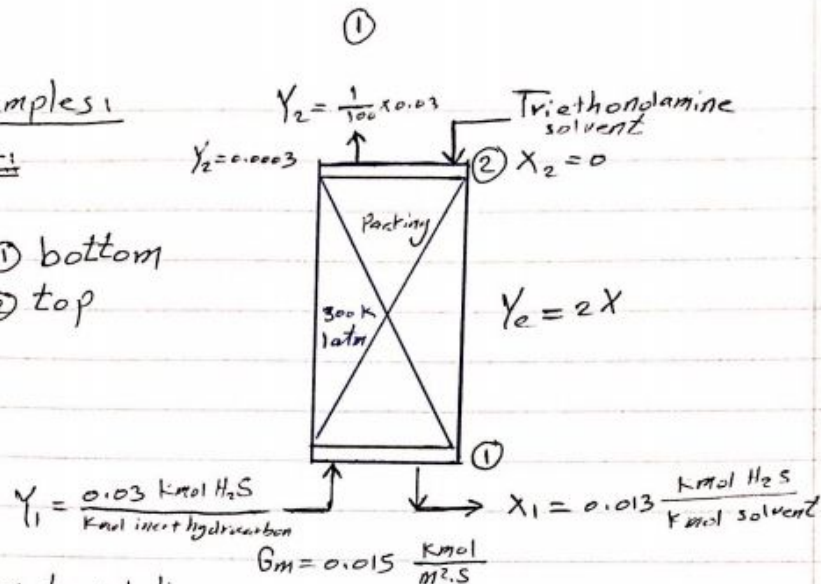


Examples

12.4:

- ① bottom
② top



solution

1- Dilute solution

2- Equilibrium is straight line from the equation $Y_c = 2X$

$$Z = \frac{G_m}{k_o G_a P_T} \int_{Y_1}^{Y_2} \frac{dY}{Y - Y_c}$$

$$Z = ?$$

$$NOG = ?$$

$$k_o G_a P_T = 0.04 \frac{\text{kmol}}{\text{s m}^3}$$

$$Y_1 = 0.03, Y_2 = 0.0003$$

$$X_2 = 0 \Rightarrow Y_{c2} = 2X_2 = 0.0$$

$$X_1 = 0.013 \Rightarrow Y_{c1} = 2X_1 = 2 \times 0.013 = 0.026$$

Bottom driving force

$$= Y_1 - Y_{c1}$$

$$= 0.03 - 0.026 = 0.004$$

Top driving force

$$= Y_2 - Y_{c2}$$

$$= 0.0003 - 0 = 0.0003$$

$$Z = \frac{G_m}{k_o G_a P_T} \times \frac{Y_1 - Y_2}{(Y - Y_c)_{LM}}$$

$$(Y - Y_c)_{LM} = \frac{(Y - Y_{c1})_1 - (Y - Y_{c1})_2}{\ln \frac{(Y - Y_{c1})_1}{(Y - Y_{c1})_2}} = \frac{0.004 - 0.0003}{\ln \frac{0.004}{0.0003}} = 0.0014$$

$$Z = \frac{\text{HOG}}{0.04} \times \frac{\text{NOG} \text{ (2)}}{0.0014}$$

$$Z = \frac{0.015}{0.04} \times \frac{0.03 - 0.0003}{0.0014}$$

$$\text{HOG} = \frac{0.015}{0.04} = 0.375 \text{ m}$$

$$\text{NOG} = \frac{0.03 - 0.0003}{0.0014} = 21.21$$

$$\left. \begin{array}{l} \text{HOG} = 0.375 \text{ m} \\ \text{NOG} = 21.21 \end{array} \right\} \begin{array}{l} Z = \text{HOG} \times \text{NOG} \\ = 0.375 \times 21.21 \\ Z = 7.95 \text{ m} \end{array}$$

