CENTRIFUGAL SEPARATIONS

There is now a wide range of situations where centrifugal force is used in place of the gravitational force in order to effect separations. The resulting accelerations may be several thousand times that attributable to gravity.

Centrifugal fields can be generated in two distinctly different ways:

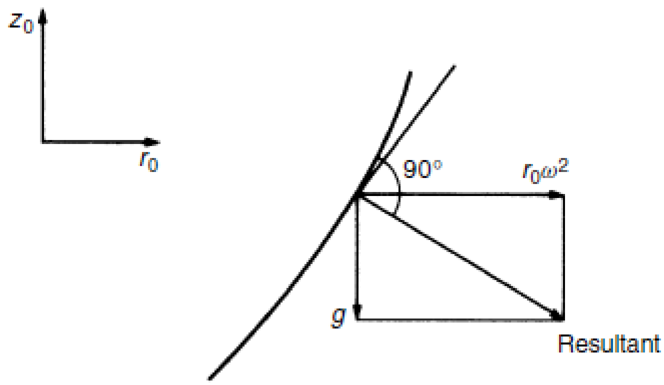
(a) By introducing a fluid with a high tangential velocity into a cylindrical or conical vessel, as in the hydrocyclone and in the cyclone separator.

(b) By the use of the centrifuge. In this case the fluid is introduced into some form of rotating bowl and is rapidly accelerated.

Using of centrifuge

(a) For separating particles on the basis of their size or density.
(b) For separating immiscible liquids of different densities.
(c) For filtration of a suspension. In this case centrifugal force replaces the force of gravity or the force attributable to an applied pressure difference across the filter.
(d) For the drying of solids and, in particular, crystals.
(e) For breaking down of emulsions and colloidal suspensions.
(f) For the separation of gases.
(g) For mass transfer processes.

For an element of liquid in a centrifuge bowl which is rotating at an angular velocity of ω, the centrifugal acceleration is **$r\omega^2$**, compared with the gravitational acceleration of g. The ratio **$r\omega^2/g$** is one measure of the separating effect obtained in a centrifuge relative to that arising from the gravitational field.



The figure above shows an element of the free surface of the liquid in a bowl which is rotating at a radius r_0 about a vertical axis at a very low speed; the centrifugal and gravitational fields will then be of the same order of magnitude. The centrifugal force per unit mass is $r_0\omega^2$ and the corresponding gravitational force is g. These two forces are perpendicular to one another and may be combined as shown to give the resultant force which must, at equilibrium, be at right angles to the free surface. Thus, the slope at this point is given by:

$$\frac{dz_0}{dr_0} = \frac{radial\ component\ \ of\ force}{axial\ component\ \ of\ force} = \frac{r_0\omega^2}{g} \quad \text{……….. (1)}$$

Where z_0 is the axial coordinate of the free surface of the liquid . Equation (1) may be integrated to give:

$$z_0 = \frac{\omega^2}{2g} r_0^2 + constant$$

If z_a is the value of z_0 which corresponds to the position where the free surface is at the axis of rotation *(r_0 = 0)*, then:

$$z_0 - z_a = \frac{\omega^2}{2g} r_0^2 \quad \text{………….. (2)}$$

CENTRIFUGAL PRESSURE

$$\frac{\partial P}{\partial r} = \rho\omega^2 r \quad \text{…………..(3)}$$

The centrifugal pressure gradient is a function of radius of rotation r, and increases towards the wall of the basket. Integration of equation (3) at a given height gives the pressure P exerted by the liquid on the walls of the bowl of radius R when the radius of the inner surface of the liquid is r0 as:

$$\boldsymbol{P = \frac{1}{2}\rho\omega^2\left(R^2 - r_0^2\right)} \quad \text{………. (4)}$$

SEPARATION OF IMMISCIBLE LIQUIDS OF DIFFERENT DENSITIES

The problem of the continuous separation of a mixture of two liquids of different densities is most readily understood by first considering the operation of a gravity settler, as shown in Figure (2). For equilibrium, the hydrostatic pressure exerted by a height $\boldsymbol{z}$ of the denser liquid must equal that due to a height $\boldsymbol{z_2}$ of the heavier liquid and a height $\boldsymbol{z_1}$ of the lighter liquid in the separator.
Thus

