Lecture Notes on Advanced

Adaptive Control

4th Year Petroleum Systems

and Control Engineering

Tikrit University

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Example 1: The plant to be controlled has the transfer function

$$G(S) = \frac{(S+0.5)}{(S+1)(S-1)}$$

1) stabilize it with feedback
2) Draw block diagram model-reference adaptive inverse control of a stabilized minimum-phase plant.

Solution:

This plant is minimum-phase and is unstable. The first step is to stabilize it with feedback. A root-locus diagram is shown in Fig. 1. It is clear from this diagram that the plant can be stabilized by making use of the simple unity feedback system of Fig. 2, by setting the loop gain within the stable range $\infty > k > 2$. The loop gain was set to $k = 4$ for this control experiment. The closed loop transfer function is minimum-phase and has two poles in the left half of the s-plane.

The plant and its stabilization are continuous (analog) systems. The adaptive inverse control part, as it would be in the real world, is discrete (digital). A diagram of the complete system is shown in Fig. 3, including the necessary analog-to-digital conversion (ADC) and digital-to-analog conversion (DAC) components. The command input is sampled and is fed to both the inverse controller and the reference model. The controller output is converted to analog form, using a zero-order hold, to drive the plant and its stabilization loop. The error signal used to adapt the inverse controller is discrete. This is the difference between the reference model output and the sampled plant output