$$\begin{array}{ccc} \text{KOG} & \cdot & \alpha & \cdot & P_T \\ \downarrow & & \downarrow & & \downarrow \\ \frac{\text{S}}{\text{m}} & \cdot & \frac{\text{m}^2}{\text{m}^3} & \cdot & \frac{\text{KN}}{\text{m}^2} \\ & & \downarrow & & \downarrow \\ \frac{\text{S}}{\text{m}} & \cdot & \frac{\text{m}^2}{\text{m}^3} & \cdot & \frac{\text{kg} \cdot \text{m}}{\text{s}^2 \cdot \text{m}^2} \end{array}$$

$$\downarrow$$

$$\frac{\text{kg}}{\text{s} \cdot \text{m}^3} \text{ or } \frac{\text{kmol}}{\text{s} \cdot \text{m}^3}$$

$$\left(\text{KN} = \frac{\text{kg} \cdot \text{m}}{\text{s}^2} \right)$$

$$\downarrow$$

$$(F = M \times A)$$

$$\downarrow$$

$$\text{kg} \times \frac{\text{m}}{\text{s}^2}$$

كمفحة: في حالة إعطائنا الوحدة $\rightarrow \text{KOG} \cdot \alpha \cdot P_T$ بـ $\frac{\text{kg}}{\text{s} \cdot \text{m}^3}$

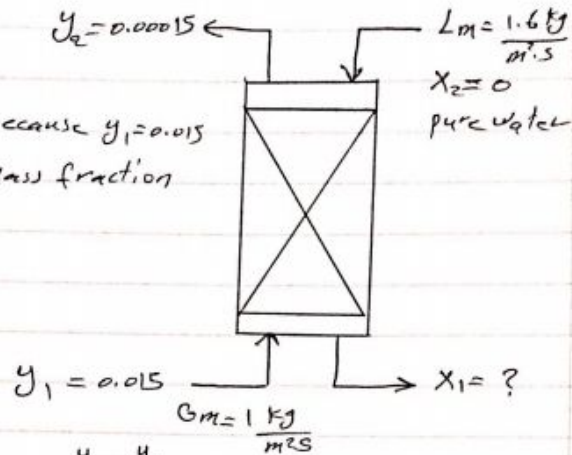
$$\downarrow \left[\frac{\text{kmol}}{\text{s} \cdot \text{m}^3} \times \frac{\text{KN}}{\text{m}^2} \right]$$

وعليه يجب ان يتم ضرب القيمة بالمفحة P_T

(3)

EX: 12.2

- 1) Dilute solution because $y_1 = 0.015$
 2) mole fraction $\approx$ mass fraction
 $y_e = 1.75 X$
 NOG = ?



$$NOG = \int_{y_2}^{y_1} \frac{dy}{y - y_e} = \frac{y_1 - y_2}{(y - y_e) LM}$$

$$(y - y_e) LM = \frac{(y_1 - y_{e1}) - (y_2 - y_{e2})}{\ln \frac{(y_1 - y_{e1})}{(y_2 - y_{e2})}}$$

$$G_m = \frac{1}{29} = 0.0345 \text{ (kmol/m}^2\text{s)}$$

$$L_m = \frac{1.6}{18} = 0.0889 \text{ (kmol/m}^2\text{s)}$$

overall Material Balance (O.M.B):

$$G_m (y_1 - y_2) = L_m (X_1 - X_2^{\rightarrow 0})$$

$$0.0345 (0.015 - 0.00015) = 0.0889 (X_1 - 0)$$

$$X_1 = 0.00576 \text{ \& } y_{e1} = 1.75 X_1 = 1.75 \times 0.00576 = 0.0101$$

$$\text{bottom driving force} = (y_1 - y_{e1}) = (0.015 - 0.0101) = 0.0049$$

$$\text{Top driving force} = (y_2 - y_{e2}) = (0.00015 - 0) = 0.00015$$

$$y_{e2} = 1.75 X_2^{\rightarrow 0} = 0$$

$$(y - y_e) LM = \frac{0.0049 - 0.00015}{\ln \frac{0.0049}{0.00015}} = 0.00136$$

$$NOG = \frac{0.015 - 0.00015}{0.00136} = 10.92$$

EX: 12.10:

