Lecture Notes on Advanced

Adaptive Control

4^{th} Year Petroleum Systems

and Control Engineering

Tikrit University

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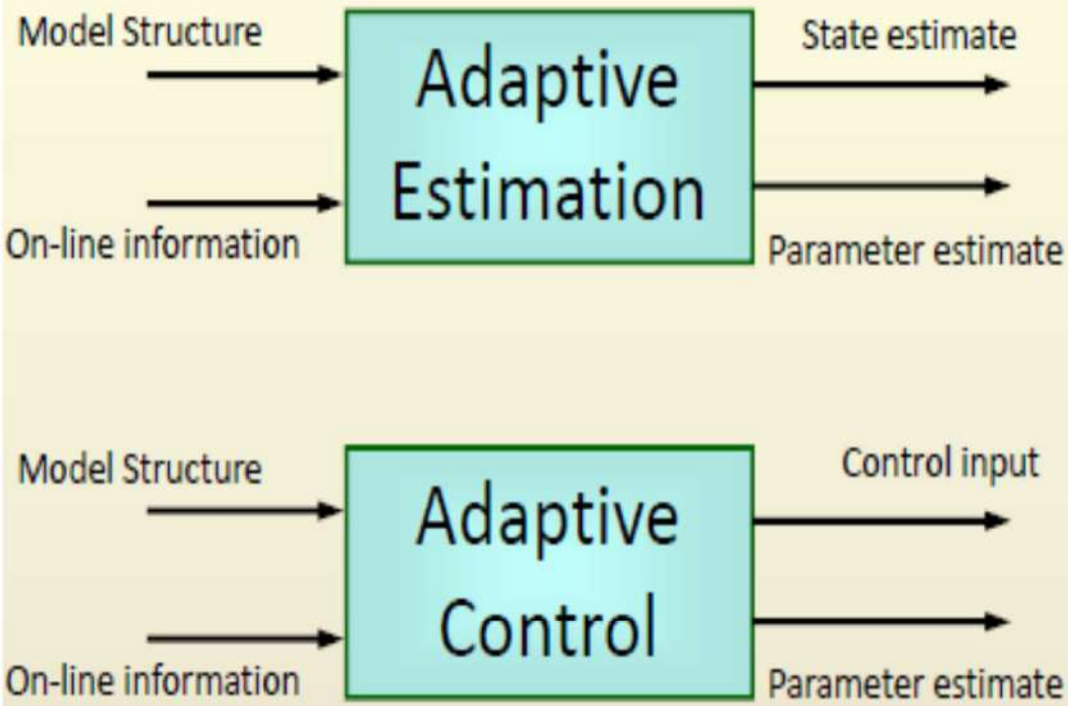
Learning in Adaptive Systems

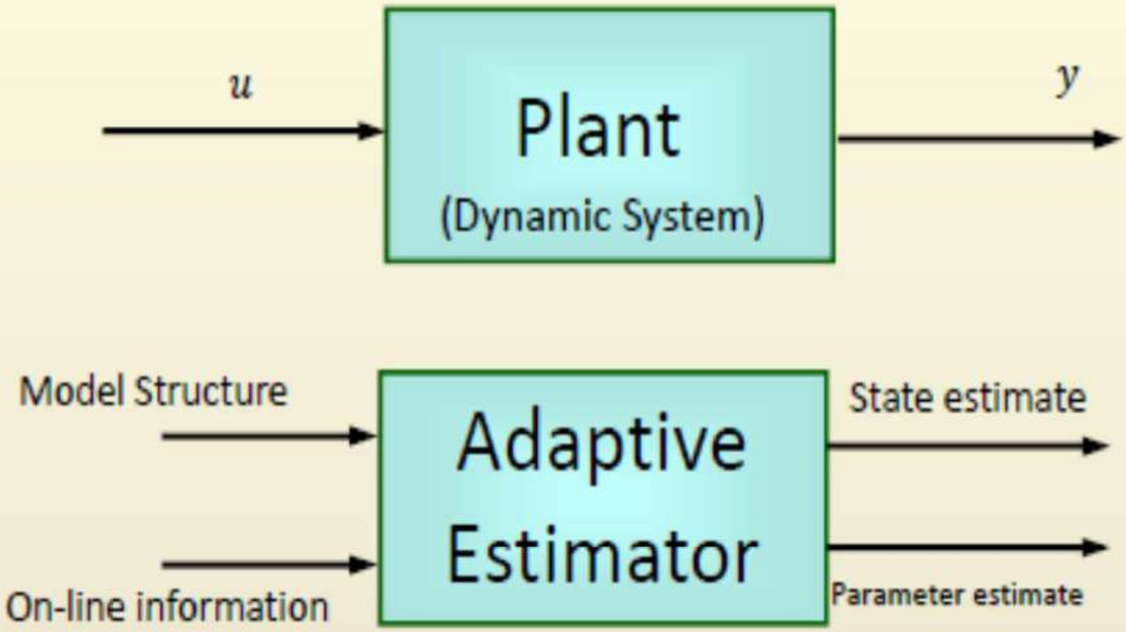


Generate real-time estimates of parameters and states using on-line data

The design of a self-tuning controller in the presence of parametric uncertainties