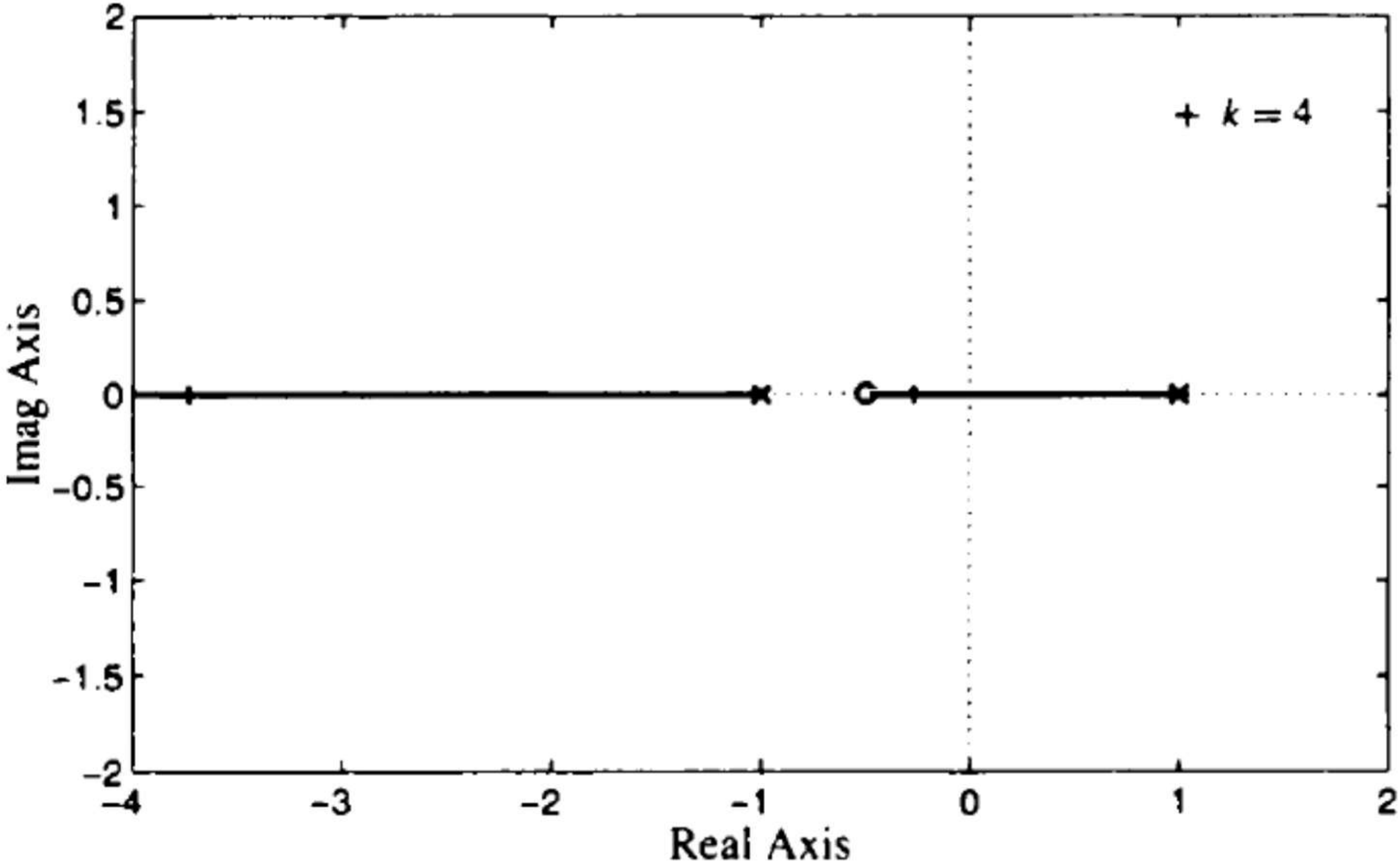


Fig. 1: Root-locus of minimum-phase plant with proportional feedback

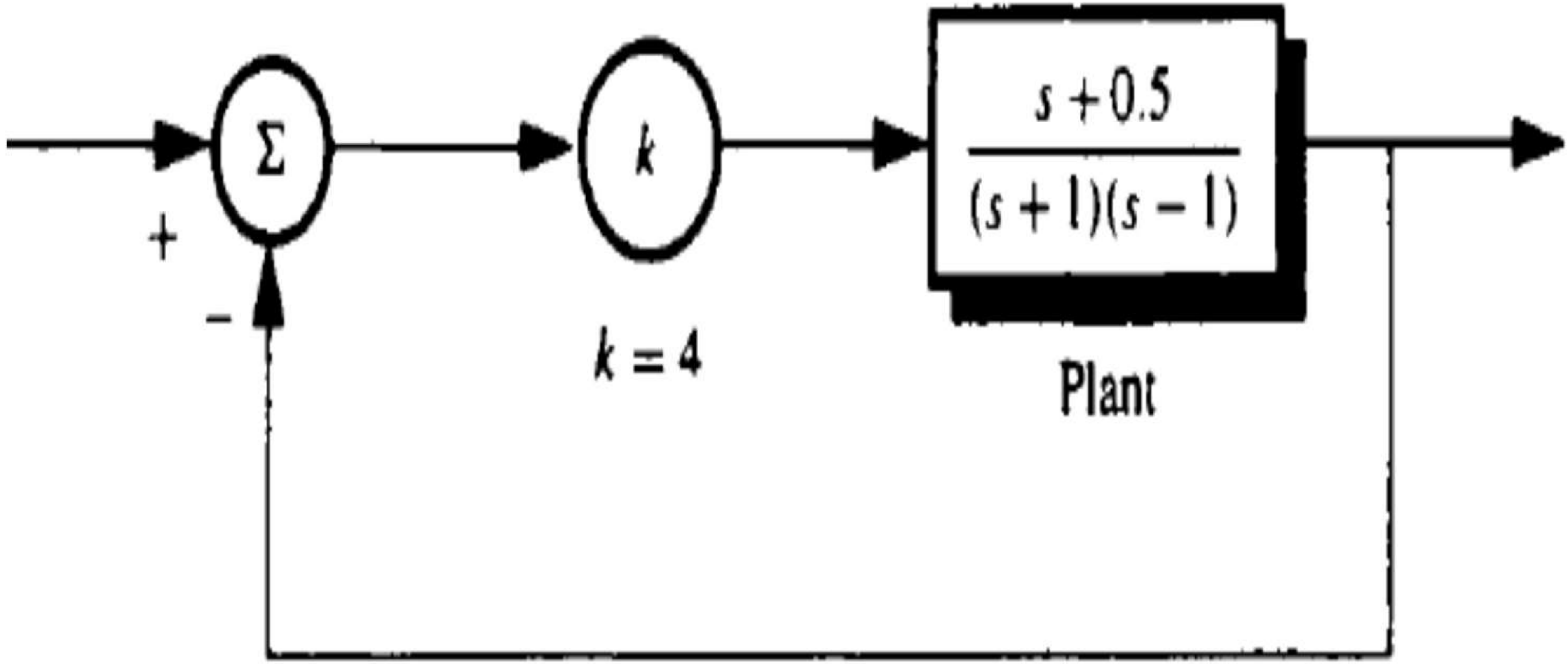


Fig. 2: Minimum-phase plant stabilized with proportional feedback, k=4

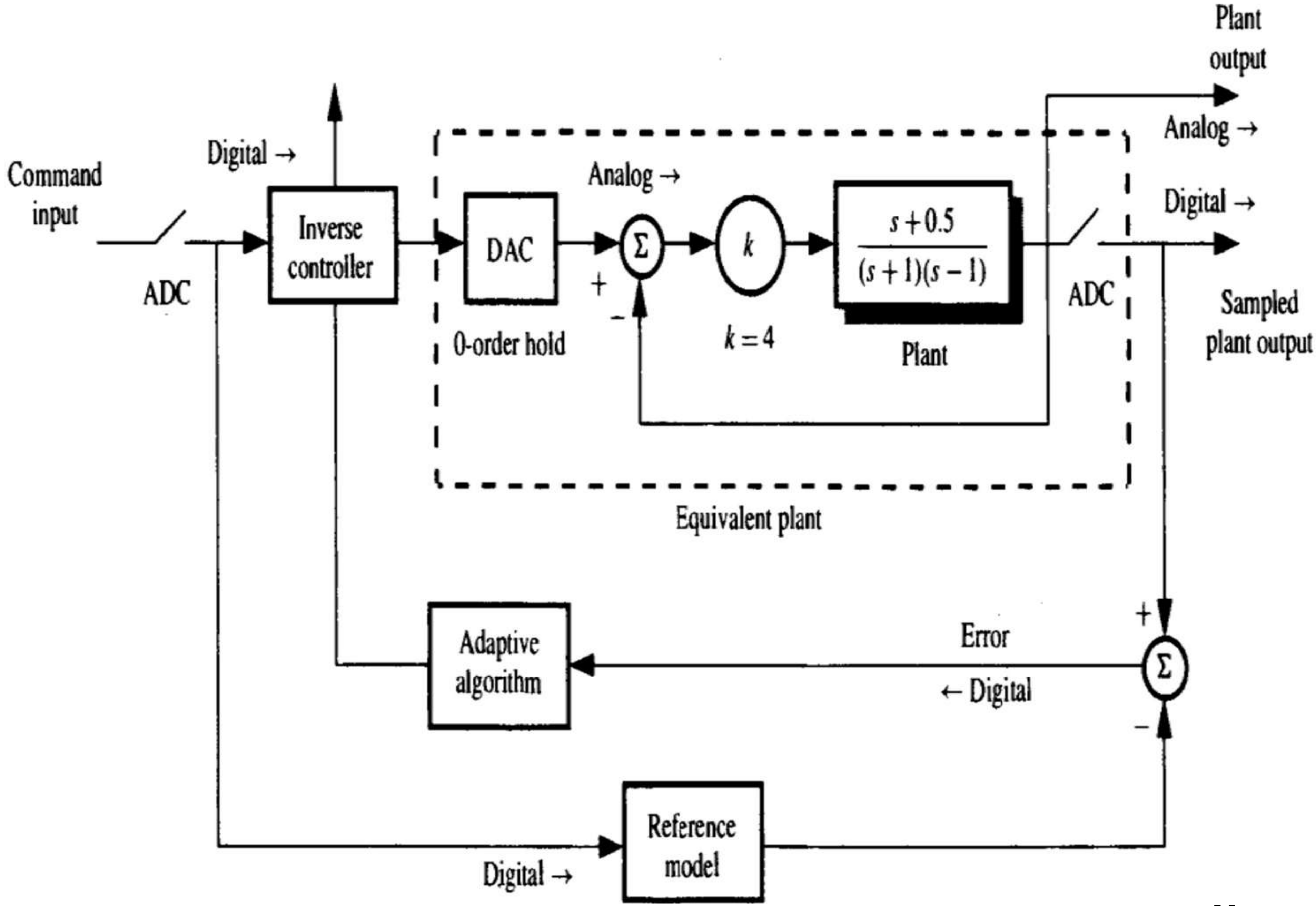


Fig. 3: Model-reference adaptive inverse control of a stabilized minimum-phase plant.

