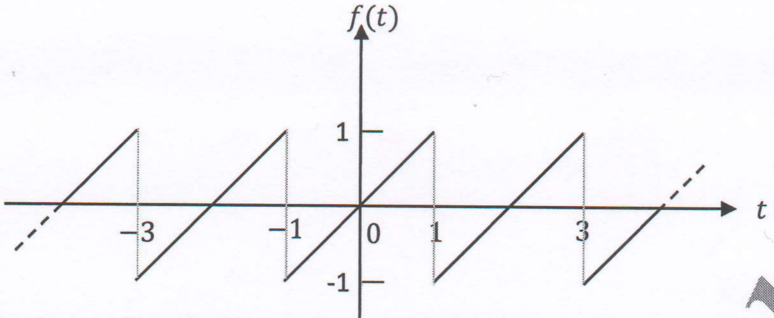
Graph the given function, then from the graph the half-period of the given function is $p = 1$



Since $f(t) = -f(-t)$ the given function is odd function then $a_n = 0$

$$b_n = \frac{2}{1}\int_0^1 f(t)\sin\frac{n\pi t}{1}\,dt = 2\int_0^1 t\cdot\sin n\pi t\ dt$$

$$b_n = 2\left[-t\cdot\frac{1}{n\pi}\cos n\pi t + \left(\frac{1}{n\pi}\right)^2\sin n\pi t\right]_0^1$$

$$b_n = 2\left[-\frac{1}{n\pi}\cos n\pi\right]$$

D		I
t	$+$	$\sin n\pi t$
1	$-$	$-\frac{1}{n\pi}\cos n\pi t$
0		$-\left(\frac{1}{n\pi}\right)^2\sin n\pi t$

Remember that $\cos n\pi = (-1)^n$

Hence $b_n = \frac{2(-1)^{n+1}}{n\pi}$

Substituting these coefficients into the series, we obtain

$$f(t) = \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}\sin n\pi t}{n}$$

PROBLMES

Find the Fourier expansion of the periodic function whose definitions on one period is

1- $f(t) = \begin{cases} 1 & 0 < t < 1 \\ 0 & 1 < t < 3 \\ -1 & 3 < t < 4 \end{cases}$ **Answers:** $a_n = 0$ $b_n = \begin{cases} 2/\pi n & n = 1,3,5,\cdots \\ 4/\pi n & n = 2,6,10,\cdots \\ 0 & n = 4,8,12,\cdots \end{cases}$

2- $f(t) = |t| \qquad -2 \le t \le 2$

3- $f(t) = \begin{cases} 0 & -2 \le t \le -1 \\ \cos\frac{\pi t}{2} & -1 \le t \le 1 \\ 0 & 1 \le t \le 2 \end{cases}$ **Answers:** $\frac{1}{\pi} + \frac{1}{2}\cos\frac{\pi t}{2} + \frac{2}{\pi}\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\cos n\pi t}{4n^2-1}$

<u>Half-Range Expansion</u>

When $f(t)$ will be defined on an interval $0 \le t \le p$, and on this interval we want to represent $f(t)$ by a Fourier. Then, if we represents $f(t)$ an even periodic function which is called **half-range cosine series**

$$f(t) = \frac{1}{2}a_o + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi t}{p} \qquad b_n = 0$$

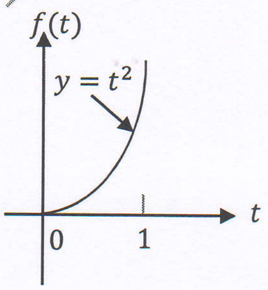
and if we represents $f(t)$ an odd periodic function which is called **half-range sine series,** then

$$f(t) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi t}{p} \qquad a_n = 0$$

Collectively, we speak of such series as **half-range expansion**

<u>Example</u>:- Find the Fourier coefficients in the half-range sine expansion of the function

$$f(t) = t^2 \qquad 0 \le t < 1$$



<u>Solution</u>

From the graph $\qquad p = 1$

The half-range sine expansion of the function is the Fourier series of the odd function ,then