Learning ≡ Parameter Estimation in Real-time

Problem Statement – Adaptive Estimation



$$\dot{x} = f(x, \theta, u)$$
$$y = g(x, \theta, u)$$

θ: parameter, unknown

x, y, u: state, output, input

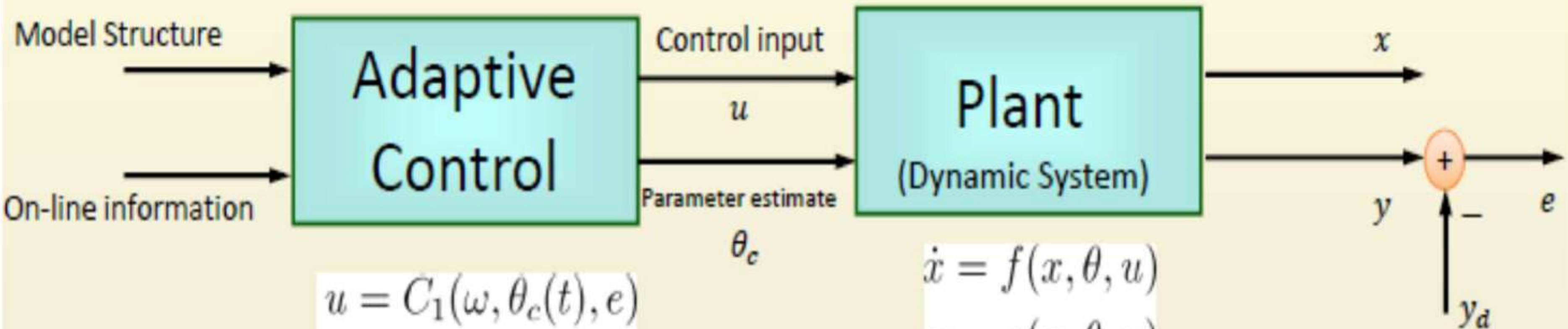
e: state error

$\hat{\theta}$: parameter estimate

$$\dot{\hat{x}} = F(\hat{x}, \hat{\theta}) \qquad \dot{\hat{\theta}} = G(\hat{x}, \hat{\theta})$$

Goal: Find F, G so that $\hat{\theta} \to \theta,\ \hat{x} \to \Gamma x$

Problem Statement – Adaptive Control



$$u = C_1(\omega, \theta_c(t), e)$$
$$\dot{\theta}_c = C_2(\omega, \theta_c(t), e)$$

$$\dot{x} = f(x, \theta, u)$$
$$y = g(x, \theta, u)$$

θ: parameter, unknown

e: tracking error

x, y, u: state, output, input

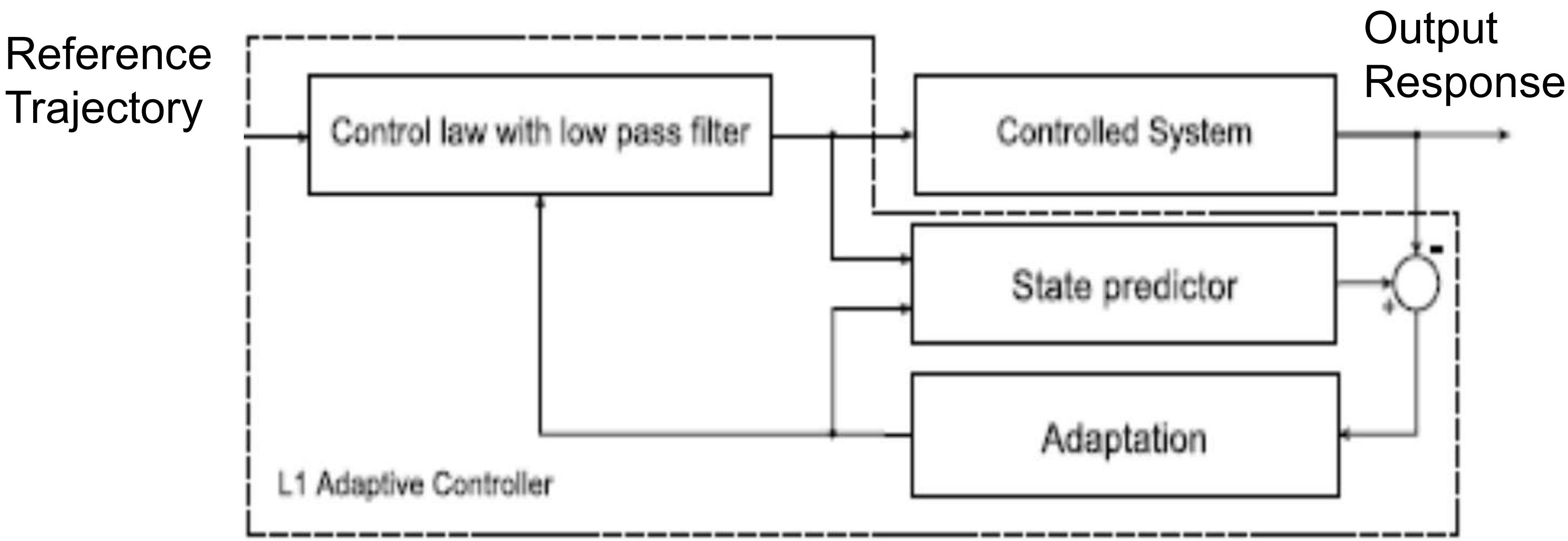
θ_c: control parameter estimate

ω: real-time data

Goal: Find u, C_1, C_2 so that regulation and tracking occur

L1 ADAPTIVE CONTROL SCHEME

The establishment of the framework of the L1 adaptive controller lies in the decoupling between the adaptation and the robustness. For that, the architecture of this scheme is constructed of 4 main parts as shown in the following figure to be explained hereafter: *the controlled system*, *the state predictor*, *the adaptation phase* and *the control law* formulated with a low pass filter.



- *Controlled System:* We will start by considering the following class of nonlinear systems described by the following dynamics:

$$\begin{aligned} \dot{x}_1(t) &= x_2(t), & x_1(0) &= x_{1_0} \\ \dot{x}_2(t) &= f(t,x(t)) + B_2\omega u, & x_2(0) &= x_{2_0} \\ y(t) &= Cx(t) \end{aligned} \quad (1)$$

where $x_1 \in \mathbb{R}^n$ and $x_2 \in \mathbb{R}^n$ are the states of the system forming the complete state vector: $x(t) = [x_1(t)^T, x_2(t)^T]^T$. $u(t) \in \mathbb{R}^m$ is the control input ($m \le n$) and $\omega \in \mathbb{R}^{m\times m}$ is the uncertainty on the input gain. $B_2 \in \mathbb{R}^{n\times m}$ is a constant full rank matrix. $C \in \mathbb{R}^{m\times n}$ is a known full rank constant matrix, $y \in \mathbb{R}^m$ is the measured output and $f(t,x(t))$ is an unknown nonlinear function representing the nonlinear therefore get $A_m = A - B_m k_m$ with $B_m = \begin{bmatrix} 0_{n\times m} \\ B_2 \end{bmatrix}$. The system can then be finally rewritten in a compact form as:

$$\dot{x}(t) = A_m x(t) + B_m\Big(\omega u_a + \theta(t)\|x(t)\|_{\mathcal{L}_\infty} + \sigma(t)\Big), \quad x(0) = x_0$$
$$y(t) = Cx(t) \qquad (2)$$

Given their structure, the vectors θ and σ can be summed to the control input as shown above. u_a is the control input used for adaptation after the transformation of the matrix A into A_m. The whole control input applied to the system is $u = u_m + u_a$ with $u_m = -k_m\, x$.