Example 2: The plant to be controlled has the transfer function

$$G(S) = \frac{(S+0.5)}{(S+1)(S-1)}$$

1) stabilize it with feedback
2) Draw block diagram of Model-reference inverse control system for minimum-phase plant with adaptive plant disturbance canceler.

Solution:

This plant is minimum-phase and is unstable. The first step is to stabilize it with feedback. A root-locus diagram is shown in Fig. 4. It is clear from this diagram that the plant can be stabilized by making use of the simple unity feedback system of Fig. 5, by setting the loop gain within the stable range $\infty > k > 2$. The loop gain was set to $k = 4$ for this control experiment. The closed loop transfer function is minimum-phase and has two poles in the left half of the s-plane.

The plant and its stabilization are continuous (analog) systems. The adaptive inverse control part, as it would be in the real world, is discrete (digital). A diagram of Model-reference inverse control system for minimum-phase plant with adaptive plant disturbance canceler is shown in Fig. 6, including the necessary analog-to-digital conversion (ADC) and digital-to-analog conversion (DAC) components. The command input is sampled and is fed to both the inverse controller and the reference model. The controller output is converted to analog form, using a zero-order hold, to drive the plant and its stabilization loop. The error signal used to adapt the inverse controller is discrete. This is the difference between the reference model output and the sampled plant output