$$\boldsymbol{z\rho_2 g = z_2\rho_2 g + z_1\rho_1 g}$$

$$\boldsymbol{z = z_2 + z_1\frac{\rho_1}{\rho_2}} \quad \text{……. (5)}$$

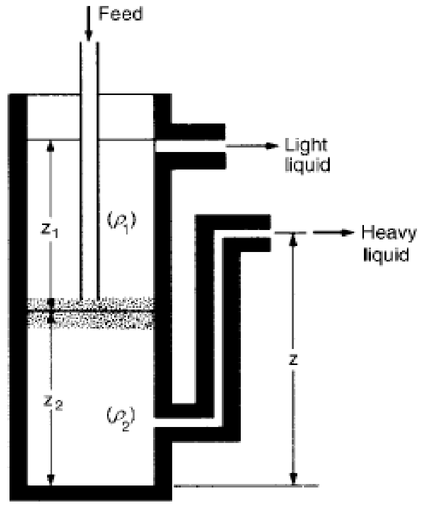
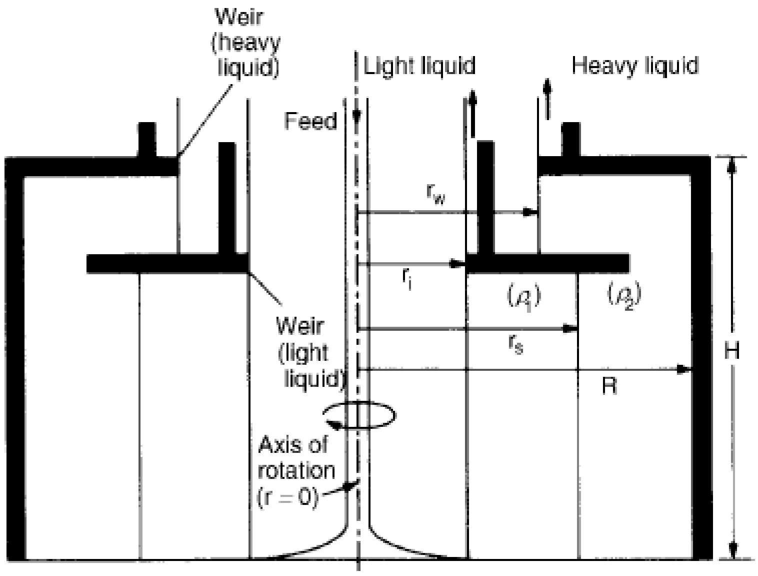


Figure (2): Gravity separation of two immiscible liquids

For the centrifuge it is necessary to position the overflow on the same principle, as shown in Figure 3. In this case the radius r_i of the weir for the less dense liquid will correspond approximately to the radius of the inner surface of the liquid in the bowl. That



Figure(3): Separation of two immiscible liquids in a centrifuge

of the outer weir r_w must be such that the pressure developed at the wall of the bowl of radius R by the heavy liquid alone as it flows over the weir is equal to that due to the two liquids within the bowl. Thus, applying equation 4 and denoting the densities of the light and heavy liquids by ρ_1 and ρ_2 respectively and the radius of the interface between the two liquids in the bowl as r_s :

$$\frac{1}{2}\rho_2\omega^2(R^2 - r_w^2) = \frac{1}{2}\rho_2\omega^2(R^2 - r_s^2) + \frac{1}{2}\rho_1\omega^2(r_s^2 - r_i^2)$$

or

$$\frac{r_s^2 - r_i^2}{r_s^2 - r_w^2} = \frac{\rho_2}{\rho_1} \qquad \ldots\ldots\ldots (6)$$

If $Q1$ volumetric rates of feed of the light liquid.

$Q2$ volumetric rates of feed of the heavy liquids .

on the assumption that there is no slip between the liquids in the bowl and that the same, then residence time is required for the two phases, then:

$$\frac{Q_1}{Q_2} = \frac{r_s^2 - r_i^2}{R^2 - r_s^2} \qquad \ldots\ldots\ldots\ldots (7)$$

Equation 7 enables the value of r_s to be calculated for a given operating condition. The retention time is given by:

$$t_R = \frac{V'}{Q_1 + Q_2} = \frac{V'}{Q} \qquad \ldots\ldots\ldots (8)$$

Where Q is the total feed rate of liquid

V' is the volumetric holdup of liquid in the bowl.

Approximately: $\boldsymbol{V' \approx \pi(R^2 - r_i^2)H}$ ………..(9)

where H is the axial length (or clarifying length) of the bowl.

