By taking the inverse of both sides $z = \cosh^{-1}(-2)$

but $\cosh^{-1} z = \ln(z \pm \sqrt{z^2 - 1})$

where $z = -2 + 0i$ $\cosh^{-1}(-2) = \ln(-2 \pm \sqrt{(-2)^2 - 1})$ remember that $\ln z = \ln|z| + i\,(\theta_1 + 2n\pi)$

$$z = \ln(-2 \pm \sqrt{3}) = \cosh^{-1} 2 + i(2n+1)\pi$$

<u>Example</u>:- Show that $\frac{d(\cos^{-1} z)}{dz} = -\frac{1}{\sqrt{1-z^2}}$

<u>Solution</u>

Let $w = \cos^{-1} z \Rightarrow \cos w = z$

differentiate both side with respect to z $\frac{d}{dz}\cos w = 1 \Rightarrow -\sin w \frac{dw}{dz} = 1$

or $\frac{dw}{dz} = -\frac{1}{\sin w}$ but $\sin^2 w + \cos^2 w = 1 \Rightarrow \sin w = \sqrt{1 - \cos^2 w}$

$\frac{dw}{dz} = -\frac{1}{\sqrt{1-\cos^2 w}}$ $\frac{d(\cos^{-1} z)}{dz} = -\frac{1}{\sqrt{1-z^2}}$

<u>Example</u>:- Show that $\frac{d(\sinh^{-1} z)}{dz} = \frac{1}{\sqrt{1+z^2}}$

<u>Solution</u>

Let $w = \sinh^{-1} z \Rightarrow \sinh w = z$

differentiate both side with respect to z $\frac{d}{dz}\sinh w = 1 \Rightarrow \cosh w \frac{dw}{dz} = 1$

or $\frac{dw}{dz} = \frac{1}{\cosh w}$ but $\cosh^2 w - \sinh^2 w = 1 \Rightarrow \cosh w = \sqrt{1 + \sinh^2 w}$

$\frac{dw}{dz} = \frac{1}{\sqrt{1+\sinh^2 w}}$ $\frac{d(\sinh^{-1} z)}{dz} = \frac{1}{\sqrt{1+z^2}}$

<u>PROBLMES</u>

1- Express the principle value of each of the following in the form $a + ib$

(a) $\ln(-10)$ **(b)** $\ln(1 - i\sqrt{3})$ **(c)** $\sin(1+i)$ **(d)** $\cosh(1 - i)$

(e) i^i **(f)** $\cos^{-1} 2$

Answers (a) $1+i\pi$ (b) $\ln 2 - i(\pi/3)$ (c) $1.299+0.635i$ (d) $0.834-2.164i$

(e) $e^{-\pi/2}$ (f) $-1.317i$

2- Express the principle value of each of the following in the form $a+ib$

(a) $\ln(-3+4i)$ (b) 2^i (c) $\tan i$ (d) $\tanh^{-1} 2$

(e) $(1-i)^{2+i}$ (f) $(2+i)^{1-i}$

3- Prove that

(a) $\cos^2 z + \sin^2 z = 1$ (b) $\cos(z_1 \mp z_2) = \cos z_1 \cos z_2 \pm \sin z_1 \sin z_2$

(c) $\sin(z_1 \mp z_2) = \sin z_1 \cos z_2 \mp \cos z_1 \sin z_2$ (d) $d(\cos z)/dz = -\sin z$

(e) $d(\sin z)/dz = \cos z$ (f) $d(\tan z)/dz = \sec^2 z$

4- What is the derivative of

(a) $\cosh z$ (b) $\sinh z$ (c) $\tanh z$

Answers (a) $\frac{d\cosh z}{dz} = \sinh z$ (b) $\frac{d\sinh z}{dz} = \cosh z$ (c) $\frac{d\tanh z}{dz} = \text{sech}^2 z$

5- Show that

(a) $d(\cos^{-1} z)/dz = -1/\sqrt{1-z^2}$ (b) $d(\sin^{-1} z)/dz = 1/\sqrt{1-z^2}$

