

*Lecture Notes on*  
**Adaptive Control**  
*4<sup>th</sup> Year Petroleum Systems*  
*and Control Engineering*  
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**Adaptive Control:** covers a set of techniques which provide a systematic approach for automatic adjustment of controllers in real time, **in order to** achieve or to maintain a desired level of control system performance when the parameters of the plant dynamic model are unknown and/or change in time.

For example, as an aircraft flies, its mass will slowly decrease as a result of fuel consumption; a control law is needed that adapts itself to such changing conditions.

**The foundation of adaptive control** is parameter estimation. Common methods of estimation include gradient descent.

**Note::** *gradient descent is a first – order iterative optimize algorithm for find the minimum function.*

## **Adaptive controller:**

Is a controller that can modify its behavior in response to changes in the dynamics of the process and the disturbances.

## **Adaptive system:**

Is any physical system that has been designed with an adaptive view point.

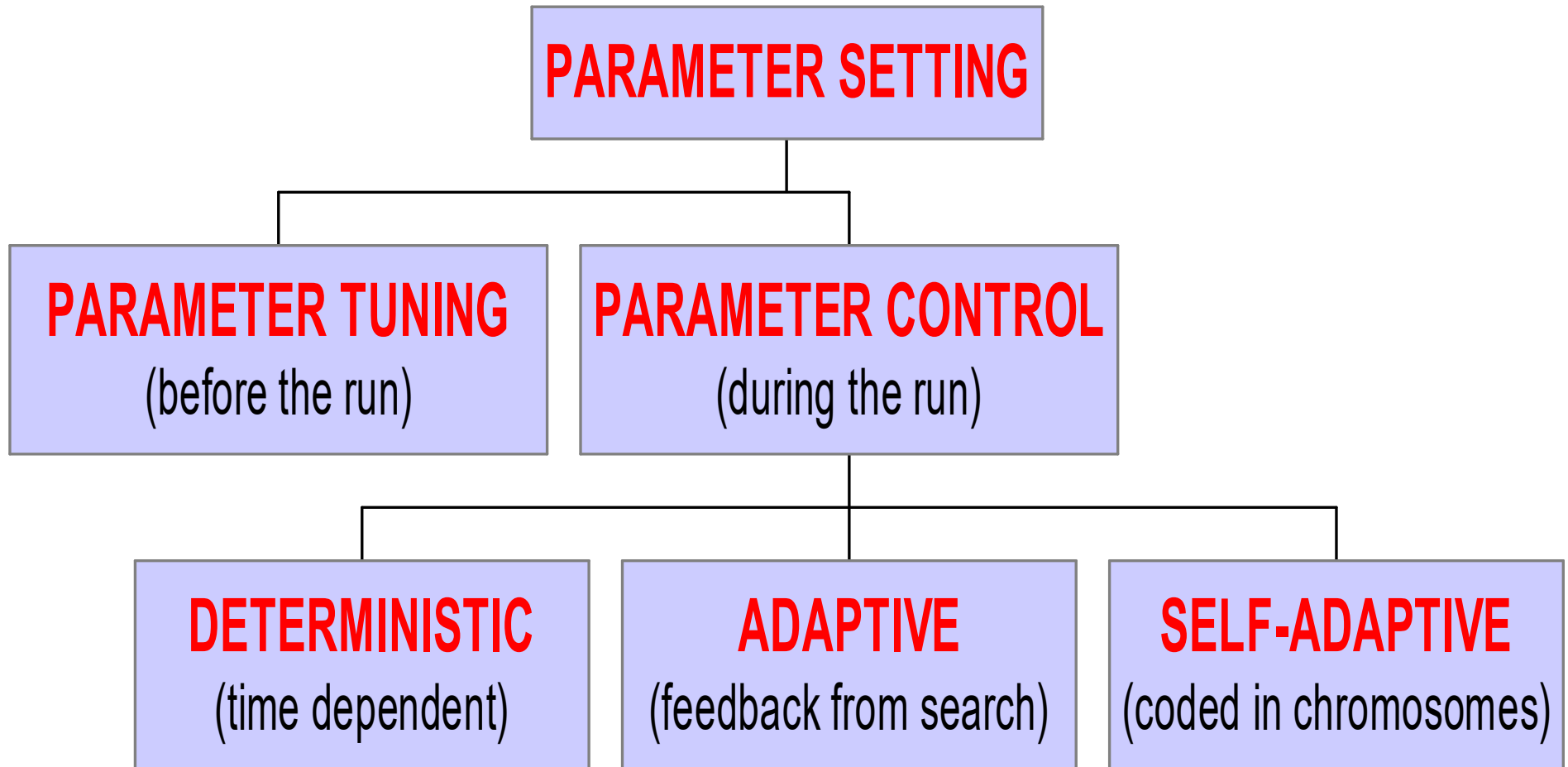
There are strong ties to nonlinear system theory with adaptive control , because adaptive systems are inherently nonlinear.

Adaptive control is different from **robust control** in that it does not need a **priori** information about the **bounds** on these uncertain or time-varying parameters; robust control **guarantees** that if the changes are within given bounds the control law need not be **changed**, while adaptive control is concerned with control law **changing itself**.

# Adaptation and tuning

- It is customary to separate the tuning and adaptation problems.
- In the **tuning problem** it is assumed that the process to be controlled has constant but unknown parameters.
- In the **adaptation problem** it is assumed that the parameters are changing.
- Many issues are much easier to handle in the tuning problem.
- **The convergence problem** is to investigate where the parameters converge to their true values.
- **The corresponding problem** is much more difficult in the adaptive case, because true values are changing.

# Adaptation and tuning



## Reasons for using Adaptive Control

There are many reasons for adaptive control. The key factors are:

- **A - Variations in process dynamics.** Parameters may vary due to nonlinear actuators, changes in the operating conditions of the process, and non-satisfactory disturbances acting on the process.
- **B - Variations in the character of the disturbances.**
- **C - Engineering efficiency.**
- High performance control systems may require precise tuning of the controller but plant (disturbance) model parameters may be unknown or time-varying.
- “Adaptive Control” techniques provide a systematic approach for automatic on-line tuning of controller parameters

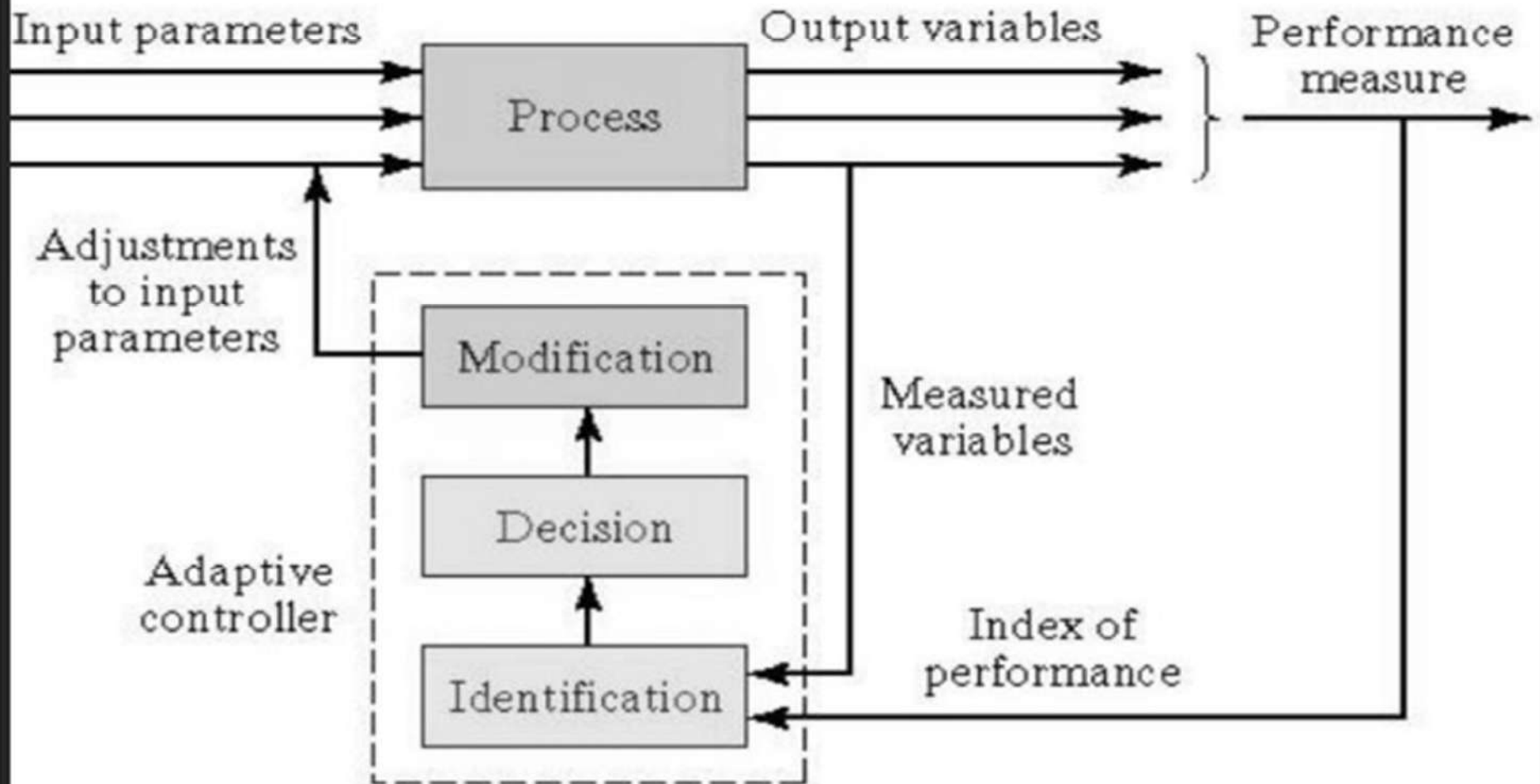
- “Adaptive Control” techniques can be viewed as approximations of some nonlinear stochastic control problems (not solvable in practice)
- **Objective of “Adaptive Control”** : to achieve and to maintain acceptable level of performance when plant (disturbance) model parameters are unknown or vary.
- The key reason is that most of the processes are nonlinear. The control loops are generally designed to maintain the controlled variable at its set point by compensating for all disturbance occurring in the process. The controller performance is optimum only for a particular range in which the process is linearized. Once the process starts to operate beyond the linearized range, the controller fails to produce desired performance. It is because of the fact that the parameters of the controller is not suitable for the current operating conditions.

- The changes in transfer function of process which occurs due to parameter variations or variation in the coefficients or wear and tear of important components.
- The nature and magnitude of disturbances vary with time. There may be an occurrence of an unpredictable and unknown disturbance in the process.
- There may be a change in nature of inputs to the process and the properties of raw materials.

In all the above cases, a conventional controller cannot perform at a satisfactory level. This demands the need for a special type of controller that adapt in accordance with the uncertainties in the process and in turn Adaptive Control.



# Adaptive Control System



# Three Functions in Adaptive Control

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1. Identification function – current value of IP is determined based on measurements of process variables
2. Decision function – decide what changes should be made to improve system performance
  - Change one or more input parameters
  - Alter some internal function of the controller
3. Modification function – implement the decision function
  - Concerned with physical changes (hardware rather than software)

## **Applications of Adaptive Control**

1. Adaptive control is used in the robotic manipulators of robotic systems which demands high positioning accuracy.
2. Adaptive control is used for altitude control of satellites. The observation satellites should be operated at lower altitudes where the air-drag makes the quick reorientation of satellite is necessary to increase the observation time.
3. Adaptive control is used in the autopilot of air crafts and steering control of ships.
4. Adaptive control is used in the control of strip temperature for the continuous annealing and processing in metallurgical processes.

5. Adaptive control is used in distillation columns to provide high product quality and a considerable reduction of thermal energy usage.

6. Adaptive control is used to stabilize PID based pH control system in chemical industries.

Without adaptive control, the process gain ( $K_p$ ) increases as the pH value becomes neutral and it leads to change in total loop gain and finally to instability of loop. The adaptive control keeps the total loop gain at the desired value (usually 0.5) by lowering the controller gain ( $K_c$ ).

