***Example(1):***

A plate and frame press gave a total of **8 m³** of filtrate in **1800 s** and **11m³** in **3600 s** when filtration was stopped. Estimate the washing time if **3 m³** of wash water is used. The resistance of the cloth may be neglected and a constant pressure is used throughout.

***Solution:***

For constant pressure filtration with no cloth resistance:

$$t = \frac{r\mu v}{2A^2(-\Delta P)}V^2$$

At $t_1 = 1800$ s, $V_1 = 8$ m$^3$, and when $t_2 = 3600$ s, $V_2 = 11$ m$^3$

Thus : $$(3600 - 1800) = \frac{r\mu v}{2A^2(-\Delta P)}(11^2 - 8^2)$$

$$\frac{r\mu v}{2A^2(-\Delta P)}(11^2 - 8^2) = 1800$$

$$\frac{r\mu v}{2A^2(-\Delta P)} = 31.6$$

$$\frac{dV}{dt} = \frac{A^2(-\Delta P)}{r\mu v V}$$

$$= \frac{1}{(2\times 31.6V)} = \frac{0.0158}{V}$$

The final rate of filtration = (0.0158 / 11) = **$1.44 \times 10^{-3}$ m$^3$/s**.

For thorough washing in a plate and frame filter, the wash water has twice the thickness of cake to penetrate and half the area for flow that is available to the filtrate. Thus the flow of wash water at the same pressure will be one-quarter of the filtration rate.

Hence: rate of washing = $(1.44 \times 10^{-3}) / 4 = 3.6 \times 10^{-4}$ m$^3$/s

and: time of washing = $3 / (3.6 \times 10^{-4})$ = **8400 s** (**2.3 h**)

***Example(2):***

In the filtration of a sludge, the initial period is effected at a constant rate with the feed pump at full capacity, until the pressure differences reaches **400 kN/m²**. The pressure is then maintained at this value for a remainder of the filtration. The constant rate operation requires **900 s** and **one-third** of the total filtrate is obtained during this period. Neglecting the resistance of the filter medium, determine **(a)** the total filtration time and **(b)** the filtration cycle with the existing pump for a maximum daily capacity, if the time for removing the cake and reassembling the press is **1200 s**. The cake is not washed.

***Solution:***

For a filtration carried out at a constant filtration rate for time $t_1$ in which time a volume $V_1$ is collected and followed by a constant pressure period such that the total filtration time is $t$ and the total volume of filtrate is $V$, then:

$$(V^2 - V_1^2) = \frac{2A^2(-\Delta P)}{r\mu v}(t - t_1)$$

Assuming *no cloth resistance*, then:

for the constant rate period: $t_1 = \frac{r\mu v}{A^2(-\Delta P)} V_1^2$

Using the data given: $t_1$ = **900 s**, **volume** = $V_1$

Thus: $\frac{r\mu v}{A^2(-\Delta P)} = \frac{900}{V_1^2}$

**(a)** *For the constant pressure period*: $V = 3V_1$ or $V1=(1/3)V$ and $(t - t_1) = t_p$

Thus: $3^2 V_1^2 - V_1^2 = \frac{2V_1^2}{900} t_P$

$$8V_1^2 = \frac{2V_1^2}{900} t_P$$

$t_p$ = **3600 s**

Thus: total filtration time = (900 + 3600) = **4500 s**

and: total cycle time = (4500 + 1200) = **5700 s**

**(b)** *For the constant rate period*:

$$t_1 = \frac{r\mu v}{A^2(-\Delta P)} V_1^2 = \frac{V_1^2}{K}$$

For the constant pressure period:

$$t - t_1 = \frac{r\mu v}{2A^2(-\Delta P)}(V^2 - V_1^2) = \frac{V^2 - V_1^2}{2K}$$

Total filtration time, $t = \frac{1}{K}\left(V_1^2 + \frac{V^2 - V_1^2}{2}\right) = \frac{(V^2 + V_1^2)}{2K}$

Rate of filtration = $\frac{V}{t + t_d}$ where $t_d$ is the downtime

$$= \frac{2KV}{V^2 + V_1^2 + 2Kt_d}$$

For the rate to be a maximum,

$$\frac{d(rate)}{dV} = 0 \quad or \quad V_1^2 - V^2 + 2Kt_d = 0$$

Thus: $t_d = \frac{1}{2K}(V^2 - V_1^2) = (t - t_1)$

But: $t_d = 1200 = (t - 900)$ and $t$ = **2100 s**

Thus: total cycle time = (2100 + 1200) = **3300 s**

***Example(3):***

A rotary filter, operating at 0.03 Hz, filters at the rate of 0.0075 $m^3/s$. Operating under the same vacuum and neglecting the resistance of the filter cloth, at what speed must the filter be operated to give a filtration rate of 0.0160 $m^3/s$?

