: $f^* = \frac{(-\Delta P)}{H}\frac{x}{\rho_f U^2}\frac{\varepsilon^3}{(1-\varepsilon)}$

$$f^* = \frac{150}{R_e^*} + 1.75$$

$f^* = \frac{150}{Re^*}$ for $Re^* < 10$

$f^* = 1.75$ for $Re^* > 2000$



# Non – spherical particles

Diameter of a sphere having the same surface to volume ratio as the non-spherical particles.

Surface area of particles per unit volume of particles.

Using surface-volume diameter $x_{SV}$

$$\left(\frac{-\Delta P}{H}\right) = 150\frac{\mu U(1-\varepsilon)^2}{x_{SV}^2\,\varepsilon^3} + 1.75\frac{\rho_f U^2}{x_{SV}}\frac{(1-\varepsilon)}{\varepsilon^3}$$

$$\left(\frac{-\Delta P}{H}\right) = 180\frac{\mu U(1-\varepsilon)^2}{x_{SV}^2\,\varepsilon^3}$$

## *Example(1):*

Water flows through 3.6 kg of glass particles of density 2590 kg/m$^3$ forming a packed bed of depth 0.475 m and diameter 0.0757 m. The variation in frictional pressure drop across the bed with water flow rate in the range 200 – 1200 cm$^3$/min is shown in the table.

a. Demonstrate that the flow is laminar.

b. Estimate the mean surface volume diameter of the particles.

c. Calculate the relevant Reynolds number.

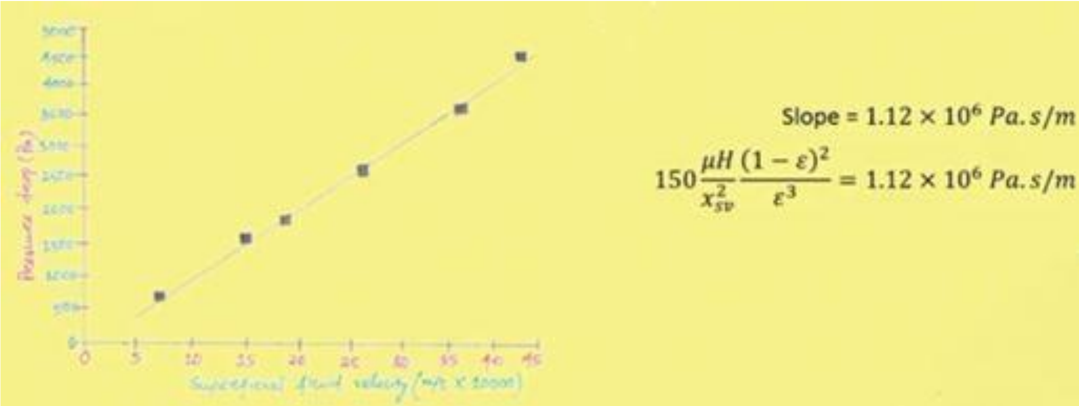
| Water flow rate (cm$^3$/min) | Pressure drop (mm Hg) |
|---|---|
| 200 | 5.5 |
| 400 | 12.0 |
| 500 | 14.5 |
| 700 | 20.5 |
| 1000 | 29.5 |
| 1200 | 36.5 |

## *Solution :*

$$\left(\frac{-\Delta P}{H}\right) = 150\frac{\mu U\,(1-\varepsilon)^2}{x_{SV}^2\,\varepsilon^3}$$

$$150\frac{\mu U\,(1-\varepsilon)^2}{x_{SV}^2\,\varepsilon^3}$$

| Water flow rate (cm$^3$/min) | Pressure drop (mm Hg) | U (*m/s**10$^4$) | Pressure drop (*Pa*) |
|---|---|---|---|
| 200 | 5.5 | 7.41 | 734 |
| 400 | 12.0 | 14.81 | 1600 |
| 500 | 14.5 | 18.52 | 1935 |
| 700 | 20.5 | 25.92 | 2735 |
| 1000 | 29.5 | 37.00 | 3936 |
| 1200 | 36.5 | 44.40 | 4870 |




*Mass of bed* = $AH(1 - \varepsilon)\rho_P$

$\varepsilon = 0.3497$

substituting $\varepsilon = 0.3497$ , $H = 0.475\ m$ and $\mu = 0.001\ Pa.s$

$$150\frac{\mu H}{x_{SV}^2}\frac{(1-\varepsilon)^2}{\varepsilon^3} = 1.12\times10^6 Pa.s/m$$

$$x_{SV} = 792$$

$$Re^* = \frac{xU\rho_f}{\mu(1-\varepsilon)} = 5.4\ (with\ maximum\ velocity)$$

## Example 2:

A packed bed of solid particles of density 2500 $kg/m^3$ occupies a depth of 1 $m$ in a vessel of cross-sectional area 0.04 $m^2$.The mass of solids in the bed is 50 $kg$ and the surface –volume mean diameter of the particles is 1$mm$. A liquid of density 800 $kg / m^3$ and viscosity 0.002 $Pa.s$ flows upwards through the bed , which is restrained at its upper surface.