## The adaptive Control Problem

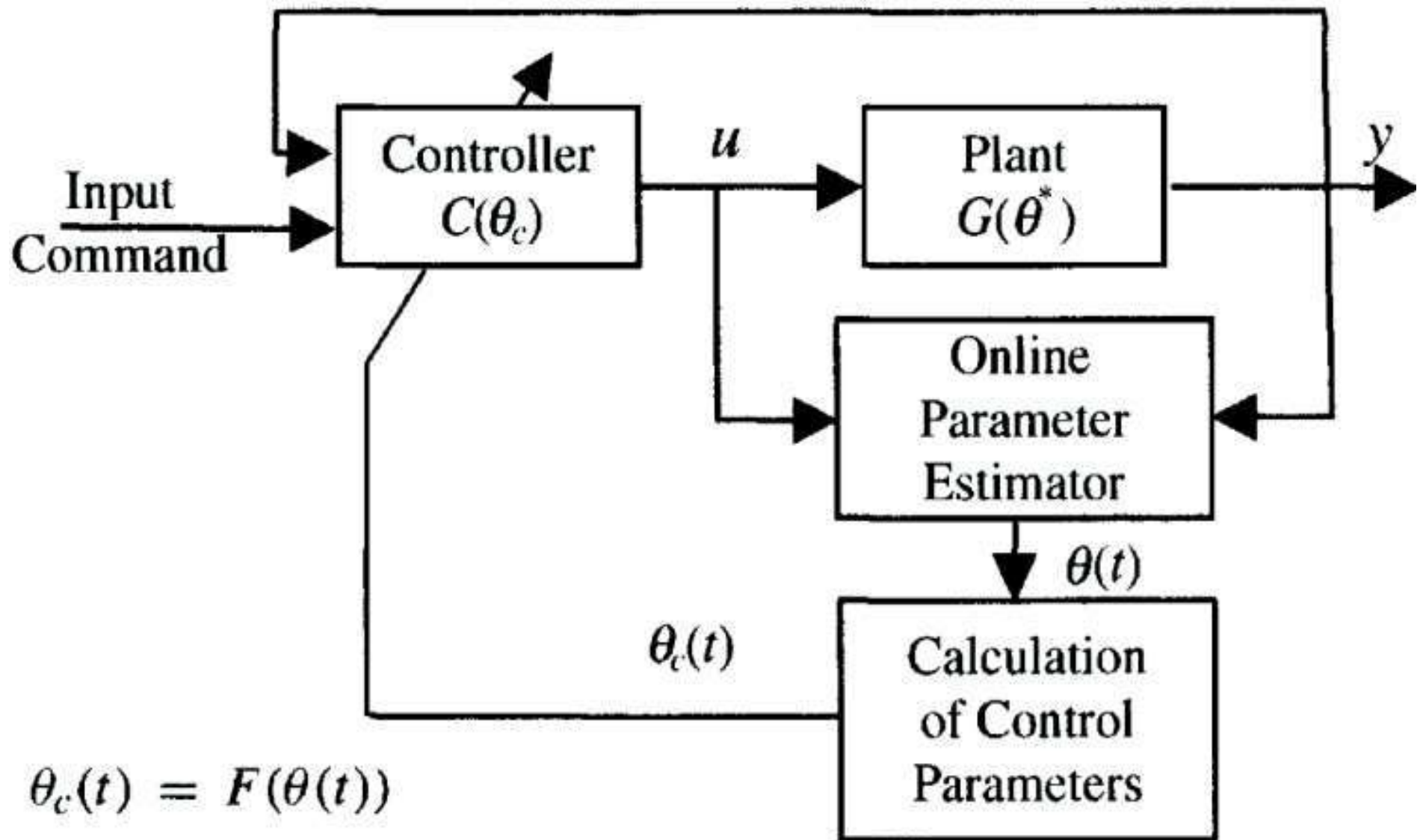
- An adaptive controller has been defined as a controller with adjustable parameters and mechanism for adjusting the parameters.
- The construction of an adaptive controller thus contains the following steps:
  - ❖ Characterize the desired behaviors of the close loop system.
  - ❖ Determine the suitable control law with adjustable parameters.
  - ❖ Find a mechanism for adjusting the parameters.
  - ❖ Implement the control law.

## **Adaptive Control Schemes**

The choice of the parameter estimator, the choice of the control law, and the way they are combined leads to different classes of adaptive control schemes. Adaptive control as defined above has also been referred to as identifier-based adaptive control in order to distinguish it from other approaches referred to as non-identifier-based, where similar control problems are solved without the use of an online parameter estimator. The class of adaptive control schemes studied in this course is characterized by the combination of an online parameter estimator, with a control law. The way the parameter estimator, also referred to as adaptive law, is combined with the control law gives rise to two different approaches:

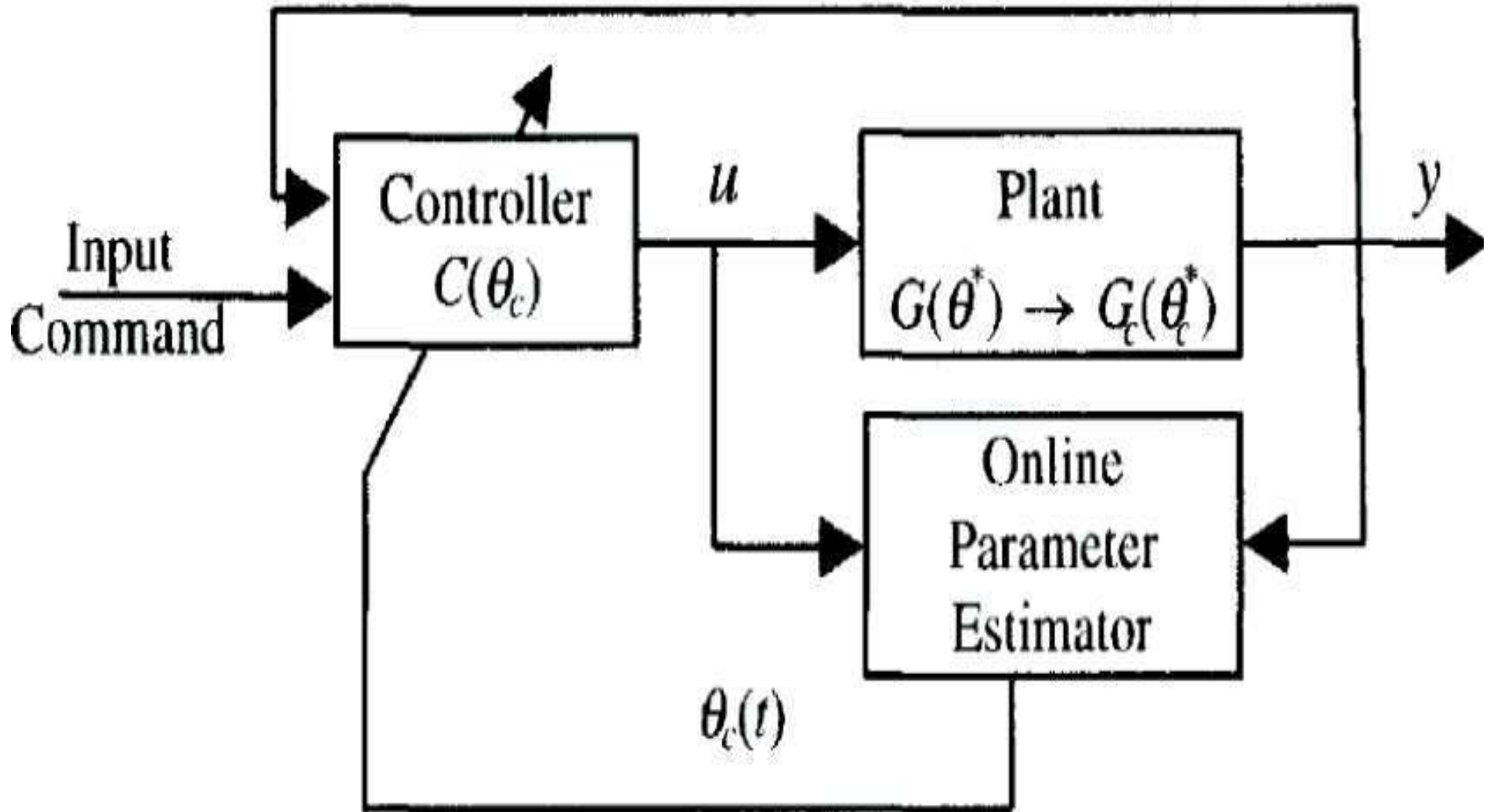
**1- In the first approach**, referred to as indirect adaptive control, the plant parameters are estimated online and used to calculate the controller parameters. In other words, at each time  $t$ , the estimated plant is formed and treated as if it is the true plant in calculating the controller parameters. This approach has also been referred to as explicit adaptive control, because the controller design is based on an explicit plant model.

**2- In the second approach**, referred to as direct adaptive control, the plant model is parameterized in terms of the desired controller parameters, which are then estimated directly without intermediate calculations involving plant parameter estimates. This approach has also been referred to as implicit adaptive control because the design is based on the estimation of an implicit plant model. The basic structure of indirect adaptive control is shown in following Figure. The plant model  $G(\boldsymbol{\theta}^*)$  is parameterized with respect to some unknown parameter vector  $\boldsymbol{\theta}^*$ .



*Indirect adaptive control structure.*





*Direct adaptive control structure.*

In general, direct adaptive control is applicable to SISO linear plants which are **minimum phase**, since for this class of plants the parameterization of the plant with respect to the controller parameters for some controller structures is possible.

Indirect adaptive control can be applied to a wider class of plants with different controller structures, but it suffers from a problem known as the **stabilizability problem** explained as follows:

The controller parameters are calculated at each time  $t$  based on the estimated plant. Such calculations are possible, provided that the estimated plant is controllable and observable or at least stabilizable and detectable.

# Types of Adaptive Control -Identifier-Based

## 1. **Self-Adaptive** Control

The Self-Adaptive Control is comparable to feedback compensation because the adaptation of controller parameters is based on the measurement of closed loop performance and aim to optimize it.

## 2. **Model-reference** adaptive control (MRAC).

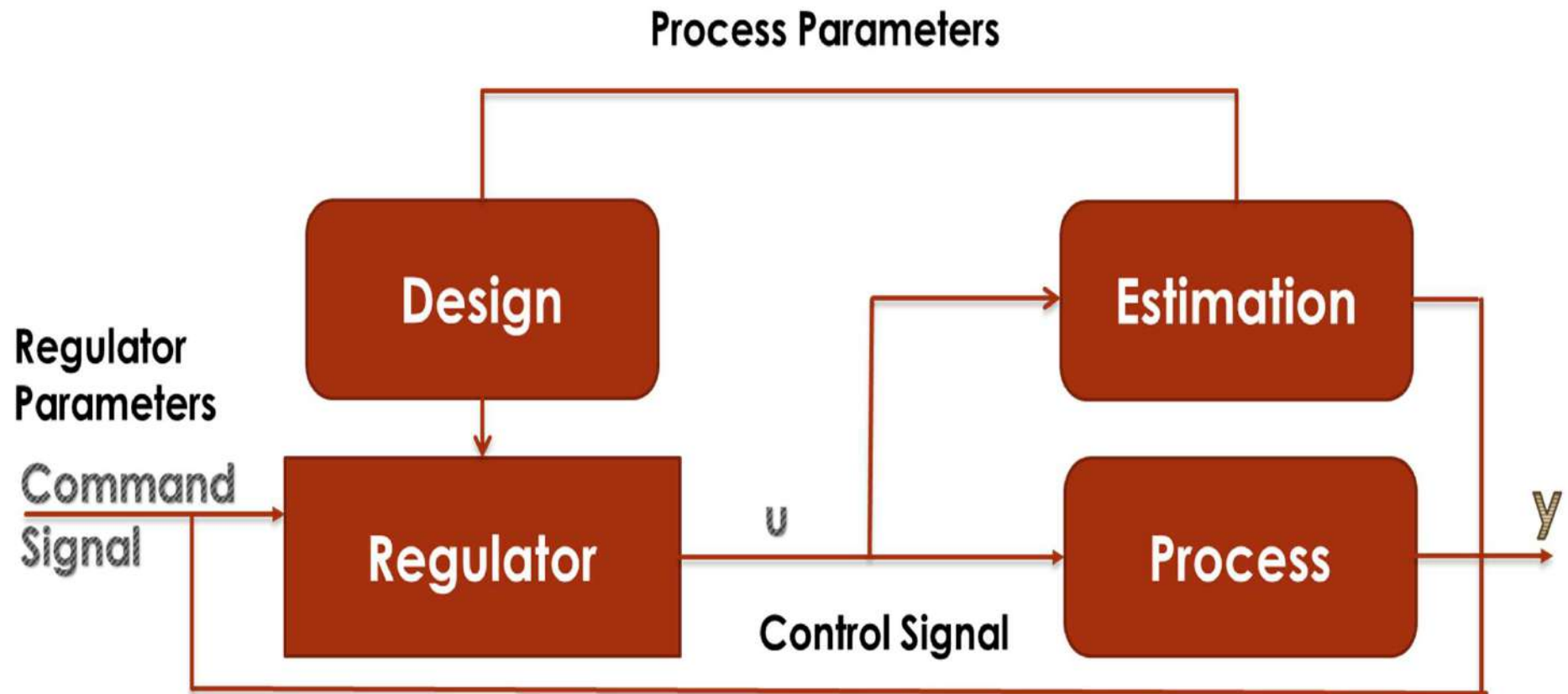
In this adaptive system, two ideas were introduced that: First, the performance of the system is specified by a model. Second, the parameters of the regulator are adjusted based on the error between the reference model and the system.