***Solution:***

For constant pressure filtration in a rotary filter:

$$V^2 = \frac{2A^2(-\Delta P)t}{r\mu v}$$

or: $V^2 \propto t \propto 1/N$ where $N$ is the speed of rotation. As $V \propto 1/N^{0.5}$ and the rate of filtration is $V/t$, then:

$$V/t \propto (1/N^{0.5})(1/t) \propto (N/N^{0.5}) \propto N^{0.5}$$

Thus: $(V/t)_1/(V/t)_2 = N_1^{0.5}/N_2^{0.5}$

$$0.0075/0.016 = 0.03^{0.5}/N_2^{0.5}$$

and: $N_2 =$ **<u>0.136 Hz</u>** ( **7.2 rpm**)

***<u>Example(4):</u>***

A rotary drum with a filter area of **3 m$^3$** operates with an internal pressure of **71.3 kN/m$^2$** below atmospheric and with **30 per cent** of its surface submerged in the slurry. Calculate the rate of production of filtrate and the thickness of cake when it rotates at **0.0083 Hz**, if the filter cake is incompressible and the filter cloth has a resistance equal to that of **1 mm** of cake.

It is desired to increase the rate of filtration by raising the speed of rotation of the drum. If the thinnest cake that can be removed from the drum has a thickness of **5 mm**, what is the maximum rate of filtration which can be achieved and what speed of rotation of the drum is required? The voidage of the cake = **0.4**, the specific resistance of cake = **2 × 10$^{12}$ m$^{-2}$** the density of solids = **2000 kg/m$^3$**, the density of filtrate = 1000 kg/m$^3$, the viscosity of filtrate = **10$^{-3}$ N s/m$^2$** and the slurry concentration = **20 per cent** by mass solids.

***<u>Solution:</u>***

A 20 per cent slurry contains 20 kg solids / 80 kg solution.

Volume of cake = 20/ [2000(1 − 0.4)] = 0.0167 m$^3$

Volume of liquid in the cake = (0.167 × 0.4) = 0.0067 m$^3$.

Volume of filtrate = (80/1000) − 0.0067 = 0.0733 m$^3$.

Thus: $v = (0.0167/0.0733) = 0.23$

The rate of filtration is given by: $$\frac{dV}{dt} = \frac{A^2(-\Delta P)}{r\mu v[v+(LA/v)]}$$

In this problem:

**$A$ = 3 m$^2$ , $(-\Delta P)$ = 71.3 kN/m$^2$ or (71.3 × 10$^3$) N/m$^2$, r = 2 × 10$^{12}$ m$^{-2}$, $\mu$ = 1 × 10$^{-3}$ Ns/m$^2$, $v$ = 0.23 and $L$ = 1 mm or 1 × 10$^{-3}$ m**

Thus: $$\frac{dV}{dt} = \frac{(3^2 \times 71.3 \times 10^3)}{0.23 \times 2 \times 10^{12} \times 1 \times 10^{-3}[V+(1 \times 10^{-3} \times 3/0.23)]} = \frac{(1.395 \times 10^3)}{(V+0.013)}$$

From which: $V_2/2 + 0.013V = (1.395 \times 10^{-3})\, t$

If the rotational speed = **0.0083 Hz**

**1 revolution** takes (1/0.0083) = **<u>120.5 s</u>**

Given element of surface is immersed for (120.5 × 0.3) = **<u>36.2 s</u>**

When **$t$ = 36.2 s**

***V*** may be found by substitution to be (**0.303 m$^3$**).

Hence: rate of filtration = (0.303/120.5) = **<u>0.0025 m$^3$/s</u>** .

Volume of filtrate for 1 revolution = 0.303 m$^3$.

Volume of cake = (0.23 × 0.303) = 0.07 m$^3$.

Thus: cake thickness = (0.07/3) = 0.023 m or **<u>23 mm</u>**

As the thinnest cake = 5 mm, volume of cake = (3 × 0.005) = 0.015 m3.

As $v = 0.23$, volume of filtrate = (3 × 0.005) / 0.23 = 0.065 $m^3$.

Thus: $\mathbf{(0.065)^2/2 + (0.013 \times 0.065) = 1.395 \times 10^{-3} t}$

and: $\mathbf{t = 2.12\ s}$

Thus: time for 1 revolution = (2.12/ 0.3) = **7.1 s**

and: speed = **0.14 Hz** (8.5 r.p.m)

Maximum filtrate rate = **0.065 $m^3$** in 7.1 s

or: (0.065/7.1) = **0.009 $m^3$/s**

***Example(5):***

A slurry containing 100 kg of whiting, of density 3000 kg/$m^3$, per $m^3$ of water, and, is filtered in a plate and frame press, which takes 900 s to dismantle, clean, and re-assemble. If the cake is incompressible and has a voidage of 0.4, what is the optimum thickness of cake for a filtration pressure *(-ΔP)* of 1000 kN/$m^2$? The density of the whiting is 3000 kg/$m^3$. If the cake is washed at 500 kN/$m^2$ and the total volume of wash water employed is 25 per cent of that of the filtrate, how is the optimum thickness of the cake affected? The resistance of the filter medium may be neglected and the viscosity of water is 1 mNs/$m^2$. In an experiment, a pressure difference of 165 kN/$m^2$ produced a flow of water of 0.02 $cm^3$/s through a centimetre cube of filter cake.