(a) Calculate the voidage (volume fraction occupied by voids) of the bed.

(b) Calculate the pressure drop across the bed when the volume flow rate of liquid is 1.44 $m^3/h$.

## Solution:

$$\boldsymbol{M = AH\ (1 - \varepsilon)\ \rho_P}$$

$$\varepsilon = 1 - \frac{50}{2500\times0.04\times1} = 0.5$$

Liquid flow rate of 1.44 $m^3/h$

$$\boldsymbol{U} = \frac{\boldsymbol{Q}}{\boldsymbol{A}} = \frac{1.44}{3600\times0.04} = 0.01\ m/s$$

Using the ***Ergun eqution***:

$$\left(\frac{-\Delta P}{H}\right) = 150\frac{\mu U(1-\varepsilon)^2}{x_{SV}^2\,\varepsilon^3} + 1.75\frac{\rho_f U^2}{x_{SV}}\frac{(1-\varepsilon)}{\varepsilon^3}$$

$\mu = 0.002\ Pa.\ s$ , $\rho_f = 800\ kg\,/\,m^3$ ,$x_{SV} = 1\ mm$ and $H = 1m$,

$$(-\Delta P) = 600\times10^3 U + 5.6\times10^6 U^2 = 6560\ Pa$$

$$Re^* = \frac{U\rho_f\,x_{SV}}{\mu(1-\varepsilon)} = 8$$

Using the ***Carman-Kozeny*** equation:

$$\left(\frac{-\Delta P}{H}\right) = 180\frac{\mu U\,(1-\varepsilon)^2}{x_{SV}^2\ \ \varepsilon^3} = 7200\ Pa$$

## ***Example( 3):***

A gas absorption tower of diameter 2 *m* contains ceramic Raschig rings randomly packed to a height of 5 *m*. Air containing a small proportion of $SO_2$ passes upwards through the absorption tower at a flow rate of 6 $m^3/s$. The viscosity and density of the gas are $1.80\times10^{-5}$ *Pa.s* and 1.2 $kg/m^3$ ,respectively. Details of the packing are: $S = 190\ m^2/m^3$ , voidage = 0.71.

- Calculate the diameter of a sphere with the same surface-volume ratio as the Raschig rings.
- Calculate the frictional pressure drop across the packing in the tower.
- Discuss how this pressure drop will vary with flow rate of the gas within ±10% of flow rate.
- Discuss how this pressure drop across the packing would vary with gas pressure and temperature.

### ***Solution :***

$S(1-\varepsilon) = S_B$   where **S** is the surface area per unit volume of rings

$$S_B = \frac{S}{(1-\varepsilon)} = \frac{190}{(1-0.71)} = 655.2\ m^2/m^3$$

If $x_{SV}$ is the diameter of a sphere with the same surface-volume ratio as the rings , then,

$$S_B = \frac{\pi x_{SV}^2}{\frac{\pi}{6}x_{SV}^3} = 655.2\ m^2/m^3$$

$x_{sv}$ =0.00916 $m$ = 9.16 $mm$

$$U = \frac{Q}{\frac{\pi D^2}{4}} = \frac{6}{\frac{\pi \times 2^2}{4}} = 1.91\ m/s$$

$\mu$=1.80 × $10^{-5}$ $Pa.\ s$ , $\rho_f$ = 1.2 $kg / m^3$ ,$x_{SV}$ = 9.16 × $10^{-3}$ $m$ and $H$ = 5$m$,

$$\left(\frac{-\Delta P}{H}\right) = 150\frac{\mu U(1-\varepsilon)^2}{x_{SV}^2\ \varepsilon^3} + 1.75\frac{\rho_f U^2}{x_{SV}}\frac{(1-\varepsilon)}{\varepsilon^3}$$

$$\left(\frac{-\Delta P}{H}\right) = 150\frac{1.8\times10^{-5}\times1.91\times(1-0.71)^2}{(9.16\times10^{-3})^2\times0.71^3} + 1.75\frac{1.2\times1.91^2}{9.16\times10^{-3}}\times\frac{(1-0.71)}{0.71^3}$$

$$(-\Delta P) = 72.0 + 3388.4 = 3460.4\ Pa$$

- Turbulent component contributes 98% of the total.
- Within ± 10% of the quoted flow rate ,the pressure drop across the bed will increase with the square of the superficial velocity and hence with the square of the flow rate:

$$(-\Delta P) \propto Q^2$$

- Pressure increase affects only the gas density
- gas density is directly proportional to absolute gas pressure (ideal gas behavior)

$$(-\Delta P) \propto \textbf{absolute gas pressure}$$

- gas viscosity has no influence

$$\left(\frac{-\Delta P}{H}\right) = 150\frac{\mu U\,(1-\varepsilon)^2}{x_{SV}^2\ \varepsilon^3} + 1.75\frac{\rho_f U^2}{x_{SV}}\frac{(1-\varepsilon)}{\varepsilon^3}$$

- variation in gas temperature will influence only the gas density
- assuming ideal gas behavior,

$$\rho_f \propto \frac{1}{T}$$

Where T is the absolute temperature

$$(\Delta P) \propto \frac{1}{T}$$

***Example (4):***

A solution of density 1100 $kg/m^3$ and viscosity 2× $10^{-3}$ $Pa.s$ is flowing under gravity at a rate of 0.24 $kg/s$ through a bed of catalyst particles. The bed diameter is 0.2 m and the depth is 0.5 $m$. The particles are cylindrical, with a diameter of 1 mm and length of 2 $mm$. They are packed to give a voidage of 0.3. Calculate the depth of liquid above the top of the bed?