## Self-Tuning Regulator

**Basic idea:** In an adaptive system, it is assumed that the regulator parameters are adjusted all the time. This implies that the regulator parameters follow changes in process; It is difficult to analyze the convergence and stability properties of such systems. To simplify the problem it can assume that the process has **constant** but **unknown** parameters. When the process is known, the design procedure specifies a set of desired controller parameters.

The adaptive controller should converge to these parameter values even when the process is known. A regulator with this property is called Self-Tuning, since it automatically tunes the controller to the desired performance.

**The Self- Tuning Regulator (STR)** is based on the idea of separating the estimation of unknown parameters from the design of the controller. The basic idea is illustrated in figure below:



In the block diagram, the design block represents an on-line solution to the design problem for a system with unknown parameters.

The design method is chosen depending on the specifications of the closed loop systems. Different combinations of estimation methods lead to regulators with different properties.

**In order to form a Self tuning or adaptive control system, three forms of controls can be used:**

- 1- Pole placement (assignment) control.
- 2- Minimum variance control.
- 3- Multistage predictive control.

## Self-Tuning cycle

At each sample interval  $T_s$ , the following sequence of action is taken:

### **Step (1) Data capture:**

The system output  $y(KT)$ , reference input  $r(KT)$  and any other variables of importance are measured.

### **Step (2) Estimator update:**

The data acquired in (1) is used together with past data and the previous control signal to update the parameter estimates in a model of the system using an appropriate recursive estimator.

### **Step (3) Controller synthesis:**

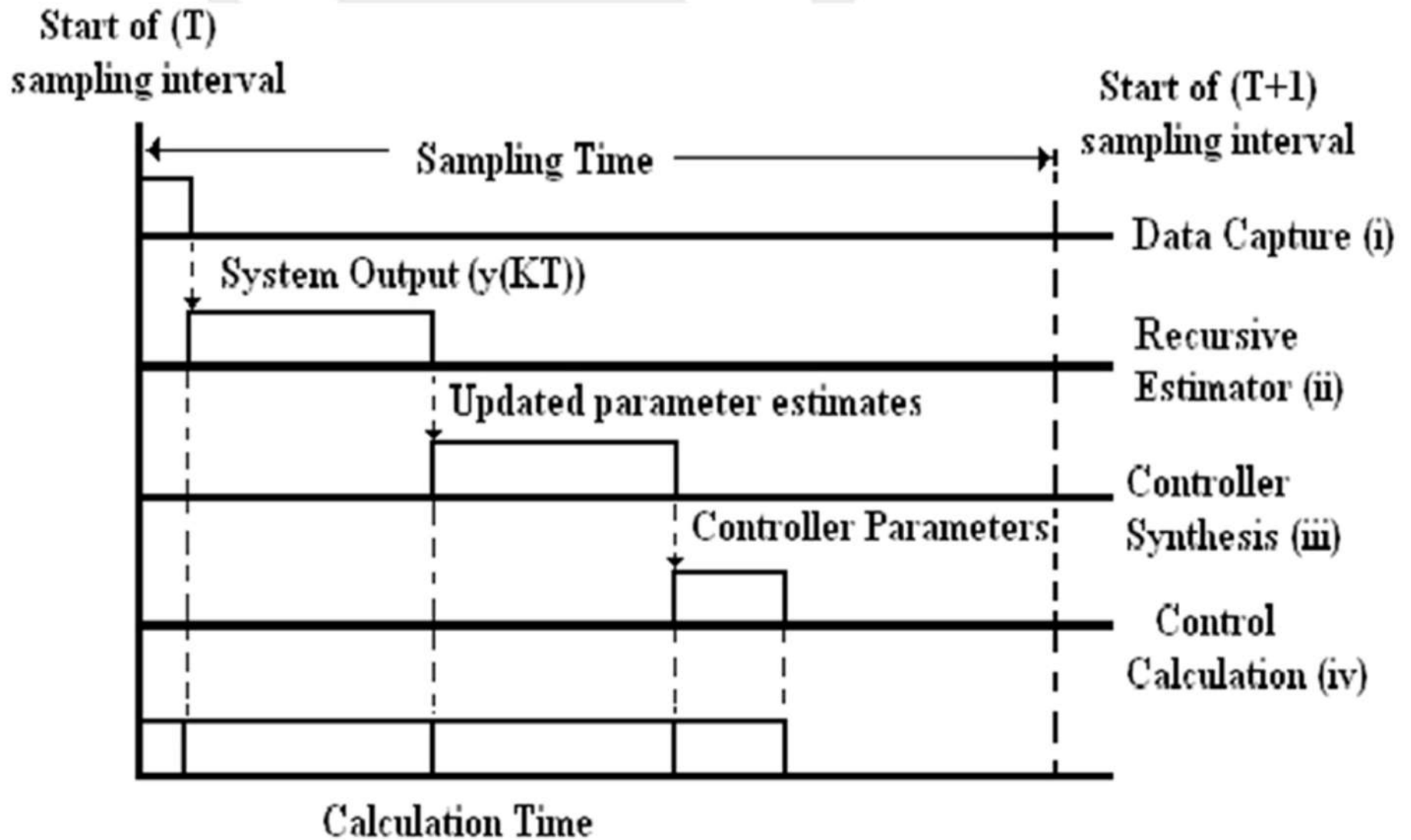
The updated parameters from (2) are used in a pole assignment identity to synthesize the parameters of the desired controller.

### **Step (4) Control algorithm:**

The controller parameters synthesized in (3) are used in a controller to calculate and input the next control signal  $u(KT)$ .

- At the end of the cycle the control computer waits until the end of sample interval  $T$  and then repeats the cycle for interval  $T+1$ , and so on.
- Steps in the self-tuning cycle have computed sequentially. However, figure below illustrates this in terms of a timing and sequence diagram.
- The total computation time must be less than the sample interval and is generally assumed to be much less.





**Fig. Timing and sequence diagram for self-tuning controller.**

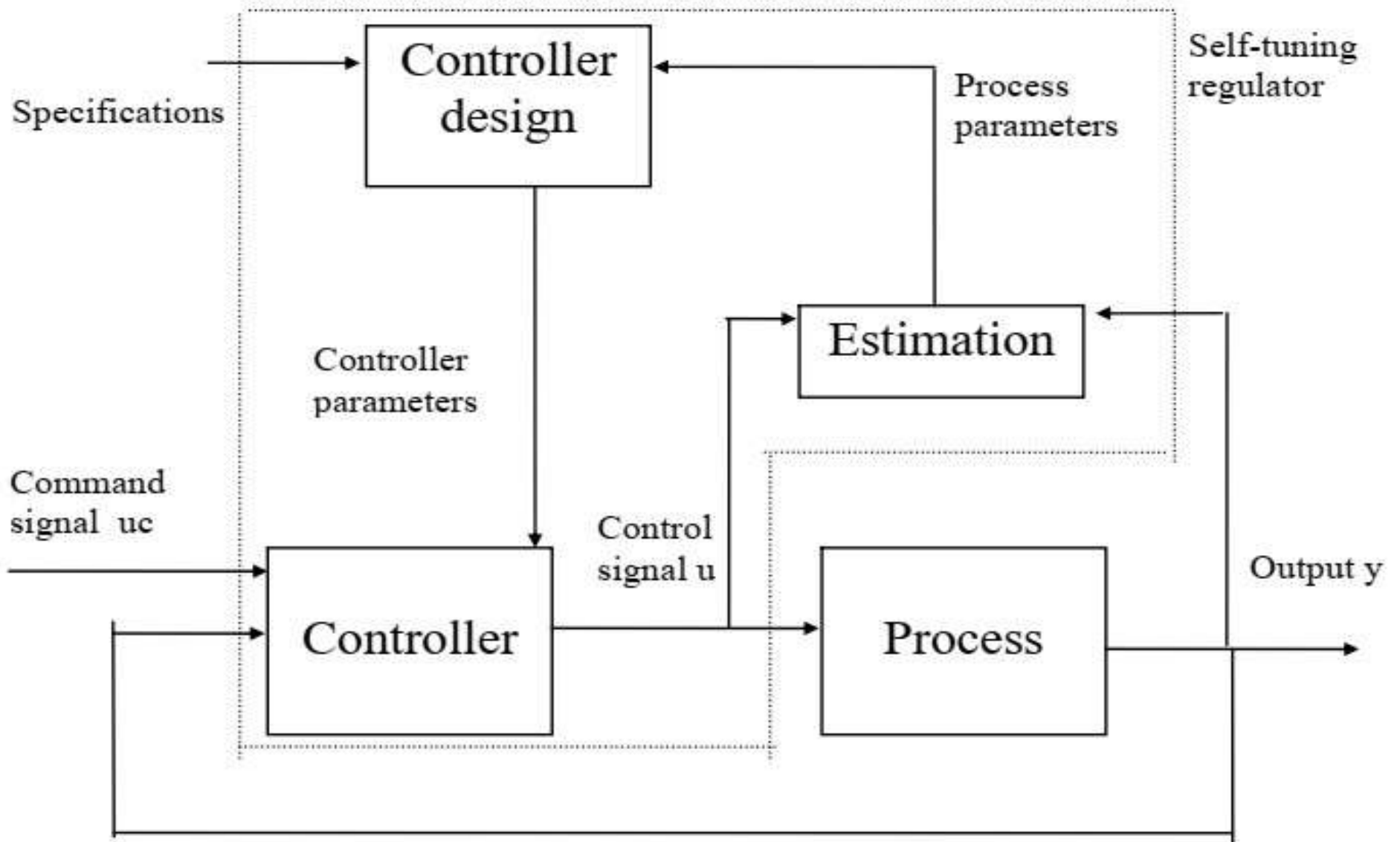


Fig. Block diagram for self-tuning controller.

## The Self Tuning Regulator is composed of two loops

- The *inner loop*, which contains the process and an ordinary feedback controller.
- The *outer loop*, which is composed by a recursive parameter estimator and design calculations. This loop adjusts the controller parameters *Indirect adaptive algorithm*: two steps: (1) estimate process model parameters; (2) update controller parameters as if estimates were correct (The Certainty Equivalence Principle).

Out of the several possible parameter estimation techniques we will use the Recursive Least Squares algorithm.

Out of several possible controller design methods we will study LQ tracking optimal control, using state space models.

## LQ tracking optimal control design

System model:

$$x(k+1) = Ax(k) + Bu(k)$$

$$y(k) = Cx(k)$$

where  $x$  is the state vector,  $u$  is the input vector,  $k$  is an integer discrete time index,  $A$ ,  $B$  and  $C$  are matrices.

Performance Index:

$$J = \frac{1}{2} \sum_{k=0}^{\infty} ((y(k) - u_c)^T Q (y(k) - u_c) + u(k)^T R u(k))$$

where  $J$  is a scalar performance index,  $Q$  is a positive semi-definite matrix,  $R$  is a positive definite matrix, and the command signal  $u_c$  is assumed to be constant.

The optimal control sequence that minimises  $J$  is:

$$u(k) = -K_f x(k) + K_r u_c$$

where the feedback gain matrix  $K_f$  is given by

$$K_f = (B^T S B + R)^{-1} B^T S A$$

where  $S$  is the solution to the algebraic Riccati equation

$$S = A^T [S - S B (B^T S B + R)^{-1} B^T S] A + Q$$

The feed-forward gain matrix  $K_r$  is given by:

$$K_r = (B^T S B + R)^{-1} B^T [I - (A - B K_f)^T]^{-1} C^T Q$$

## Questions:

- When is there a bounded solution  $S$  to the algebraic Riccati Equation?
- When is the closed loop plant asymptotically stable?