- *Adaptation Phase:* This stage uses the error between the measured and the estimated states to update the parameters. The adapation law for each estimated parameter vector is then given by:

$$
\begin{aligned}
\dot{\hat{\theta}}(t) &= \Gamma Proj(\hat{\theta}(t), -(\tilde{x}^T(t) P B_m)^T \|x(t)\|_{\mathcal{L}_\infty}) \\
\dot{\hat{\sigma}}(t) &= \Gamma Proj(\hat{\sigma}(t), -(\tilde{x}^T(t) P B_m)^T) \\
\dot{\hat{\omega}}(t) &= \Gamma Proj(\hat{\omega}(t), -(\tilde{x}^T(t) P B_m)^T u_a^T(t))
\end{aligned} \qquad (3)
$$

The parameter P is the solution to the algebraic Lyapunov equation: $A_m^T P + P A_m^T = -Q$ for any arbitrary symmetric $Q = Q^T > 0$. Γ is the adaptation gain and $\tilde{x}(t)$ the error between the predicted state and the measured one. The term *Proj* refers to the projection operator which is a robust technique that bounds the estimated parameters by abiding to the Lyapunov stability rules.

- ***State Predictor:*** The states of the system are calculated at each iteration using the estimated parameters obtained from the adaptation phase (cf. below) along with the control input. Based on equation (2) the state predictor is then given by:

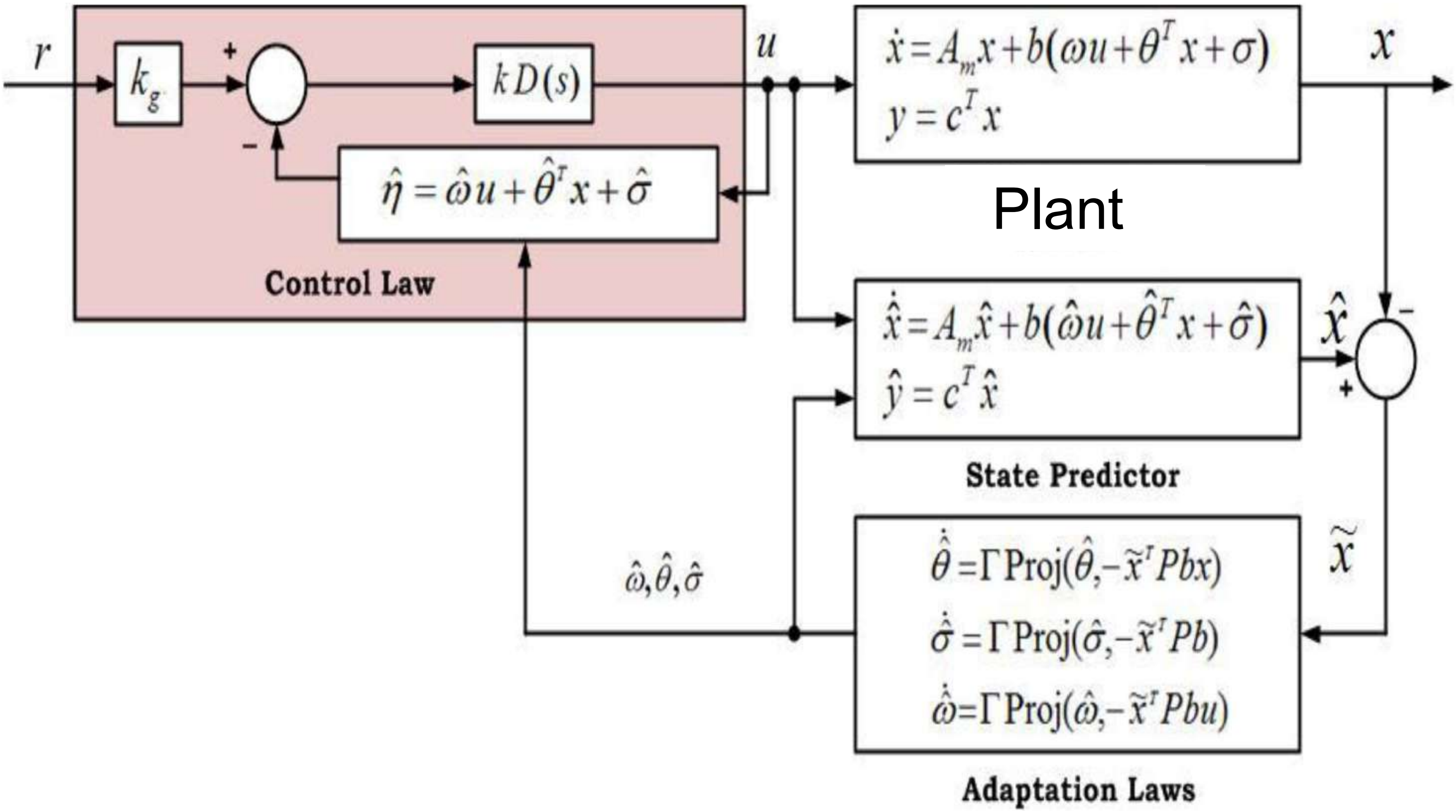
$$\dot{\hat{x}}(t) = A_m\hat{x}(t) + B_m\left(\hat{\omega}(t)u_a(t) + \hat{\theta}(t)\|x(t)\|_{\mathcal{L}_\infty} + \hat{\sigma}(t)\right) \quad (4)$$

