Lecture Notes on Fuzzy Logic Control

4th Year Petroleum Systems and Control Engineering

Tikrit University

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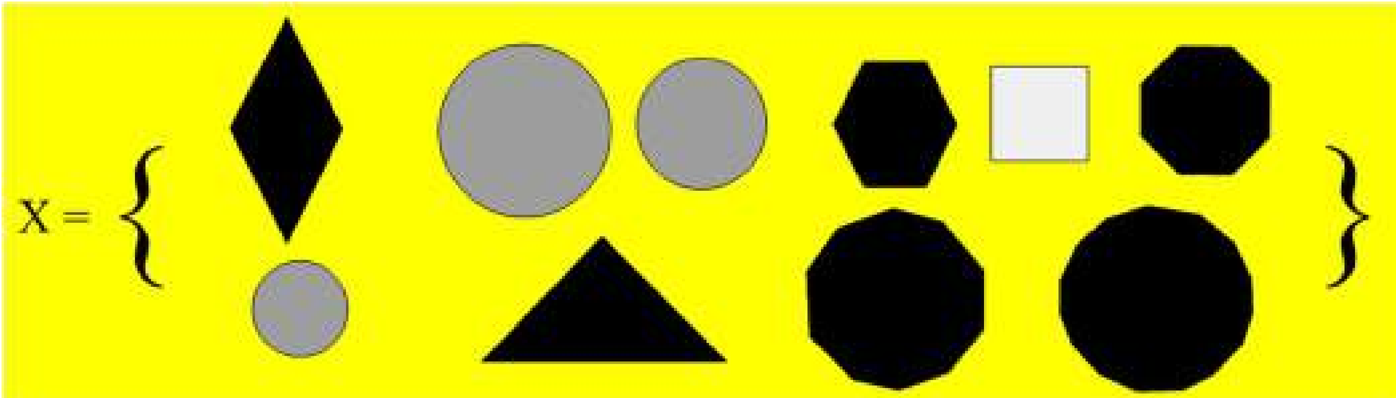
Why Fuzzy Logic

- Based on intuition and judgment
- No need for a mathematical model
- Provides a smooth transition between members and nonmembers
- Relatively simple, fast and adaptive
- Less sensitive to system fluctuations
- Can implement design objectives, difficult to express mathematically, in linguistic or descriptive rules.
- Complex, ill-defined processes difficult for description and analysis by exact mathematical techniques
- Approximate and inexact nature of the real word; vague concepts easily dealt with by humans in daily life
- Tolerance of imprecision in return for tractability, robustness,
- and short computation time

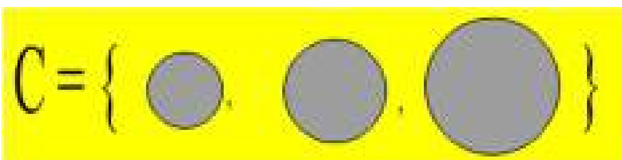
Fuzzy system applications

- Pattern recognition and classification
- Fuzzy clustering
- Image and speech processing
- Fuzzy systems for prediction
- Fuzzy control
- Monitoring
- Diagnosis
- Optimization and decision making
- Group decision making

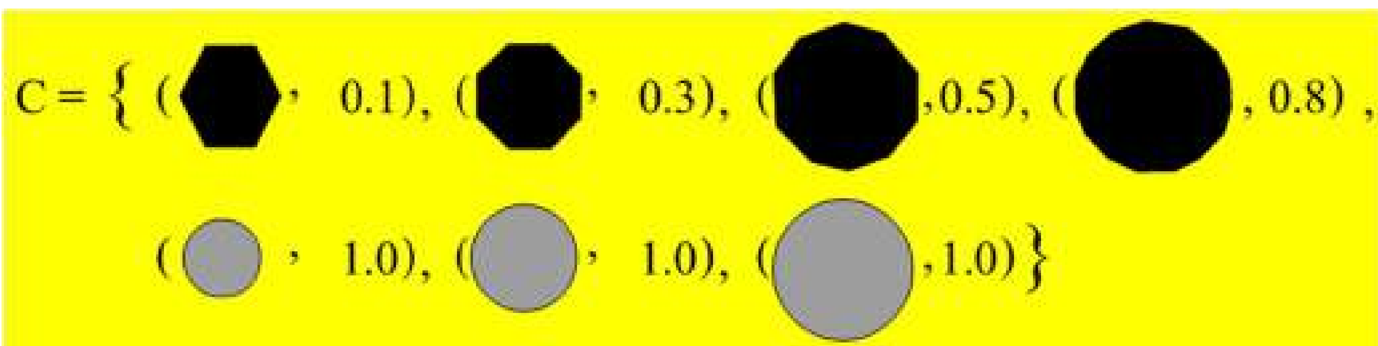
Crisp and Fuzzy example



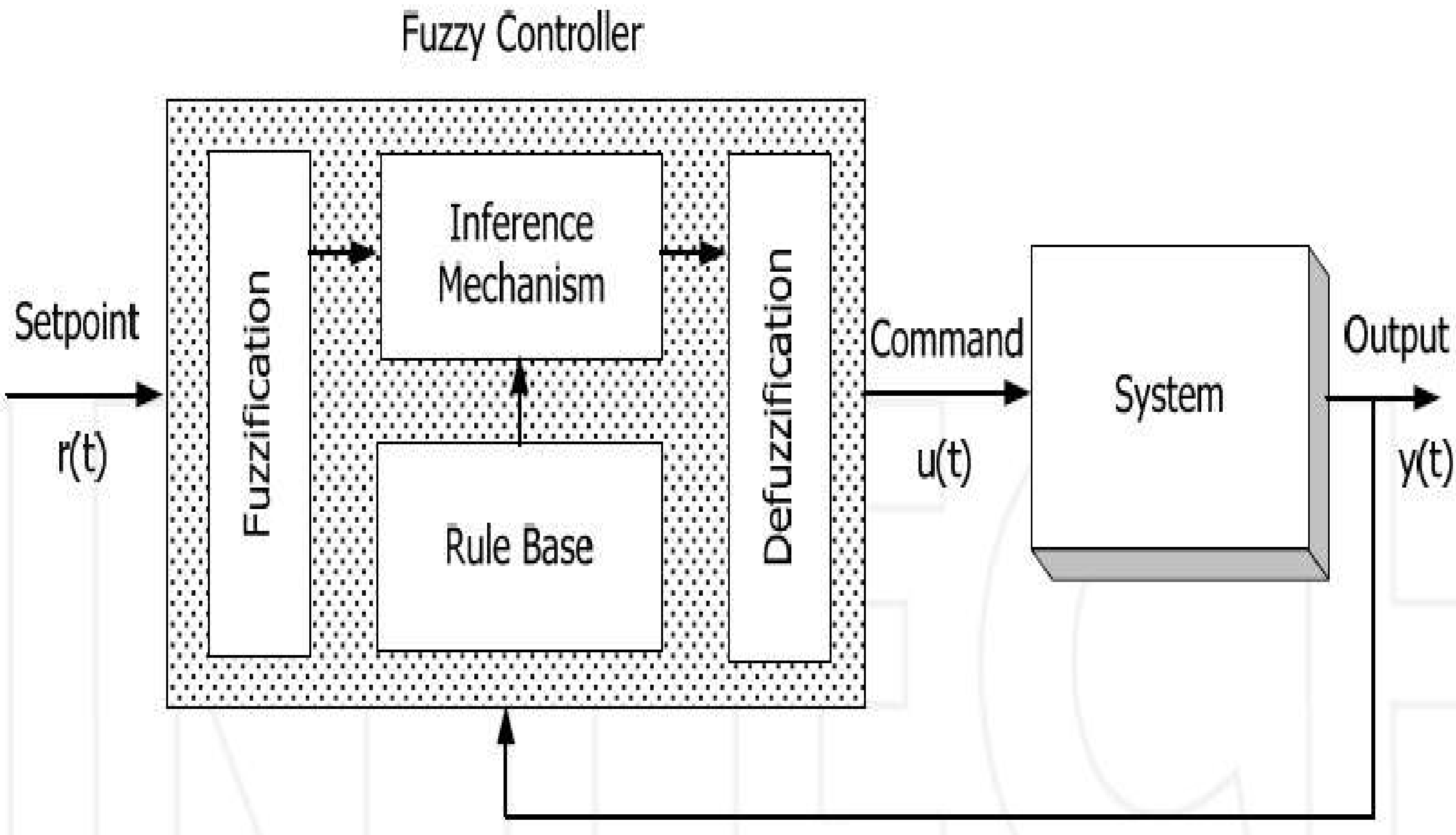
One can define the crisp set “circles” as:



The fuzzy set “circles can be defined as:

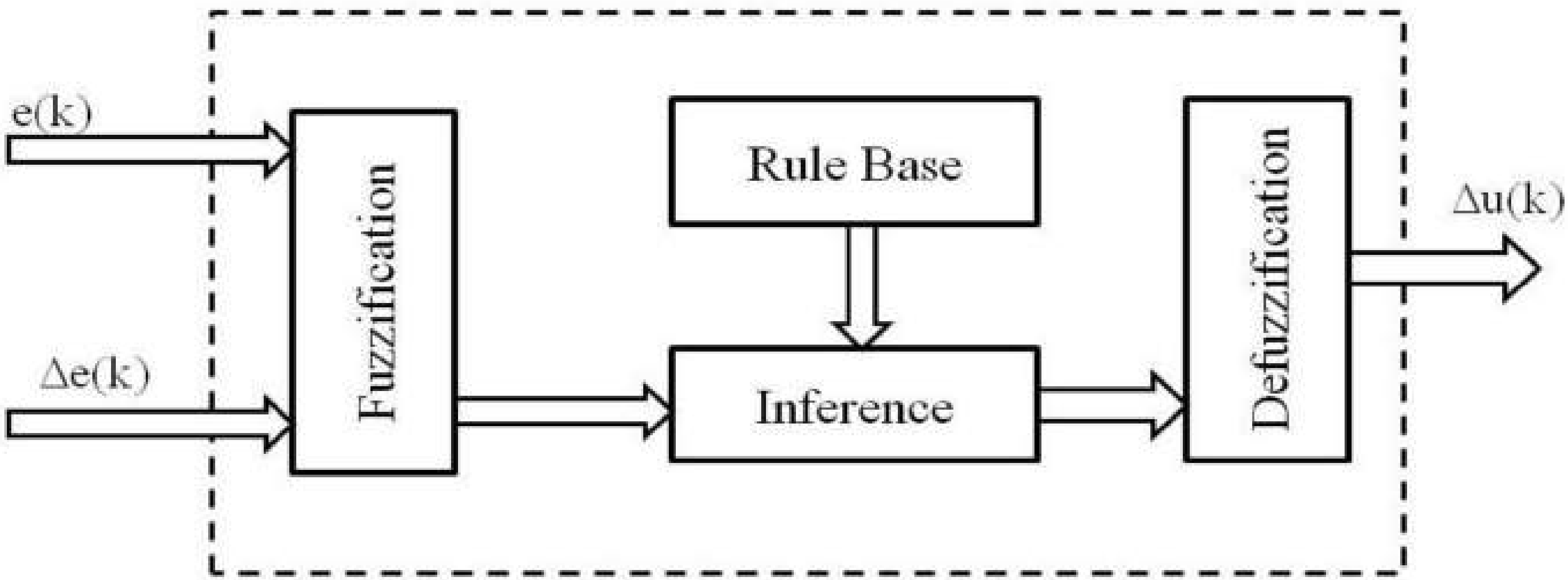


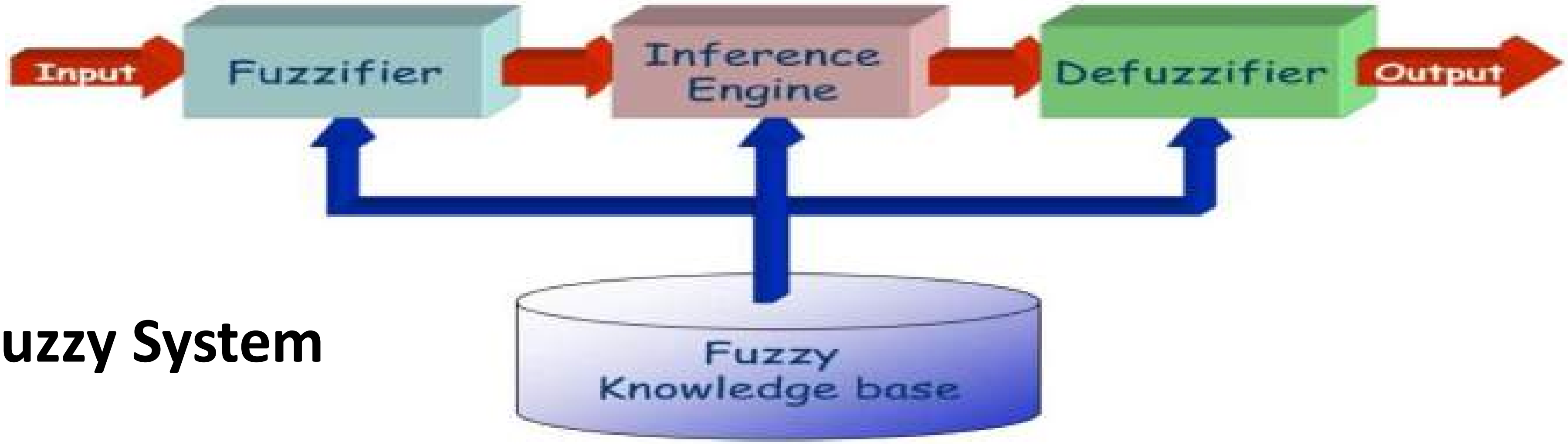
General Structure of Fuzzy Control Systems



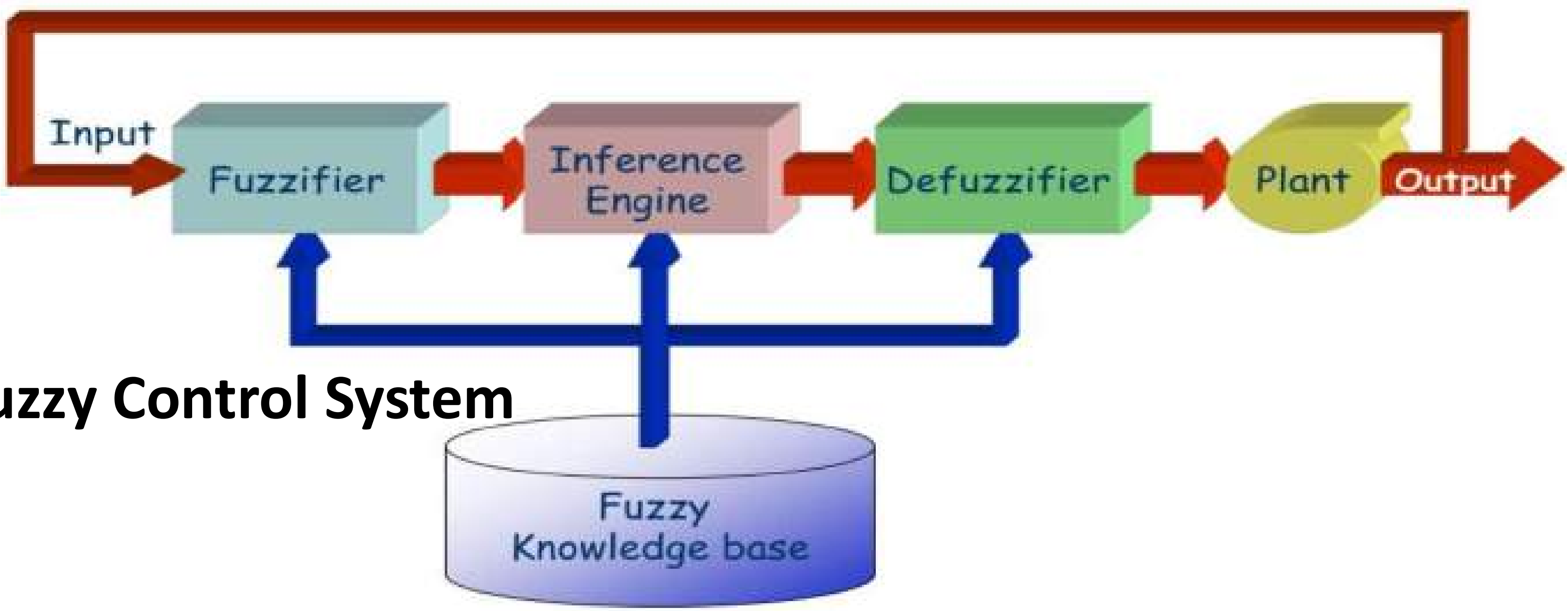
Basic Components in Fuzzy control

1. Fuzzification (Translate input into truth values).
2. Fuzzy Rule base (Compute output truth values.
4. Inference — Choice of AND, OR, NOT. Determines how to interpret (implement) IF-THEN statements
3. Defuzzification (Transfer truth values into output).





Fuzzy System



Fuzzy Control System

Classical Set Theory

A set is a collection of objects with a common property.

Examples:

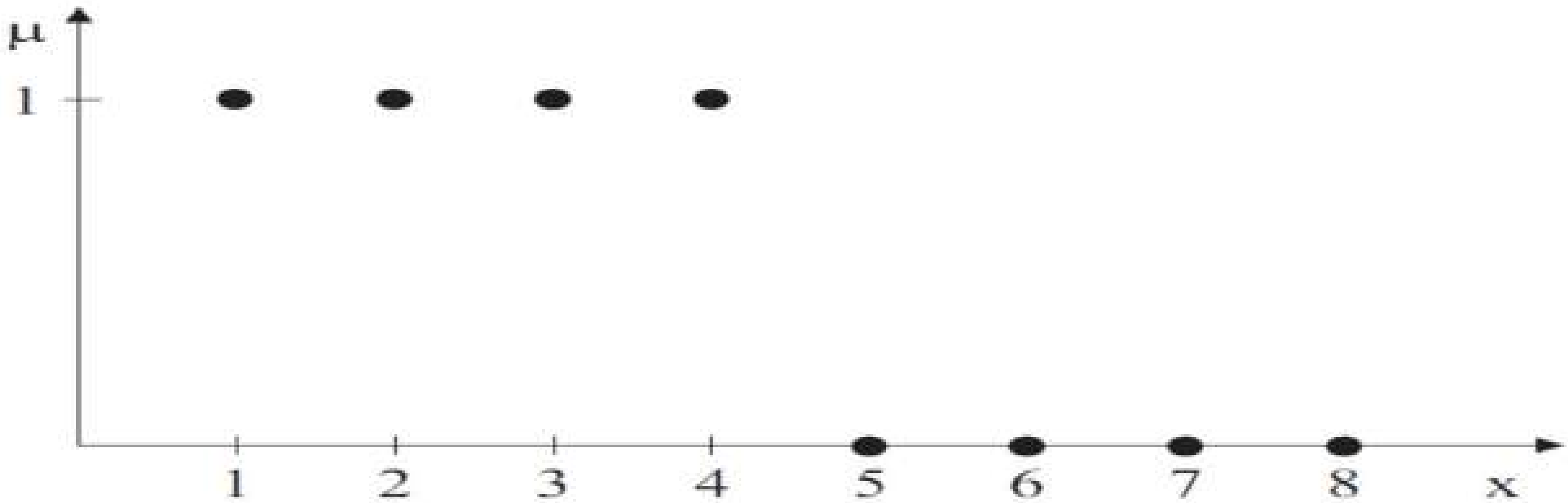
- Set of natural numbers smaller than 5: $A = \{1, 2, 3, 4\}$
- Unit disk in the complex plane: $A = \{z | z \in \mathbb{C}, |z| \leq 1\}$
- A line in $\mathbb{R}^2$: $A = \{(x, y) | ax + by + c = 0, (x, y, a, b, c) \in \mathbb{R}\}$

Representation of Sets

- Enumeration of elements: $A = \{x_1, x_2, \ldots, x_n\}$
- Definition by property: $A = \{x \in X | x \text{ has property } P\}$
- Characteristic function: $\mu_A(x) : X \to \{0, 1\}$

$$\mu_A(x) = \begin{cases} 1 & x \text{ is member of } A \\ 0 & x \text{ is not member of } A \end{cases}$$

Set of natural numbers smaller than 5

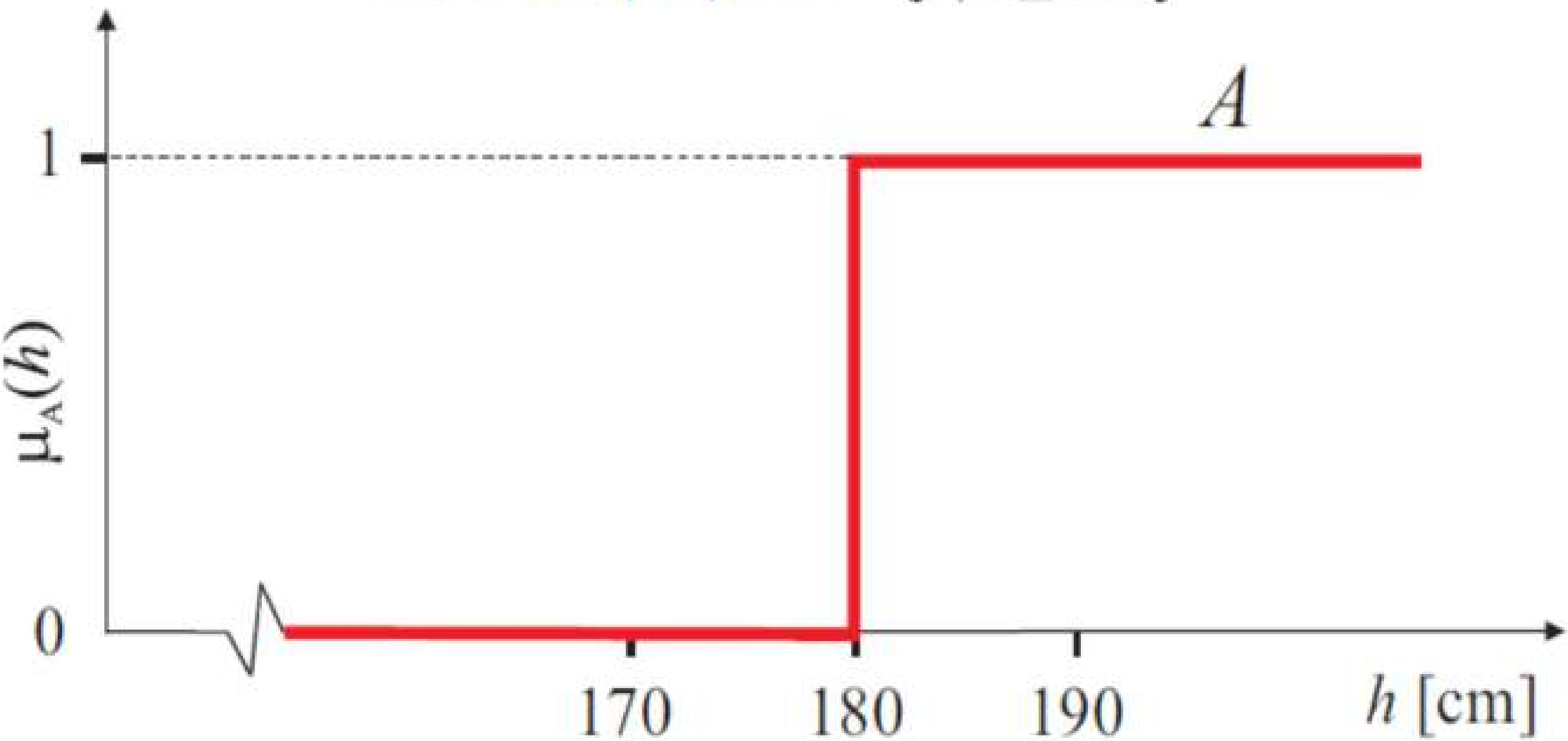


Why Fuzzy Sets?