*Answer:*  $(A,B)$  should be stabilizable under linear state feedback and  $(A, \sqrt{Q})$  should be observable.

The ARE can be solved by different methods, such as the iteration method, the eigenvalue method, etc.

In Matlab, the following command can be used to obtain  $K_f$  and  $S$

$$[K_f, S] = dlqry(A, B, C, D, Q, R)$$

## ***Finding the state space model***

We intend to use the Recursive Least Squares algorithm for our self tuning controller.

Assume, for simplicity, that the system is single-input single-output

Assume that the following autoregressive exogenous (ARX) model structure is used by the Recursive Least Squares algorithm:

$$A^*(q^{-1})y(k) = B^*(q^{-1})u(k) + \varepsilon(k)$$

where

$$A^*(q^{-1}) = 1 + a_1q^{-1} + \dots + a_{n_a}q^{-n_a}$$

$$B^*(q^{-1}) = b_1q^{-1} + b_2q^{-2} + \dots + b_{n_b}q^{-n_b}$$

An equivalent non-minimal state space realization of the above ARX model is given below.

$$x(k+1) = Ax(k) + Bu(k)$$

$$y(k) = Cx(k) + \varepsilon(k)$$

where the state vector is defined as:

$$x(k) = [y(k) \ y(k-1) \ \dots \ y(k-n_a+1) \quad u(k-1) \ \dots \ u(k-n_b+1)]^T$$

$$\dim x = n_a + n_b - 1$$

Note that the state vector at time  $k$  is simply formed using past values of the input and output variables. No state observer is required. In contrast, if minimal state space realizations are used, then a state *observer* is usually required.

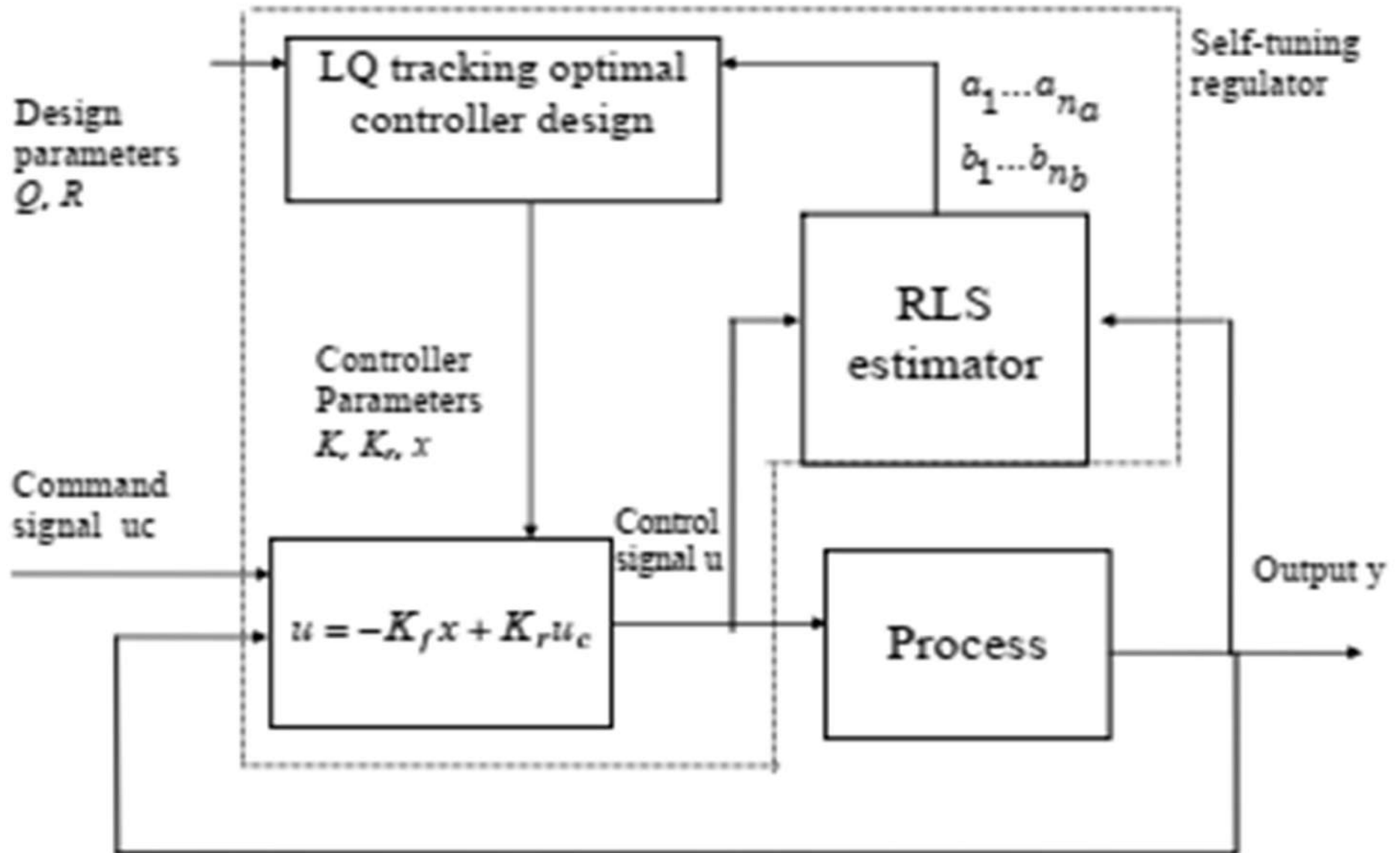


$$A = \begin{bmatrix} -a_1 & -a_2 & \dots & -a_{n_a} & b_1 & \dots & b_{n_b} \\ 1 & 0 & \dots & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 & 0 & \dots & 1 & 0 \end{bmatrix}_{n \times n}$$

$$B = \begin{bmatrix} b_1 \\ 0 \\ \vdots \\ 0 \\ 1 \\ \vdots \\ 0 \end{bmatrix}_{n \times 1}$$

$$C = [1 \quad 0 \quad \dots \quad 0 \quad 0 \quad \dots \quad 0]_{1 \times n}$$

# The self tuning controller



Recall the RLS algorithm:

$$K(k) = P(k-1)\varphi(k) \left[ I + \varphi(k)^T P(k-1)\varphi(k) \right]^{-1}$$

$$\varepsilon(k) = y(k) - \varphi(k)^T \hat{\theta}(k-1)$$

$$\hat{\theta}(k) = \hat{\theta}(k-1) + K(k)\varepsilon(k)$$

$$P(k) = \left[ I - K(k)\varphi(k)^T \right]^{-1} P(k-1)$$

Note: this type of controller may be thought of as an *infinite horizon adaptive predictive controller*. This type of controller, with minor modifications, is well suited for multivariable systems.

**Example 1:** Consider the plant:

$$G(s) = \frac{0.5}{s^2 + s + 1}$$

The z- transform equivalent of this plant under zero order hold is:

$$G(z) = \frac{0.00935z + 0.00875}{z^2 - 1.7826z + 0.8187}$$

The plant is to be controlled by a self-tuning LQ controller, using the least squares parameter estimation algorithm.

The RLS model has the following ARX structure:

$$y(k) = -a_1 y(k-1) - a_2 y(k-2) + b_1 u(k-1) + b_2 u(k-2) + \varepsilon(k)$$

The parameter and regression vectors are

$$\theta = [a_1 \ a_2 \ b_1 \ b_2]^T \quad \varphi(k) = [-y(k-1) \ -y(k-2) \ u(k-1) \ u(k-2)]^T$$

The forgetting factor used was 0.98, the sampling time was 0.2 s, the initial parameter vector was  $\theta = [0.1 \ 0.1 \ 0.1 \ 0.1]^T$  and the initial covariance matrix was  $P = 100 \times I_4$ .

The non-minimal state space realization used was given by:

$$x(k) = [y(k) \ y(k-1) \ u(k-1)]^T$$

*Controller:*

The control law was:

$$u(k) = -K_f x(k) + K_r u_c$$

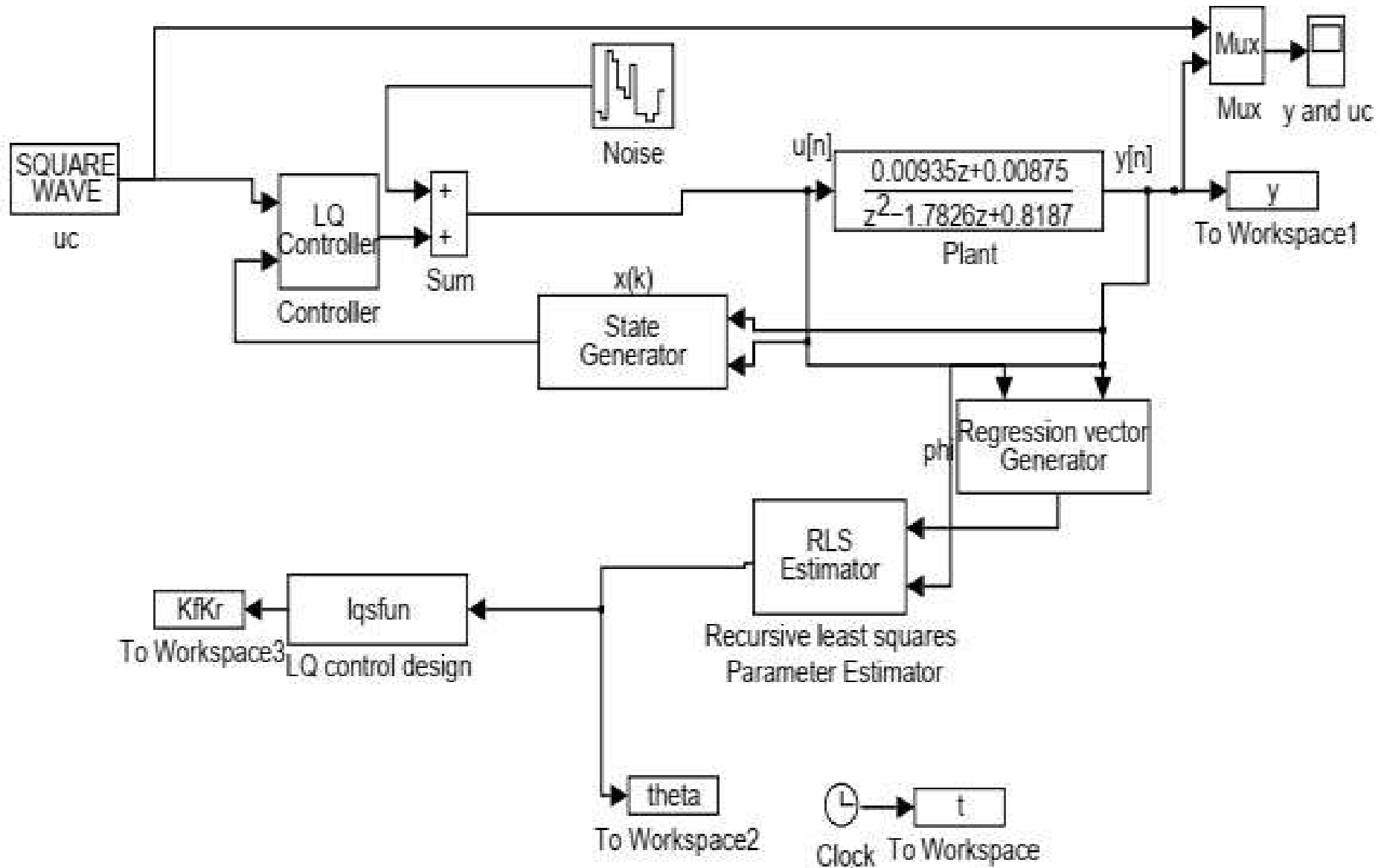
The following tuning parameters were used:

$$Q = 1.0, \quad R = 0.03$$

The command signal  $u_c$  was a square wave with amplitude  $[+1, -1]$  and period 50 s.