$$P_G = y_2 P_{total}$$

$$y_2 = \frac{P_G}{P_{total}} = \frac{1.14}{101.3} = 0.0113$$

$$y_1 = \frac{y_1}{1-y_1} = \frac{0.035}{1-0.035} = 0.036$$

$$KOLa = 0.19 \text{ kmol SO}_2 / \text{s m}^2$$

$$Z = \frac{L_m}{KOLa CT} \int_{x_2}^{x_1} \frac{dx}{x_e - x}$$

$$\text{at } y_2 = 0.0113$$

from drawing

$$x_{e2} = 0.54 \times 10^{-3}$$

$$\text{at } y_1 = 0.036 \Rightarrow x_{e1} = 1.41 \times 10^{-3}$$

$$(x_{e1} - x_1) = 1.41 \times 10^{-3} - 1.145 \times 10^{-3} = 0.265 \times 10^{-3} \frac{\text{kmol SO}_2}{\text{kmol H}_2\text{O}}$$

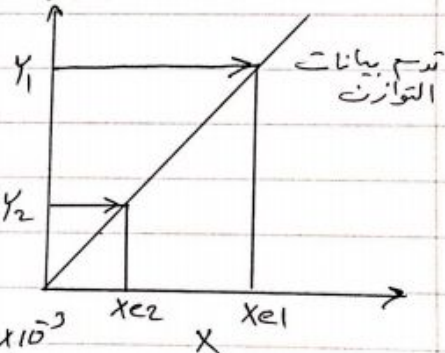
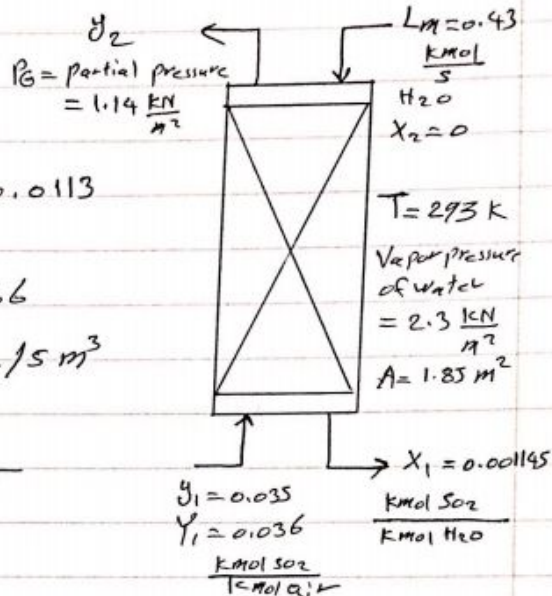
$$(x_{e2} - x_2) = 0.54 \times 10^{-3} - 0.0 = 0.54 \times 10^{-3}$$

$$(x_e - x) LM = \frac{(0.54 - 0.265) \times 10^{-3}}{\ln \frac{0.54}{0.265}} = 3.86 \times 10^{-4} \frac{\text{kmol SO}_2}{\text{kmol H}_2\text{O}}$$

$$Z = \frac{L_m}{KOLa CT} \times \frac{x_1 - x_2}{(x_e - x) LM}$$

$$Z = \frac{0.43}{1.85 \times 0.19} \times \frac{0.001145 - 0}{3.86 \times 10^{-4}}$$

$$Z = 3.64 \text{ m}$$



$$KOLa CT = \frac{m}{s} \cdot \frac{m^2}{m^3} \cdot \frac{\text{kmol}}{m^3} = \frac{\text{kmol}}{m^3 s}$$

$$L_m = 0.43 \frac{\text{kmol}}{s} \times \frac{1}{1.85 \text{ m}^2}$$

$$L_m = \frac{\text{kmol}}{s \text{ m}^2}$$

(5)