- Classical sets are good for well-defined concepts (maths, programs, etc.)
- Less suitable for representing commonsense knowledge in terms of vague concepts such as:
- want to buy a big car with moderate consumption
- If the temperature is too low, increase heating a lot

Classical Set Approach

set of tall people $A = \{h | h \geq 180\}$

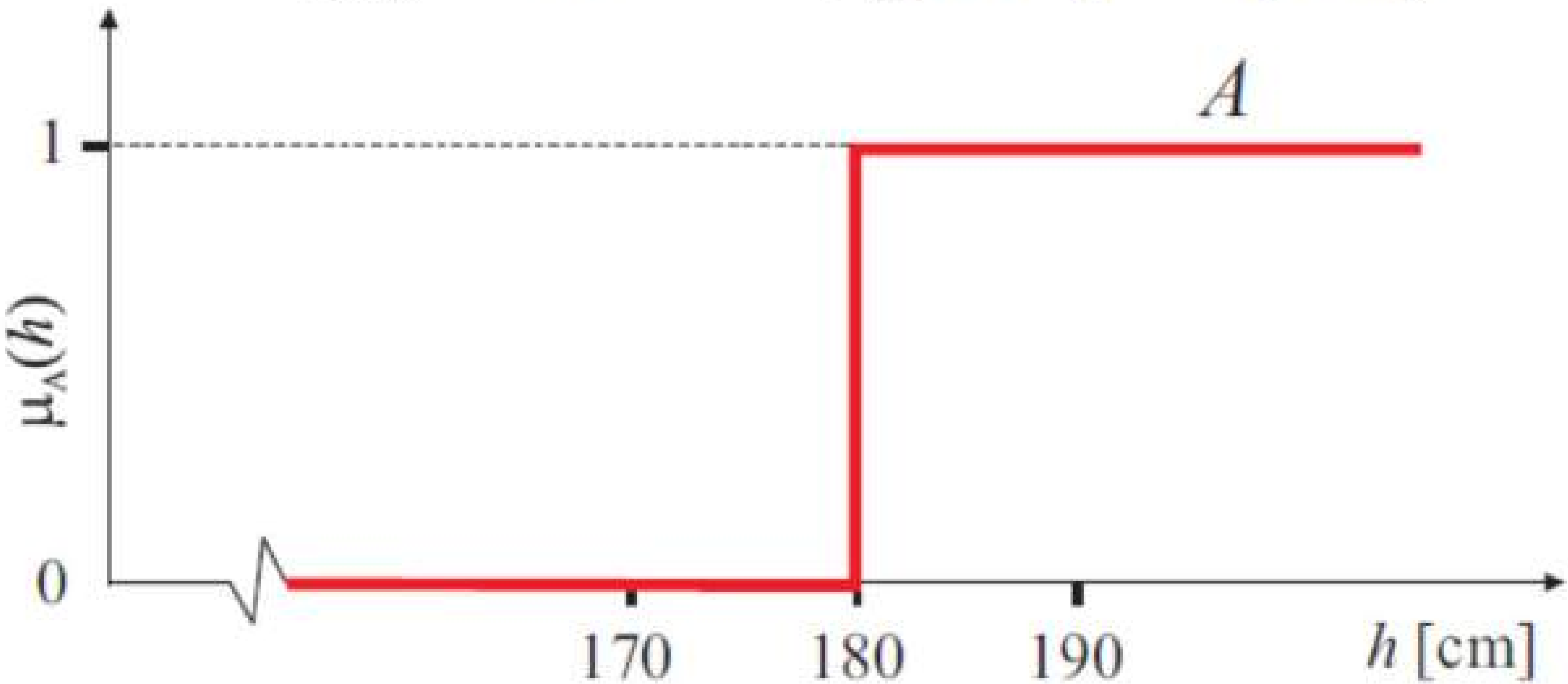


Logical Propositions

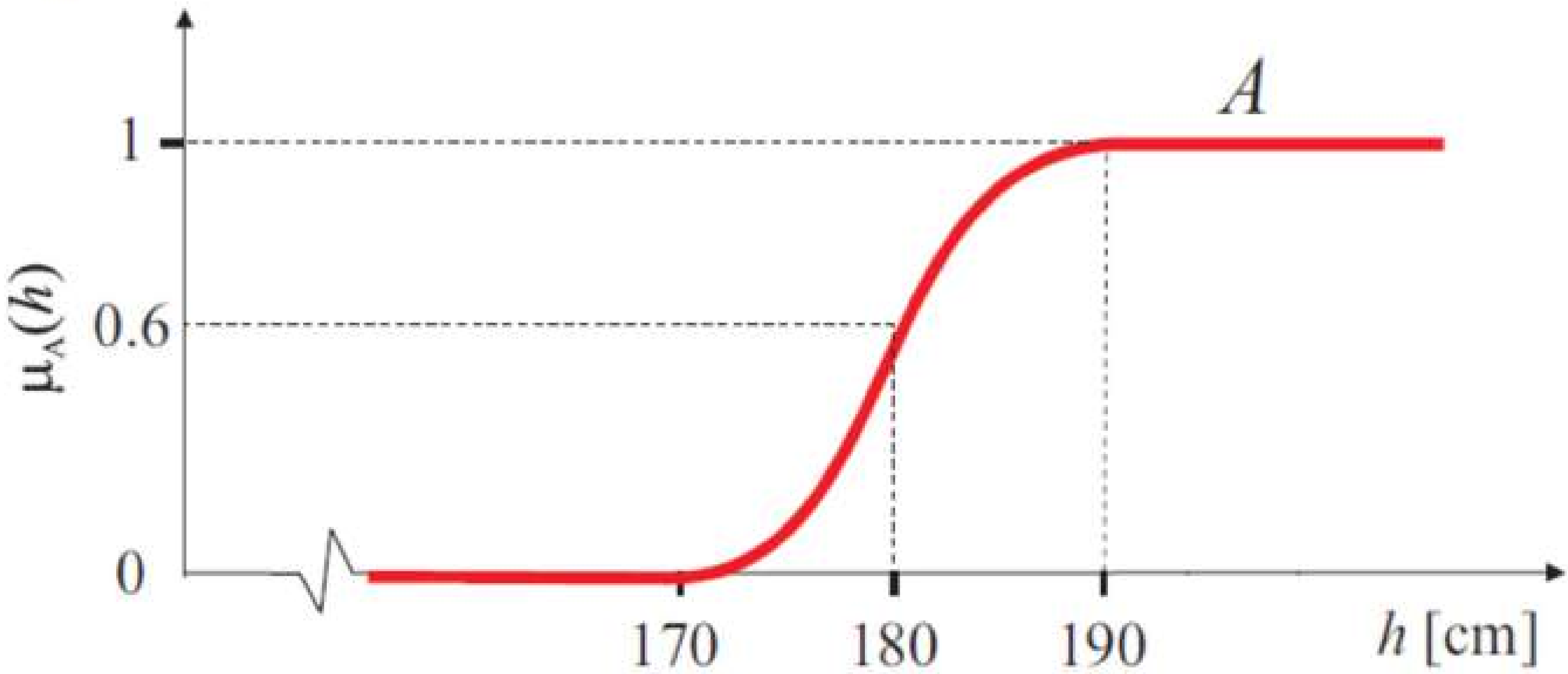
"John is tall" . . . true or false

John's height: $h_{John} = 180.0$ $\mu_A(180.0) = 1$ (true)

$h_{John} = 179.5$ $\mu_A(179.5) = 0$ (false)



Fuzzy Set Approach

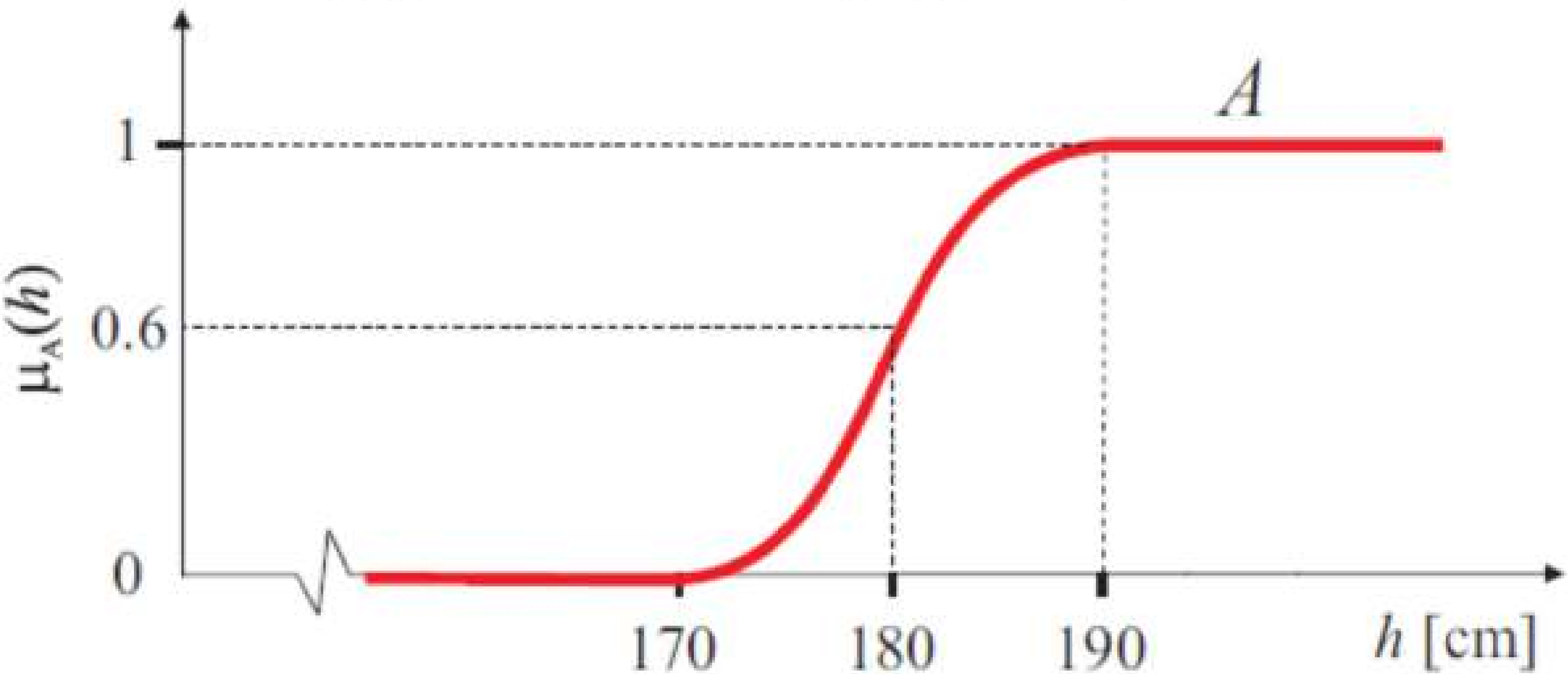


$$\mu_A(h) = \begin{cases} 1 & h \text{ is full member of } A \quad (h \geq 190) \\ (0,1) & h \text{ is partial member of } A \quad (170 < h < 190) \\ 0 & h \text{ is not member of } A \quad (h \leq 170) \end{cases}$$

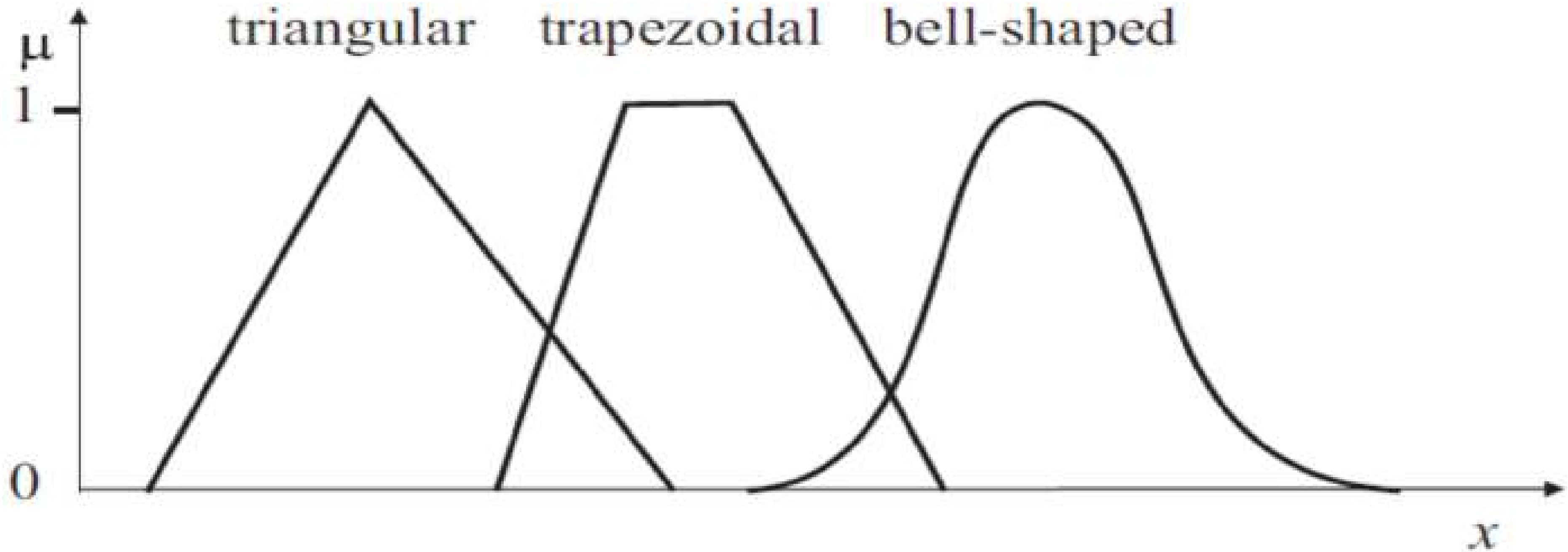
Fuzzy Logic Propositions

"John is tall" ...degree of truth

John's height:	$h_{John} = 180.0$	$\mu_A(180.0) = 0.6$
	$h_{John} = 179.5$	$\mu_A(179.5) = 0.56$
	$h_{Paul} = 201.0$	$\mu_A(201.0) = 1$



Shapes of Membership Functions



- Pointwise as a list of membership/element pairs:

$$A = \{\mu_A(x_1)/x_1, \ldots, \mu_A(x_n)/x_n\} = \{\mu_A(x_i)/x_i | x_i \in X\}$$

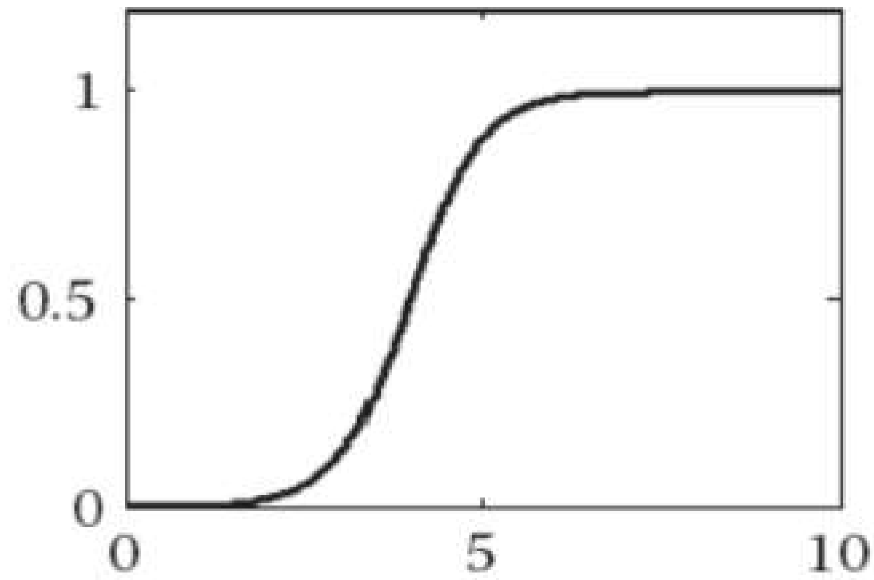
- As a list of α-level/α-cut pairs:

$$A = \{\alpha_1/A_{\alpha_1},\ \alpha_2/A_{\alpha_2},\ \ldots, \alpha_n, A_{\alpha_n}\} = \{\alpha_i/A_{\alpha_i} | \alpha_i \in (0,1)\}$$

Sigmoidal Membership Function

A sigmoidal MF depends on two parameters $\{a, c\}$ as

$$\mathrm{sigmf}(x; a, c) = \frac{1}{1 + e^{-a(x-c)}}$$



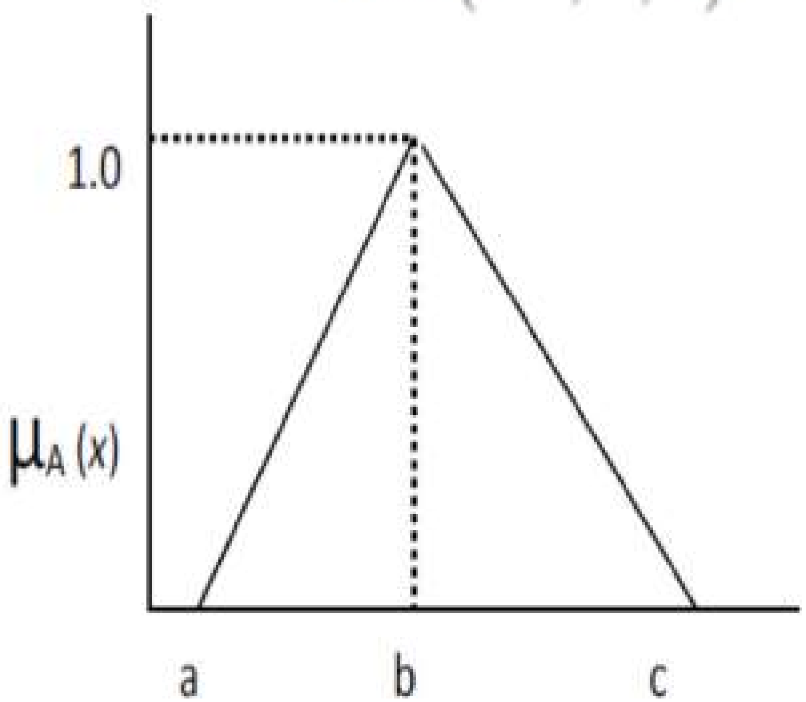
sigmf (x: [2, 4]).

where the sign of the parameter a determines the open-end direction of the sigmoidal MF. If a is a positive number, the MF will open to the right. If a is negative, the MF will open to the left. By this property, it is convenient to represent the fuzzy concepts such as extreme positive or extreme negative.

Triangular Membership function:

Let a, b and c represent the x coordinates of the three vertices of $\mu_A(x)$ in a fuzzy set A (a: lower boundary and c: upper boundary where membership degree is zero, b: the centre where membership degree is 1). or more compactly as

$$\text{trimf}(x: a, b, c) = \max\left(\min\left(\frac{x-a}{b-a}, \frac{c-x}{c-b}\right), 0\right)$$



$$\mu_A(x) = \begin{cases} 0 & \text{if } x \le a \\ \dfrac{x-a}{b-a} & \text{if } a \le x \le b \\ \dfrac{c-x}{c-b} & \text{if } b \le x \le c \\ 0 & \text{if } x \ge c \end{cases}$$

Trapezoidal membership function:

Let a, b, c and d represents the x coordinates of the membership function. then

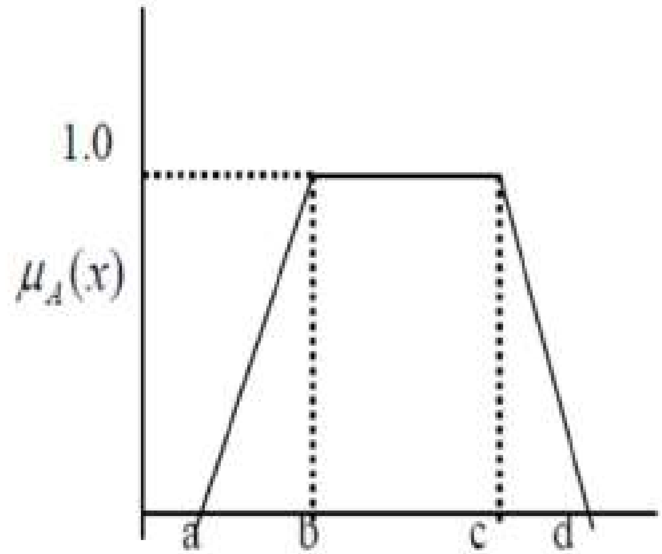
Trapezoid(x; a, b, c, d) = 0 if x ≤ a;

= (x-a)/ (b-a) if a ≤ x ≤ b

= 1 if b ≤ x ≤ c;

= (d-x)/ (d-c) 0 if c ≤ x ≤ d;

= 0, if d ≤ x.