- *Control law formulation:* The last stage pertains to the formulation of the control input characterized by the addition of a low pass filter. It is written as:

$$u_a(s) = -kD(s)(\hat{\eta}_l(s) - k_g r) \qquad (5)$$

$D(s) \in \mathbb{R}^{m\times m}$ is a strictly proper transfer matrix leading to the stable closed-loop filter: $C(s) = \frac{\omega k D(s)}{\mathbb{I}_m + \omega k D(s)}$. k is a positive feedback gain, $k_g = -(CA_m^{-1}B_m)^{-1}$ is a feedforward prefilter to the reference signal $r(t)$ and $\hat{\eta}_l = \hat{\omega}(t)u_a(t) + \hat{\theta}\|x(t)\|_{\mathcal{L}_\infty}$. To ensure the stability of the closed-loop, the feedback gain k and the filter $D(s)$ must be chosen in order to fulfill the $\mathcal{L}_1$ norm condition. The

L1 Adaptive Control (shown in the following figure) has been proposed and widely advertised in aerospace control for achieving fast and robust adaptation and better performance than the existing Model Reference Adaptive Control (MRAC) Schemes.

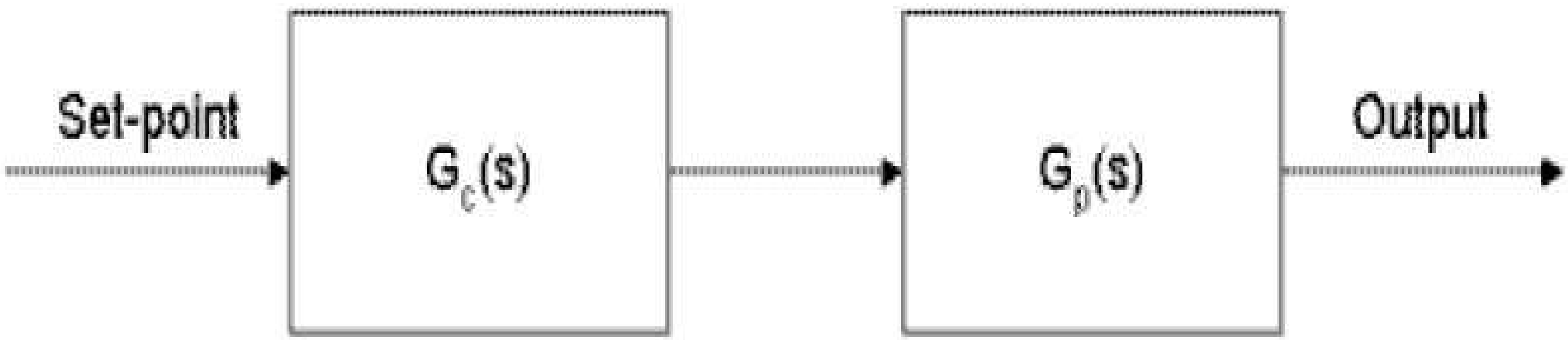


Internal Model Control

On the basis of the internal model control (IMC), a two-degrees-of-freedom internal model control was developed. Compared with the conventional feedback control, the internal model control structure mainly embeds an internal model consistent with the control object in the control object, so the deviation between the internal model and the control object determines the quality of the control effect.

The IMC configuration is based on the model of the process, hence, any unknown parameters of the model need to be estimated online in order to evaluate the gains of an adaptive IMC (AIMC).