EX: 12.9 (not dilute solution)
مثال تصميم للخاليل الكتلة (في الخففة)

① يجب تحديد G و L_m الى $K_m a$

$$M.wt \text{ of air} = 29$$

$$M.wt \text{ of } NH_3 = 17$$

$$\text{Average } M.wt = 0.1 \times 17 + 0.9 \times 29 = 27.8$$

$$\frac{0.95}{27.8} = 0.034 \frac{\text{kmol}}{\text{m}^2 \cdot \text{s}}$$

$$G_m (\text{inert gas}) = 0.034 \times 0.9 = 0.0307 \frac{\text{kmol}}{\text{m}^2 \cdot \text{s}}$$

$$L_m = \frac{0.65}{18} = 0.036 \frac{\text{kmol}}{\text{s} \cdot \text{m}^2}$$

$$Y_1 = \frac{y_1}{1-y_1} = \frac{0.1}{1-0.1} = 0.111$$

$$Y_2 = \frac{1}{100} \times 0.111 = 0.00111$$

② Find The equation of equilibrium line
box I. Material balance

$$G_m (Y - Y_2) = L_m (X - X_2^{\text{sat}})$$

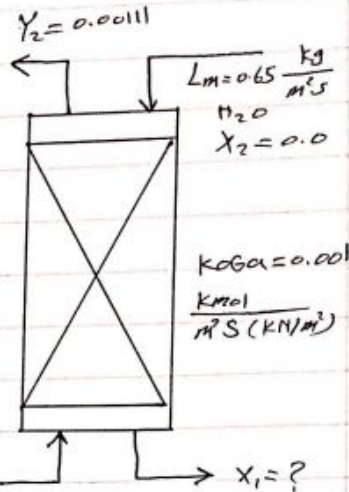
$$Y = \frac{L_m}{G_m} X + Y_2$$

$$Y = \frac{0.036}{0.037} X + 0.001$$

$$\boxed{Y = 1.137 X + 0.001} \quad \text{operating line equation}$$

معادلة خط التشغيل

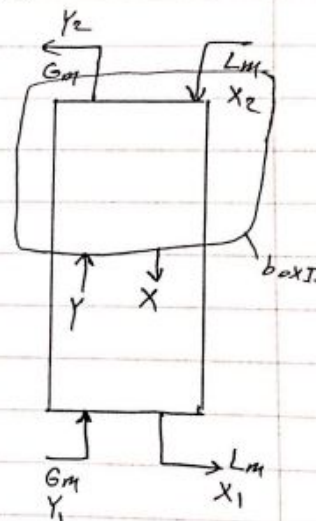
$$Z = \frac{G_m}{K_o G a P_T} \int_{Y_2}^{Y_1} \frac{(1+Y)(1+Y_i)}{(Y-Y_i)} dY$$



$$y_1 = 10\% NH_3$$

$$G = 0.95 \frac{\text{kg}}{\text{m}^2 \cdot \text{s}}$$

$$y_1 = 0.1$$



⑥

③ Find the equilibrium data يتم تحديد بيانات التوازن
 $P_G = y P_T$ لكن نحتاج منحنى التوازن بين X و Y ، وكما يلي:

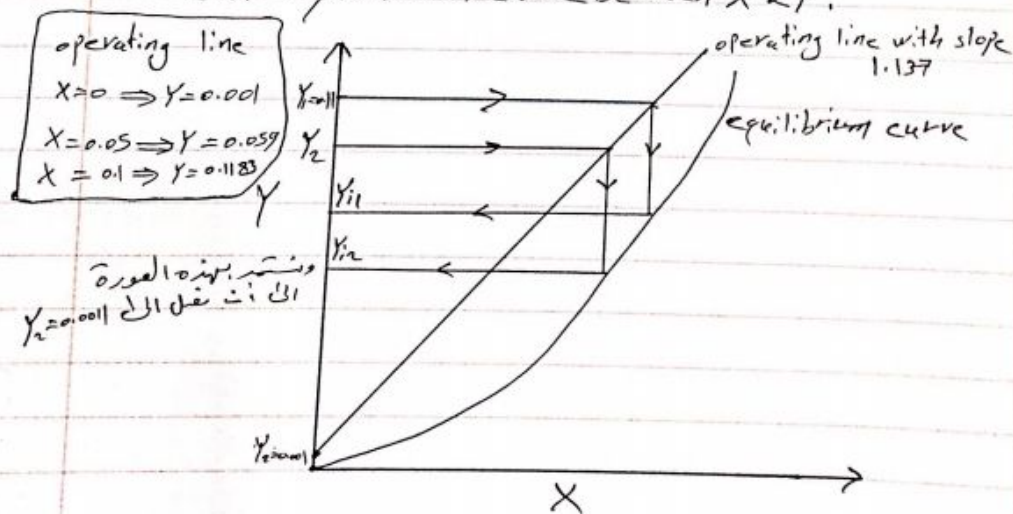
$$P_G = \frac{Y}{Y+1} P_T$$

$$P_G + P_G Y = P_T Y$$

$$Y = \frac{P_G}{P_T - P_G}$$

محطات $\rightarrow X$	0.21	0.031	0.042	0.053	0.079	0.106	0.15
$\frac{\text{kmol NH}_3}{\text{kmol H}_2\text{O}}$							
P_G	12	18.2	24.7	31.7	50	67.6	114
mm Hg							
Partial pressure							
$P_T - P_G$	748	741.8	735.1	728.3	710	690.4	646
Atmosphere							
$Y = \frac{P_G}{P_T - P_G}$	0.016	0.0245	0.0339	0.0473	0.0704	0.101	0.178

④ Drawing operating line with equation $Y = 1.137X + 0.001$ and equilibrium curve between X & Y .

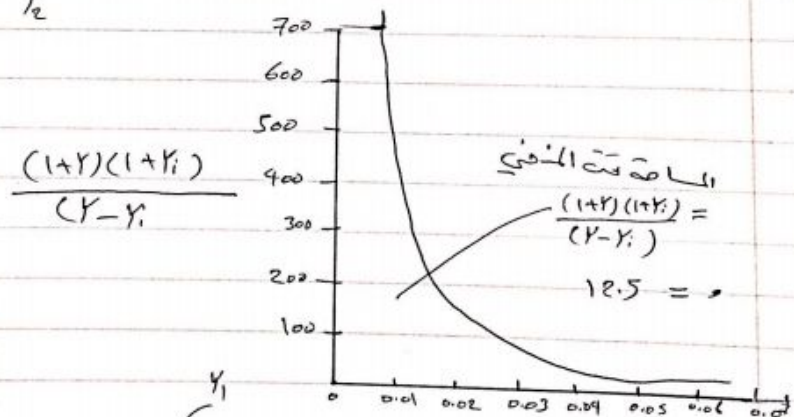


(7)

Y	Y_i	$\frac{(1+Y)(1+Y_i)}{(Y-Y_i)}$
$Y_1 = 0.111$	0.084	55
0.1	0.078	53.8
0.08	0.059	54.3
0.06	0.042	61.4
0.04	0.027	82.0
0.02	0.013	184
0.01	0.006	25.4
0.005	0.0026	421
$Y_2 = 0.0011$	0.0	1000