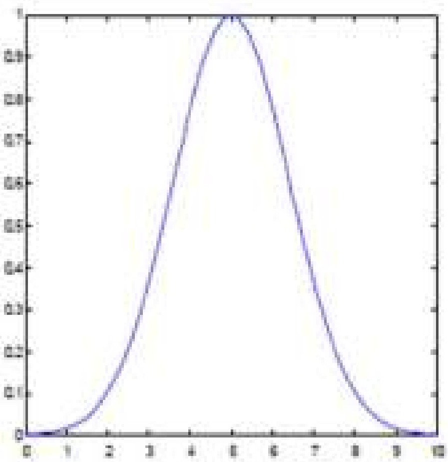
$$\mu_{trapezoid} = \max\left(\min\left(\frac{x-a}{b-a}, 1, \frac{d-x}{d-c}\right), 0\right)$$

Gaussian membership function:

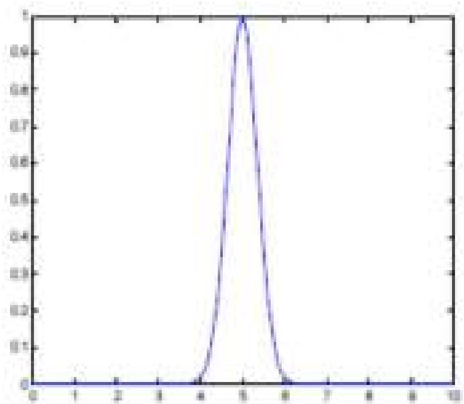
The Gaussian membership function is usually represented as Gaussian(x:c,s) where c, s represents the mean and standard deviation.

$$\mu_A(x,c,s,m) = \exp\left[-\frac{1}{2}\left|\frac{x-c}{s}\right|^m\right]$$

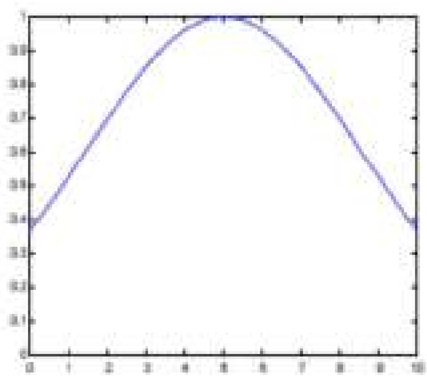
Here c represents centre, s represents width and m represents fuzzification factor.



c=5, s=0.5, m=2



c=5, s=2, m=2

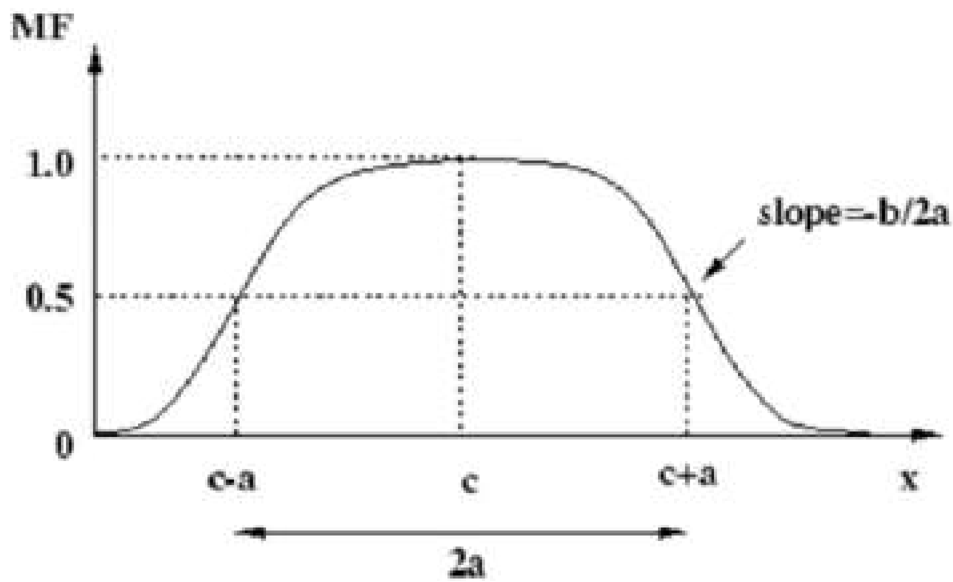


c=5, s=5, m=2

Generalized Bell membership function:

A generalized bell membership function has three parameters: a –responsible for its width, c responsible for its center and b –responsible for its slopes. Mathematically,

$$gbellmf(x;a,b,c) = \frac{1}{1+\left|\frac{x-c}{b}\right|^{2b}}$$



Features of Membership Function

Core: comprises of elements x of the universe, such that

$\mu_A(x) = 1$

Support: comprises of elements x of universe, such that

$\mu_A(x) > 0$

Boundaries: comprise the elements x of the universe

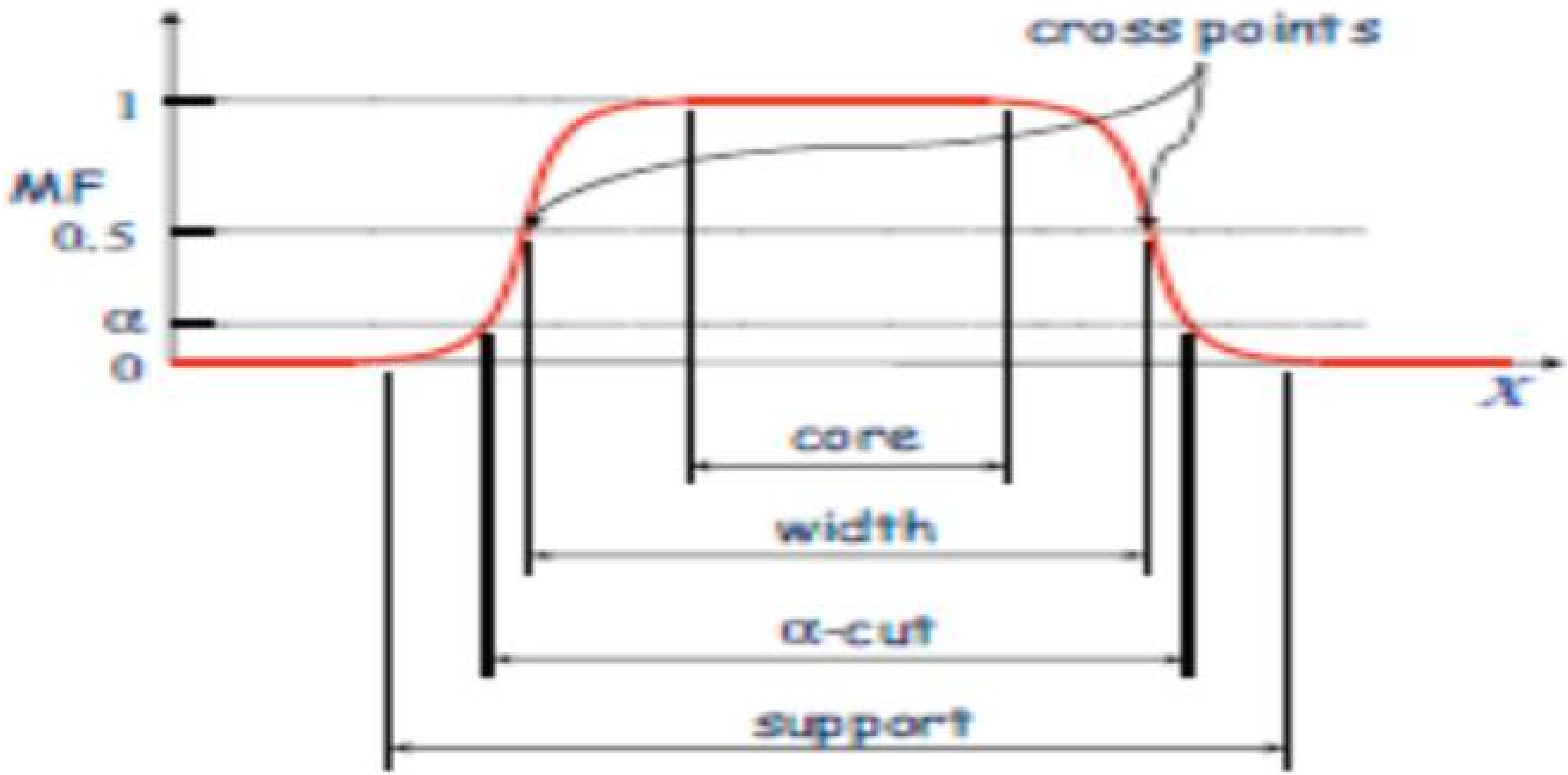
$0 < \mu_A(x) < 1$

A normal fuzzy set has at least one element with membership 1

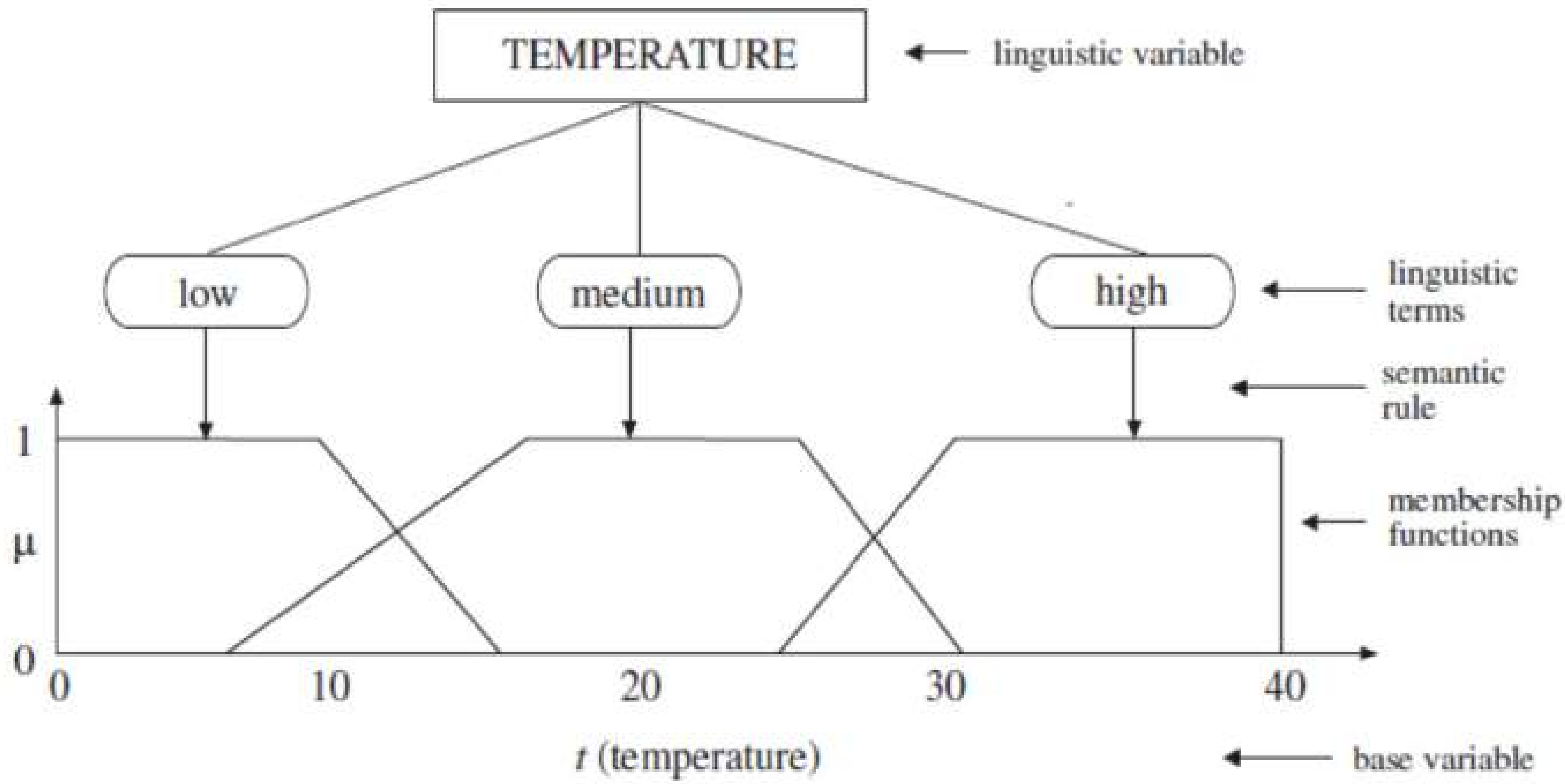
For fuzzy set, if one and only one element has a membership = 1, this element is called as the prototype of set.

A subnormal fuzzy set has no element with membership=1.

MF Terminology



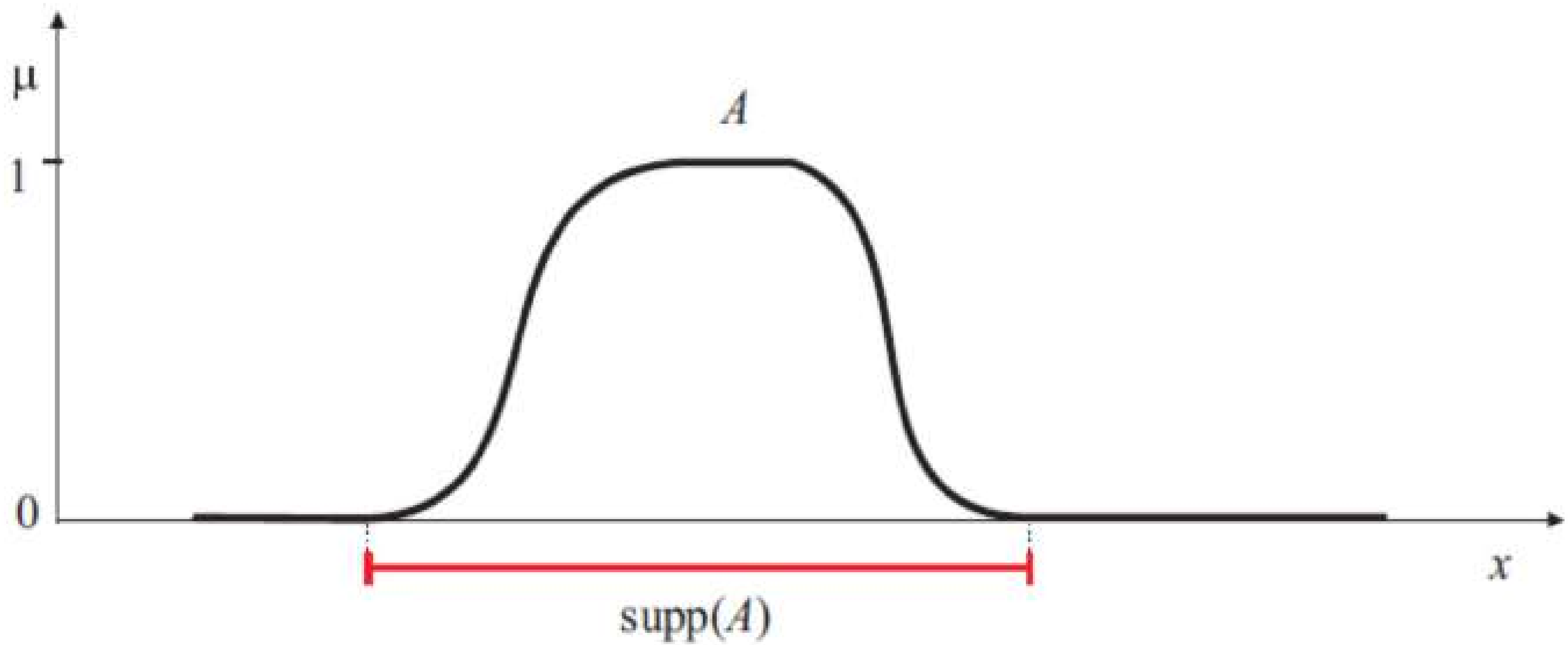
Linguistic Variable



Basic requirements: coverage and semantic soundness

Support of a Fuzzy Set

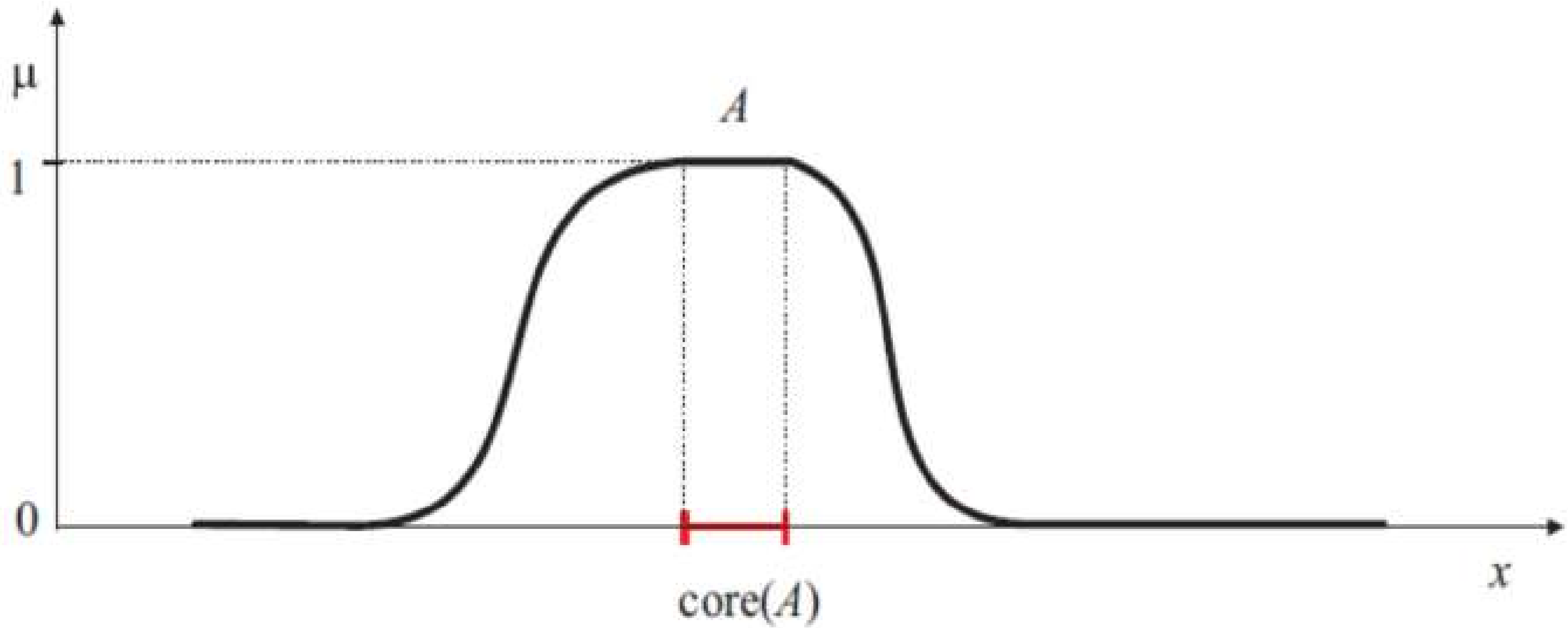
$$\text{supp}(A) = \{x | \mu_A(x) > 0\}$$



support is an *ordinary set*

Core (Kernel) of a Fuzzy Set

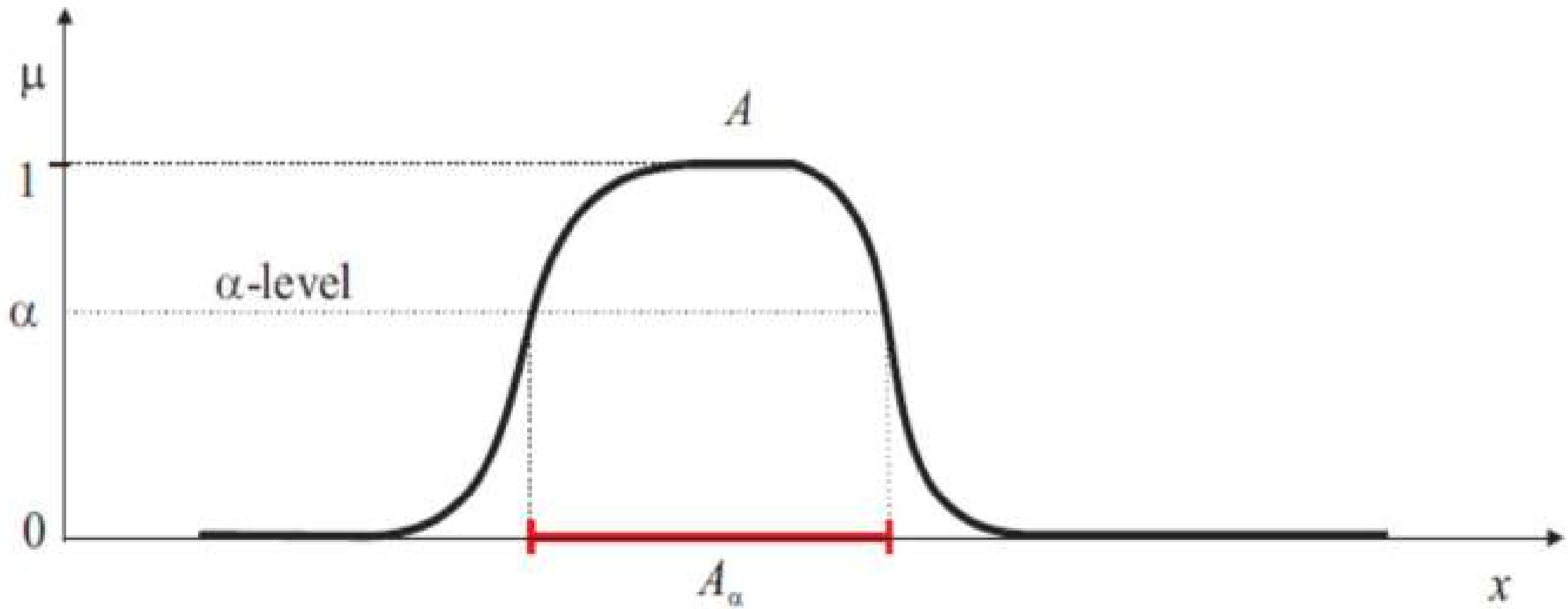
$$\text{core}(A) = \{x | \mu_A(x) = 1\}$$



core is an *ordinary set*

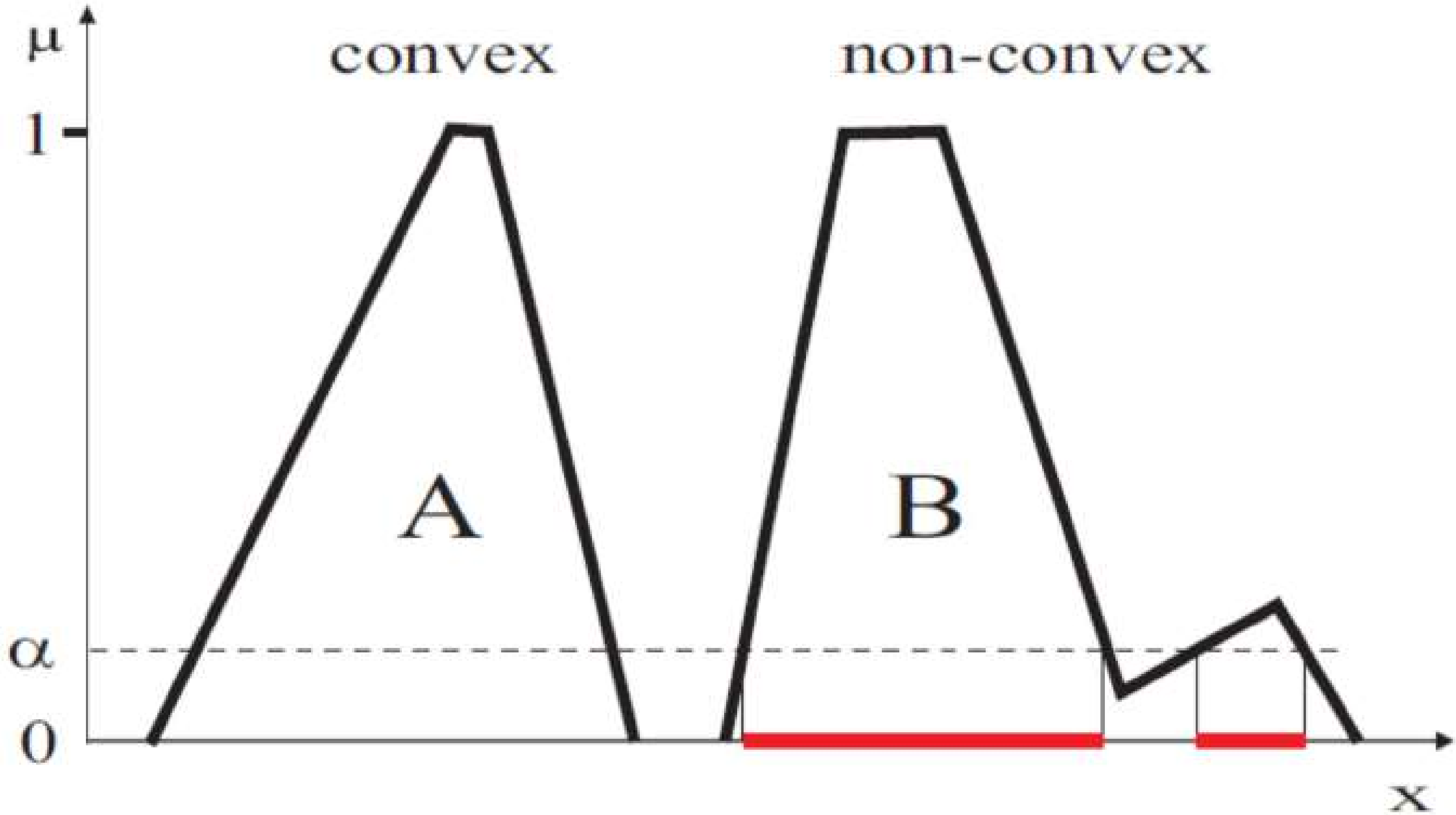
α-cut of a Fuzzy Set

$$A_\alpha = \{x | \mu_A(x) > \alpha\} \quad \text{or} \quad A_\alpha = \{x | \mu_A(x) \geq \alpha\}$$