- Internal Model Control Open-Loop

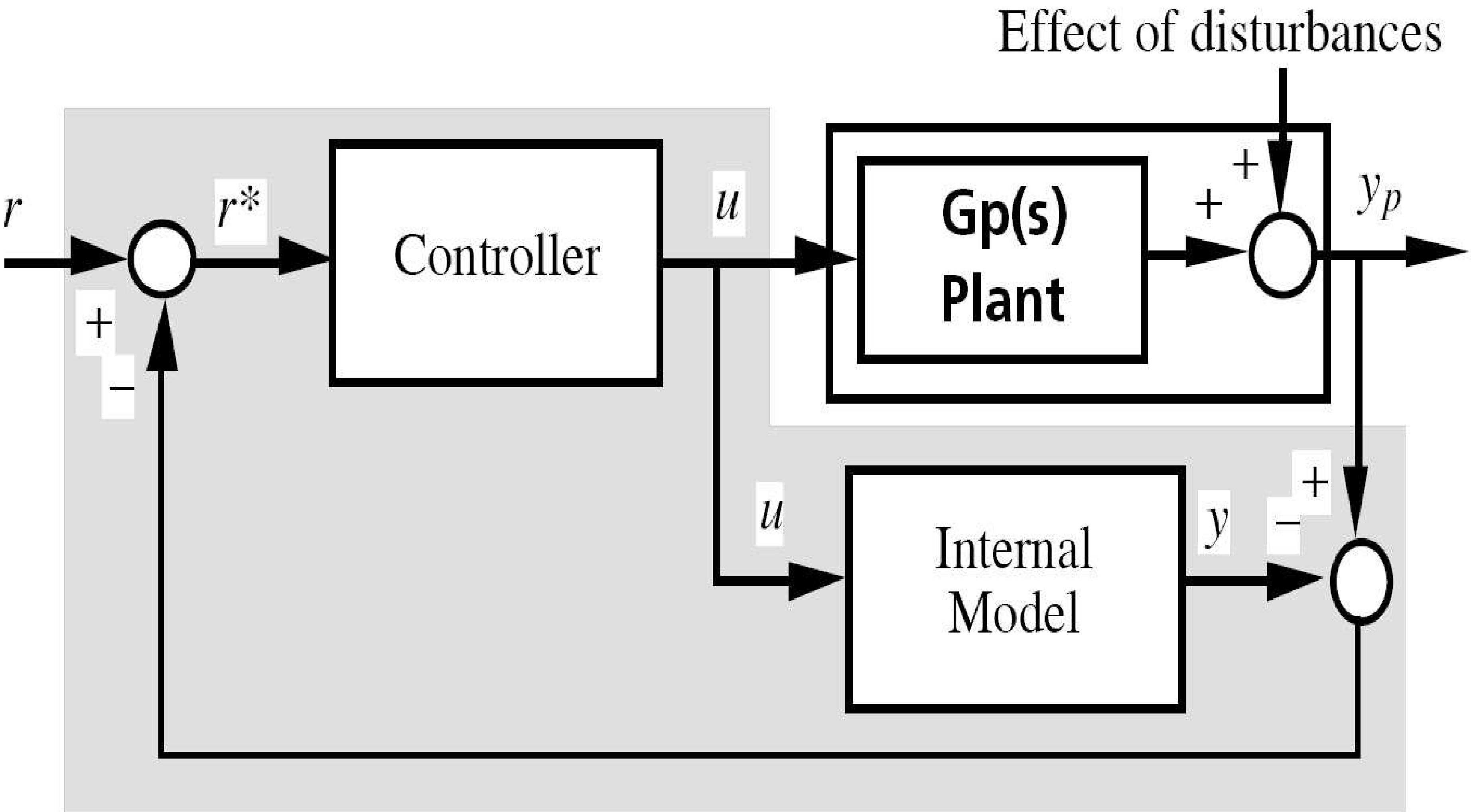


Let Gp(s) = approx(Gp(s))

And Gc(s) = approx(Gp(s)) ^ -1

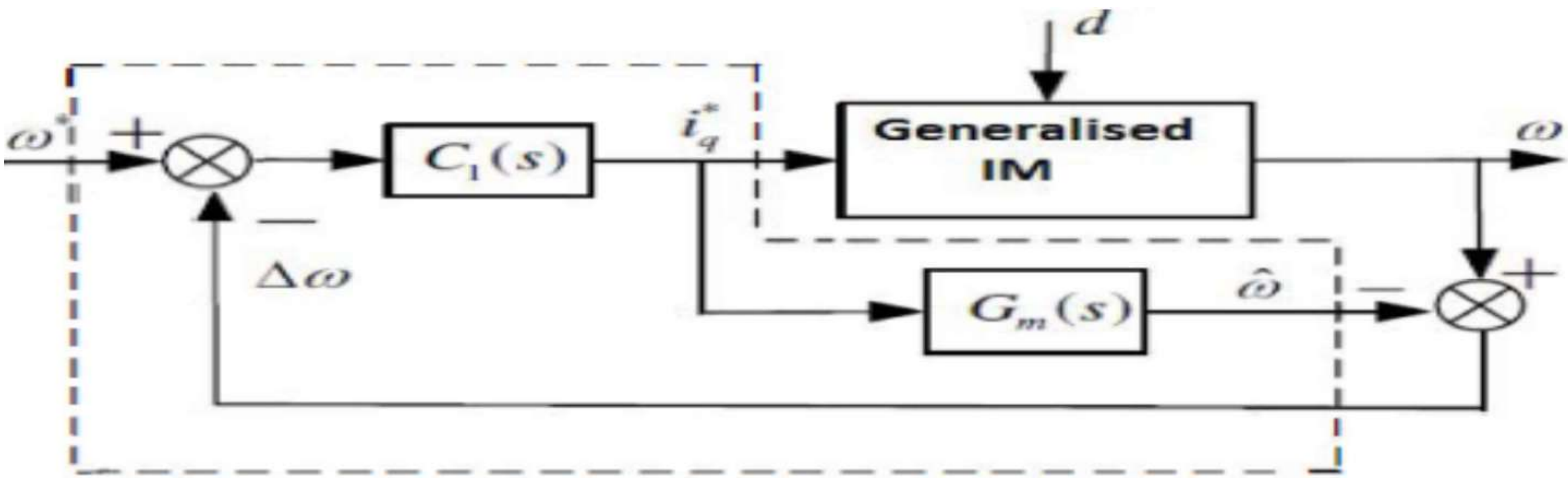
Then Gp(s)*Gc(s) = approx(Gp(s)) * approx(Gp(s)) ^ -1 = 1

Internal Model Control Closed-Loop



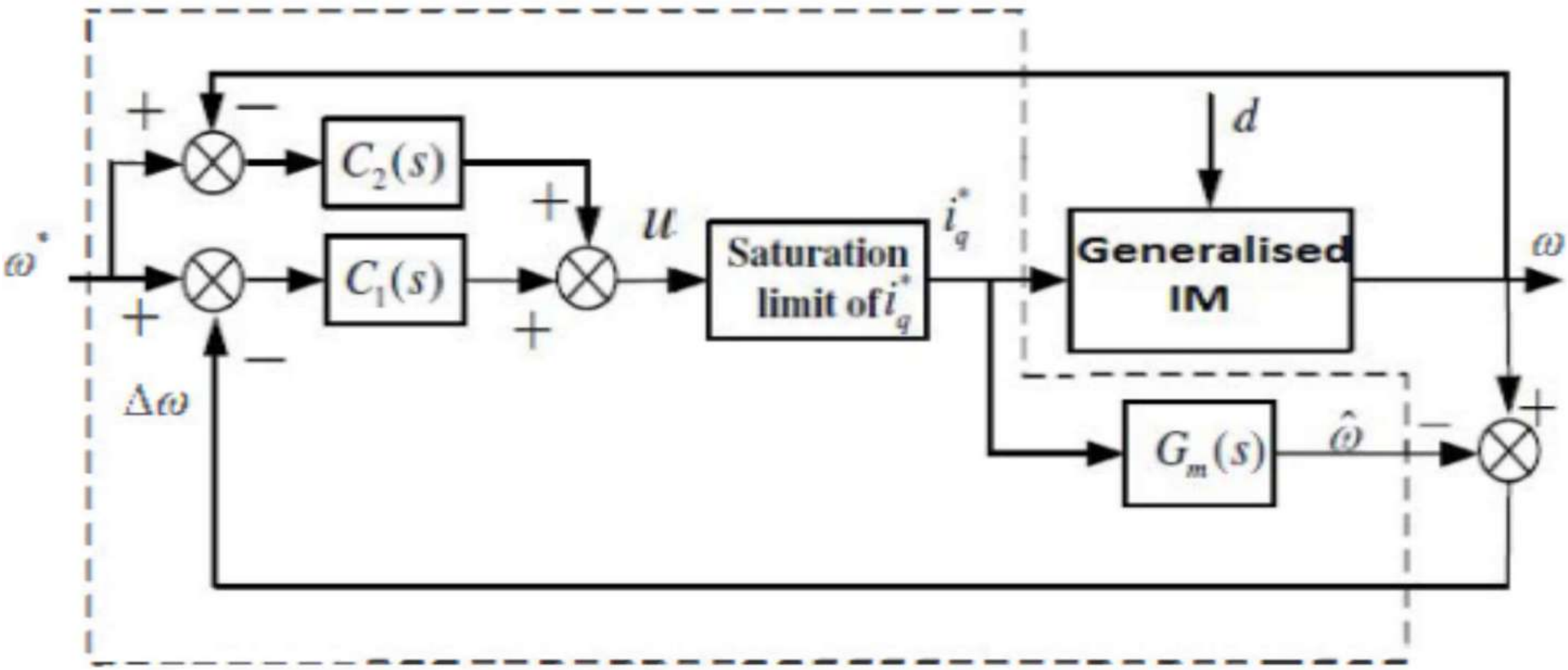
- Internal Model Control Advantages
  - Provides time-delay compensation
  - At steady-state, the controller will give offset free responses(perfect control at S.S)
  - The controller can be used to shape both the input tracking and disturbance rejection responses
  - The controller is the inverse of the plant without non-invertible components(time-delay)
  - Perfect Tracking is achieved despite model-mismatch, as long as the controller is the perfect inverse of the model.