(c) $d(\tan^{-1} z)/dz = 1/(1+z^2)$

6- What is

(a) $d(\cosh^{-1} z)/dz$ (b) $d(\sinh^{-1} z)/dz$ (c) $d(\tanh^{-1} z)/dz$

Answers (a) $\frac{d\cosh^{-1} z}{dz} = \frac{1}{(z^2-1)^{1/2}}$ (b) $\frac{d\sinh^{-1} z}{dz} = \frac{1}{(z^2+1)^{1/2}}$ (c) $\frac{d\tanh^{-1} z}{dz} = \frac{1}{1-z^2}$

7- Find all solutions of the equation $e^z = -2$ **Answers** $\ln 2 + i(2n+1)\pi$

8- Find all solutions of the equation $\sin z = 3$

9- Find all solutions of the equation $\cosh z = -2$ **Answers** $\cosh^{-1} 2 + i(2n+1)\pi$

10- Prove that $\overline{e^z} = e^{\bar{z}}$

11- Prove that $\overline{\cos z} = \cos \bar{z}$

12- Is $\overline{\ln z} = \ln \bar{z}$?

13- Is $\overline{\sin z} = \sin \bar{z}$?

14- Show that $\tan z = \frac{\sin 2x + i \sinh 2y}{\cos 2x + \cosh 2y}$

15- Show that $\tanh z = \frac{\sinh 2x + i \sin 2y}{\cosh 2x + \cos 2y}$

16- If $w = \sin z$, what are the equations of the curves in the uv plane into which the lines $x = c$ and $y = k$ are transformed by w?

17 Work Exercise 16 if $w = \cosh z$

18 Work Exercise 16 if $w = \sinh z$

Fourier Series and Fourier Integral

JOSEPH FOURIER (1768-1830). French physicist and mathematician

Definition A function f is **periodic** if and only if if there exists a positive number $2p$ such that for every t in the domain of f, $f(t+2p)=f(t)$. The number $2p$ is called a **period** of f

Notes:-

1- If $f(t)$ and $g(t)$ have the period $2p$, then the function $h(t)=af(t)+bg(t)$.has the period $2p$ also.

2- $f(t)=$ constant is periodic.

The Euler Coefficients

Let $f(t)$ be an arbitrary periodic function of period $2p$, then $f(t)$ has formal expansion of the form

$$f(t)=\frac{1}{2}a_o+a_1\cos\frac{\pi t}{p}+a_2\cos\frac{2\pi t}{p}+\cdots\cdots\cdots+a_n\cos\frac{n\pi t}{p}+\cdots\cdots+b_1\sin\frac{\pi t}{p}+$$
$$b_2\sin\frac{2\pi t}{p}+\cdots\cdots+b_n\sin\frac{n\pi t}{p}+\cdots \qquad \cdots\cdots\cdots(1)$$

The introduction of the factor $\frac{1}{2}$ is a conventional device to render more symmetric the final formulas for the coefficients.

To determine the coefficients a_o , a_n and b_n, we need the following definite integrals, which are valid for values of d

1- $\int_d^{d+2p}\cos\frac{n\pi t}{p}dt=0 \qquad n\neq 0$

2- $\int_d^{d+2p}\sin\frac{n\pi t}{p}dt=0$

3- $\int_d^{d+2p}\cos\frac{n\pi t}{p}\cos\frac{m\pi t}{p}dt=0 \qquad n\neq m$

4- $\int_d^{d+2p}\cos^2\frac{n\pi t}{p}dt=p \qquad n\neq 0$

5- $\int_d^{d+2p}\cos\frac{m\pi t}{p}\sin\frac{n\pi t}{p}dt=0$

6- $\int_d^{d+2p}\sin\frac{m\pi t}{p}\sin\frac{n\pi t}{p}dt=0 \qquad n\neq m$

7- $\int_d^{d+2p}\sin^2\frac{n\pi t}{p}dt=p \qquad n\neq 0$

Now, to find a_o integrate both sides of equation (1) from $t=d$ to $t=d+2p$