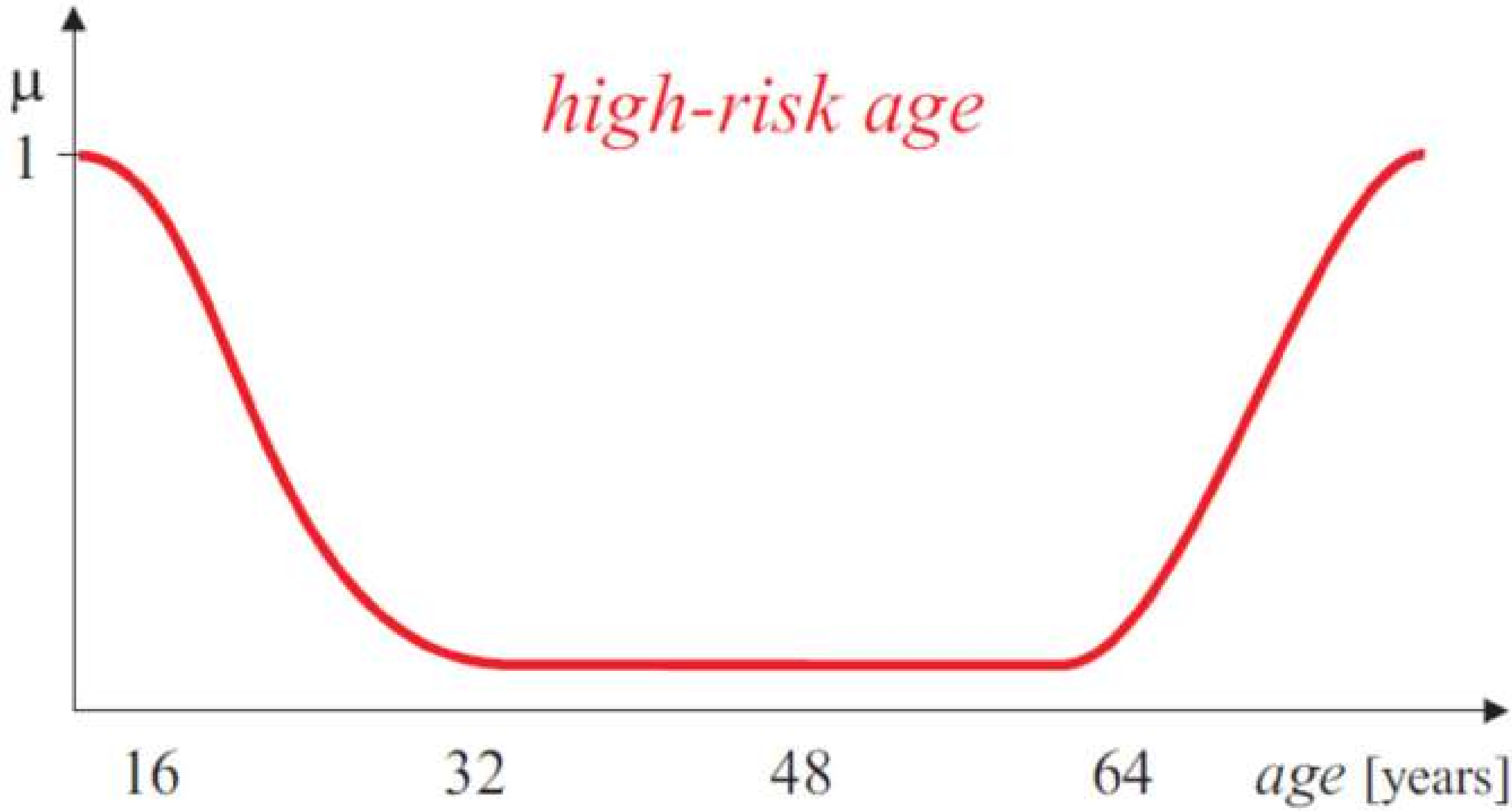
A_α is an *ordinary set*

Convex and Non-Convex Fuzzy Sets



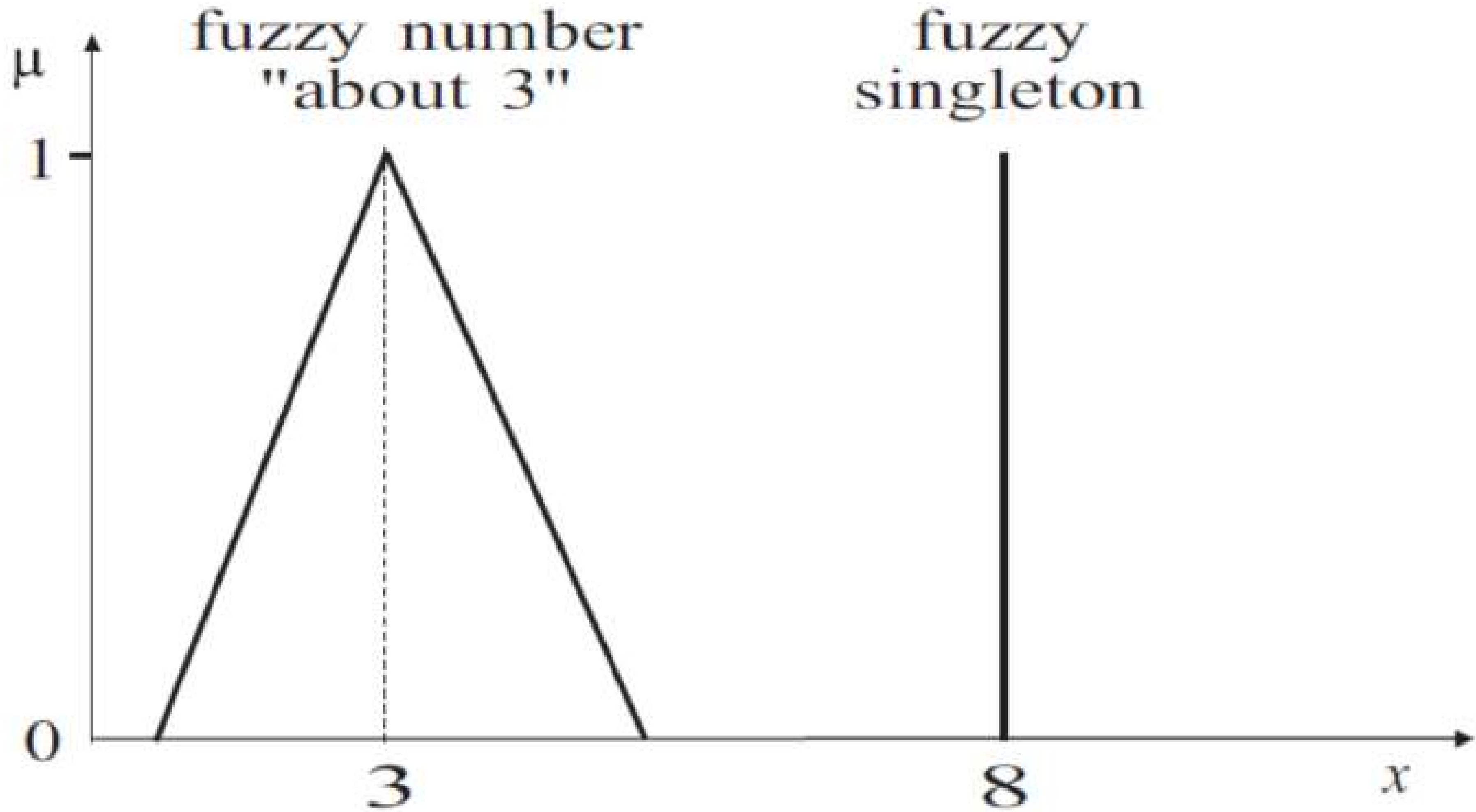
A fuzzy set is **convex** ⇔ all its α-cuts are convex sets.

Non-Convex Fuzzy Set: an Example



High-risk age for car insurance policy.

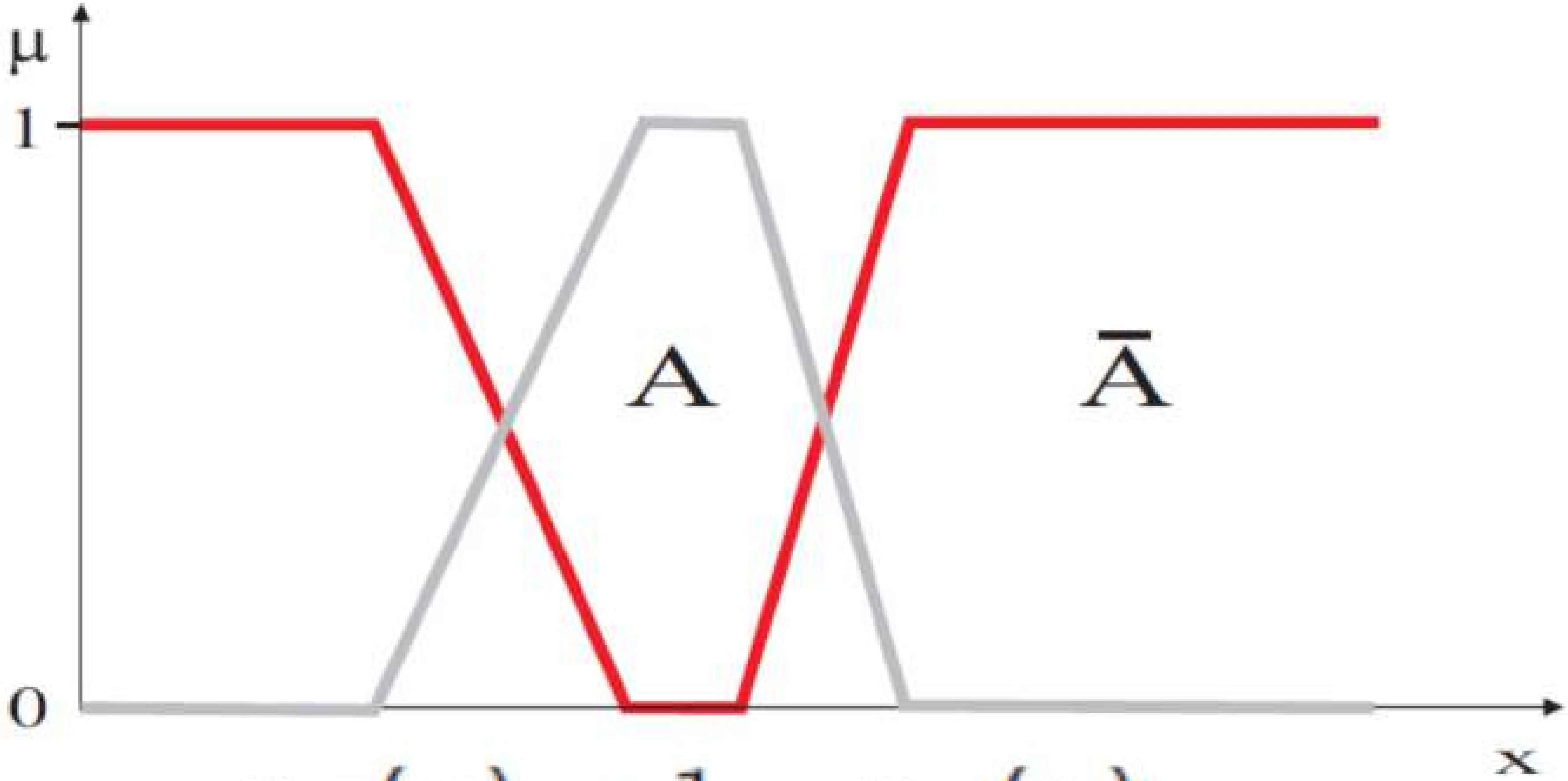
Fuzzy Numbers and Singletons



Fuzzy linear regression: $y = \tilde{3}x_1 + \tilde{5}x_2$

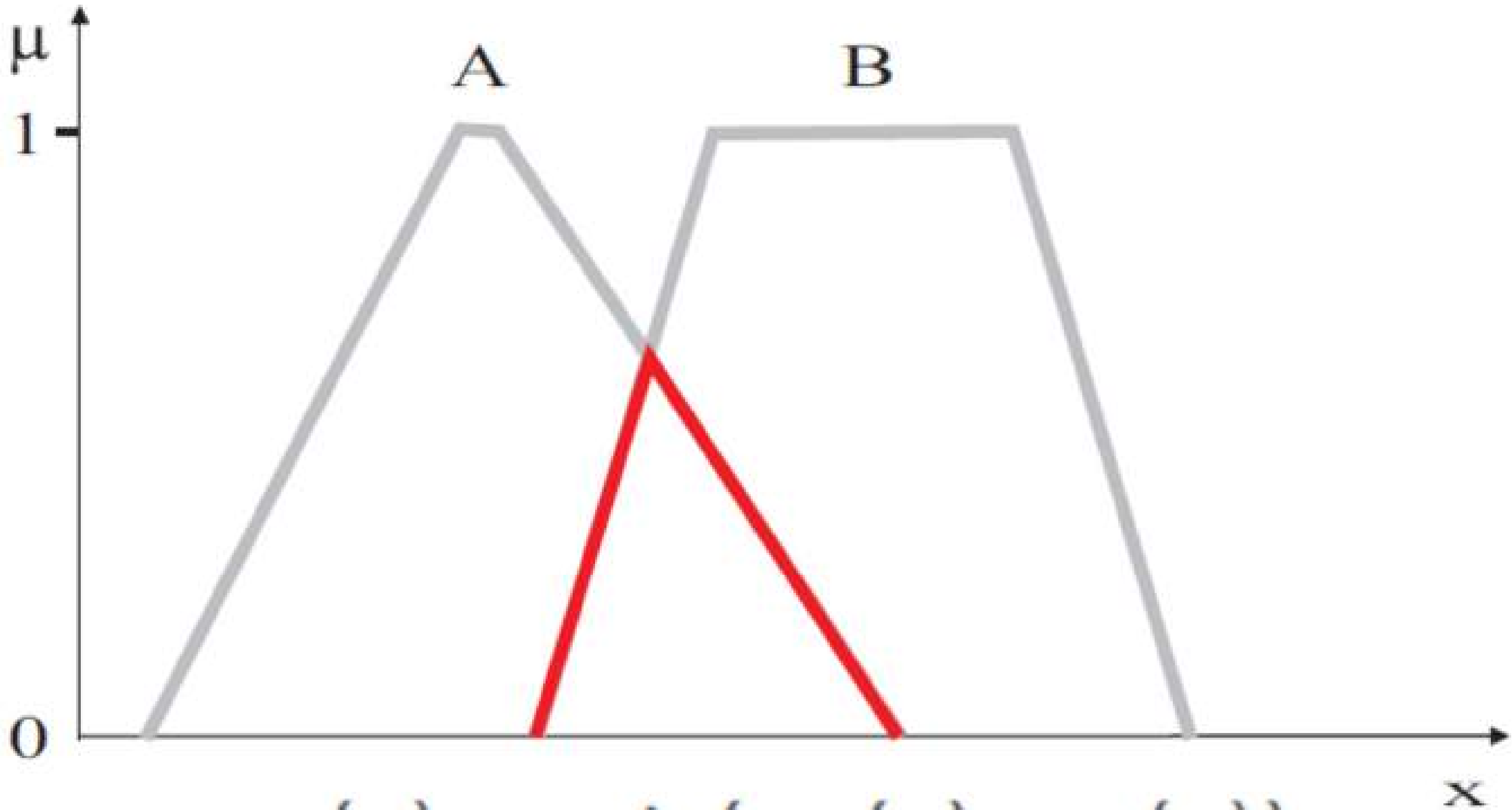
Fuzzy set-theoretic operations

Complement (Negation) of a Fuzzy Set



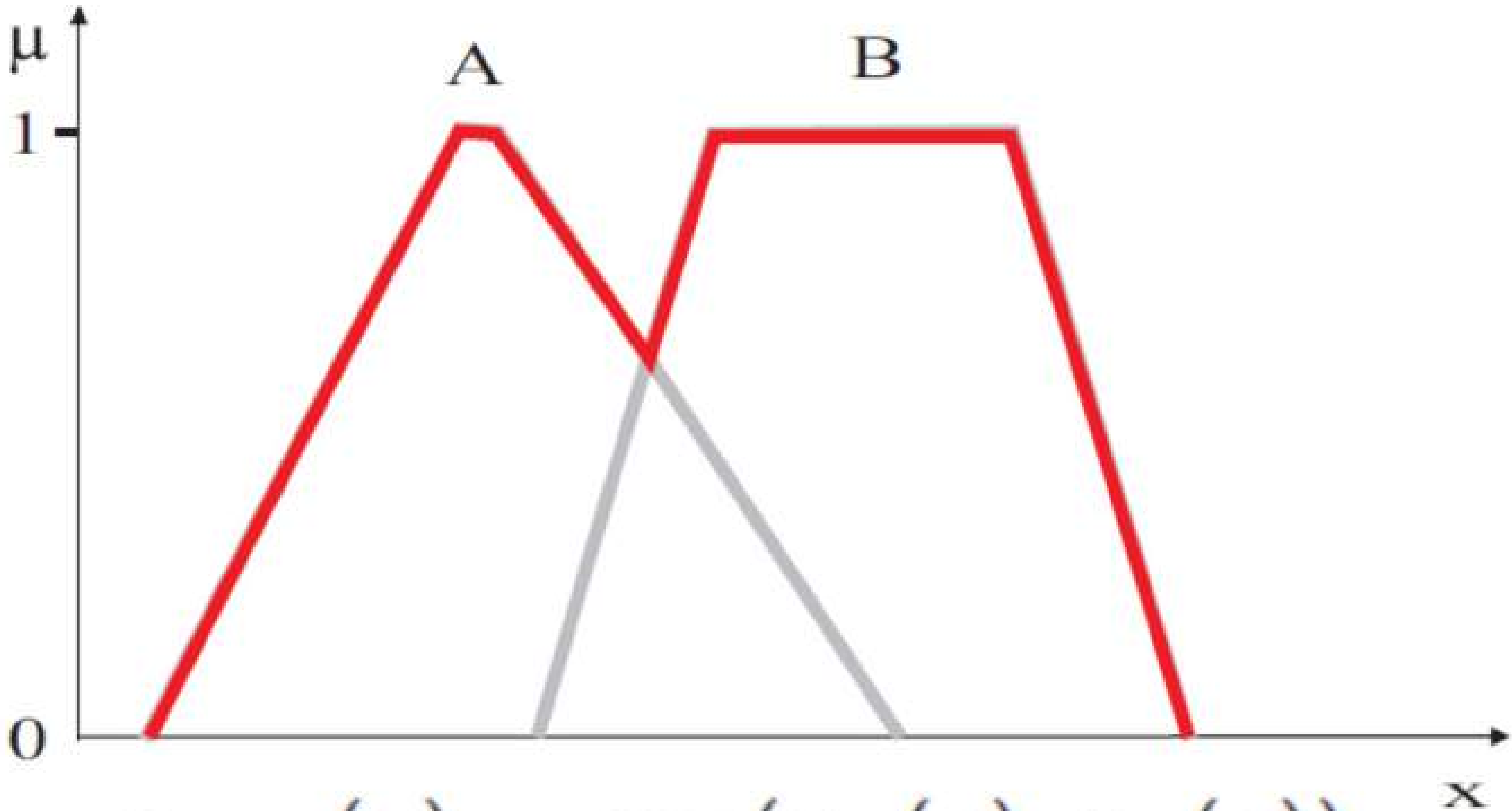
$$\mu_{\bar{A}}(x) = 1 - \mu_A(x)$$

Intersection (Conjunction) of Fuzzy Sets



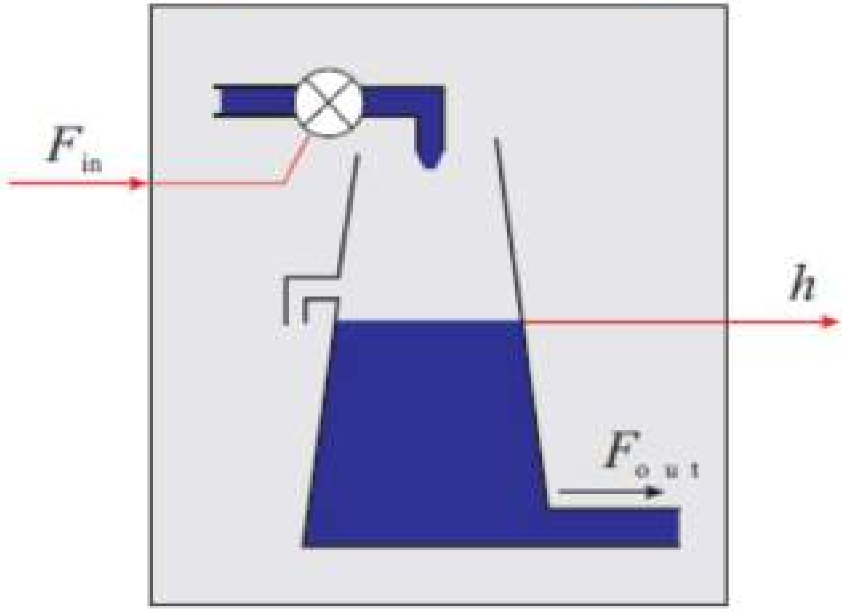
$$\mu_{A\cap B}(x) = \min(\mu_A(x), \mu_B(x))$$