عمل التكميل التالي بواسطة المساحة تحت المنحنى (5)

$$\int_{Y_2}^{Y_1} \frac{(1+Y)(1+Y_i)}{(Y-Y_i)} \cdot dY$$

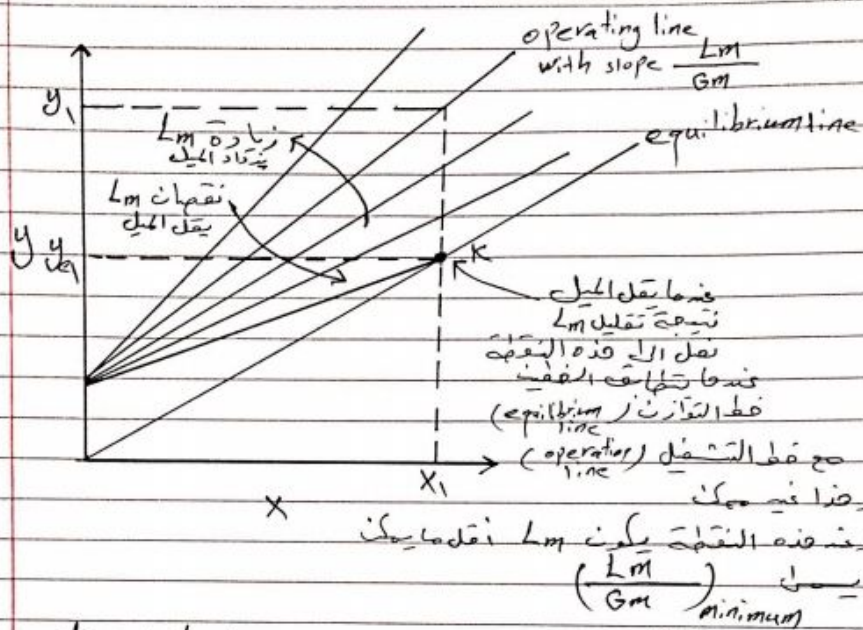


$$Z = \frac{G_m}{K_O G_A P_T} \int_{Y_2}^{Y_1} \frac{(1+Y)(1+Y_i)}{(Y-Y_i)} dY$$

$$= \frac{0.0307}{0.001 \times 101.3} \times 12.5 = 3.82 \text{ m}$$

(8)

Effect of L_m and $\frac{L_m}{G_m}$ on the absorption design:



at point K $\Rightarrow y_1 = y_{e1}$
 at $(\frac{L_m}{G_m})_{\text{minimum}} \Rightarrow$

overall M.B

$$G_m(y_1 - y_2) = L_m(X_1 - X_2^*)$$

$$\left(\frac{L_m}{G_m}\right)_{\text{operating}} = \frac{y_1 - y_2}{X_1}$$

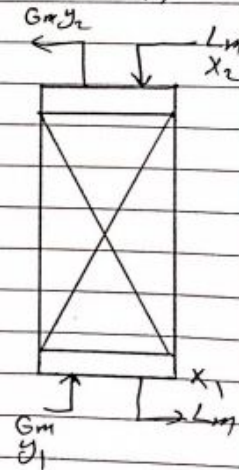
$$y_{e1} = mX_1$$

$$X_1 = \frac{y_{e1}}{m}$$

but at $(\frac{L_m}{G_m})_{\text{min.}} \Rightarrow y_1 = y_{e1}$

$$\therefore X_1 = \frac{y_1}{m}$$

$$\left(\frac{L_m}{G_m}\right)_{\text{min}} = \frac{y_1 - y_2}{X_1 - X_2^*} = \frac{y_1 - y_2}{X_1} = \frac{y_1 - y_2}{\frac{y_1}{m}} = \frac{y_1 - y_2}{\frac{y_1}{m} \times \frac{1}{y_1}}$$



②

$$\left(\frac{L_m}{G_m}\right)_{min} = \frac{1 - \frac{y_2}{y_1}}{\frac{1}{m}} = \left(1 - \frac{y_2}{y_1}\right)m$$

$$\therefore \left(\frac{L_m}{G_m}\right)_{min} = m \left(1 - \frac{y_2}{y_1}\right)$$

$$\text{but } \left(\frac{L_m}{G_m}\right)_{operating} = \frac{y_1 - y_2}{x_1}$$

وعليه يجب أن يكون $\left(\frac{L_m}{G_m}\right)_{operating}$ أكبر من $\left(\frac{L_m}{G_m}\right)_{min}$