Thus: $t_R = \frac{Q}{\pi(R^2 - r_i^2)H}$ ………. (10)

SEDIMENTATION IN A CENTRIFUGAL FIELD

Because centrifuges are normally used for separating fine particles and droplets, it is necessary to consider only the Stokes' law region in calculating the drag between the particle and the liquid.

$$\frac{dr}{dt} = \frac{d^2(\rho_s - \rho)r\omega^2}{18\mu} \quad …….. (11)$$

$$= u_0 \frac{r\omega^2}{g} \quad …….. (12)$$

At the walls of the bowl of radius r, dr/dt is given by:

$$\left(\frac{dr}{dt}\right)_{r=R} = \frac{d^2(\rho_s - \rho)r\omega^2}{18\mu} \quad ……. (13)$$

The time taken to settle through a liquid layer of thickness h at the walls of the bowl is given by integration of equation 11 between the limits $r = r_0$ (the radius of the inner surface of the liquid), and $r = R$. This equation may be simplified where $R - r_0$ $(= h)$ is small compared with R then.

$$t_R = \frac{18\mu h}{d^2(\rho_s - \rho)R\omega^2} \quad ………. (14)$$

t_R is then the minimum retention time required for all particles of size greater than d to be deposited on the walls of the bowl. Thus, the maximum throughput Q at which all particles larger than d will be retained is given by substitution for t_R to give:

$$Q = \frac{d^2(\rho_s - \rho)R\omega^2 V'}{18\mu h} \quad ……. (15)$$

or

$$Q = \frac{d^2(\rho_s - \rho)g}{18\mu h} \frac{R\omega^2 V}{hg} \quad ………(16)$$

From eq for settling velocity:

$$\frac{d^2(\rho_s - \rho)g}{18\mu} = u_0$$

where u_0 is the terminal falling velocity of the particle in the gravitational field and hence:

$$Q = u_0 \frac{R\omega^2 V'}{hg} \quad ………… (17)$$

Writing the capacity term as:

$$\sum = \frac{R\omega^2 V'}{hg}$$

$$= \frac{\pi R(R^2 - r_0^2)H\omega^2}{hg}$$

$$= \pi R(R + r_0)H\frac{\omega^2}{g} \quad \ldots\ldots. \ (18)$$

Then : $Q = u_0 \sum$ ……. (19)

For cases where the thickness *h* of the liquid layer at the walls is comparable in order of magnitude with the radius *R* of the bowl, it is necessary to use equation in place of equation 14 for the required residence time in the centrifuge or:

$$t_R = \frac{18\mu}{d^2(\rho_s - \rho)\omega^2} ln\frac{R}{r_0} \quad \ldots\ldots\ldots(20)$$

Then :

$$Q = \frac{d^2(\rho_s - \rho)\omega^2 V'}{18\mu \ ln(R/r_0)} \quad \ldots\ldots\ldots. \ (21)$$

$$= \frac{d^2(\rho_s - \rho)g}{18\mu}\frac{\omega^2 V'}{g \ ln(R/r_0)} \quad \ldots\ldots\ldots \ (22)$$

$$= u_0 \sum \quad \ldots\ldots.. \ (23)$$

In this case:

$$\sum = \frac{\omega^2 V'}{g \ ln(R/r_0)}$$

$$= \frac{\pi(R^2 - r_i^2)H}{ln(R/r_0)}\frac{\omega^2}{g} \quad \ldots\ldots. \ (24)$$

Example (1):

In a test on a centrifuge all particles of a mineral of density 2800 kg/m^3 and of size 5 μm, equivalent spherical diameter, were separated from suspension in water fed at a volumetric throughput rate of 0.25 m^3/s. Calculate the value of the capacity factor ∑ What will be the corresponding size cut for a suspension of coal particles in oil fed at the rate of 0.04 m^3/s? The density of coal is 1300 kg/m^3 and the density of the oil is 850 kg/m^3 and its viscosity is 0.01 Ns/m^2. It may be assumed that Stokes' law is applicable.

Solution:

The terminal falling velocity of particles of diameter 5 μm in water, of density ρ = 1000 kg/m^3 and, of viscosity $\mu = 10^{-3}$ Ns/m^2, is given by:

$$u_0 = \frac{d^2(\rho_s - \rho)g}{18\mu} = \frac{25\times10^{-12}\times(2800-1000)\times9.81}{18\times10^{-3}} = 2.45 \times 10^{-5} \ m/s$$

$Q = u_0 \sum$

and: $$\sum = \frac{0.25}{(2.45\times10^{-5})} = 1.02 \times 10^4 \ m^2$$

For the coal-in-oil mixture:

$$u_0 = \frac{Q}{\Sigma} = \frac{0.04}{(1.02\times10^4)} = 3.94 \times 10^{-6} m/s$$

$$d^2 = \frac{18\mu u_0}{(\rho_s - \rho)}$$

$$= \frac{18\times10^{-2}\times3.92\times10^{-6}}{(1300-850)\times9.81} = \mathbf{4 \times 10^{-6} m \ or \ 4\ \mu m}$$