Union (Disjunction) of Fuzzy Sets

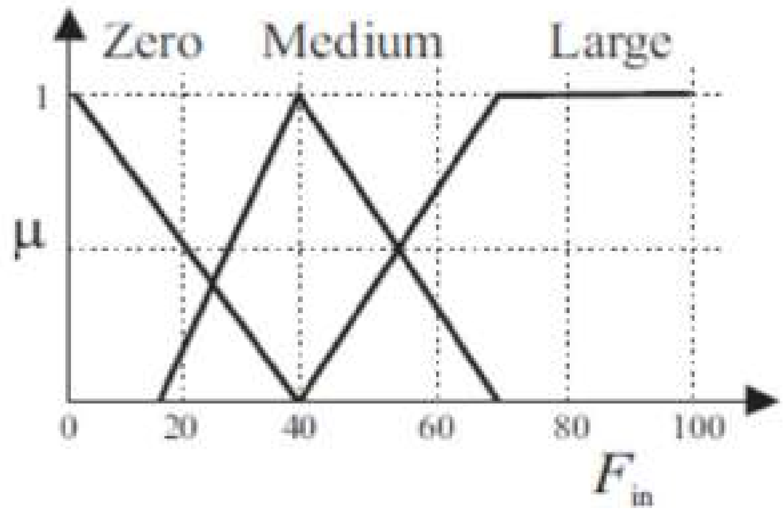


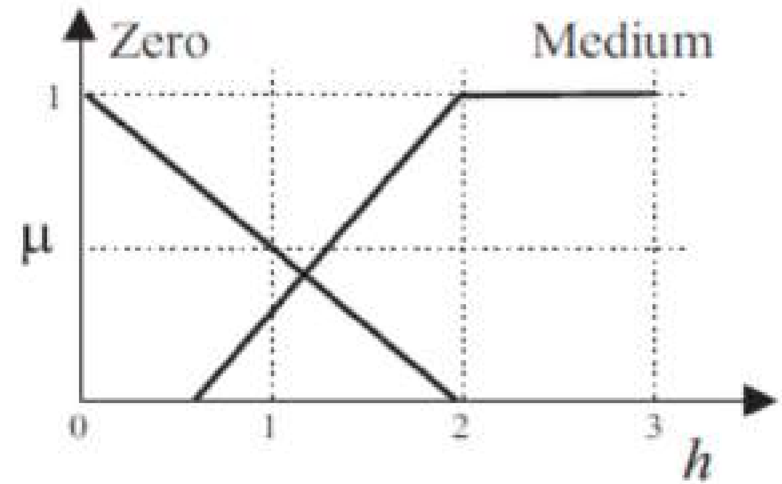
$$\mu_{A \cup B}(x) = \max(\mu_A(x), \mu_B(x))$$

Example: Modeling of Liquid Level

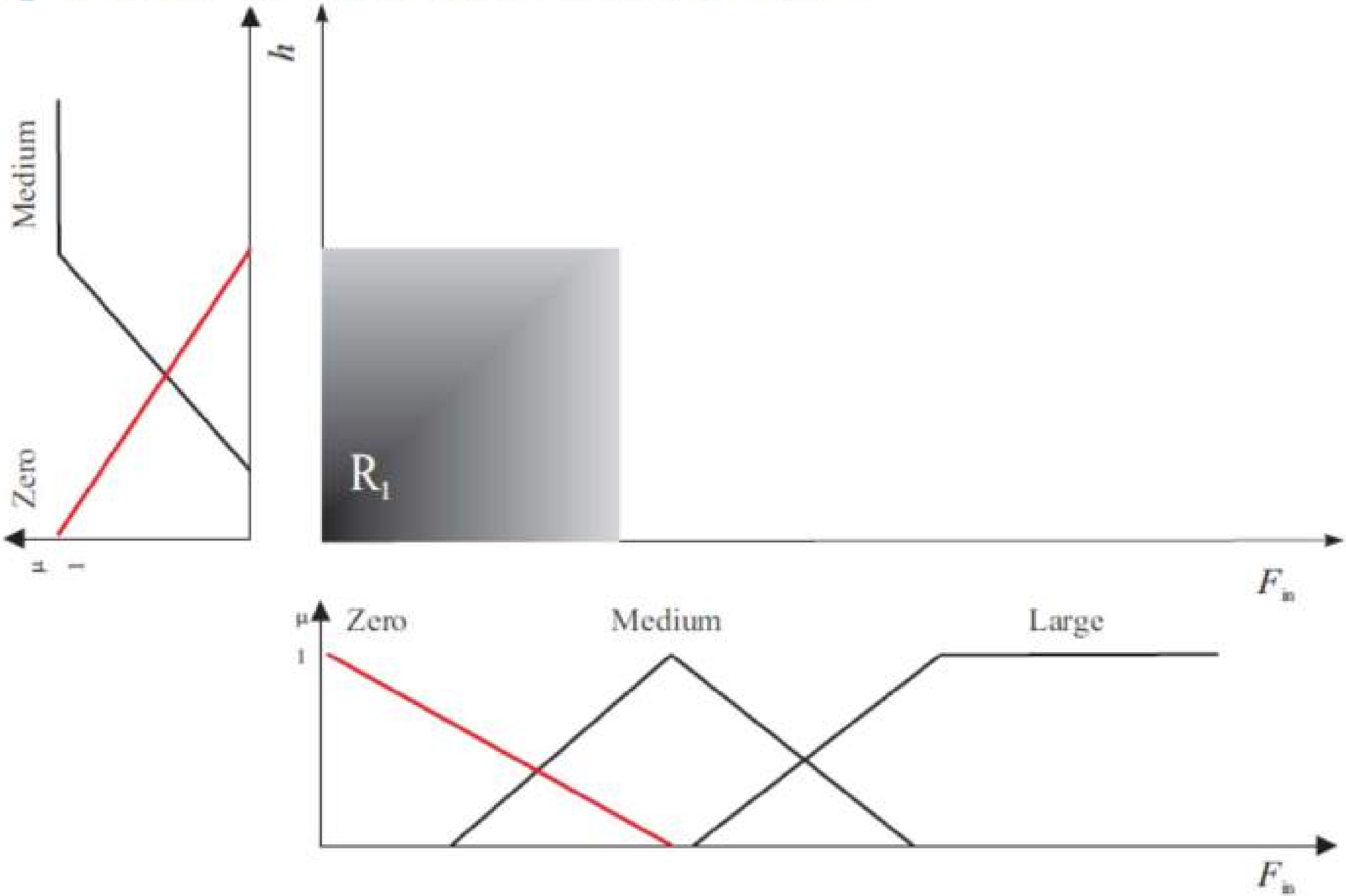


- If F_{in} is Zero then h is Zero
- If F_{in} is Med then h is Med
- If F_{in} is Large then h is Med

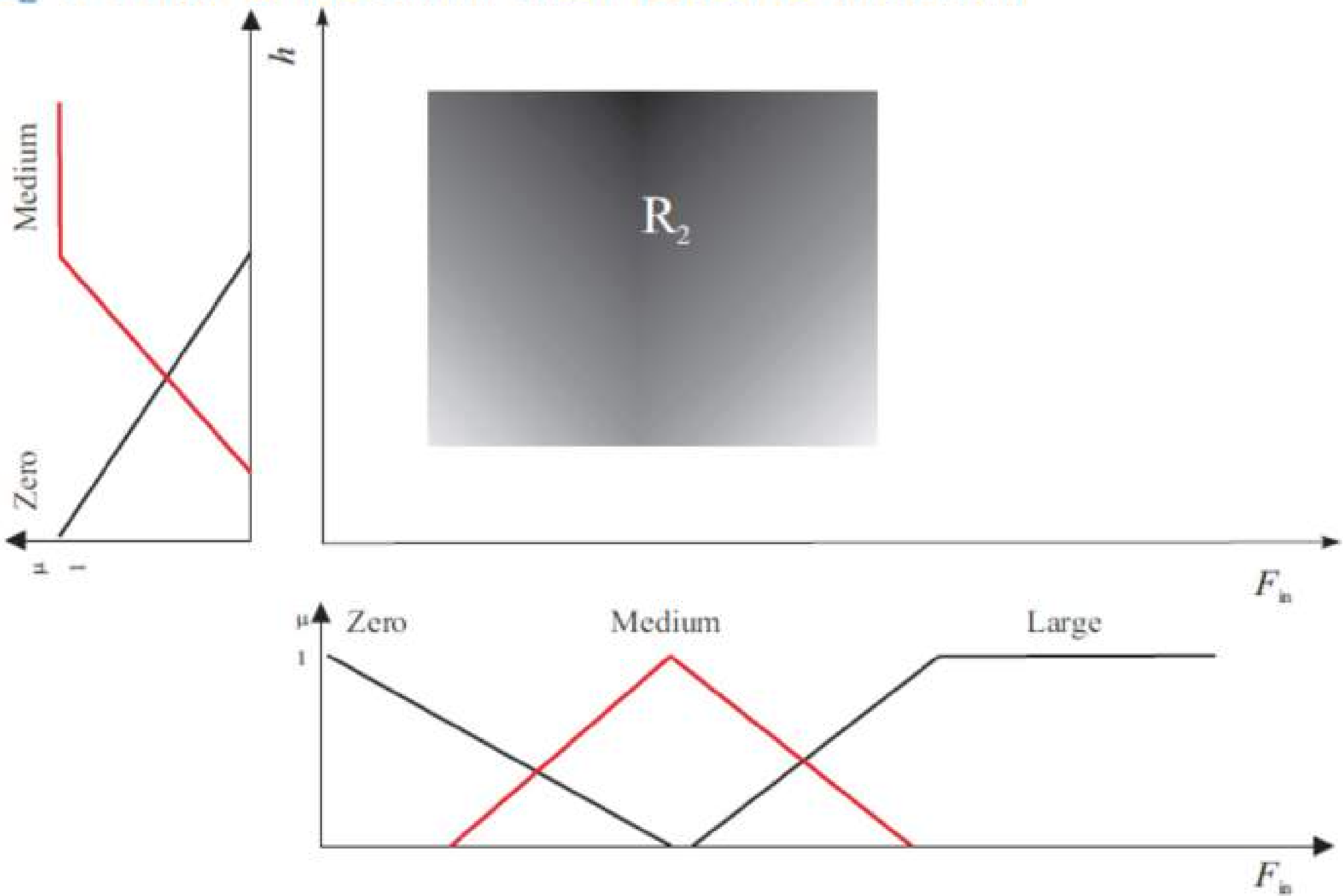




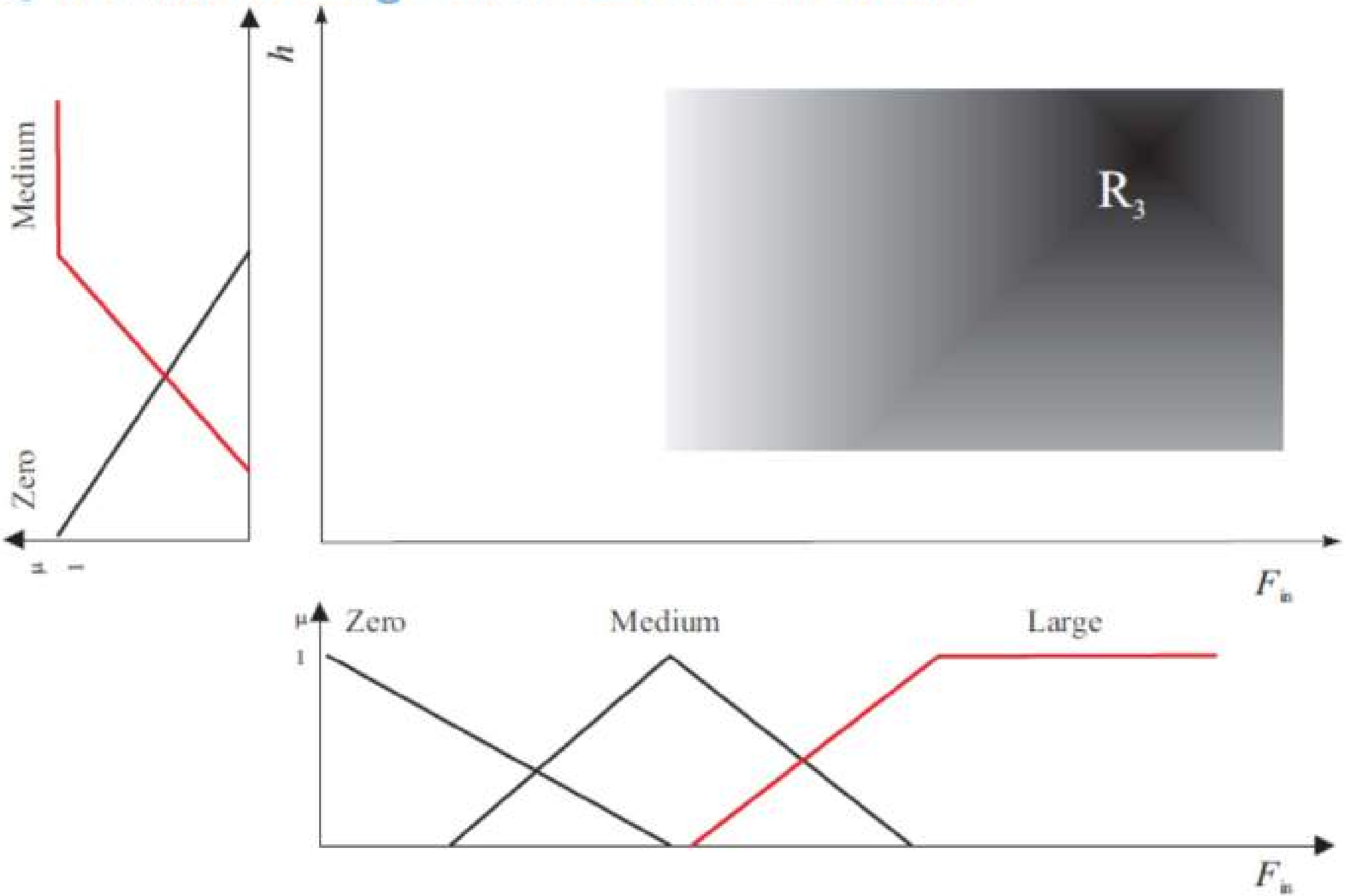
$\mathcal{R}_1$ If Flow is Zero then Level is Zero



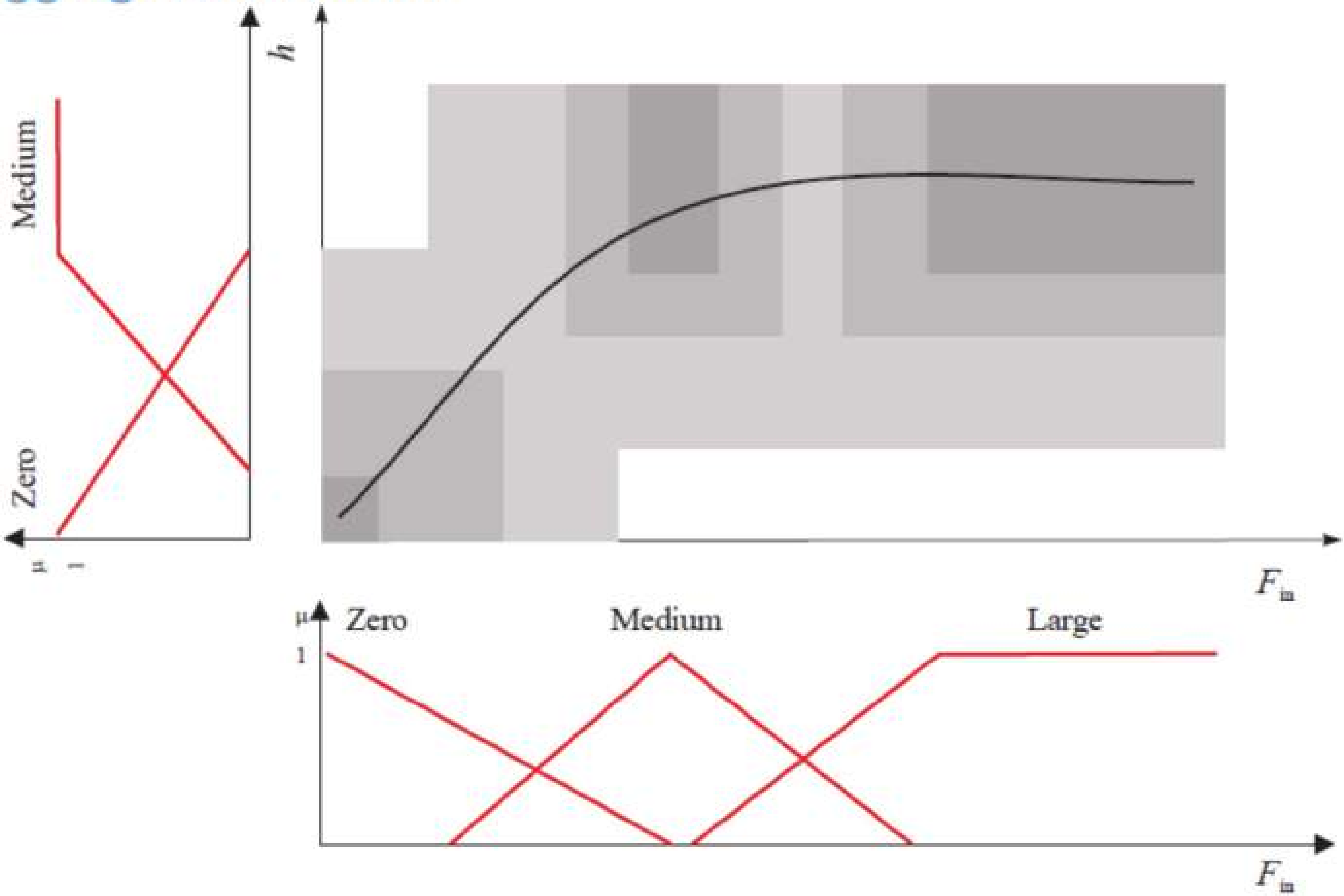
$\mathcal{R}_2$ If Flow is Medium then Level is Medium



$\mathcal{R}_3$ If Flow is Large then Level is Medium



Aggregated Relation



Simplified Approach

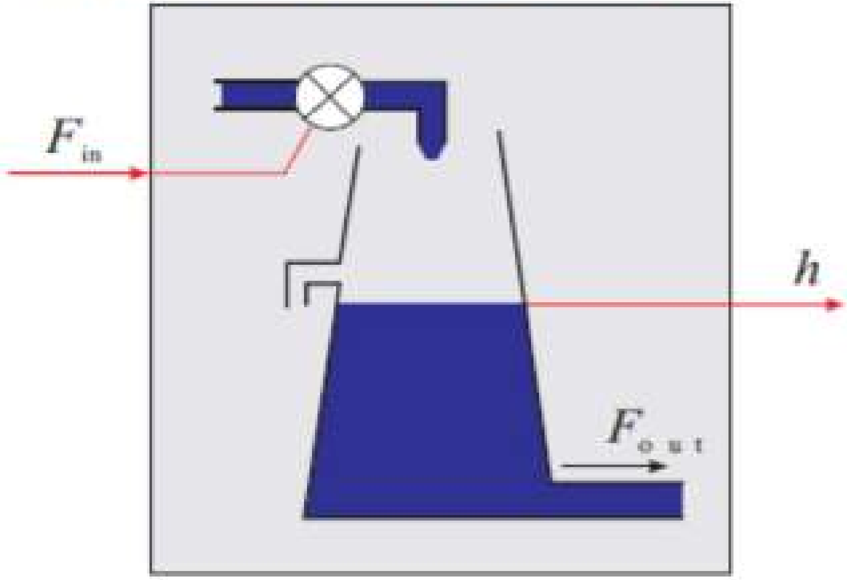
1 Compute the match between the input and the antecedent membership functions (degree of fulfillment).

2 Clip the corresponding output fuzzy set for each rule by using the degree of fulfillment.

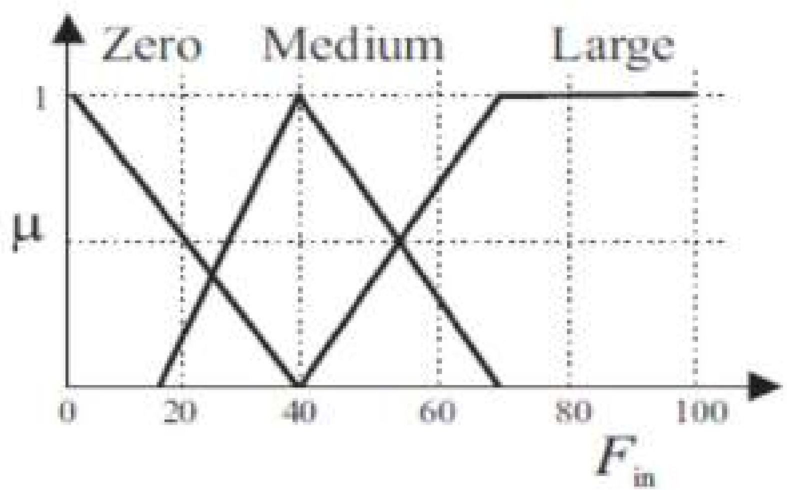
3 Aggregate output fuzzy sets of all the rules into one fuzzy set.