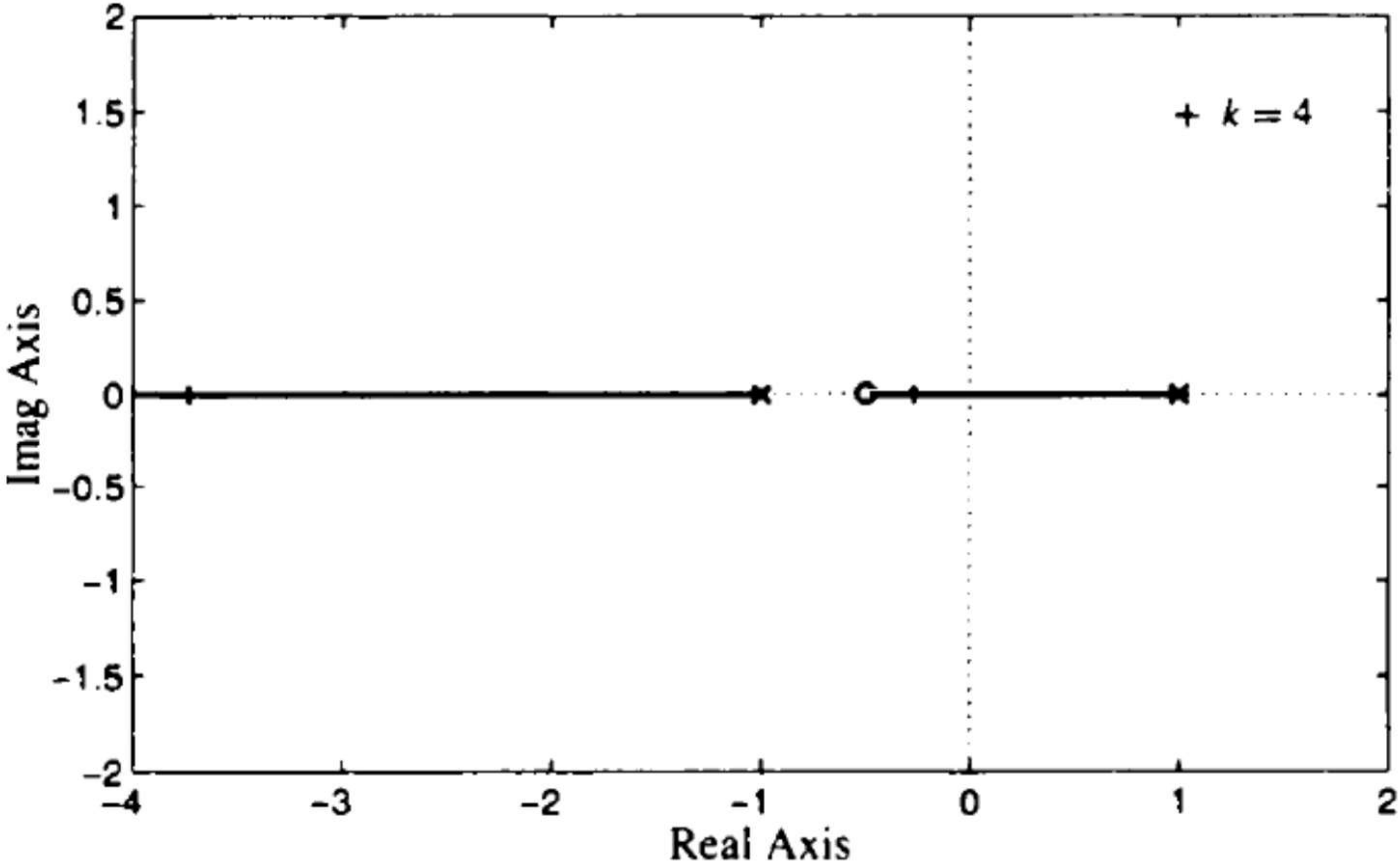


Fig. 4: Root-locus of minimum-phase plant with proportional feedback

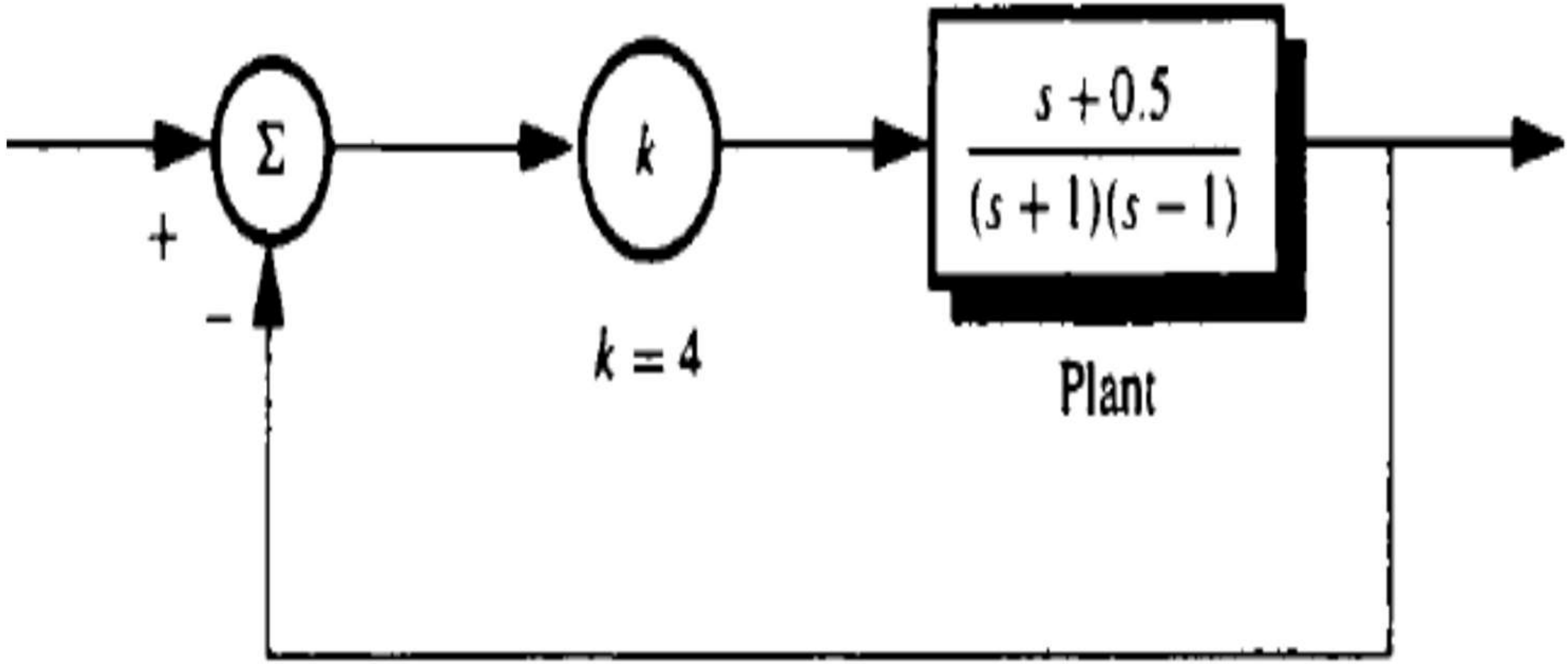


Fig. 5: Minimum-phase plant stabilized with proportional feedback, k=4

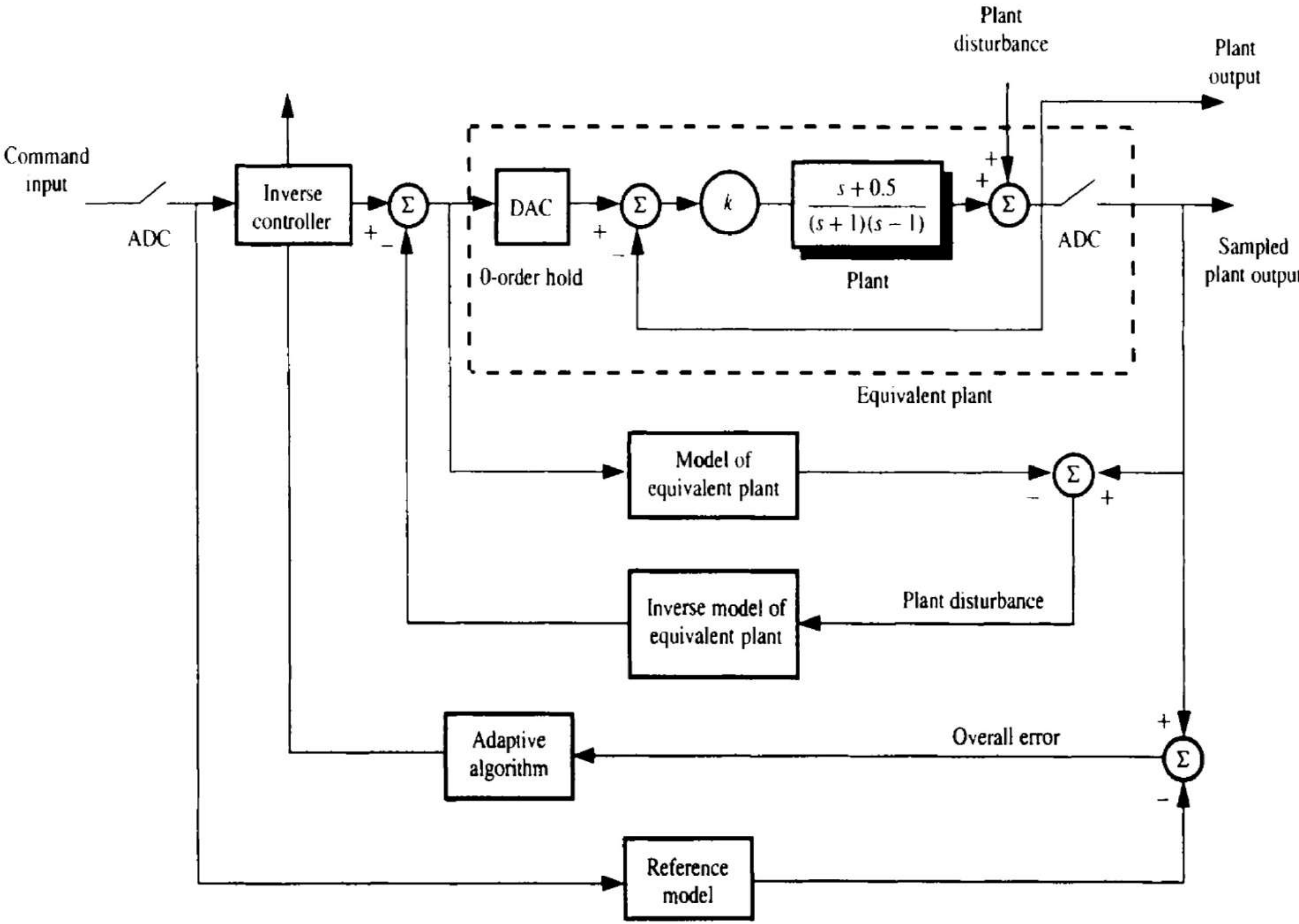


Fig. 6: Model-reference inverse control system for minimum-phase plant with adaptive plant disturbance canceler