Standard IMC method is considered as a robust control method which includes the internal model, the inverse internal model and the filter as one unity. It can guarantee the stability of the system for open loop stable plants. The standard IMC structure for Induction motor is shown in next figure, where the "generalized IM" includes the two current loops, IM and other components, Gm(S) is the internal model, and C1(S) is the internal model controller



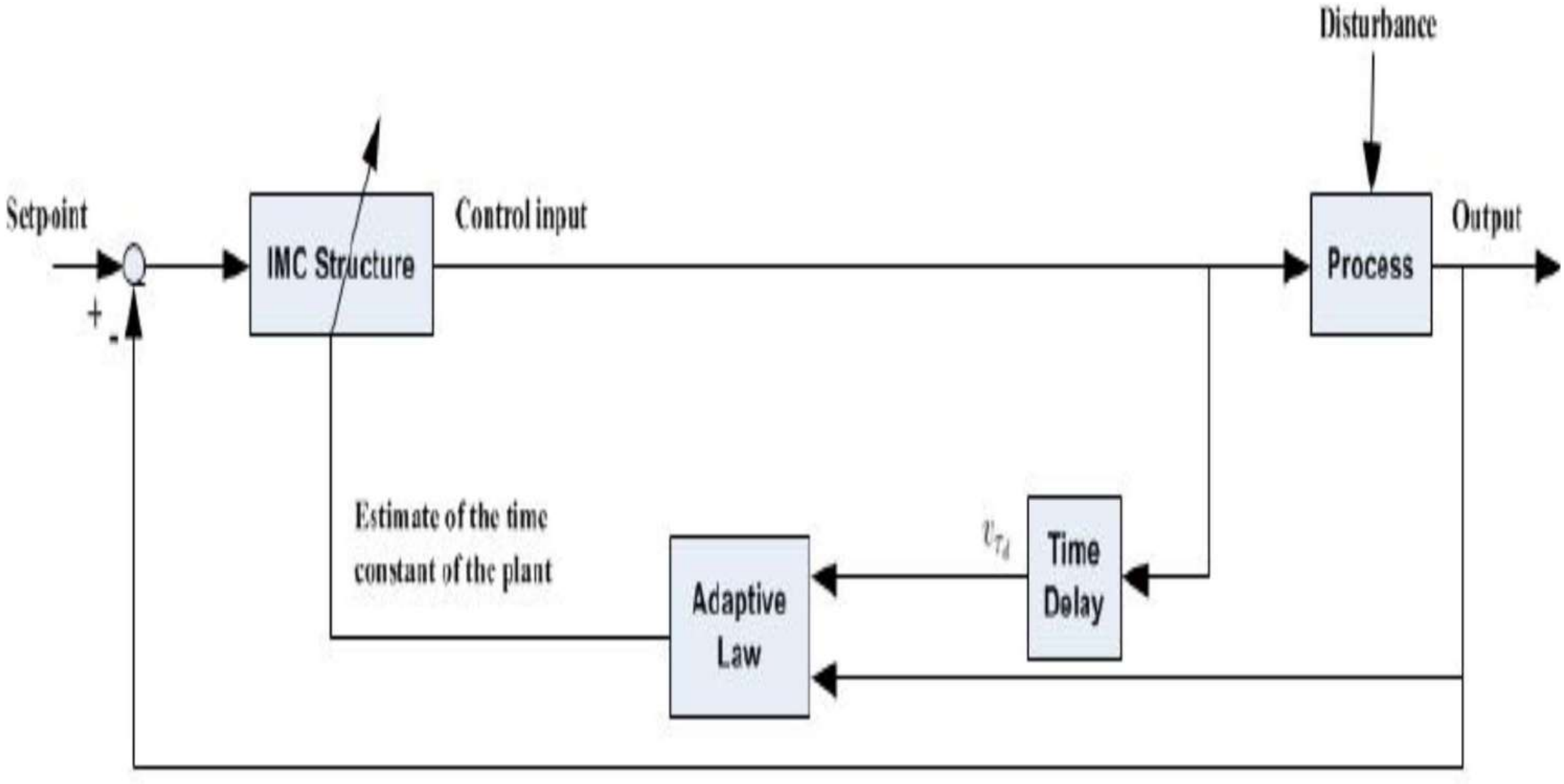
Standard IMC

In order to enhance the ability tracking and load disturbance rejection of the system, a feedback control term C1(s) is designed based on the standard internal model control framework. A modified IMC scheme for induction motor is proposed, as shown in the following figure. The output of the feedback control term C2 can compensate for the effect of control input saturation as anti-windup compensation to improve the tracking performance.