***Solution:***

$$U = \frac{0.24}{1100\times\frac{\pi}{4}(0.2)^2} = 6.94\times10^{-3} m/s$$

- Volume of one cylindrical particle: $\frac{\pi}{2} mm^3$

- Surface area of one cylindrical particle: $2.5\pi\ mm^2$
- Surface –volume ratio: $\frac{2.5\pi}{\pi/2} = 5mm^2/mm^3$
- For a sphere , surface-volume ratio:$\frac{6}{x_{SV}}$

$$x_{SV} = 1.2$$

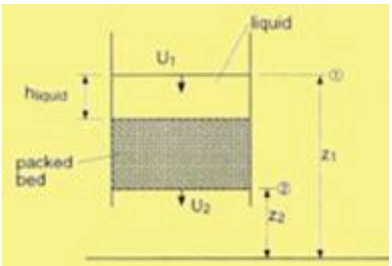
$$Re^* = \frac{x_{SV}\,U\rho_f}{\mu(1-\varepsilon)} = \frac{(6.94\times10^{-3})\times1100\times(1.2\times10^{-3})}{(2\times10^{-3})\times(1-0.3)} = 6.5$$

$$\left(\frac{-\Delta P}{H}\right) = 150\frac{\mu U(1-\varepsilon)^2}{x_{SV}^2\ \varepsilon^3}$$

$\mu$ = 0.002 *Pa. s* , $\rho_f$ = 1100 *kg / m*$^3$ ,$x_{SV}$ = 1.2 *mm* and *H* = 0.5 *m*, ε = 0.3

$$\frac{(-\Delta P)}{0.5} = 150\times\frac{2\times10^{-3}\times6.94\times10^{-3}}{(1.2\times10^{-3})^2}\times\frac{(1-0.3)^2}{0.3^3} = 26240\ Pa/m$$

- Frictional head loss: $\frac{13120}{1100\times9.81} = 1.216\ m$




$$Z_1 + \frac{U_1^2}{2g} + \frac{P_1}{\rho_f g} = Z_2 + \frac{U_2^2}{2g} + \frac{P_2}{\rho_f g} + h_{loss}$$

$$Z_1 - Z_2 = h_{loss} = 1.216\ m$$

Depth of liquid above the bed: (1.216 – 0.5) = 0.716

# Darcy's law and permeability

- For rate of flow of water through beds of sand of various thicknesses average velocity
- Proportional to the driving pressure
- Inversely proportional to the thickness of the bed

$$\boldsymbol{u_c} = \left(\frac{\mathbf{1}}{\boldsymbol{A}}\right)\left(\frac{\boldsymbol{dV}}{\boldsymbol{dT}}\right) = \boldsymbol{B}\frac{(-\Delta \boldsymbol{P})}{\boldsymbol{L}}$$

***A*** = total cross sectional area

***V*** = volume of fluid flowing in time *t*

***B*** depends on the physical properties of the bed and fluid

- Low Reynolds number flow through granular material of homogeneous permeability
- Flow resistance is mainly from viscous drag

$$\boldsymbol{u_c = B\frac{(-\Delta P)}{L} = k\frac{(-\Delta P)}{\mu L}}$$

$\boldsymbol{\mu}$ = viscosity of the fluid

$\boldsymbol{k}$ = permeability coefficient

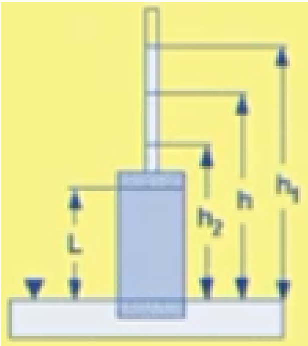
- ***K*** provides a measure of flow ability of fluid through a packed/granular bed or porous medium

## Permeability

- Circular cylinder of length, L and cross-sectional area, A
- Constant head difference (h) is applied across the bed producing a flow rate Q

$$\boldsymbol{Q = KA\frac{h}{L} \qquad K = \frac{QL}{Ah}}$$

- Falling head flow:




$$KA\frac{h}{L} = a\left(-\frac{dh}{dt}\right)$$

$$-\frac{dh}{h} = \frac{KA}{aL}dt$$

$$ln\left(\frac{h_1}{h_2}\right) = \frac{KA}{aL}(t_2 - t_1)$$

# Flow through packed bed

- Larger packings
- Hollow in nature
- Better mass transfer with relatively small pressure gradients
- High interfacial area between phases
- Uniform liquid distribution
- No accurate expression for pressure drop

# Flow operation