L1 Adaptive Control Concept

Direct Model Reference Adaptive Control (MRAC)

Let the system dynamics propagate according to the following differential equation:

$$\begin{aligned} \dot{x}(t) &= A_m x(t) + b\left(u(t) + k_x^\top x(t)\right), \quad x(0) = x_0, \\ y(t) &= c^\top x(t), \end{aligned} \tag{1.1}$$

where $x(t) \in \mathbb{R}^n$ is the state of the system (measured), $A_m \in \mathbb{R}^{n \times n}$ is a known Hurwitz matrix that defines the desired dynamics for the closed-loop system, $b, c \in \mathbb{R}^n$ are known constant vectors, $k_x \in \mathbb{R}^n$ is a vector of unknown constant parameters, $u(t) \in \mathbb{R}$ is the control input, and $y(t) \in \mathbb{R}$ is the regulated output. Given a uniformly bounded piecewise-continuous reference input $r(t) \in \mathbb{R}$, the objective is to define an adaptive feedback signal $u(t)$ such that $y(t)$ tracks $r(t)$ with desired specifications, while all the signals remain bounded.

The MRAC architecture proceeds by considering the nominal controller

$$u_{\text{nom}}(t) = -k_x^\top x(t) + k_g r(t), \tag{1.2}$$

$$k_g \triangleq \frac{1}{c^\top A_m^{-1} b}. \tag{1.3}$$

This nominal controller assumes perfect cancelation of the uncertainties in (1.1) and leads to the desired (ideal) reference system

$$\begin{aligned} \dot{x}_m(t) &= A_m x_m(t) + b k_g r(t), \quad x_m(0) = x_0, \\ y_m(t) &= c^\top x_m(t), \end{aligned} \tag{1.4}$$

where $x_m(t) \in \mathbb{R}^n$ is the state of the reference model. The choice of k_g according to (1.3) ensures that $y_m(t)$ tracks step reference inputs with zero steady-state error.

The direct model reference adaptive controller is given by

$$u(t) = -\hat{k}_x^\top(t)x(t) + k_g r(t), \tag{1.5}$$

where $\hat{k}_x(t) \in \mathbb{R}^n$ is the estimate of k_x. Substituting (1.5) into (1.1) yields the closed-loop system dynamics

$$\dot{x}(t) = (A_m - b\tilde{k}_x^\top(t))x(t) + bk_g r(t), \quad x(0) = x_0,$$

$$y(t) = c^\top x(t),$$

where $\tilde{k}_x(t) \triangleq \hat{k}_x(t) - k_x$ denotes the parametric estimation error.

Letting $e(t) \triangleq x_m(t) - x(t)$ be the tracking error signal, the tracking error dynamics can be written as

$$\dot{e}(t) = A_m e(t) + b\tilde{k}_x^\top(t)x(t), \quad e(0) = 0. \tag{1.6}$$

The update law for the parametric estimate is given by

$$\dot{\hat{k}}_x(t) = -\Gamma x(t) e^\top(t) P b, \quad \hat{k}_x(0) = k_{x0}, \qquad (1.7)$$

where $\Gamma \in \mathbb{R}^+$ is the adaptation gain and $P = P^\top > 0$ solves the algebraic Lyapunov equation

$$A_m^\top P + P A_m = -Q$$

for arbitrary $Q = Q^\top > 0$.

Consider the following Lyapunov function candidate:

$$V(e(t), \tilde{k}_x(t)) = e^\top(t) P e(t) + \frac{1}{\Gamma} \tilde{k}_x^\top(t) \tilde{k}_x(t). \qquad (1.8)$$

The block diagram of the closed-loop system is given in fig. 7.