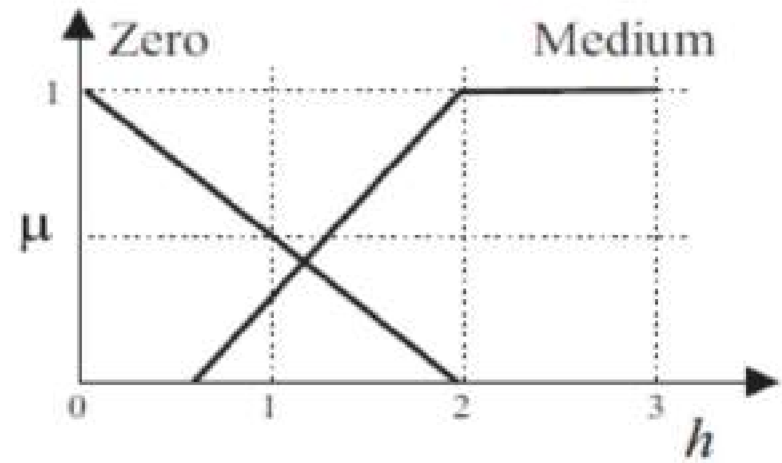
This is called the Mamdani or max-min inference method.

Water Tank Example

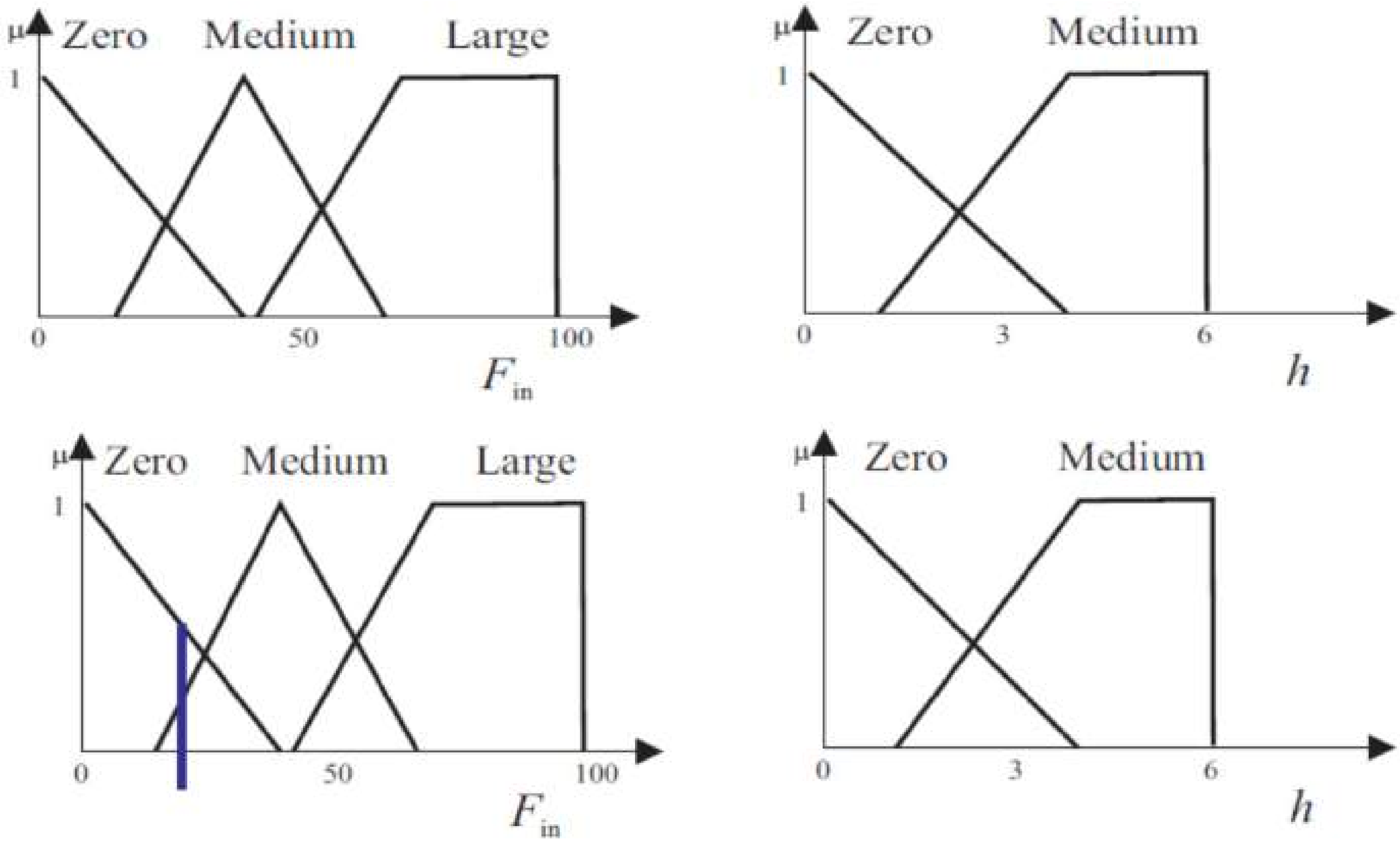


- If F_{in} is Zero then h is Zero
- If F_{in} is Med then h is Med
- If F_{in} is Large then h is Med



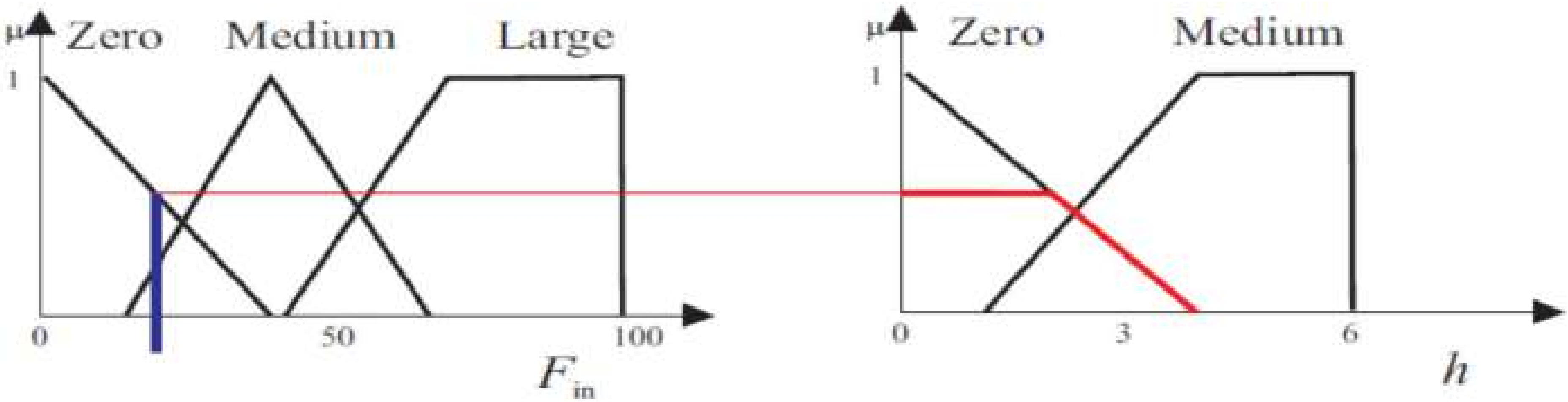


Mamdani Inference: Example



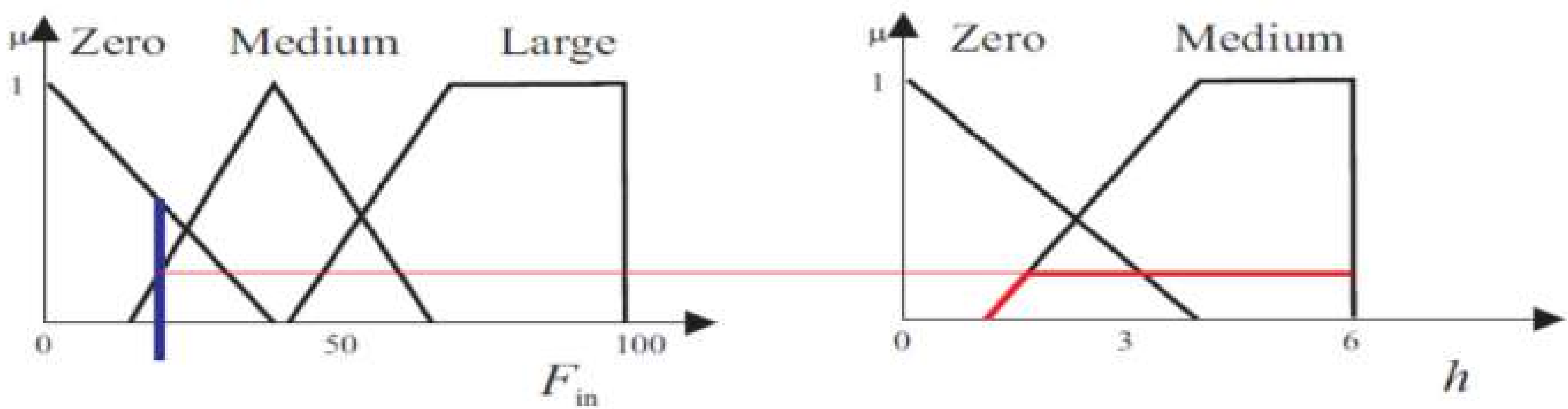
Given a crisp (numerical) input (F_{in}).

If F_{in} is Zero then h is Zero



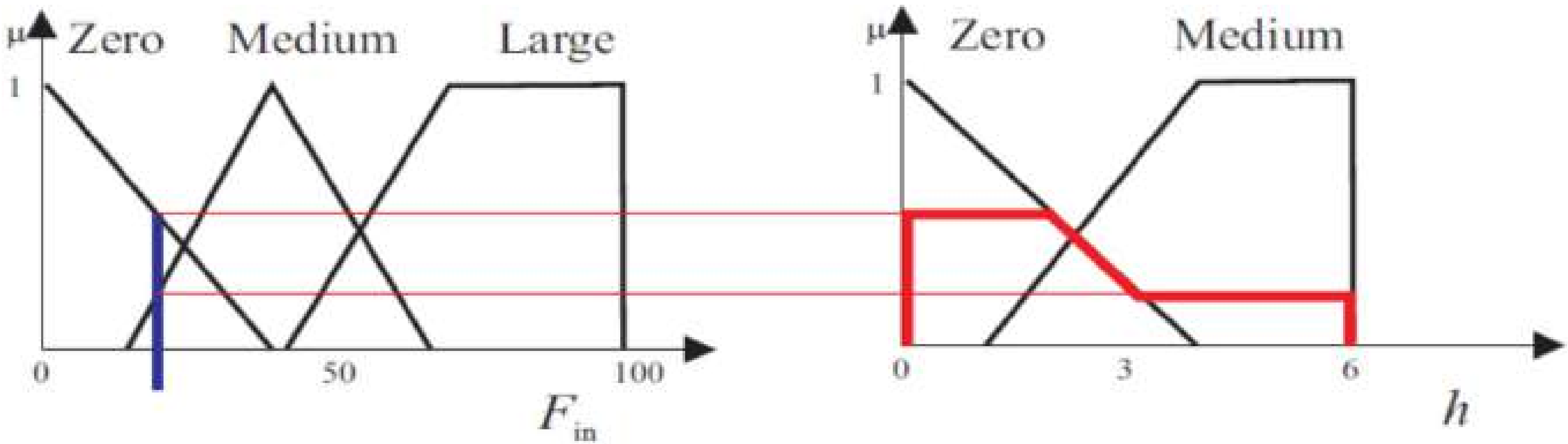
Clip consequent membership function of the first rule.

If F_{in} is Medium then h is Medium



Clip consequent membership function of the second rule.

Aggregation



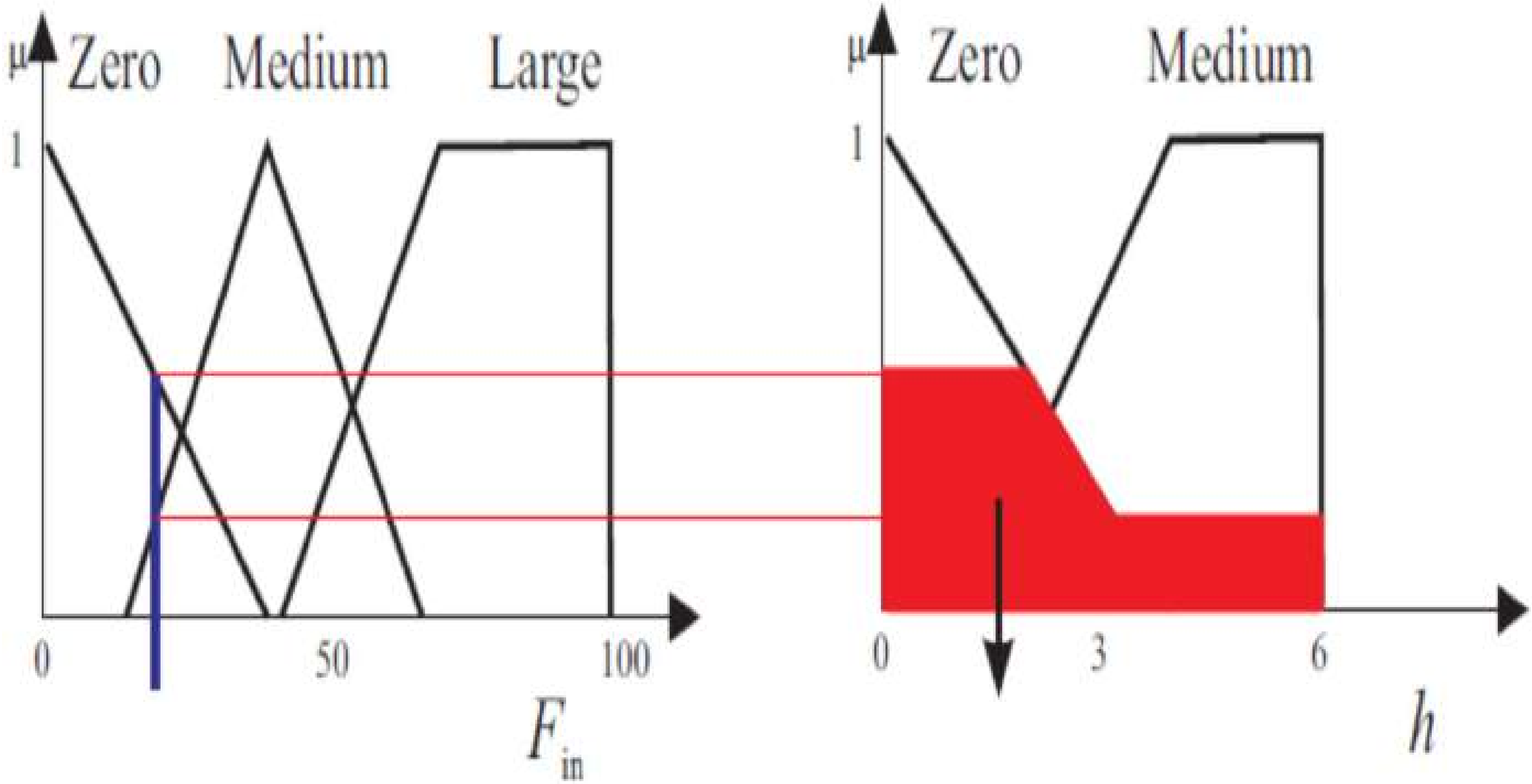
Combine the result of the two rules (union).

Defuzzification

Center-of-Gravity Method

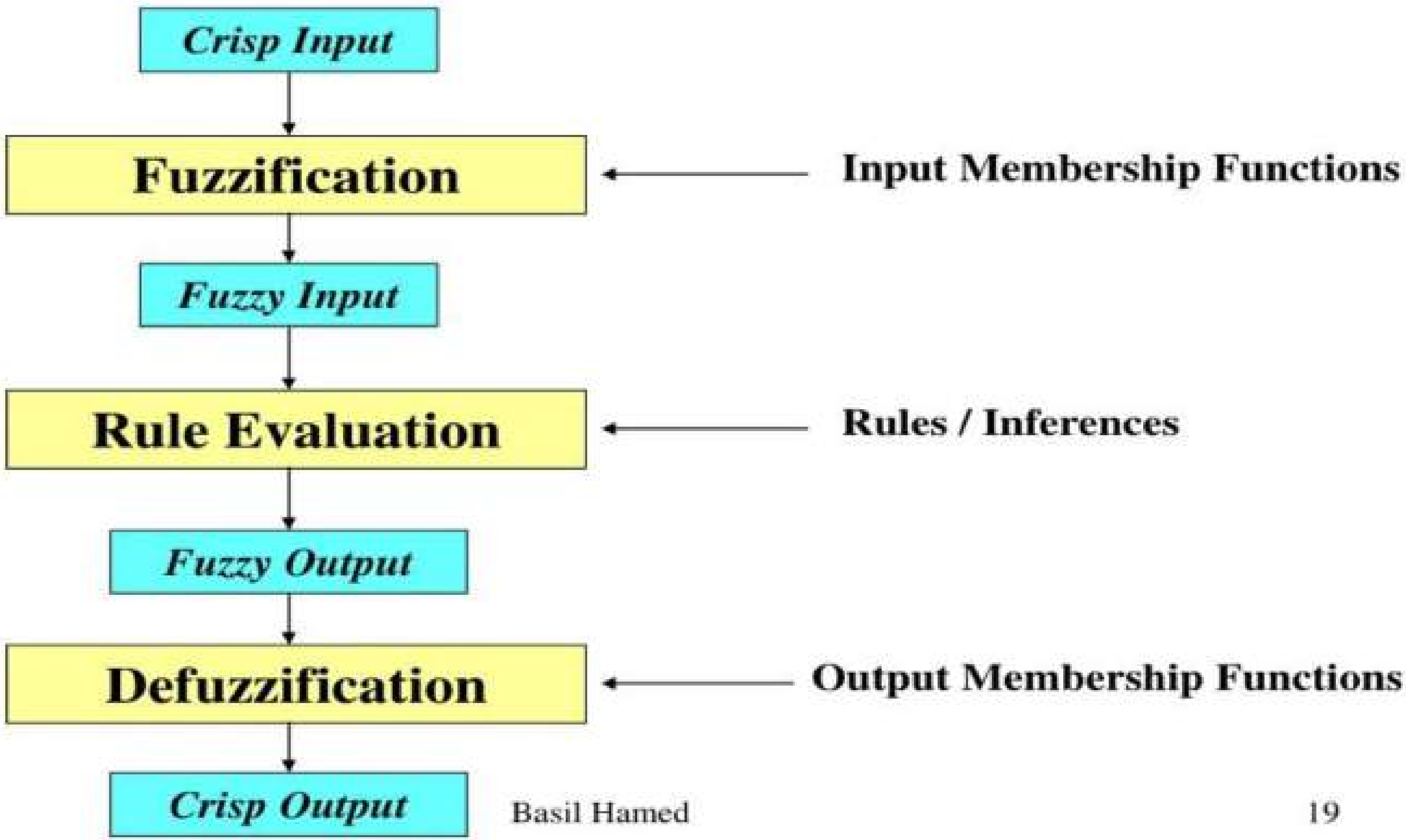
$$y_0 = \frac{\sum_{j=1}^{F} \mu_{B'}(y_j) y_j}{\sum_{j=1}^{F} \mu_{B'}(y_j)}$$

Defuzzification



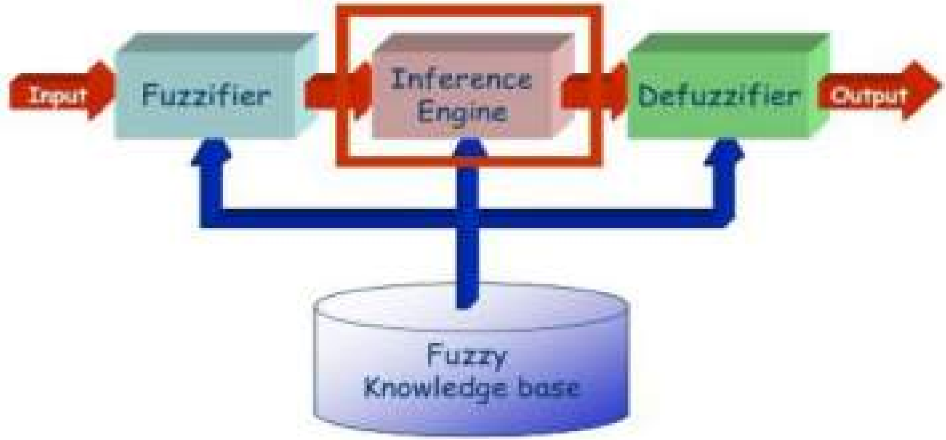
Compute a crisp (numerical) output of the model (center-of-gravity method).

Operation of Fuzzy System

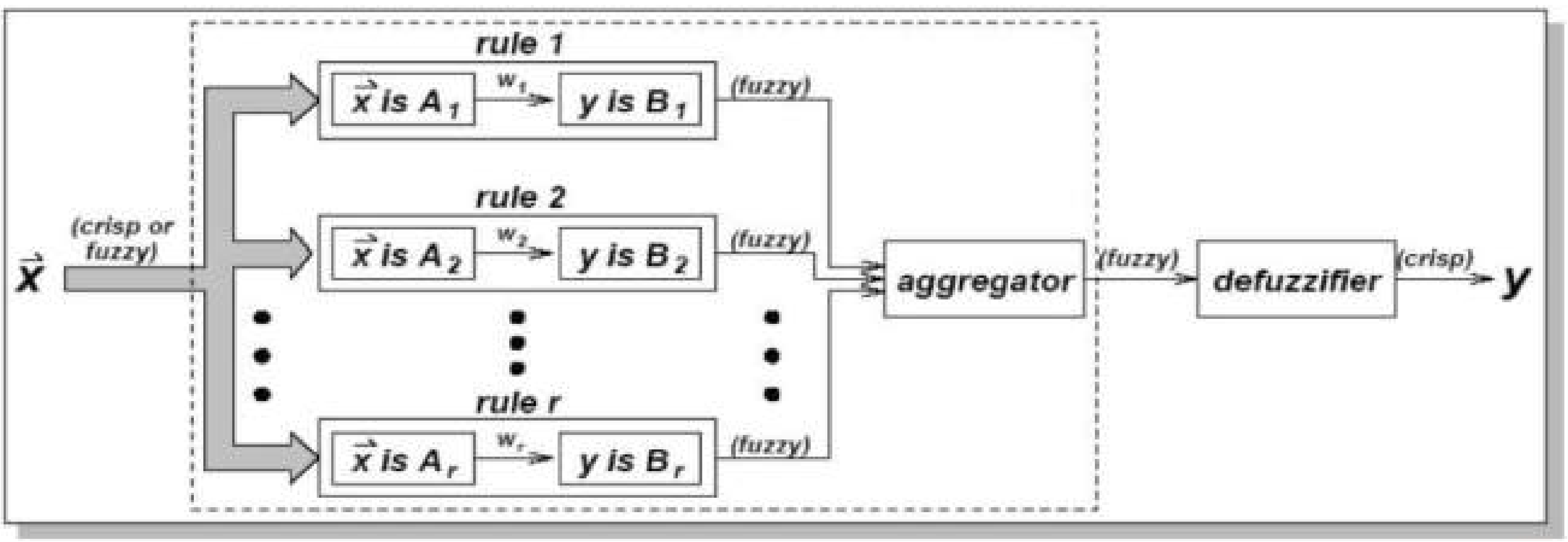


Basil Hamed 19

Inference Engine



Using If-Then type fuzzy rules converts the fuzzy input to the fuzzy output.