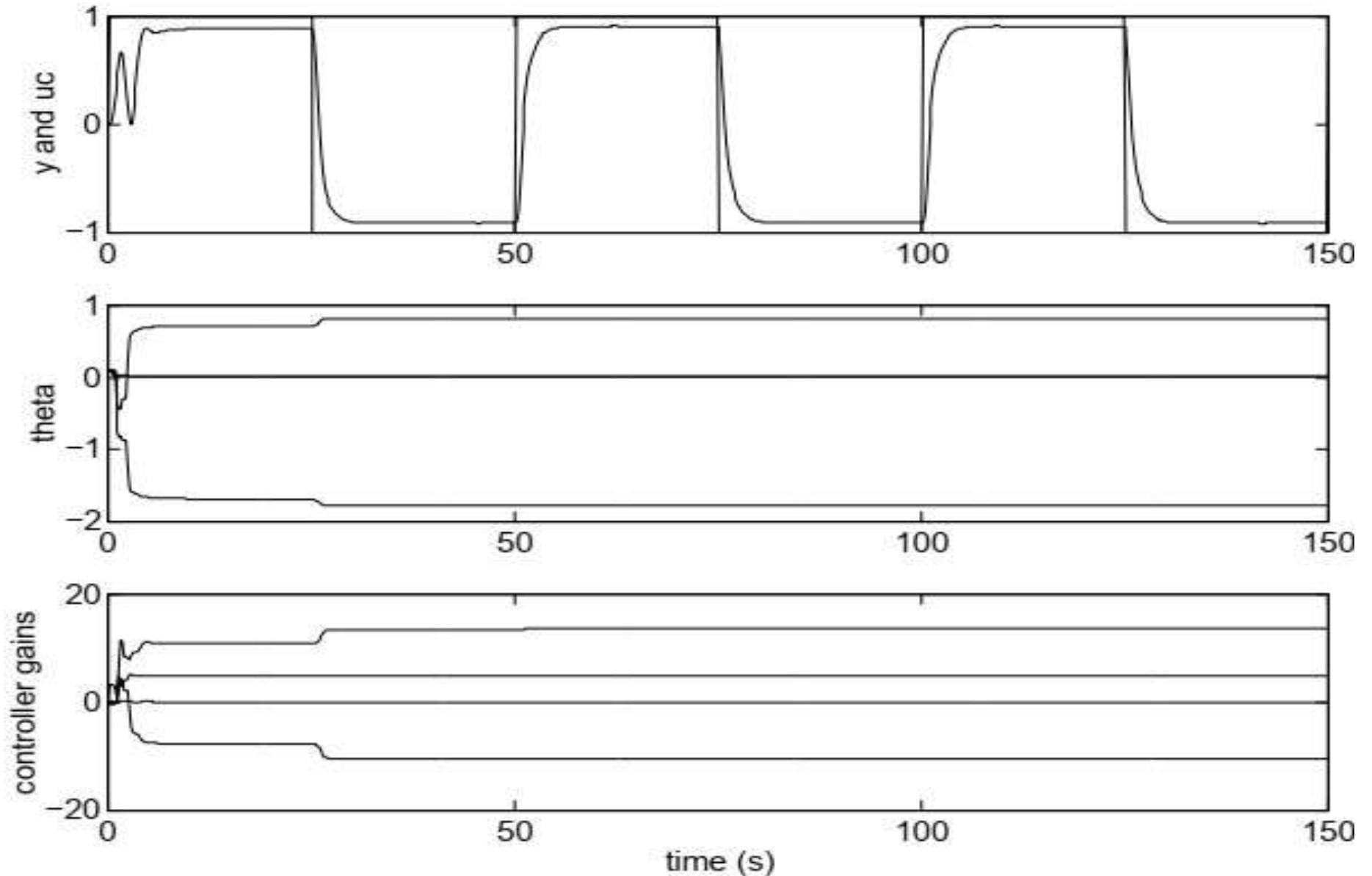
A small random signal was added to the controller output in order to enhance input excitation, which is important for identification.

# Simulink diagram, Example 1



## Simulation Results, Example 1

Notice that the controller does not have integral action, since the steady state error is non-zero.





## Integral Action

There are several ways of introducing integral action to a controller, and so eliminate steady state error. A simple way of doing it is to add a term proportional to the accumulated error to the controller output, so that the new controller output is given by:

$$u(k) = -K_f x(k) + K_r u(k) + K_i d(k)$$

where  $K_f$  and  $K_r$  are obtained using the LQ design procedure,  $K_i$  is a constant chosen by the designer and  $d(k)$  is given by the recursive formula:

$$d(k) = d(k-1) + (u_c(k) - y(k))$$

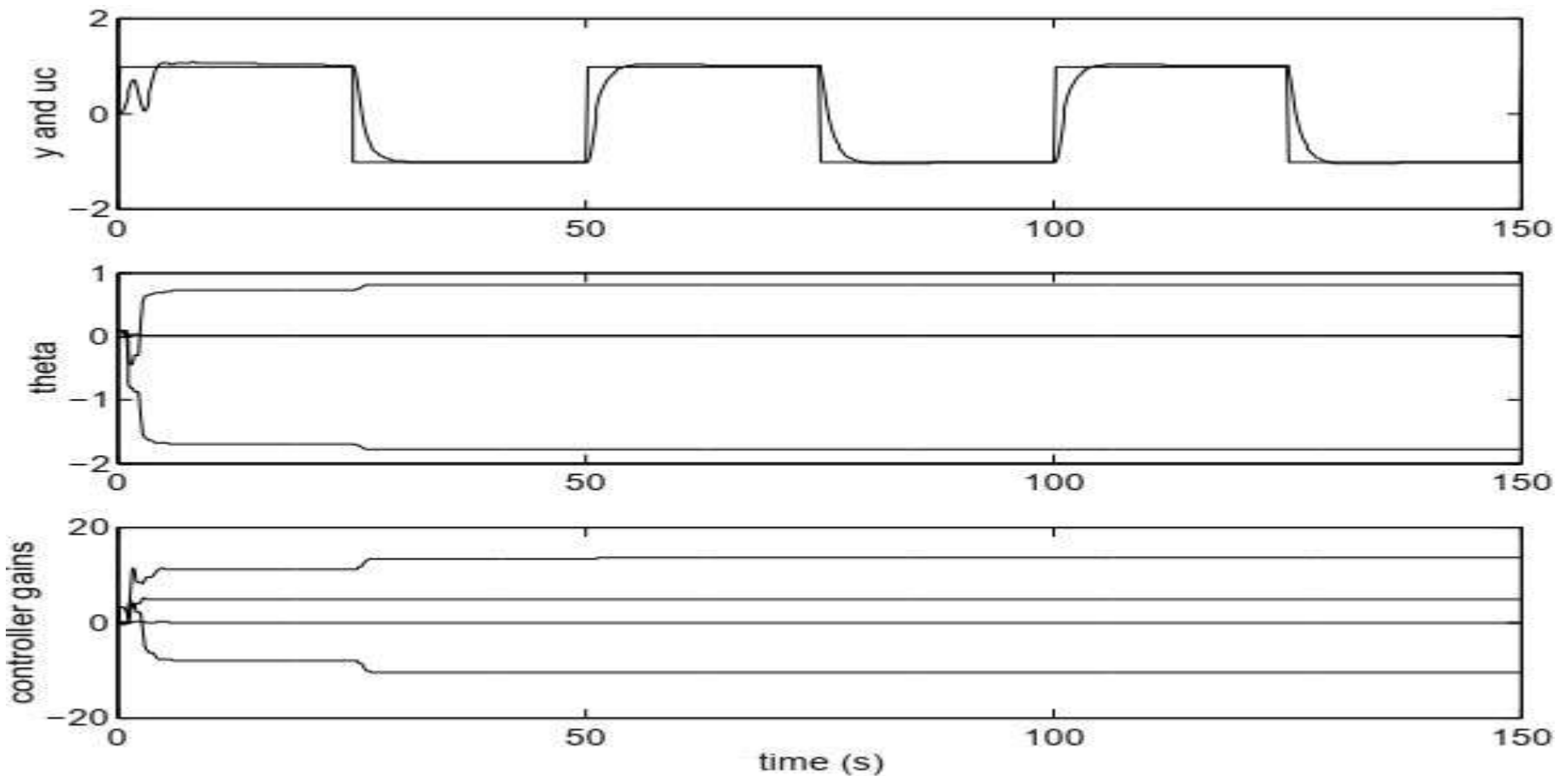
Care must be taken to stop updating  $d(k)$  when the controller or associated actuator saturate at their higher or lower limits. Otherwise an undesirable phenomenon called *integrator windup* may occur. Note that it is also possible to compute  $K_i$  optimally by using a model of the system with an extra state, where the extra state is  $d(k)$ .

**Example 2:** This example is identical to Example 1, except that the control law has been modified to:

$$u(k) = -K_f x(k) + K_r u(k) + K_i d(k)$$

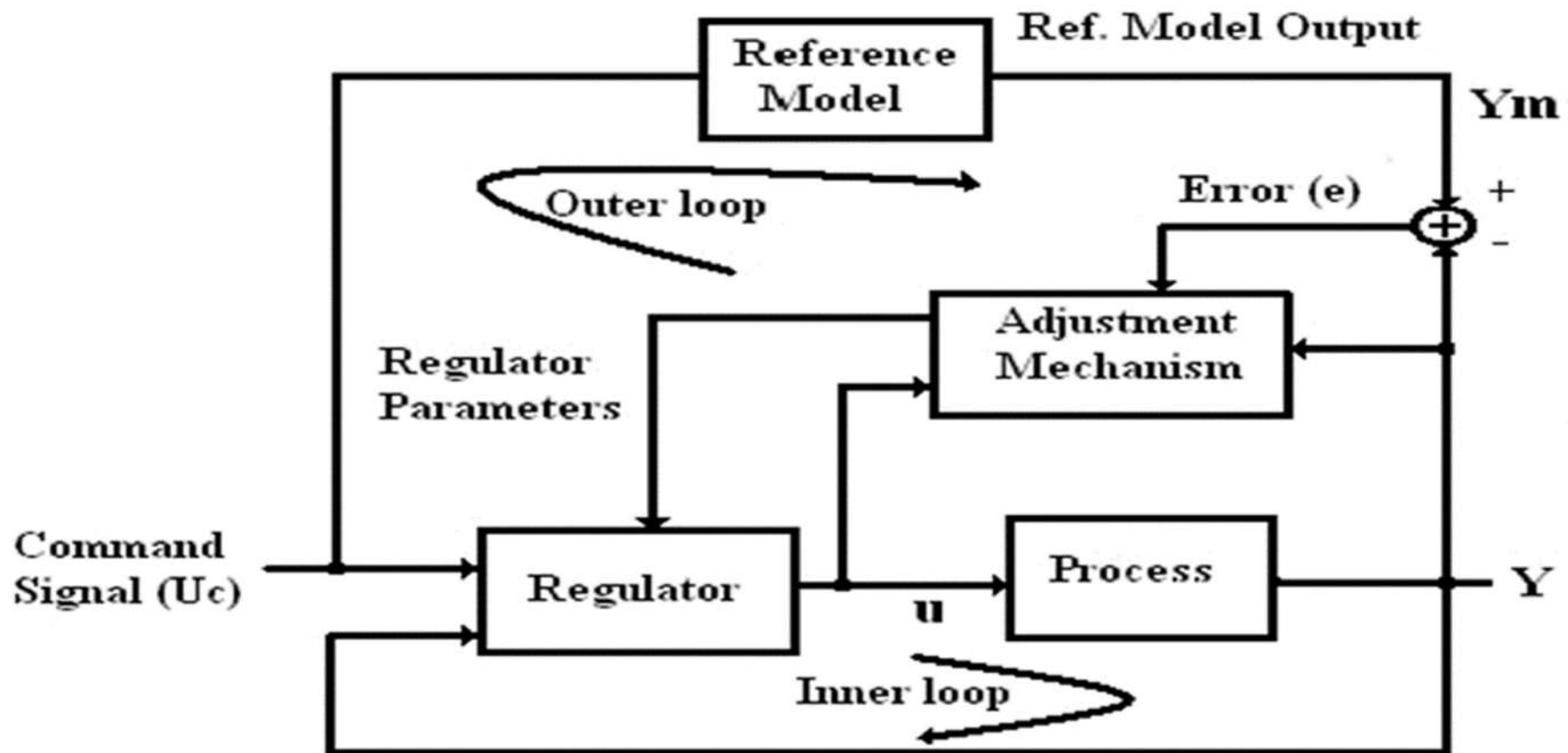
$$d(k) = d(k-1) + (u_c(k) - y(k))$$

where  $K_i = 0.1$ . This introduces integral action into the controller.