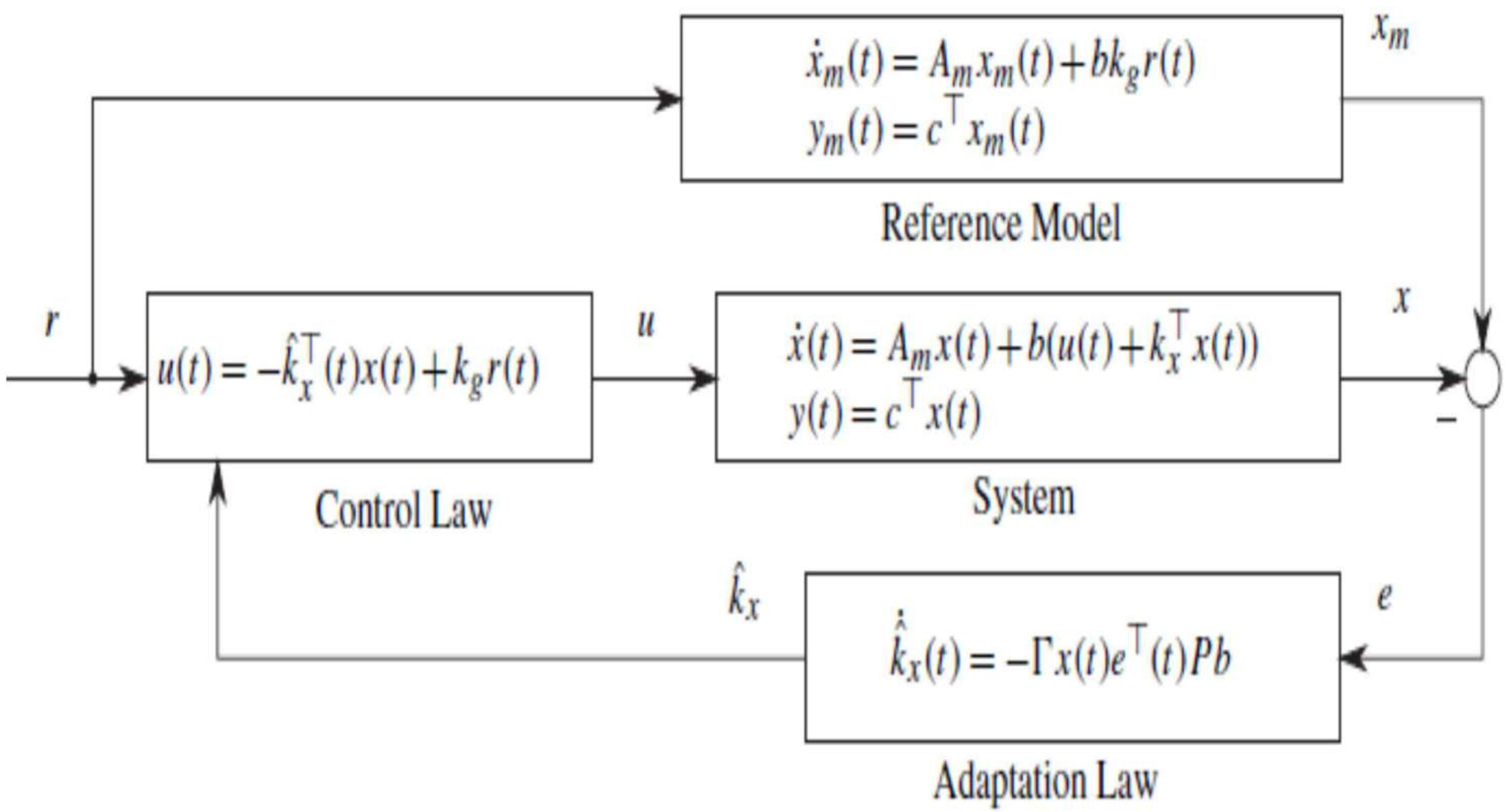


Figure 7: Closed-loop direct MRAC architecture

Direct MRAC with State Predictor

Next, we consider a re-parameterization of the above architecture using a state predictor (or identifier), given by

$$\begin{aligned} \dot{\hat{x}}(t) &= A_m \hat{x}(t) + b(u(t) + \hat{k}_x^\top(t)x(t)), \quad \hat{x}(0) = x_0, \\ \hat{y}(t) &= c^\top \hat{x}(t), \end{aligned} \tag{1.9}$$

where $\hat{x}(t) \in \mathbb{R}^n$ is the state of the predictor. The system in (1.9) replicates the system structure from (1.1) with the unknown parameter k_x replaced by its estimate $\hat{k}_x(t)$. By subtracting (1.1) from (1.9), we obtain the *prediction error dynamics* (or identification error dynamics), independent of the control choice,

$$\dot{\tilde{x}}(t) = A_m \tilde{x}(t) + b\tilde{k}_x^\top(t)x(t), \quad \tilde{x}(0) = 0,$$

where $\tilde{x}(t) \triangleq \hat{x}(t) - x(t)$ and $\tilde{k}_x(t) \triangleq \hat{k}_x(t) - k_x$. Notice that these error dynamics are identical to the error dynamics in (1.6).

Next, let the adaptive law for $\hat{k}_x(t)$ be given as

$$\dot{\hat{k}}_x(t) = -\Gamma x(t)\tilde{x}^\top(t)Pb, \quad \hat{k}_x(0) = k_{x0}, \tag{1.10}$$

where $\Gamma \in \mathbb{R}^+$ is the adaptation rate and $A_m^\top P + PA_m = -Q$, $Q = Q^\top > 0$. This adaptive law is similar to (1.7) in its structure, except that the tracking error $e(t)$ is replaced by the prediction error $\tilde{x}(t)$. The choice of the Lyapunov function candidate

$$V(\tilde{x}(t), \tilde{k}_x(t)) = \tilde{x}^\top(t)P\tilde{x}(t) + \frac{1}{\Gamma}\tilde{k}_x^\top(t)\tilde{k}_x(t)$$

The block diagram of the closed-loop system with the predictor is given in Figure 7. Figures 6 and 7 illustrate the fundamental difference between the direct MRAC and the predictor-based adaptation. In Figure 7, the control signal is provided as input to both systems, the system and the predictor, while in Figure 6 the control signal serves only as input to the system. This feature is the key to the development of L1 adaptive control architectures with quantifiable performance bounds.