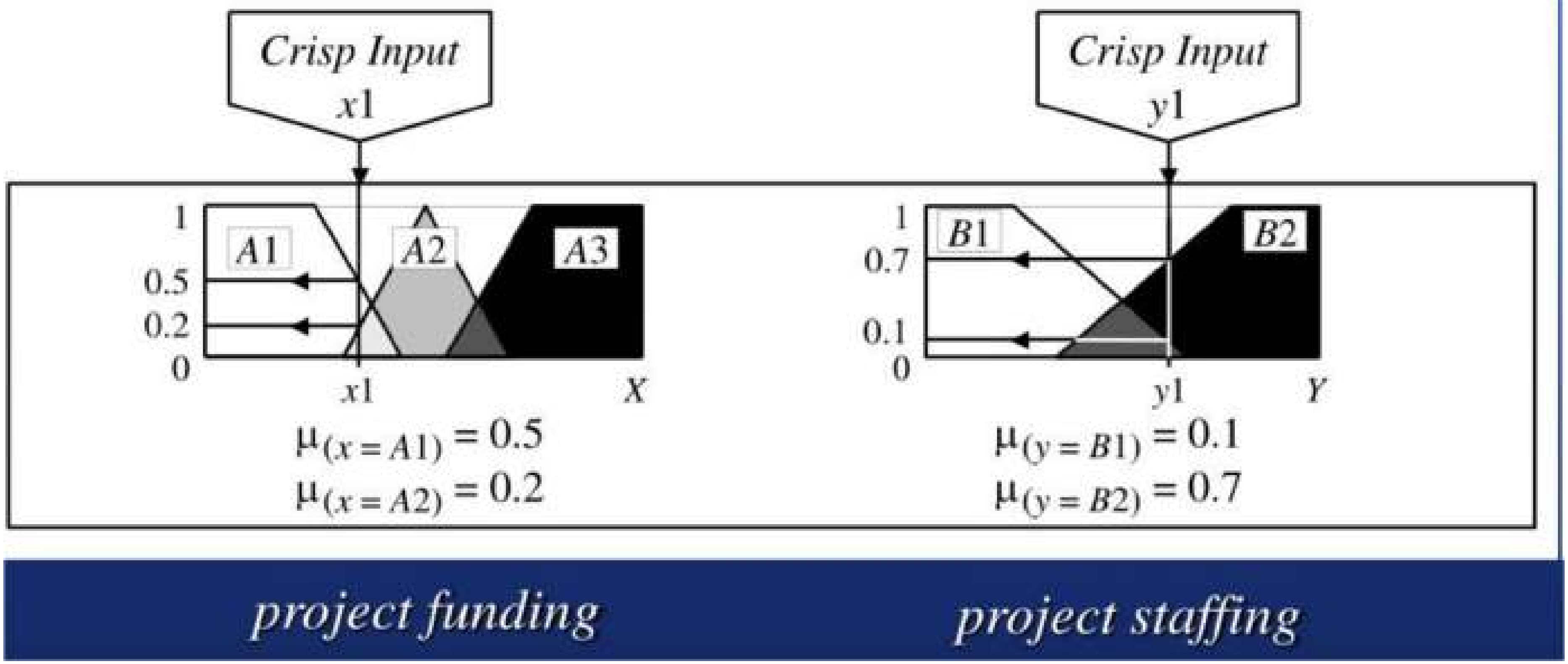


We examine a simple two-input one-output problem that includes three rules:

Rule: 1	Rule: 1
IF x is $A3$	IF *project_funding* is *adequate*
OR y is $B1$	OR *project_staffing* is *small*
THEN z is $C1$	THEN *risk* is *low*
Rule: 2	Rule: 2
IF x is $A2$	IF *project_funding* is *marginal*
AND y is $B2$	AND *project_staffing* is *large*
THEN z is $C2$	THEN *risk* is *normal*
Rule: 3	Rule: 3
IF x is $A1$	IF *project_funding* is *inadequate*
THEN z is $C3$	THEN *risk* is *high*

Step 1: Fuzzification

- Take the crisp inputs, *x*1 and *y*1 (*project funding* and *project staffing*)
- Determine the degree to which these inputs belong to each of the appropriate fuzzy sets.



Step 2: Rule Evaluation

- take the fuzzified inputs, $\mu_{(x=A1)} = 0.5$, $\mu_{(x=A2)} = 0.2$, $\mu_{(y=B1)} = 0.1$ and $\mu_{(y=B2)} = 0.7$

- apply them to the antecedents of the fuzzy rules.

- If a given fuzzy rule has multiple antecedents, the fuzzy operator (AND or OR) is used to obtain a single number that represents the result of the antecedent evaluation. This number (the truth value) is then applied to the consequent membership function.

Step 3: Rule Evaluation

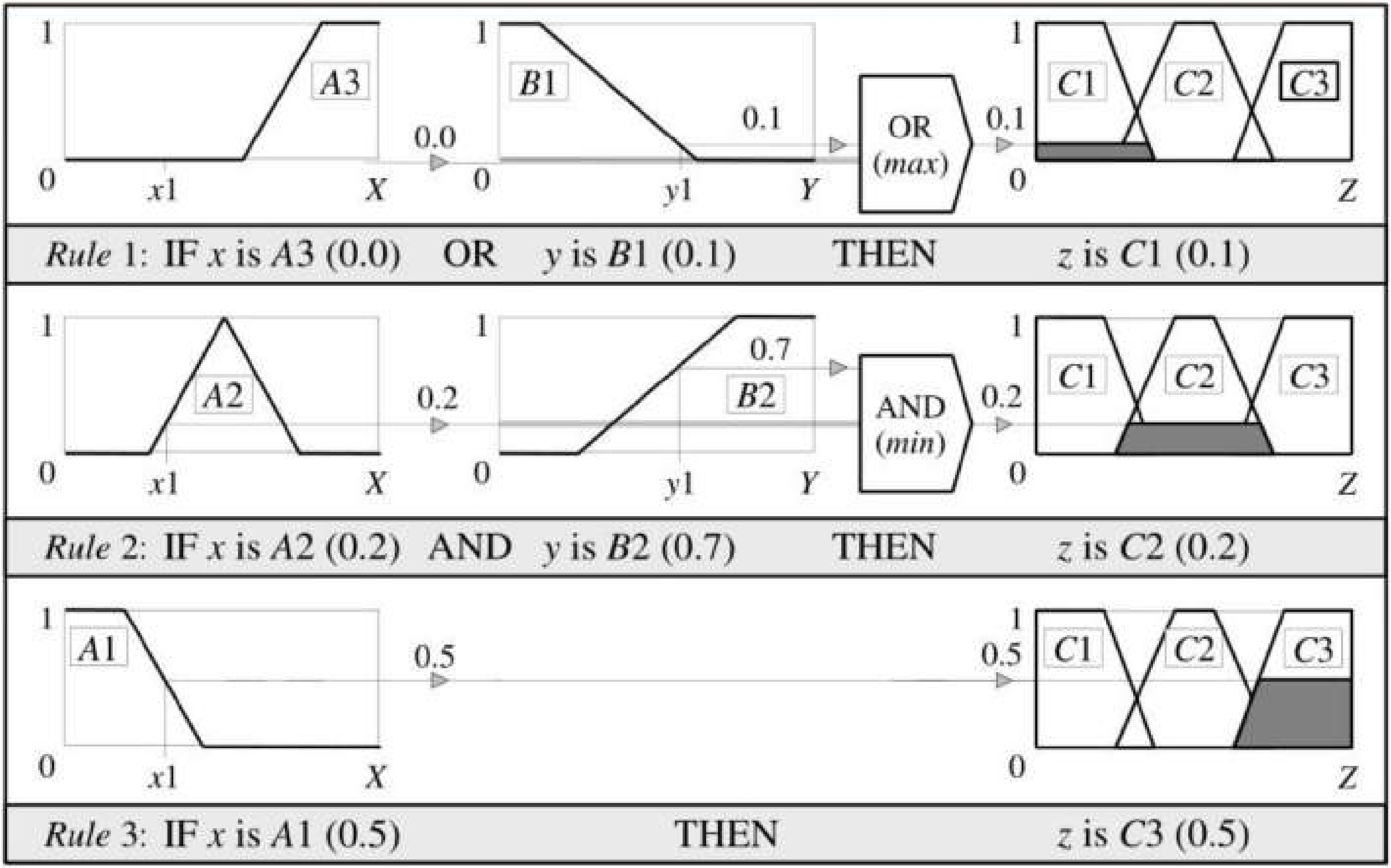
To evaluate the disjunction of the rule antecedents, we use the **OR fuzzy operation**. Typically, fuzzy expert systems make use of the classical fuzzy operation **union**:

$$\mu_{A \cup B}(x) = max\ [\mu_A(x), \mu_B(x)]$$

Similarly, in order to evaluate the conjunction of the rule antecedents, we apply the **AND fuzzy operation intersection**:

$$\mu_{A \cap B}(x) = min\ [\mu_A(x), \mu_B(x)]$$

Mamdani-style rule evaluation



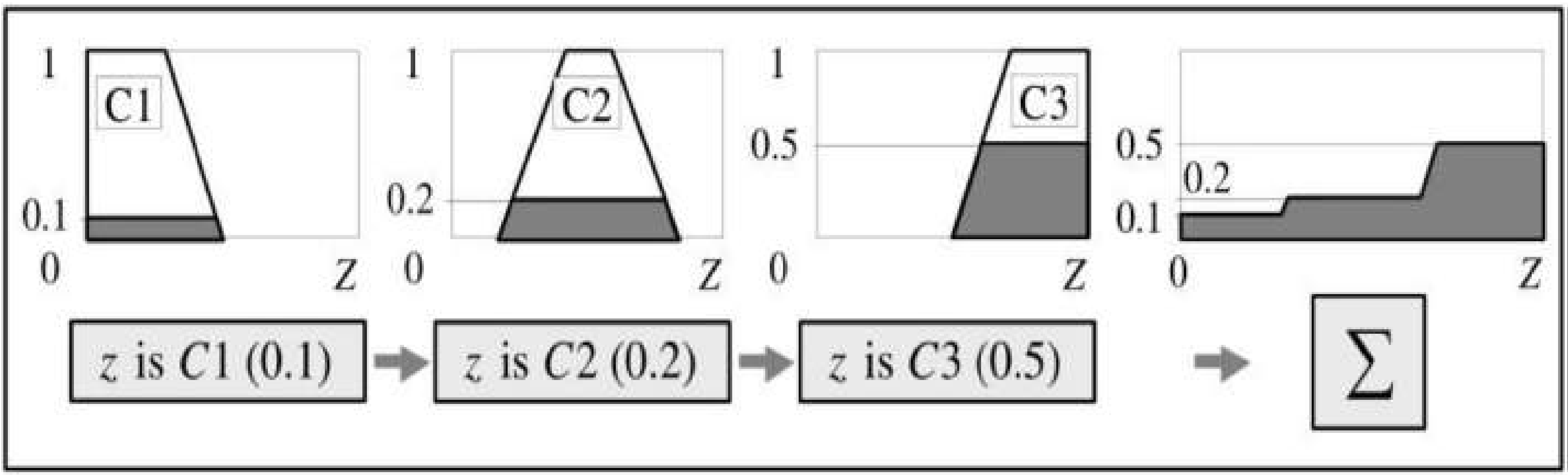
- Now the result of the antecedent evaluation can be applied to the membership function of the consequent.
- There are two main methods for doing so:
  - Clipping
  - Scaling
- The most common method is to cut the consequent membership function at the level of the antecedent truth.
- This method is called **clipping** (Max-Min Composition) .
  - The clipped fuzzy set loses some information.
  - Clipping is still often preferred because:
    - it involves less complex and faster mathematics
    - it generates an aggregated output surface that is easier to defuzzify.

Step 4: Aggregation of The Rule Outputs

- Aggregation is the process of unification of the outputs of all rules.

- We take the membership functions of all rule consequents previously clipped or scaled and combine them into a single fuzzy set.

Aggregation of the rule outputs



Step 5: Defuzzification

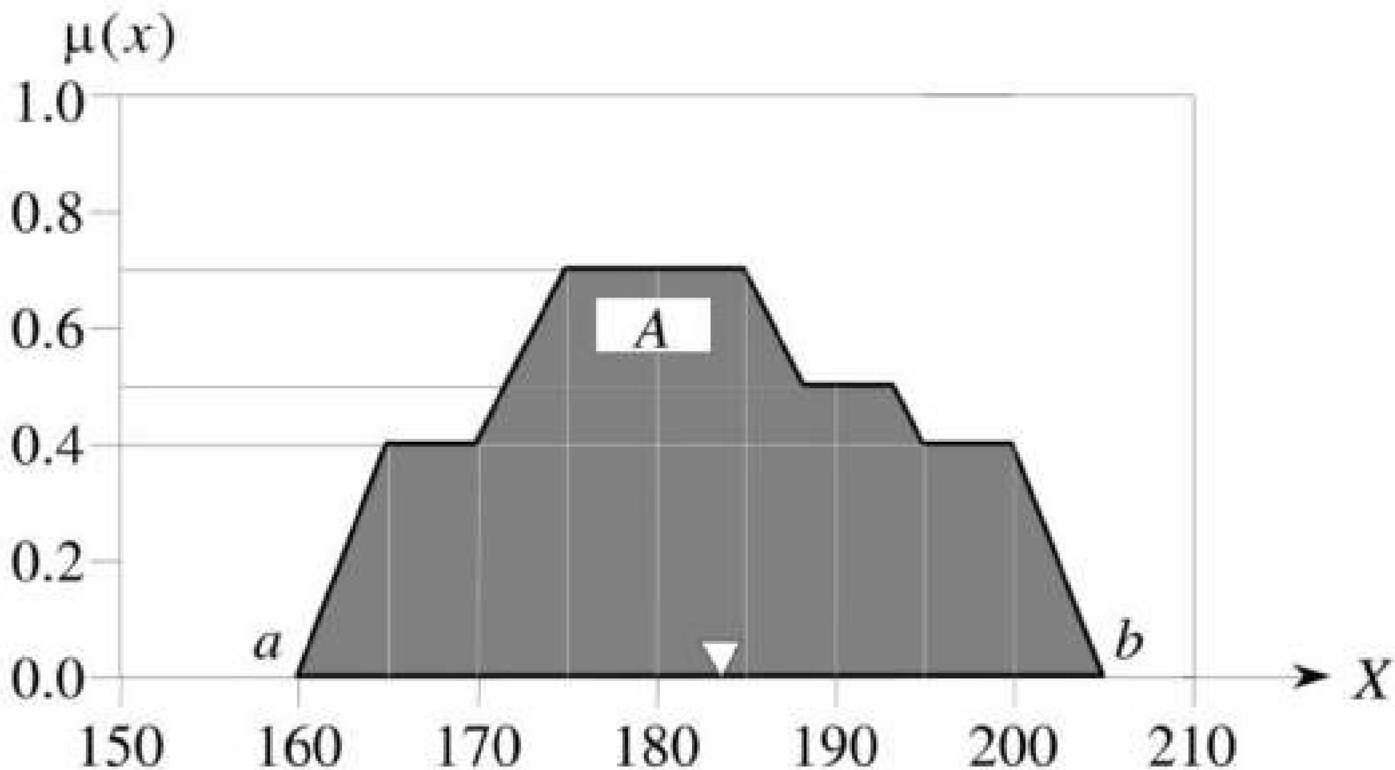
- Fuzziness helps us to evaluate the rules, but the final output of a fuzzy system has to be a crisp number.

- The input for the defuzzification process is the aggregated output fuzzy set and the output is a single number.

- There are several defuzzification methods, but probably the most popular one is the **centroid technique**.
- It finds the point where a vertical line would slice the aggregate set into two equal masses. Mathematically this **centre of gravity (COG)** can be expressed as:

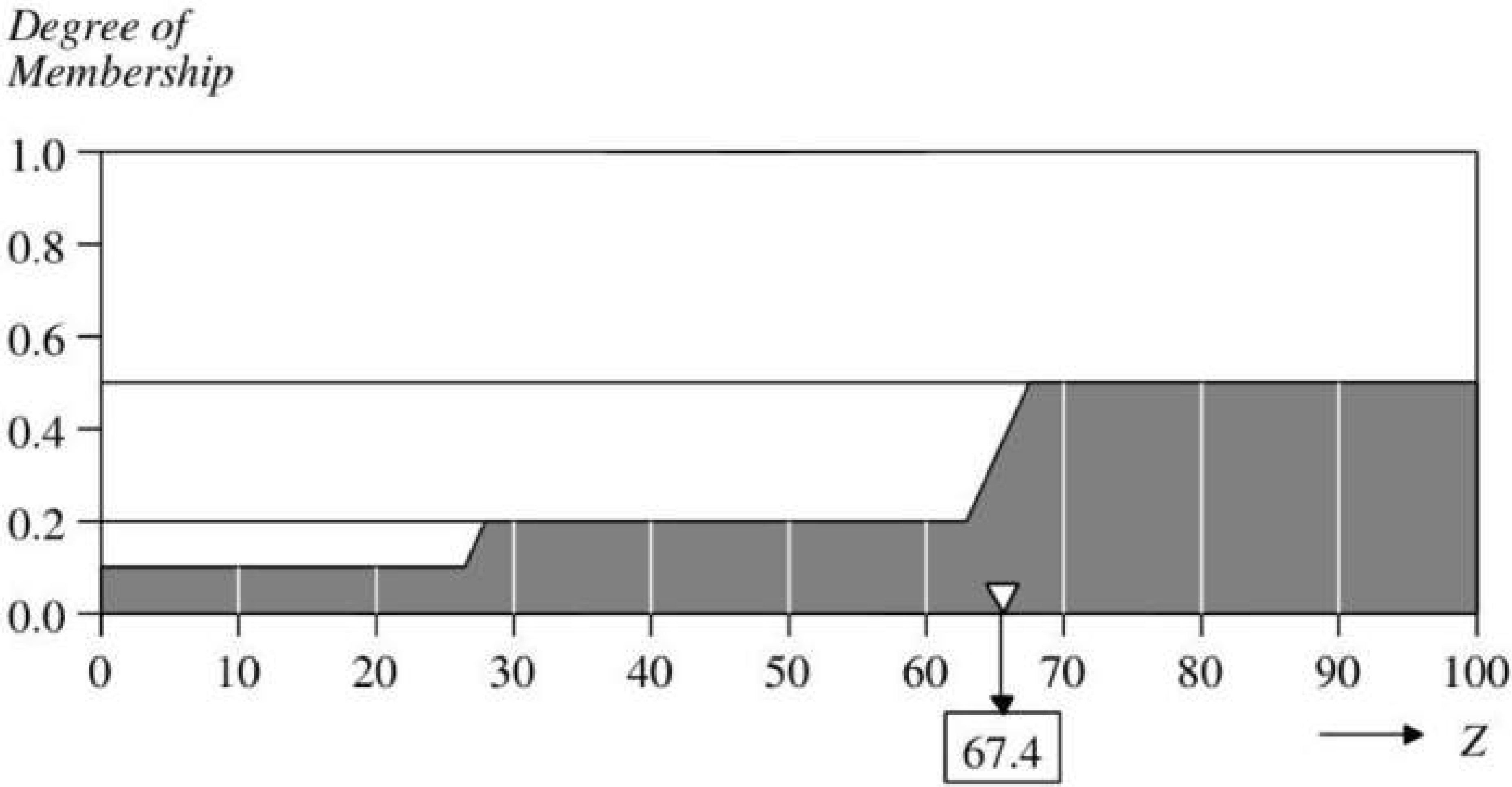
$$COG = \frac{\int_a^b \mu_A(x)\, x\, dx}{\int_a^b \mu_A(x)\, dx}$$

- Centroid defuzzification method finds a point representing the centre of gravity of the fuzzy set, *A*, on the interval, *ab*.
- ***A reasonable estimate*** can be obtained by calculating it over a sample of points.



Centre of gravity (COG)

$$COG = \frac{(0+10+20)\times 0.1 + (30+40+50+60)\times 0.2 + (70+80+90+100)\times 0.5}{0.1+0.1+0.1+0.2+0.2+0.2+0.2+0.5+0.5+0.5+0.5} = 67.4$$



Sugeno Fuzzy Control

- Mamdani-style inference, requires to find the centroid of a two-dimensional shape
  - by integrating across a continuously varying function.
  - In general, this process is not computationally efficient.
- **Michio Sugeno** suggested to use a single spike, a *singleton*, as the membership function of the rule consequent.
- A **fuzzy singleton**, is a fuzzy set with a membership function that is unity at a single particular point on the universe of discourse and zero everywhere else.
- Sugeno-style fuzzy inference is very similar to the Mamdani method.
- Sugeno changed only a rule consequent. Instead of a fuzzy set, he used a mathematical function of the input variable.

- The format of the **Sugeno-style fuzzy rule** is

  IF x is A AND y is B
  THEN z is $f(x, y)$

- where x, y and z are linguistic variables
- A and B are fuzzy sets on universe of discourses X and Y
- $f(x, y)$ is a mathematical function

The most commonly used **zero-order Sugeno fuzzy model** applies fuzzy rules in the following form:

IF x is A AND y is B
THEN z is k

where k is a constant.

- In this case, the output of each fuzzy rule is constant.
- All consequent membership functions are represented by singleton spikes.

Fuzzy Rules of TSK Model

If x is A and y is B then $z = f(x, y)$

Fuzzy Sets — Crisp Function

$f(x, y)$ is very often a polynomial function w.r.t. x and y.