***Solution:***

$$\frac{1}{A}\frac{dV}{dt} = \frac{(-\Delta P)}{r\mu}\frac{A}{vV} \quad \text{-----(1)}$$

The slurry contains 100 kg whiting/$m^3$ of water.

Volume of 100 kg whiting = (100/ 3000) = 0.0333 $m^3$.

Volume of cake = 0.0333/(1 − 0.4) = 0.0556 $m^3$.

Volume of liquid in cake = (0.0556 × 0.4) = 0.0222 $m^3$.

Volume of filtrate = (1 − 0.0222) = 0.978 $m^3$.

Thus: volume of cake / volume of filtrate **$v = 0.0569$**

In the experiment:

**$A = 10^{-4}\ m^2$, $(-\Delta P) = 1.65 \times 10^5\ N/m^2$, $l = 0.01\ m$,**

$\frac{dV}{dt} = 2 \times 10^{-8} m^3/s \quad \mu = 10^{-3}\ Ns/m^2$

Inserting these values in equation (1) gives:

$$\left(\frac{1}{10^{-4}}\right)(2 \times 10^{-8}) = \frac{1}{r}\frac{(1.65 \times 10^5)}{(10^{-3})(10^{-2})}, \textit{and } \boldsymbol{r} = 8.25 \times 10^{13} m^{-2}$$

From equation (1):

$$V^2 = \frac{2A^2(-\Delta P)t}{r\mu v}$$

But: ***L*** **= half frame thickness = V*v*/*A***

Thus: $$L^2 = \frac{2A(-\Delta P)vt}{r\mu}$$

$$L^2 = \frac{2 \times (1 \times 10^6) \times 0.0569}{(8.25 \times 10^{13})(1 \times 10^{-3})} t_f$$

$= 1.380 \times 10^{-6}\, t_f$ (where $t_f$ in the filtration time)

$\boldsymbol{L = 1.161 \times 10^{-3} t_f^{0.5}}$

If the resistance of *the filter medium is neglected*, the optimum cake thickness occurs when *the filtration time is equal to the downtime*.

Thus: $\boldsymbol{t = 900}$ **s**, $\boldsymbol{t^{1/2} = 30}$

$L_{opt} = 34.8 \times 10^{-3}$ m = 34.8 or 35 mm

and: optimum frame thickness = **70 mm**

بالنسبة لعملية الغسيل ، إذا انخفض ضغط الترشيح إلى النصف ، فإن معدل الغسيل ينخفض إلى النصف. يكون لماء الغسل ضعف سماكة اختراقها ونصف مساحة التدفق المتاحة للمرشح ، بحيث يكون معدل الغسيل ، مع مراعاة هذه العوامل ، ثُمن معدل الترشيح النهائي .

The final filtration rate: $\frac{dV}{dt} = \frac{A^2(-\Delta P)}{r\mu v V}$

$$= \frac{1\times10^6 A^2}{(8.25\times10^{13})\times10^{-3}\times0.0569\, V} = \frac{(2.13\times10^{-4}A^2)}{V}$$

and: The washing rate = (**final rate of filtration / 8**) = $2.66 \times 10^{-5} A^2/V$

The volume of wash water = ***V/4***.

Hence: washing time $t_w = (V/4) / (2.66 \times 10^{-5} A^2/V)$

That is: $t_w = 940\, V^2/A^2$

$V^2 = L^2A^2/v^2$

Therefore: $\boldsymbol{t_w = \left(\frac{L^2A^2}{(0.0569)^2}\right)\left(\frac{940}{A^2}\right) = 2.9 \times 10^5 L^2}$

The filtration time $t_f$ was shown earlier to be: $t_f = L^2/ 1.380 \times 10^{-6} = 7.25 \times 105\, L^2$

Thus: total cycle time $= L^2\,(2.90 \times 10^5 + 7.25 \times 10^5) + 900$

$= 1.015 \times 106\, L^2 + 900$

**The rate of cake production is then**:

$$= \frac{L}{1.025 \times 10^6 L^2 + 900} = R$$

For $\boldsymbol{dR/dL} = 0$, then: $1.025 \times 10^6 L^2 + 900 - 2.050 \times 10^6 L^2 = 0$

$$L^2 = \frac{900}{1.025\times10^6} \quad \text{and} \quad L = 29.6 \times 10^{-3} m = 29.6 mm$$

Thus: Frame thickness = **59.2 ≈ 60 mm**