## Model Reference Adaptive Controller (MRAC)

- The model-Reference Adaptive Controller is one of the main approaches of adaptive control. The basic principle is illustrated in figure below:

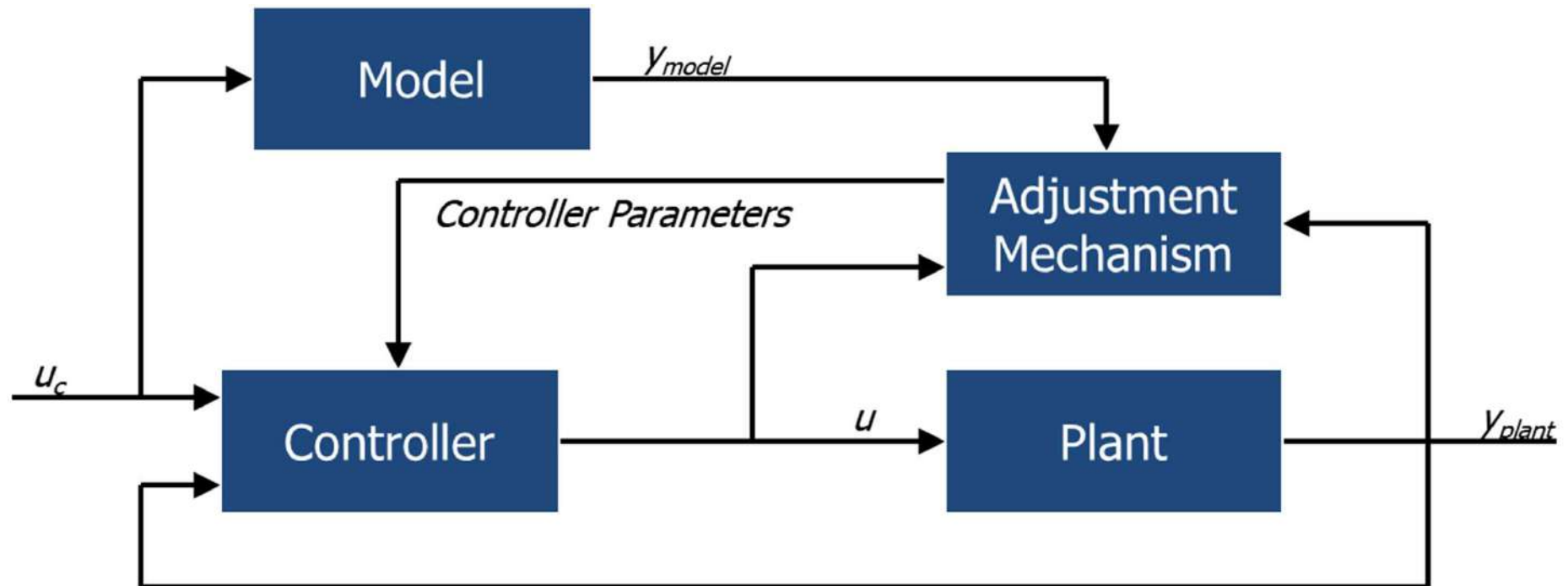


- The desired performance is expressed in terms of a reference model, which gives the desired response to a command signal.
- The system also has an ordinary feedback loop composed of the process and the regulator.
- The error ( $e$ ) is the difference between the output(s) of the system and the reference model. The regulator has parameters that are changed, based on the error.
- There are thus two loops: an inner loop, which provides the ordinary control feedback and outer loop, which adjusts the parameters in the inner loop.
- The inner loop is assumed faster than the outer loop.

- For a system with adjustable parameters, the model reference adaptive methods gives a general approach for adjusting the parameters so that the closed loop transfer function will be close to a prescribed model.
- This is called the Model-following problem.
- One important question is how small we can make the error ( $e$ ). This depends both on the model, the system and the command signal. If it is possible to make the error equal to zero for all command signals, the perfect model following is achieved.
- The MIT rule is a scalar parameter adjustment law which was proposed in 1961 for the model reference adaptive control of linear systems modeled as the cascade of a known stable plant and a single unknown gain

## MODEL REFERENCE ADAPTIVE CONTROL

- Design controller to drive plant response to model ideal response (error =  $y_{\text{plant}} - y_{\text{model}} \Rightarrow 0$ )
- Designer chooses: reference model, controller structure, and tuning gains for adjustment mechanism.
- Basic methods used to design adjustment mechanism are 1.MIT Rule 2.Lyapunov rule



Adaptation of feed forward gain

- Tracking error:  $e = y_{plant} - y_{model}$

- Form cost function:  $J(\theta) = \frac{1}{2} e^2(\theta)$

- Update rule:  $\frac{d\theta}{dt} = -\gamma \frac{\delta J}{\delta \theta} = -\gamma e \frac{\delta e}{\delta \theta}$

– Change in  $\theta$  is proportional to negative gradient of  $J$

# MIT Rule

- For system  $\frac{Y(s)}{U(s)} = kG(s)$  where  $k$  is unknown.

- Goal: Make it look like  $\frac{Y(s)}{U_c(s)} = k_o G(s)$

using reference model  $G_m(s) = k_o G(s)$



- Choose cost function:

$$J(\theta) = \frac{1}{2} e^2(\theta) \longrightarrow \frac{d\theta}{dt} = -\gamma e \frac{\delta e}{\delta \theta}$$

- Write equation for error:

$$e = y - y_m = kGU - G_m U_c = kG\theta U_c - k_o G U_c$$

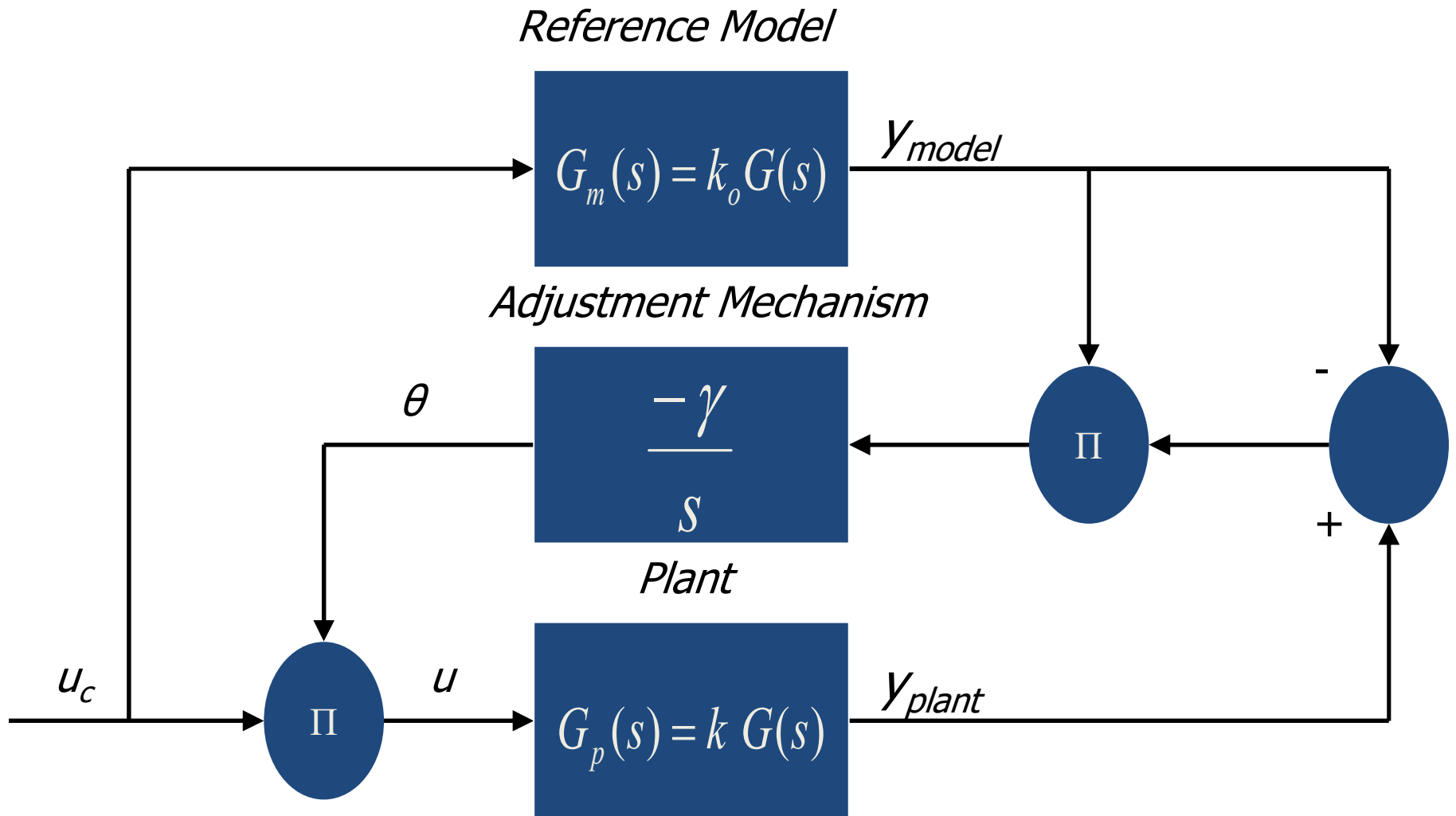
- Calculate sensitivity derivative:

$$\frac{\delta e}{\delta \theta} = kGU_c = \frac{k}{k_o} y_m$$

- Apply MIT rule:

$$\frac{d\theta}{dt} = -\gamma' \frac{k}{k_o} y_m e = -\gamma y_m e$$

- Adaptation feedforward gain



# MRAC Example

Process Model :  $\frac{dy}{dt} = -ay + ku_c$

Reference Model :  $\frac{dy_m}{dt} = -ay_m + k_m u$

Controller Equation :  $u_c = \theta u$

Equation of Error :  $e = y - y_m$

Desired Equilibrium :  $e = 0$

Derivative of Error :  $\frac{de}{dt} = -ae + (k\theta - k_m)u$

## APPLYING MIT RULE

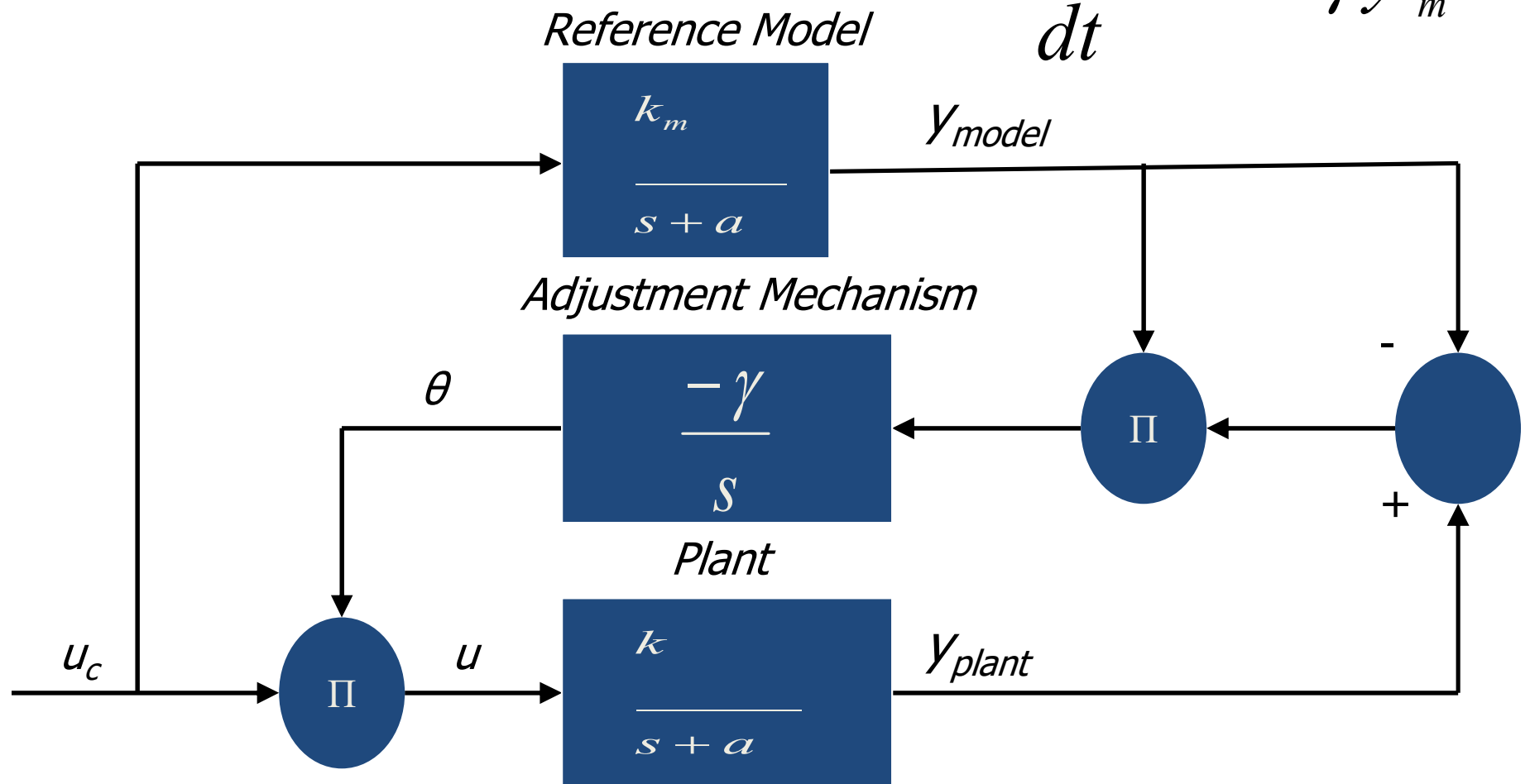
- Cost function :  $J(\theta) = \frac{1}{2} e^2(\theta)$