Modified IMC

Next we employ the adaptive IMC where the process time constant is estimated online and the controller gains are updated accordingly.



Adaptive Inverse Control

The approach to be developed, adaptive inverse control that is simple, robust, and precise. With some knowledge of the subject of adaptive filtering; adaptive inverse control is easy to understand and use in practice. The basic idea of adaptive inverse control is to drive the plant with a signal from a controller whose transfer function is the inverse of that of the plant itself. The idea is illustrated with the system of next figure. The objective of this system is to cause the plant output to follow the command input. Since the plant is generally unknown, it is necessary to adapt or to adjust the parameters of the controller in order to create a true plant inverse.

An error signal, the difference between the plant output and the command input, is used by an adaptive algorithm to adjust the controller’s parameters to minimize the mean square of this error.

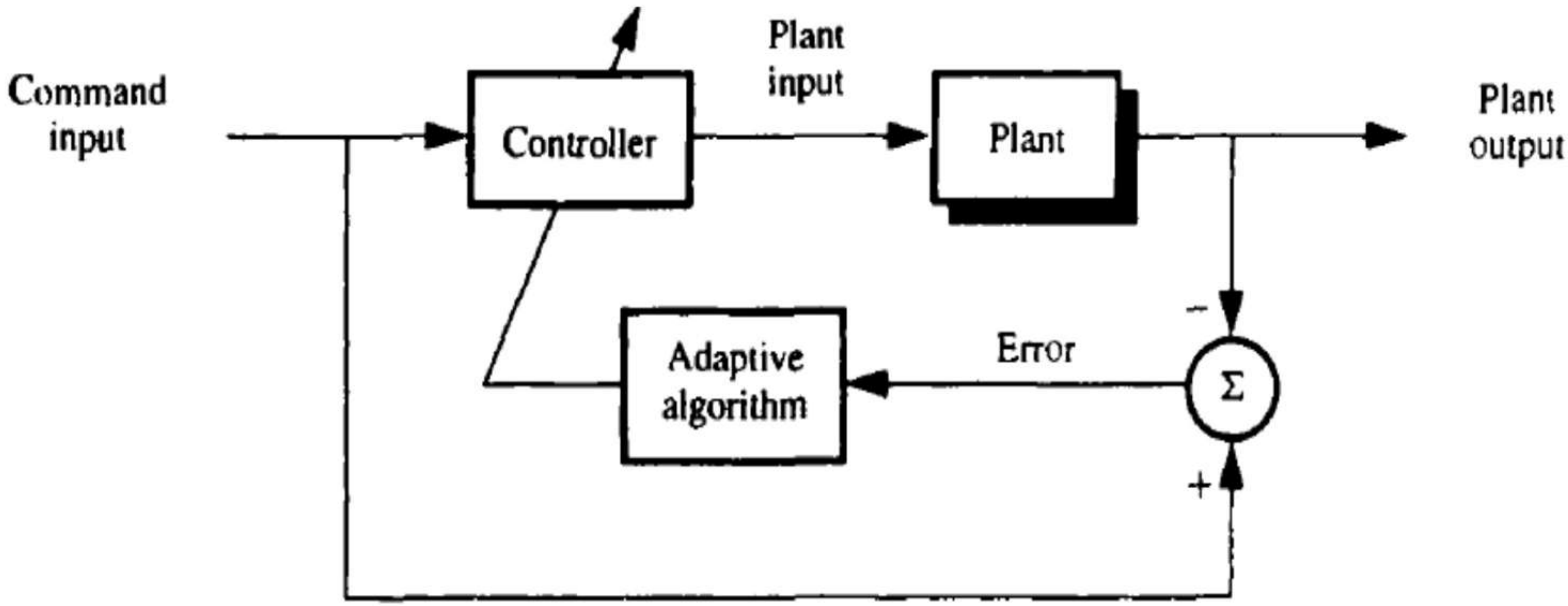
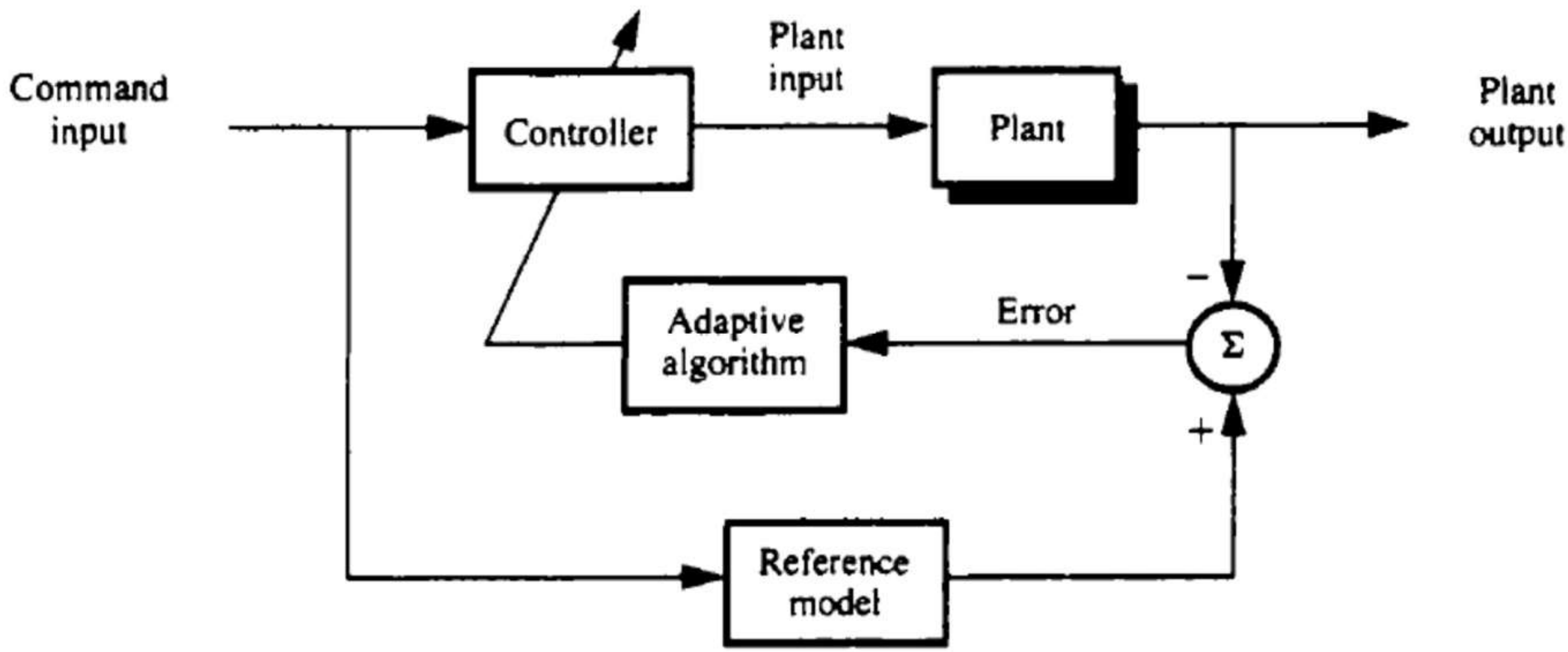
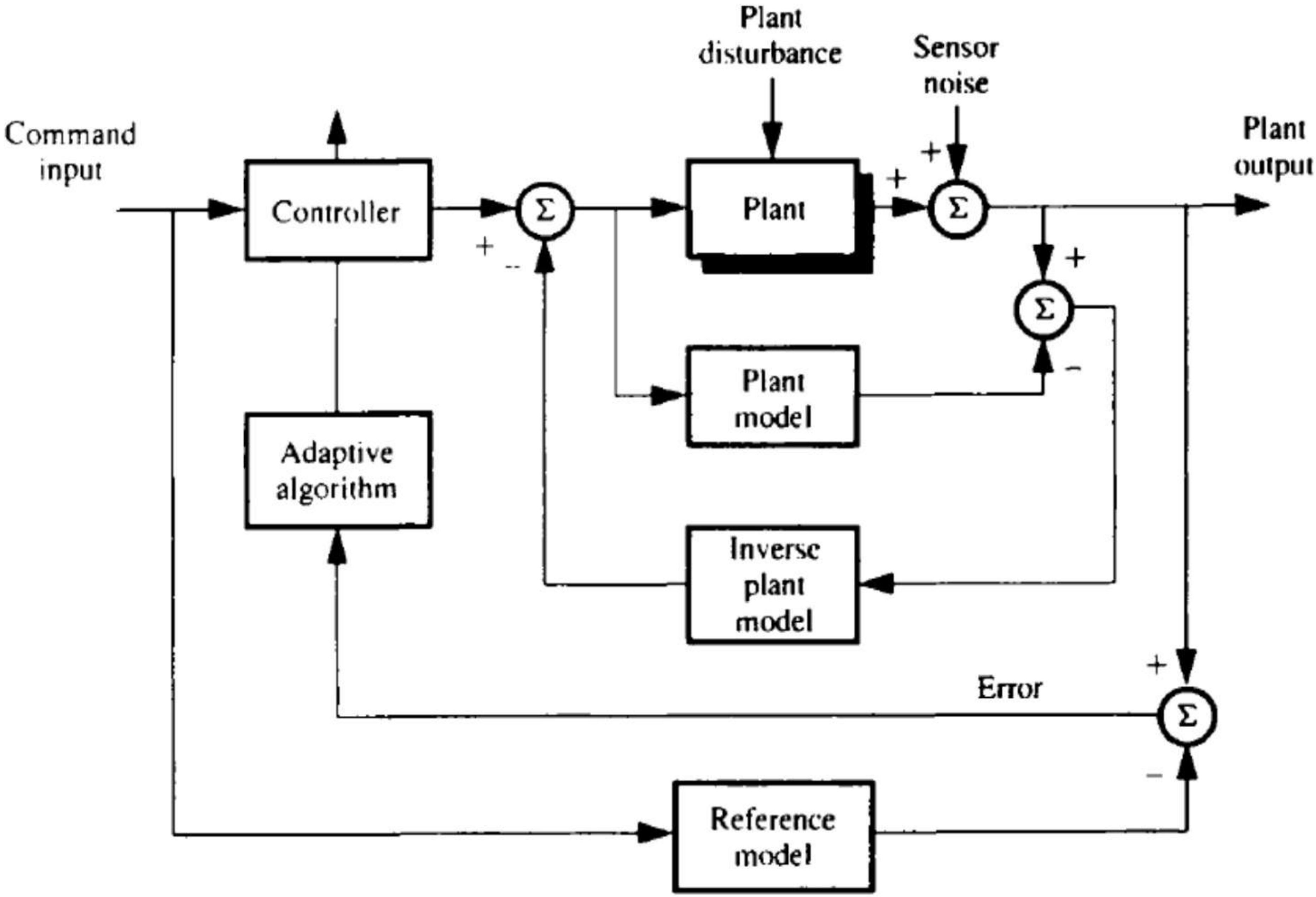


Figure 1.2 Basic concept of adaptive inverse control.

Sometimes it is desired that the plant output track not the command input itself but a delayed or smoothed version of the command input. The system designer would generally know the smoothing characteristic to be used. The smoothing model is generally designated as the reference model in the control theory field. Thus, the system of next figure may be called a model-reference adaptive inverse control system.



Plant **noise and disturbance** present a problem for the adaptive inverse control approach. Lack of **feedback** from the plant output back to the plant input permits internal plant noise and disturbance to exist **unchecked** at the plant output. Some modifications they have been applied to the cancelation of plant noise and disturbance. An adaptive inverse control system including the plant noise and disturbance canceling features and the model-reference control features are shown in next figure. Use is made of both a plant model and a plant inverse model. The plant model has the same transfer function as the plant, while the plant inverse model has a transfer function which is the reciprocal of that of the plant.


Plant
disturbance
Sensor
noise
Command
input
Controller
Plant
Plant
output
Plant
model
Adaptive
algorithm
Inverse
plant
model
Error
Reference
model