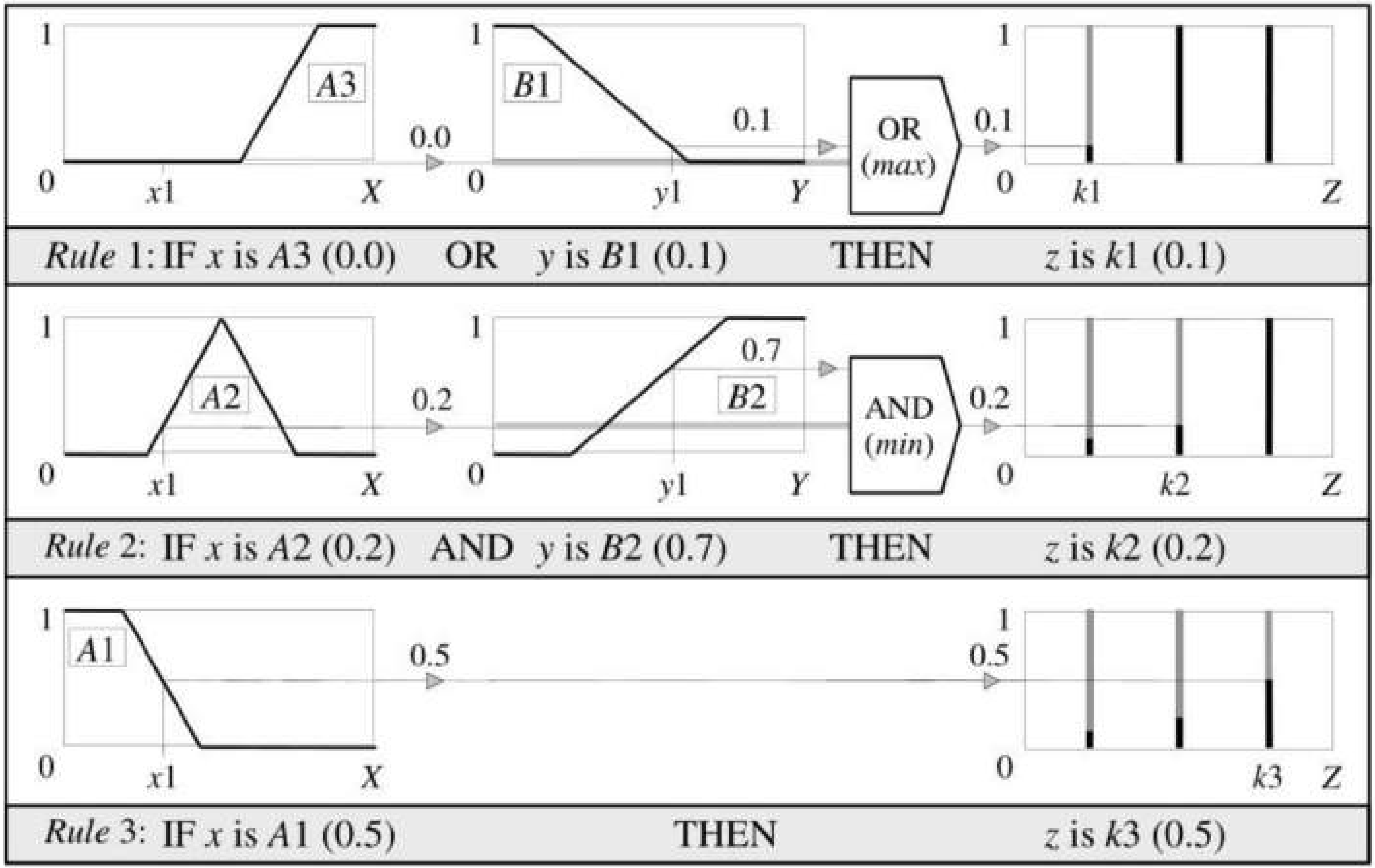
Examples

R1: if X is small and Y is small then $z = -x + y + 1$

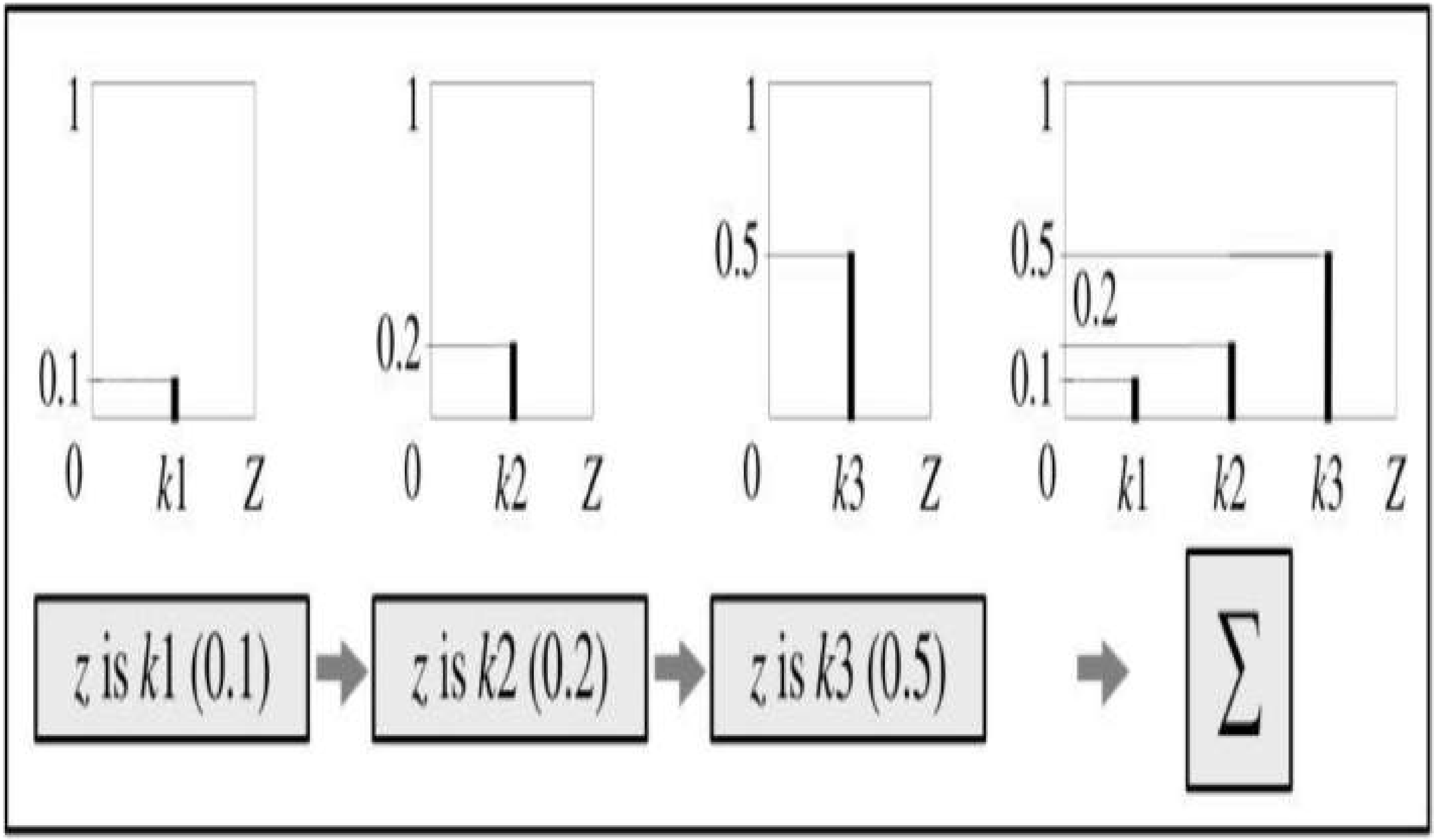
R2: if X is small and Y is large then $z = -y + 3$

R3: if X is large and Y is small then $z = -x + 3$

Sugeno-style rule evaluation



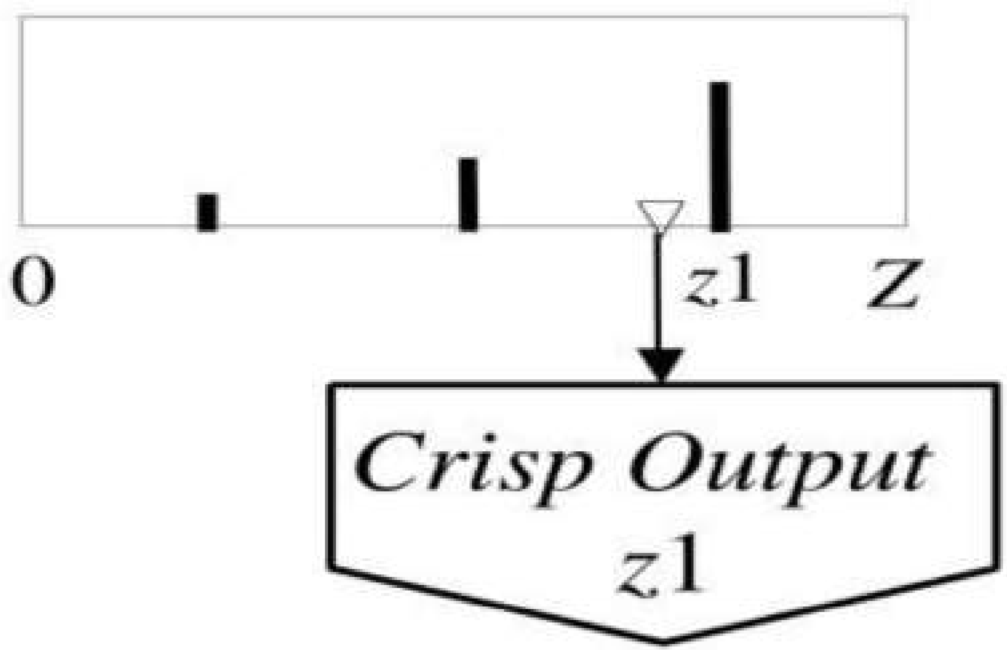
Sugeno-style aggregation of the rule outputs



Weighted average (WA)

$$WA = \frac{\mu(k1)\times k1 + \mu(k2)\times k2 + \mu(k3)\times k3}{\mu(k1)+\mu(k2)+\mu(k3)} = \frac{0.1\times 20 + 0.2\times 50 + 0.5\times 80}{0.1+0.2+0.5} = 65$$

Sugeno-style defuzzification



Review Fuzzy Models

If <antecedence> then <consequence>.

The same style for
- Mamdani Fuzzy Models
- Larsen Fuzzy Models
- Sugeno Fuzzy Models
- Tsukamoto Fuzzy Models

Different styles for
- Mamdani Fuzzy Models
- Larsen Fuzzy Models
- Sugeno Fuzzy Models
- Tsukamoto Fuzzy models

Comparisons between Mamdani and Sugeno

Advantages of the Mamdani Method

It is intuitive.

It has widespread acceptance.

It is well suited to human input.

Advantages of the Sugeno Method

It is computationally efficient.

It works well with linear techniques (e.g., PID control).

It works well with optimization and adaptive techniques.

It has guaranteed continuity of the output surface.

It is well suited to mathematical analysis.

Tuning Fuzzy Systems

1. Review model input and output variables, and if required redefine their ranges.

2. Review the fuzzy sets, and if required define additional sets on the universe of discourse.
   - The use of wide fuzzy sets may cause the fuzzy system to perform roughly.

3. Provide sufficient overlap between neighbouring sets.
   - It is suggested that triangle-to-triangle and trapezoid-to-triangle fuzzy sets should overlap between 25% to 50% of their bases.

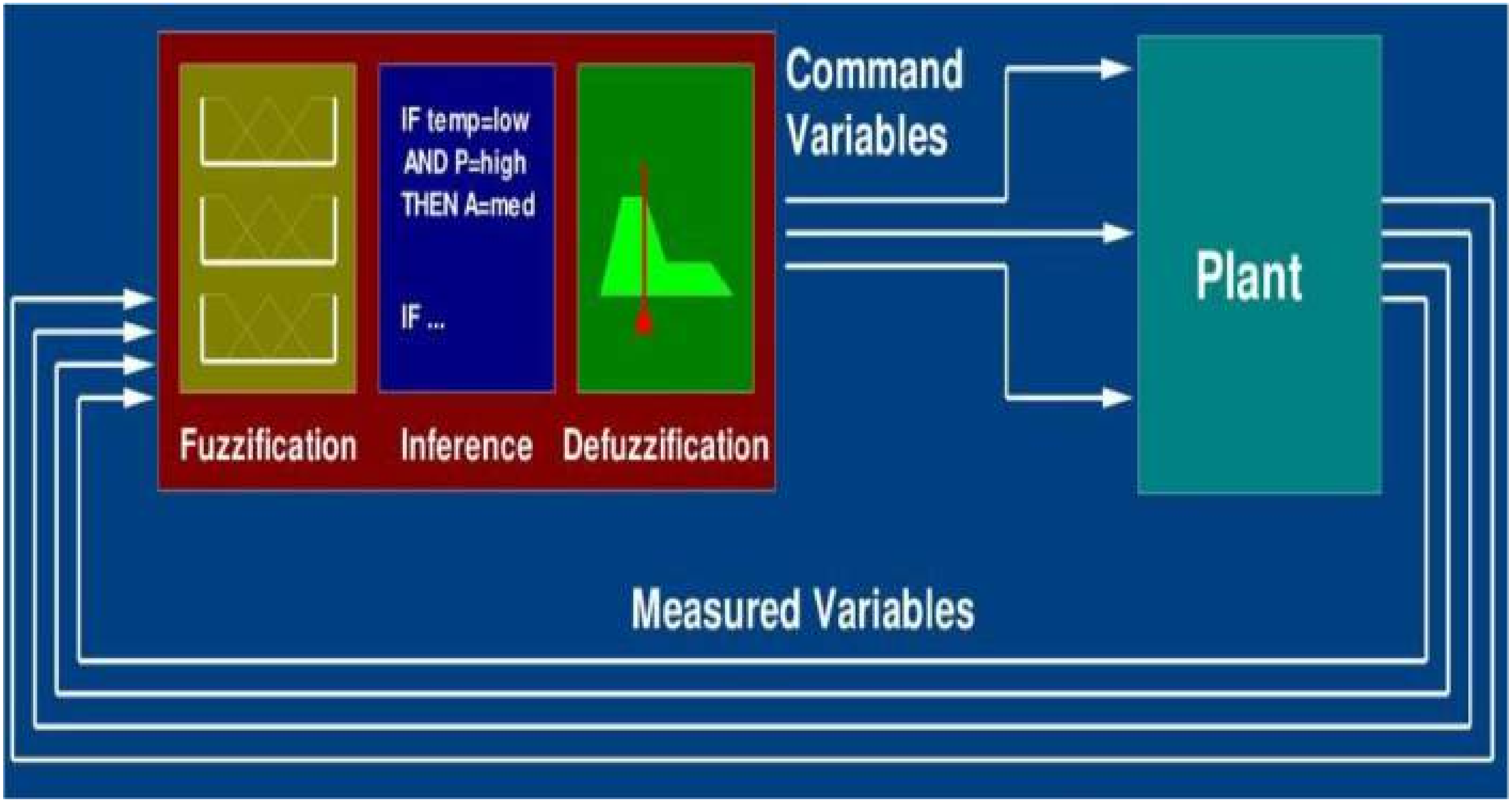
4. Review the existing rules, and if required add new rules to the rule base.

5. Adjust the rule execution weights. Most fuzzy logic tools allow control of the importance of rules by changing a weight multiplier.

6. Revise shapes of the fuzzy sets. In most cases, fuzzy systems are highly tolerant of a shape approximation.

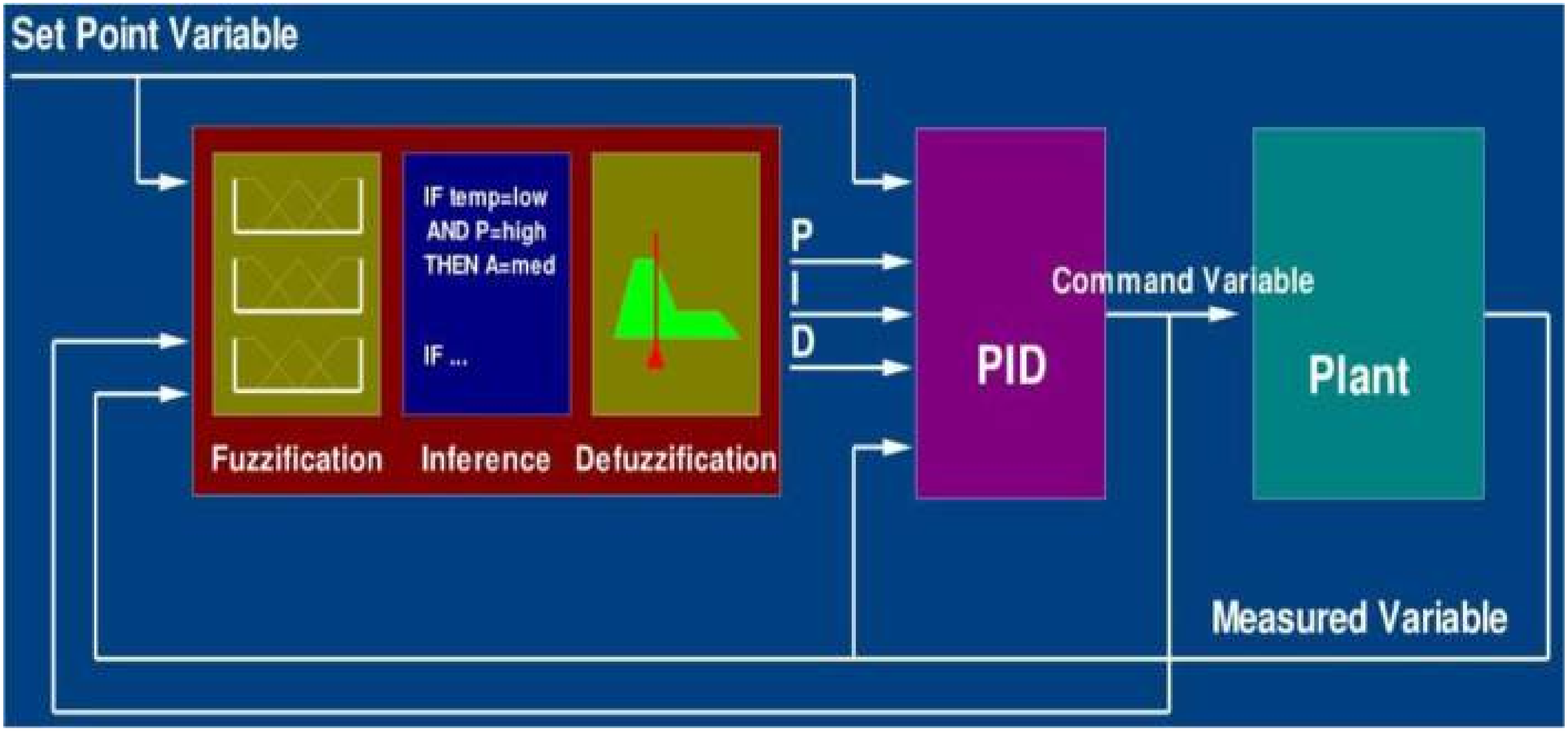
Types of Fuzzy Controllers:
- Direct Controller -

The Outputs of the Fuzzy Logic System Are the Command Variables of the Plant:



- PID Adaptation -

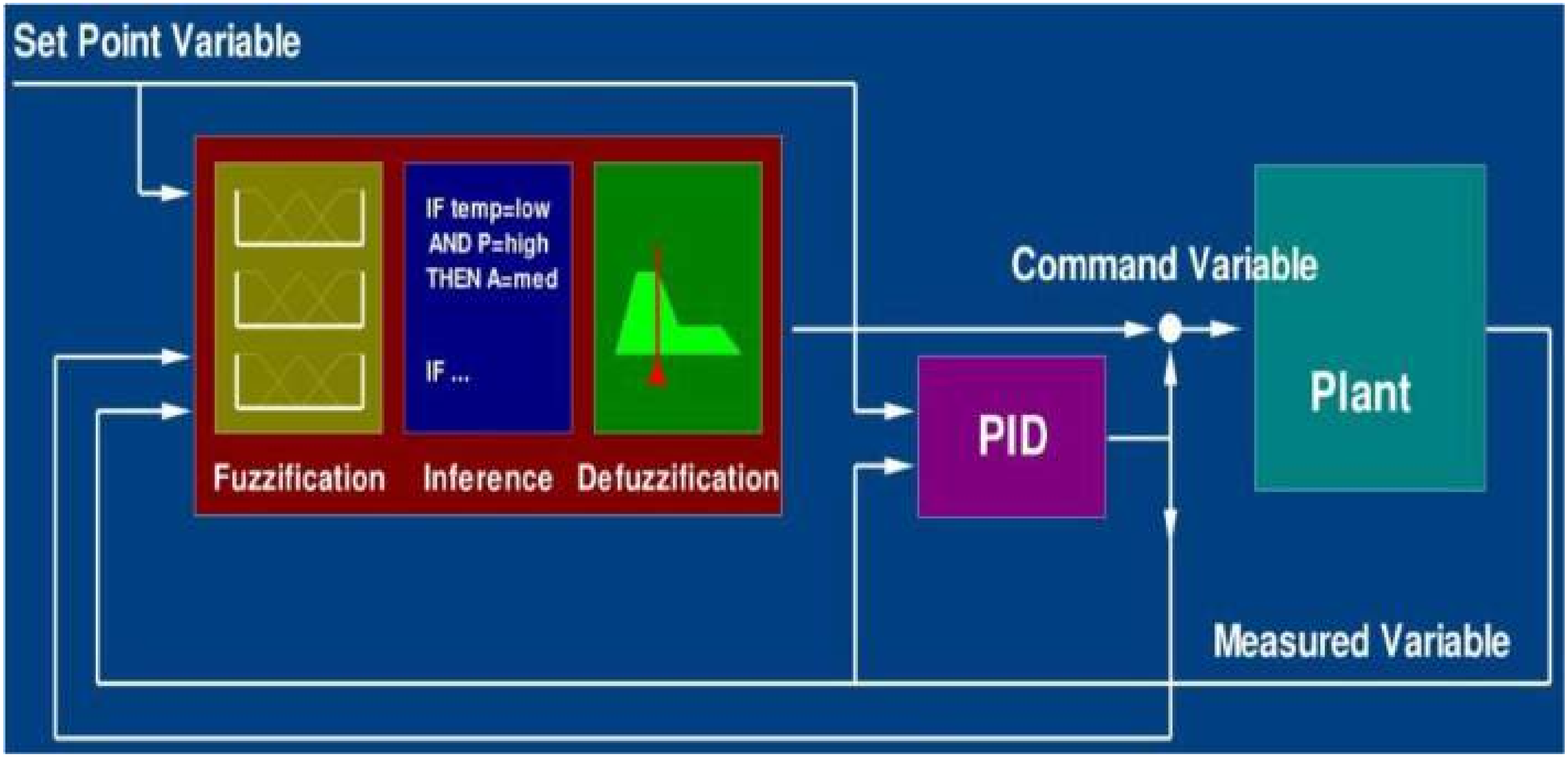
Fuzzy Logic Controller Adapts the P, I, and D Parameter of a Conventional PID Controller:



The Fuzzy Logic System Analyzes the Performance of the PID Controller and Optimizes It !

- Fuzzy Intervention -

Fuzzy Logic Controller and PID Controller in Parallel:



Intervention of the Fuzzy Logic Controller into Large Disturbances !