- Error equation :  $e = y_{plant} - y_{model}$

- Updating rule :  $\frac{d\theta}{dt} = -\gamma \frac{\delta J}{\delta \theta} = -\gamma e \frac{\delta e}{\delta \theta}$

# CONSTRUCTION OF BLOCK DIAGRAM

$$\frac{d\theta}{dt} = -\gamma y_m e$$



# MIT Rule to Lyapunov Transition

1. Several Problems were encountered in the usage of the MIT rule.
2. Also, it was not possible in general to prove closed loop stability, or convergence of the output error to zero.
3. A new way of redesigning adaptive systems using Lyapunov theory was proposed by Parks.
4. This was based on Lyapunov stability theorems, so that stable and provably convergent model reference schemes were obtained.
5. The update laws are similar to that of the MIT Rule, with the sensitivity functions replaced by other functions.
6. The theme was to generate parameter adjustment rule which guarantee stability

# LYAPUNOV STABILITY

- Lyapunov's method states that a system has a uniform asymptotically stable equilibrium  $x=0$ .if a Lyapunov function  $V(x)$ exists that satisfies:
  - $V(x) > 0$  for  $X \neq 0$  (positive definite)
  - $\dot{V}(X) < 0$  for  $x \neq 0$  (negative definite)
  - $V(\infty) \rightarrow \infty$  for  $x \rightarrow \infty$
  - $V(0) = 0$ .

## Adaption of feed forward gain using Lyapunov function

**Process Model :** 
$$\frac{dy}{dt} = -ay + ku_c$$

**Reference Model :** 
$$\frac{dy_m}{dt} = -ay_m + k_m u$$

**Controller Equation :** 
$$u_c = \theta u$$

**Equation of Error :** 
$$e = y - y_m$$

**Desired Equilibrium :** 
$$e = 0$$

**Derivative of Error :** 
$$\frac{de}{dt} = -ae + (k\theta - k_m)u$$



- Lyapunov Function:  $V = \frac{\gamma}{2} e^2 + \frac{k}{2} \left( \theta - \frac{k_m}{k} \right)^2$

- Derivative of Lyapunov Function:

$$\begin{aligned} \frac{dV}{dt} &= \gamma e(-ae + k(\theta - \frac{k_m}{k})u) + k(\theta - \frac{k_m}{k}) \frac{d\theta}{dt} \\ &= -\gamma a e^2 + k(\theta - \frac{k_m}{k}) \left( \frac{d\theta}{dt} + \gamma e u \right) \end{aligned}$$

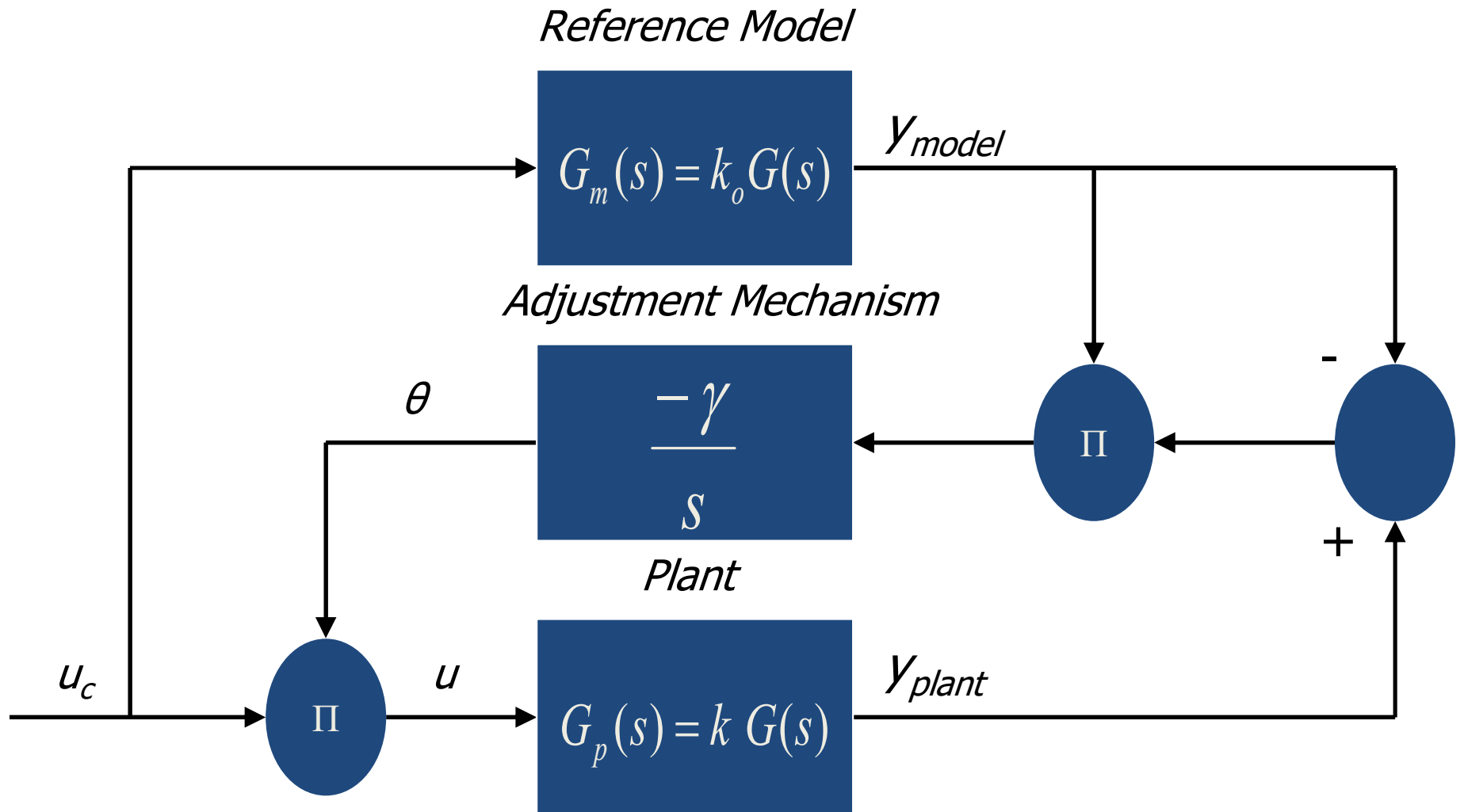
- Choosing the Adjustment Rule:

$$\frac{d\theta}{dt} = -\gamma e \Rightarrow \frac{dV}{dt} = -\gamma a e^2$$

- Adaptation Law according Lyapunov Methods:

$$\frac{d\theta}{dt} = -\gamma e$$

# Adaptive Feed Forward gain Block Diagram



# MRAC for first order closed loop system

- Process Model:  $\frac{dy}{dt} = -ay + bu_c$
- Reference Model:  $\frac{dy_m}{dt} = -ay_m + b_m u$
- Controller Structure:  $u_c = \theta_1 u - \theta_2 y$
- Error Equation:  $e = y - y_m$
- Desired Equilibrium:  $e = 0$
- Derivative of Error:  $\frac{de}{dt} = -a_m e - (b\theta_2 + a - a_m y) + (b\theta_1 - b_m) u$

- Candidate for Lyapunov Function:

$$V(e, \theta_1, \theta_2) = \frac{1}{2} (e^2 + \frac{1}{b\gamma} (b\theta_2 + a - a_m)^2 + \frac{1}{b\gamma} (b\theta_1 - b_m)^2)$$

- Derivative of Lyapunov Function:

$$\begin{aligned} \frac{dV}{dt} &= e \frac{de}{dt} + \frac{1}{\gamma} (b\theta_2 + a - a_m) \frac{d\theta_2}{dt} + \frac{1}{\gamma} (b\theta_1 - b_m) \frac{d\theta_1}{dt} \\ &= -a_m e^2 + \frac{1}{\gamma} (b\theta_2 + a - a_m) \left( \frac{d\theta_2}{dt} - \gamma y e \right) + \frac{1}{\gamma} (b\theta_1 - b_m) \left( \frac{d\theta_1}{dt} + \gamma y e \right) \end{aligned}$$

- Adaptation Law:  $\frac{d\theta_1}{dt} = -\gamma y e$   $\frac{d\theta_2}{dt} = \gamma y e$

# ADVANTAGES OF USING LYAPUNOV FUNCTION

- The Analysis of system equations is difficult in M.I.T rule. where as in Lyapunov method is easy.
- M.I.T rule does not guarantee error convergence or stability
- Lyapunov Adaptive laws gives guaranteed stability.(i.e. error  $(e)=0$ ).

## **Adaptive Control: Non-Identifier-Based**

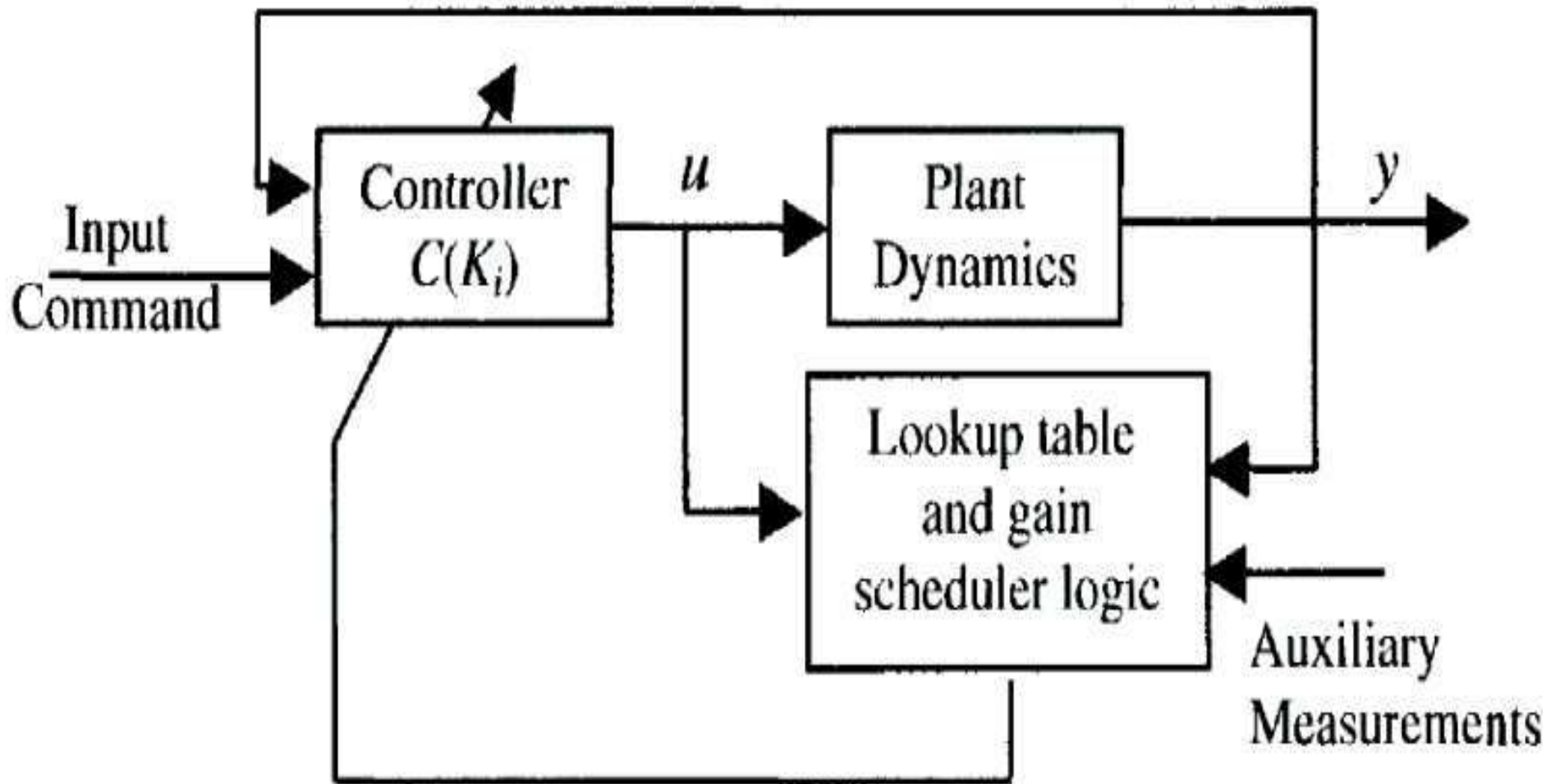
Another class of schemes that do not involve online parameter estimators is referred to as non-identifier-based adaptive control schemes. In this class of schemes, the online parameter estimator is replaced with search methods for finding the controller parameters in the space of possible parameters, or it involves switching between different fixed controllers, assuming that at least one is stabilizing or uses multiple fixed models for the plant covering all possible parametric uncertainties or consists of a combination of these methods.