$$a_n = 0 \qquad b_n = \frac{2}{p}\int_0^p f(t)\sin\frac{n\pi t}{p}\,dt$$

Or $\qquad b_n = \frac{2}{1}\int_0^1 t^2 \sin\frac{n\pi t}{1}\,dt =$

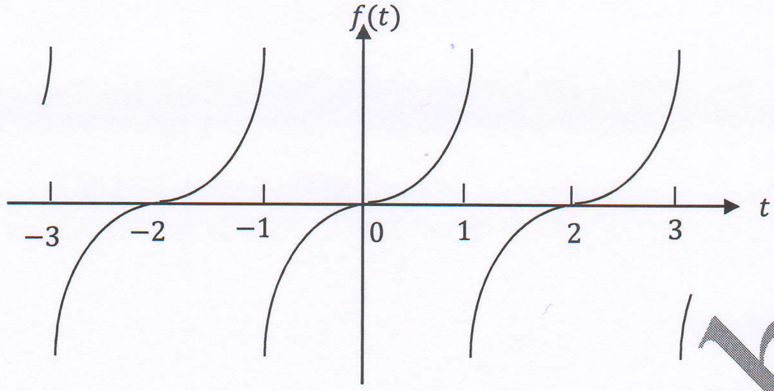
D		I
t^2	+	$\sin n\pi t$
$2t$	−	$-\frac{1}{n\pi}\cos n\pi t$
2	+	$-\left(\frac{1}{n\pi}\right)^2 \sin n\pi t$
0		$\left(\frac{1}{n\pi}\right)^3 \cos n\pi t$

$$b_n = 2\left[-\frac{t^2}{n\pi}\cos n\pi t + 2t\left(\frac{1}{n\pi}\right)^2 \sin n\pi t + 2\left(\frac{1}{n\pi}\right)^3 \cos n\pi t\right]_0^1$$

$$b_n = -\frac{2}{n\pi}\cos n\pi + 4\left(\frac{1}{n\pi}\right)^3 [\cos n\pi - 1]$$

since $\quad \cos n\pi = (-1)^n$

$$b_n = \frac{2(-1)^{n+1}}{n\pi} - \frac{4}{n^3\pi^3}\left[(-1)^{n+1} + 1\right] = \begin{cases} \frac{2}{n\pi} - \frac{8}{n^3\pi^3} & n\ odd \\ \frac{-2}{n\pi} & n\ even \end{cases}$$



The half-range cosine expansion of the function is the Fourier series of the even function ,then

$$b_n = 0 \qquad a_n = \frac{2}{p}\int_0^p f(t)\ \cos\frac{n\pi t}{p}\, dt$$

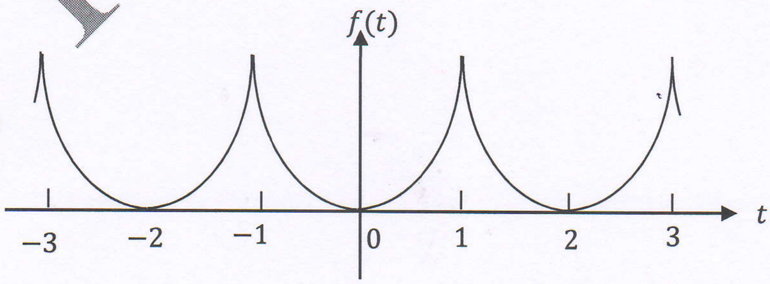
Or $$a_n = \frac{2}{1}\int_0^1 t^2\ \cos n\pi t\ dt$$

D	I
t^2	$\cos n\pi t$
$2t$ (+)	$\frac{1}{n\pi}\sin n\pi t$
2 (−)	$-\left(\frac{1}{n\pi}\right)^2 \cos n\pi t$
0 (+)	$-\left(\frac{1}{n\pi}\right)^3 \sin n\pi t$

$$a_n = 2\left[\frac{t^2}{n\pi}\sin n\pi t + 2t\left(\frac{1}{n\pi}\right)^2 \cos n\pi t - 2\left(\frac{1}{n\pi}\right)^3 \sin n\pi t\right]_0^1$$

$$a_n = \frac{2^2}{n^2\pi^2}\cos n\pi = \frac{2^2}{n^2\pi^2}(-1)^n \qquad n \neq 0$$

for $n = 0$ $$a_0 = \frac{2}{1}\int_0^1 t^2\ dt = \frac{2}{3}$$



PROBLMES

Find the half-range cosine and sine expansion of each of the following functions.

1- $f(t) = \begin{cases} 1 & 0 \le t \le \pi \\ 0 & \pi < t \le 2\pi \end{cases}$

2- $f(t) = \begin{cases} at & 0 \leq t \leq l/2 \\ a(l-t) & l/2 < t \leq l \end{cases}$