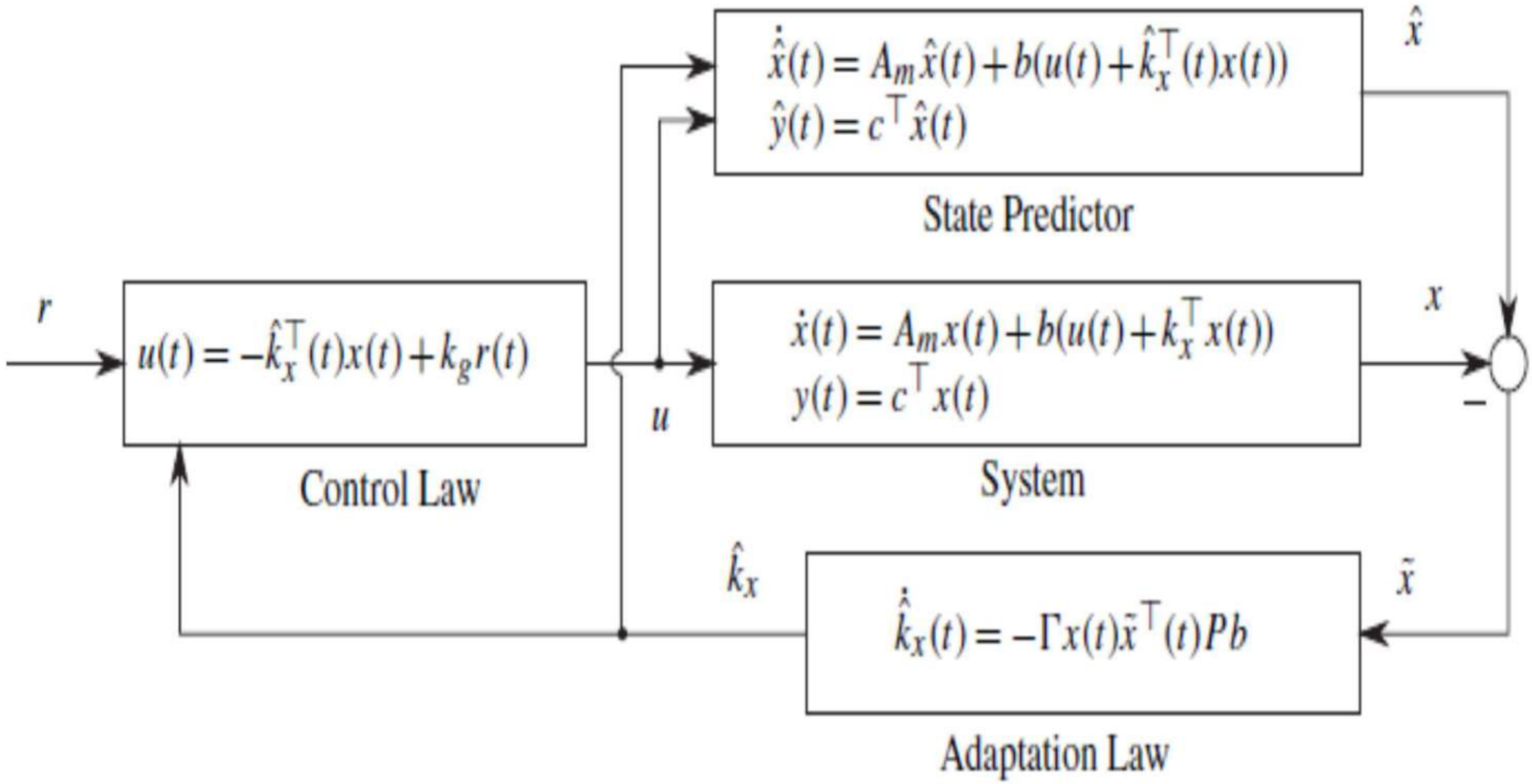


Figure 7: Closed-loop MRAC architecture with state predictor

Example 3: Drive the equations of L1 adaptive control and draw its block diagram for the following system:

$$\dot{x}(t) = Ax(t) + b(u(t) + \theta^\top x(t)), \quad x(0) = x_0,$$
$$y(t) = c^\top x(t), \tag{2.1}$$

where $x(t) \in \mathbb{R}^n$ is the system state vector (measured); $u(t) \in \mathbb{R}$ is the control signal; $b, c \in \mathbb{R}^n$ are known constant vectors; A is the known $n \times n$ matrix, with (A, b) controllable; θ is the unknown parameter, which belongs to a given compact convex set $\theta \in \Theta \subset \mathbb{R}^n$; and $y(t) \in \mathbb{R}$ is the regulated output. In this section we present an adaptive control solution, which ensures that the system output $y(t)$ follows a given piecewise-continuous bounded reference signal $r(t)$ with quantifiable transient and steady-state performance bounds.

the control structure

$$u(t) = u_m(t) + u_{ad}(t), \quad u_m(t) = -k_m^\top x(t), \tag{2.2}$$

$$u_{ad}(s) = -C(s)\left(\hat{\eta}(s) - k_g r(s)\right) \qquad \hat{\eta}(t) \triangleq \hat{\theta}^\top(t)x(t)$$

Solution: The closed-loop system:

$$\dot{x}(t) = A_m x(t) + b(\theta^\top x(t) + u_{\mathrm{ad}}(t)), \quad x(0) = x_0,$$
$$y(t) = c^\top x(t). \tag{2.3}$$

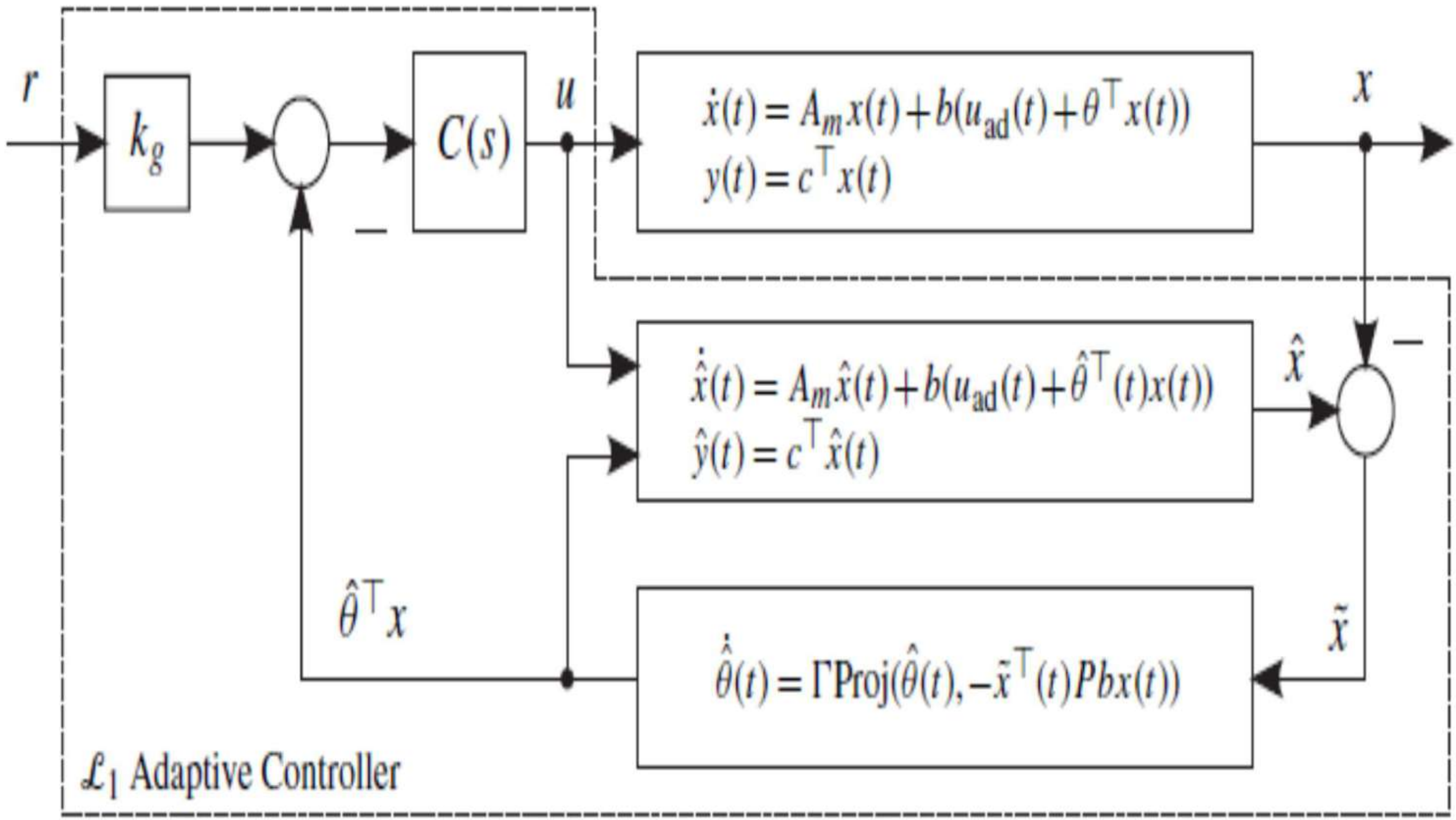
The state predictor:

$$\dot{\hat{x}}(t) = A_m \hat{x}(t) + b(\hat{\theta}^\top(t) x(t) + u_{\mathrm{ad}}(t)), \quad \hat{x}(0) = x_0,$$
$$\hat{y}(t) = c^\top \hat{x}(t), \tag{2.4}$$

where $\hat{x}(t) \in \mathbb{R}^n$ is the state of the predictor and $\hat{\theta}(t) \in \mathbb{R}^n$ is the estimate of the parameter θ, governed by the following projection-type adaptive law:

$$\dot{\hat{\theta}}(t) = \Gamma \mathrm{Proj}(\hat{\theta}(t), -\tilde{x}^\top(t) P b x(t)), \quad \hat{\theta}(0) = \hat{\theta}_0 \in \Theta, \tag{2.5}$$

where $\tilde{x}(t) \triangleq \hat{x}(t) - x(t)$ is the prediction error, $\Gamma \in \mathbb{R}^+$ is the adaptation gain, and $P = P^\top > 0$ solves the algebraic Lyapunov equation $A_m^\top P + P A_m = -Q$ for arbitrary symmetric $Q = Q^\top > 0$.



Closed-loop L1 adaptive system

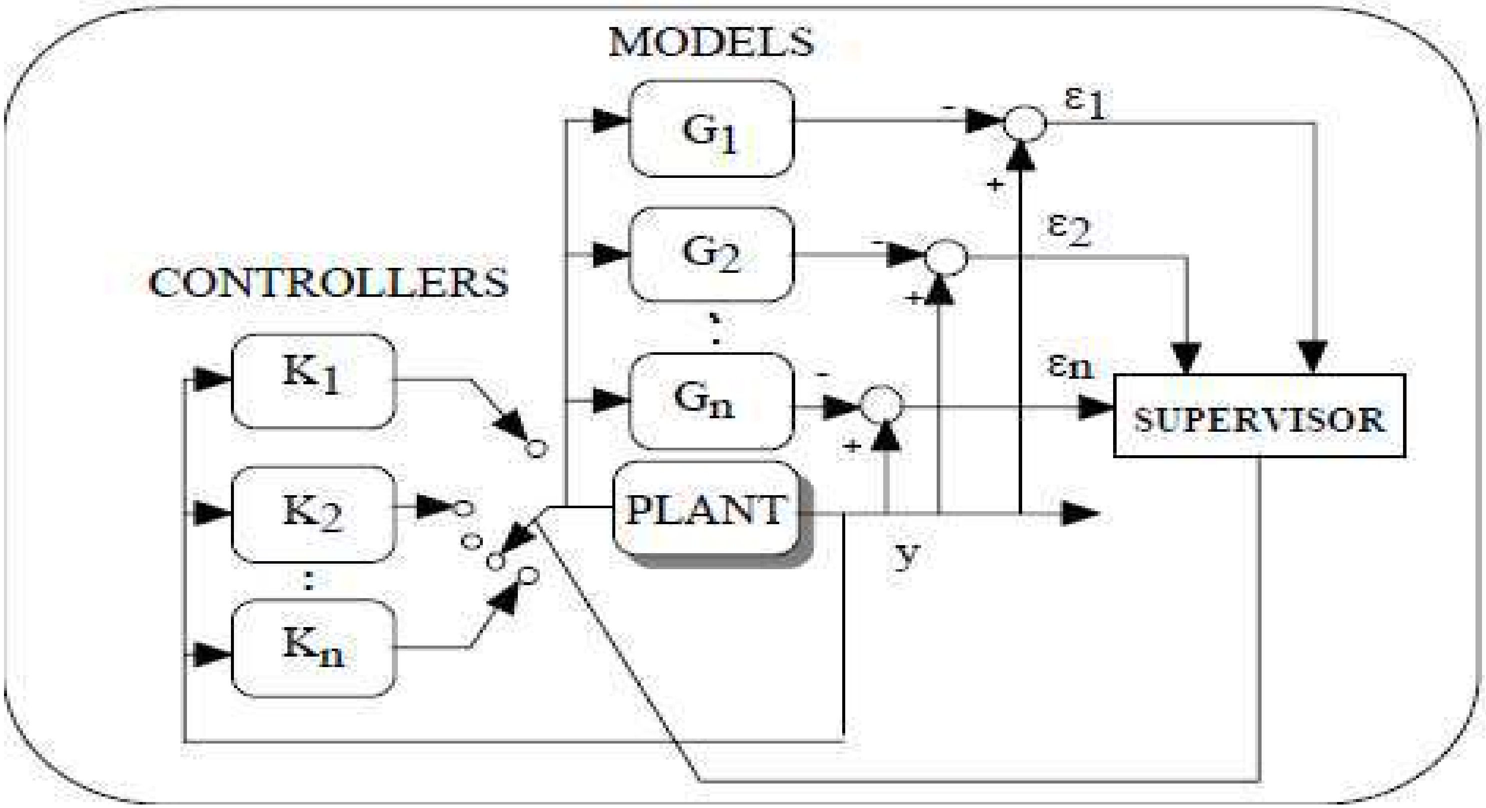
Supervisory Control

The “supervisor”:

•will check what “plant-model” error is minimum

•will switch to the controller associated with the selected model

Can provide a very fast decision (if there are not too many models) but not a fine tuning



Adaptive Control with Multiple Models

The supervisor select the best fixed model and then the adaptive model will be selected. Multiple fixed models: improvement of the adaptation transients. Adaptive plant model estimator(Closed Loop Output Error (CLOE) Estimator) :performance improvement.