We briefly describe the main features, advantages, and limitations of these non-identifier-based adaptive control schemes. Some of these approaches are relatively recent and research is still going on.

## **1. Programmed or Gain Scheduled** Adaptive Control

The Programmed Adaptive Control is compared to feed forward compensation because it adjust the controller parameters based on the measurement of an auxiliary variable and the knowledge of operating conditions of process. Also, there is no feedback to check the correctness of adaptation.

The gain scheduler consists of a lookup table and the appropriate logic for detecting the operating point and choosing the corresponding value of control gains from the lookup table. With this approach, plant parameter variations can be compensated by changing the controller gains as functions of the input, output, and auxiliary measurements. The advantage of gain scheduling is that the controller gains can be changed as quickly as the auxiliary measurements respond to parameter changes. Frequent and rapid changes of the controller gains, however, may lead to instability; therefore, there is a limit to how often and how fast the controller gains can be changed.



*Gain scheduling structure.*



One of the disadvantages of gain scheduling is that the adjustment mechanism of the controller gains is precomputed offline and provides no feedback to compensate for incorrect schedules. A careful design of the controllers at each operating point to meet certain robustness and performance measures can accommodate some uncertainties in the values of the plant parameters. However large unpredictable changes in the plant parameters, may lead to deterioration of performance or even to complete failure.

Despite its limitations, gain scheduling is a popular method for handling parameter variations in flight control and other systems. While gain scheduling falls into the generic definition of adaptive control, we do not classify it as adaptive control due to the lack of online parameter estimation which could track unpredictable changes in the plant parameters.

## Gain Scheduling

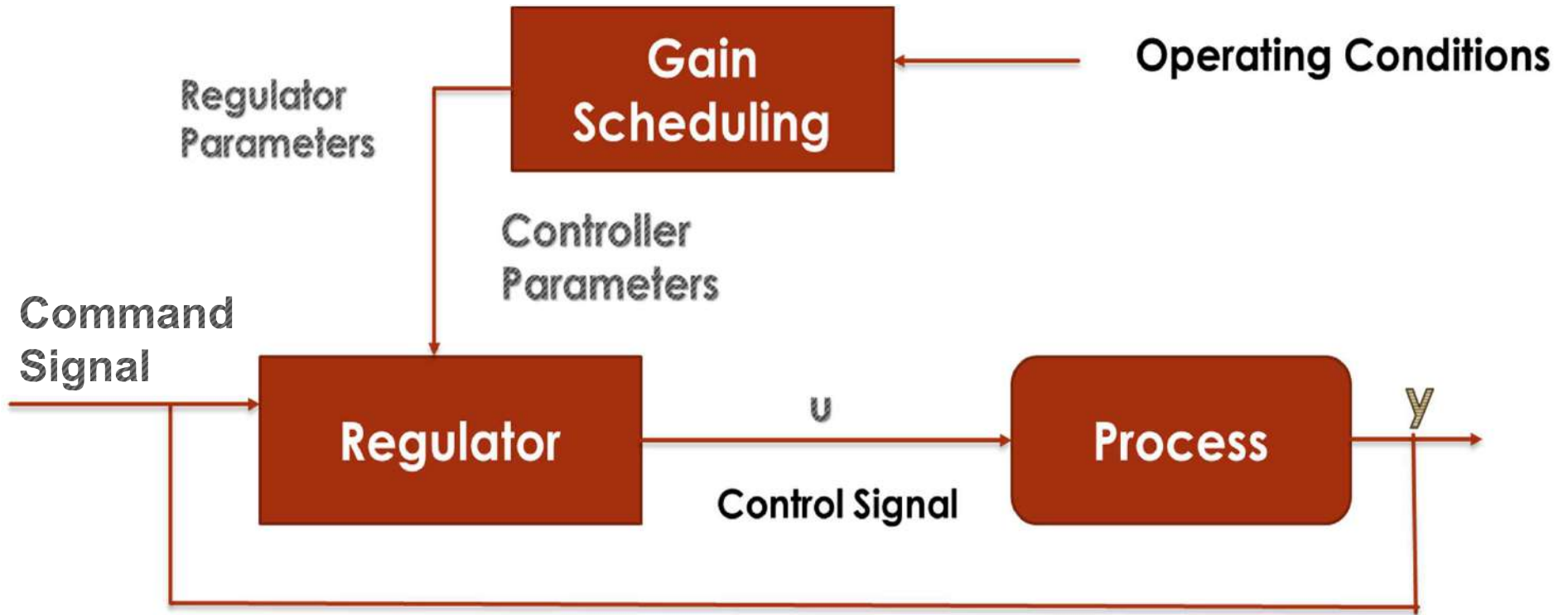
In many situations it is known how the dynamics of a process change with the operating conditions of the process. One source for the change in dynamics may be nonlinearities that are known.

It is then possible to change the parameters of the controller by monitoring the operating conditions of the process. This idea is called gain scheduling.

Its principle is to reduce the effects of parameter variations by changing the parameters of the regulator as function of auxiliary variables that correlate well with these changes in process dynamics. Gain scheduling was used in special cases: such as autopilots for high-performance air-craft.

# Principles

It is sometimes possible to find auxiliary variable that correlate well with the changes in process dynamics. It is then to reduce the effects of parameter variations simply by changing the parameters of the regulator as functions of auxiliary variables as shown in figure below:



## Design of Gain Scheduling regulators

It is difficult to give general rules for designing gain scheduling regulators. The key question is to determine the variables that can be used as scheduling variables. It is clear that these auxiliary signals must reflect the operating conditions of the plant. The following general ideas can be useful:

- 1) Linearization of nonlinear actuators.
- 2) Gain scheduling based on measurements of auxiliary variables.
- 3) Time scaling based on production rate.
- 4) Nonlinear transformation.

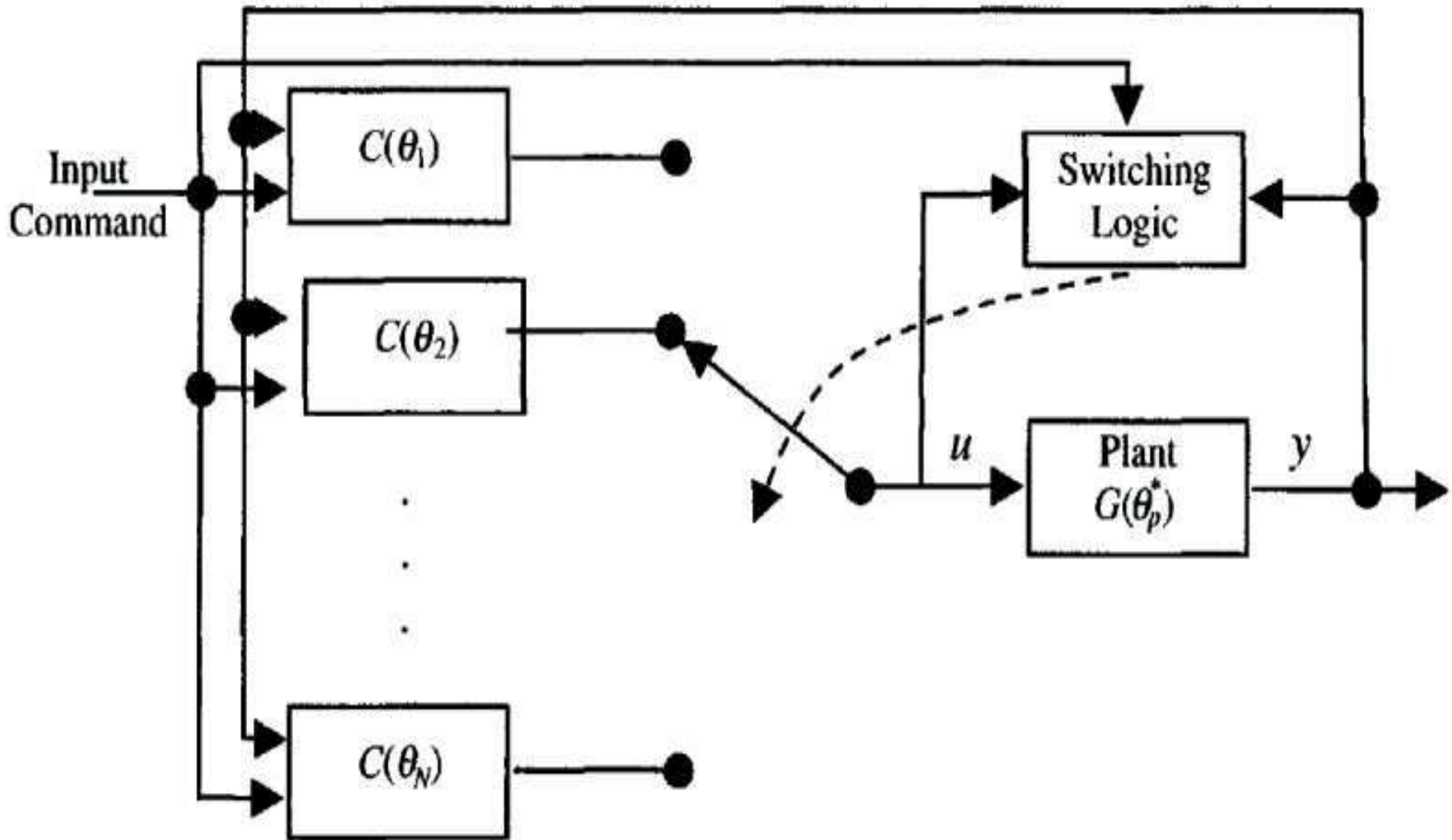
# **Adaptive Control: Non-Identifier-Based**

## **2. Search Methods**

A class of non-identifier-based adaptive control schemes emerged over the years which do not explicitly rely on online parameter estimation. These schemes are based on search methods in the controller parameter space until the stabilizing controller is found or the search method is restricted to a finite set of controllers, one of which is assumed to be stabilizing. In some approaches, after a satisfactory controller is found it can be tuned locally using online parameter estimation for better performance.

### **3. Multiple Models and Switching Schemes**

Since the plant parameters are unknown, the parameter space is parameterized with respect to a set of plant models which is used to design a finite set of controllers so that each plant model from the set can be stabilized by at least one controller from the controller set. A switching approach is then developed so that the stabilizing controller is selected online based on the I/O data measurements. Without going into specific details, the general structure of this multiple model adaptive control with switching, as it is often called, is shown in next Figure.



*Multiple models adaptive control with switching*