لكي نتجنب تقاطع الخطين

$$\left(\frac{L_m}{G_m}\right)_{operating} > \left(\frac{L_m}{G_m}\right)_{min}$$

والقيمة التقريبية لزيادة $\left(\frac{L_m}{G_m}\right)_{oper}$ هي 40% أي أكبر من

$$\therefore \left(\frac{L_m}{G_m}\right)_{operating} > 1.4 \left(\frac{L_m}{G_m}\right)_{min}$$

EX: 12.12

$$y_e = 0.1 y_1$$

$$y_e = m x$$

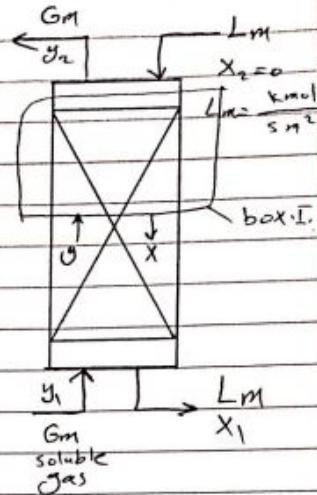
$$HOG = 0.6 m$$

$$\left(\frac{L_m}{G_m}\right)_{ope.} = 1.5 \left(\frac{L_m}{G_m}\right)_{min.}$$

Z = ?

solution

$$NOG = \int_{y_2}^{y_1} \frac{dy}{y - y_e}$$



box I M.B.

$$G_m (y - y_2) = L_m (x - x_2)$$

$$x = \frac{G_m}{L_m} (y - y_2)$$

If the equilibrium data $y_e = m x$

$$NOG = \int_{y_2}^{y_1} \frac{dy}{y - \frac{m G_m}{L_m} (y - y_2)}$$

$$NOG = \int_{y_2}^{y_1} \frac{dy}{y \left(1 - \frac{m G_m}{L_m}\right) + \frac{m G_m}{L_m} y_2}$$

$$NOG = \left[\frac{1}{1 - \frac{m G_m}{L_m}} \right] \ln \left[\left(1 - \frac{m G_m}{L_m}\right) \frac{y_1}{y_2} + \frac{m G_m}{L_m} \right]$$

$$\text{let } \phi = \frac{m G_m}{L_m}$$

$$NOG = \frac{1}{1 - \phi} \ln \left[(1 - \phi) \frac{y_1}{y_2} + \phi \right]$$

معادلة سرعة تدفق الغاز في عمود الامتصاص

1- $\left(\frac{L_m}{G_m}\right)_{min}$ نجد (11)

$$\left(\frac{L_m}{G_m}\right)_{min} = m \left(1 - \frac{y_2}{y_1}\right)$$

$$y_2 = 0.1 y_1$$

$$\left(\frac{L_m}{G_m}\right)_{min} = m \left(1 - \frac{0.1 y_1}{y_1}\right)$$

$$\left(\frac{L_m}{G_m}\right)_{min} = 0.9 m$$

but $\left(\frac{L_m}{G_m}\right)_{op.c.} = 1.5 \left(\frac{L_m}{G_m}\right)_{min}$

$$\therefore \left(\frac{L_m}{G_m}\right)_{op.c.} = 1.5 \times 0.9 m$$

$$\left(\frac{L_m}{G_m}\right)_{op.c.} = 1.35 m \Rightarrow \boxed{\frac{m G_m}{L_m} = \frac{1}{1.35}}$$

$$\phi = \frac{m G_m}{L_m} = \frac{1}{1.35} = 0.74$$

$$\frac{y_1}{y_2} = \frac{y_1}{0.1 y_1} = 10$$

$$\begin{aligned} \therefore NOG &= \frac{1}{1-\phi} \ln \left[(1-\phi) \frac{y_1}{y_2} + \phi \right] \\ &= \frac{1}{1-0.74} \ln \left[(1-0.74) \times 10 + 0.74 \right] \end{aligned}$$

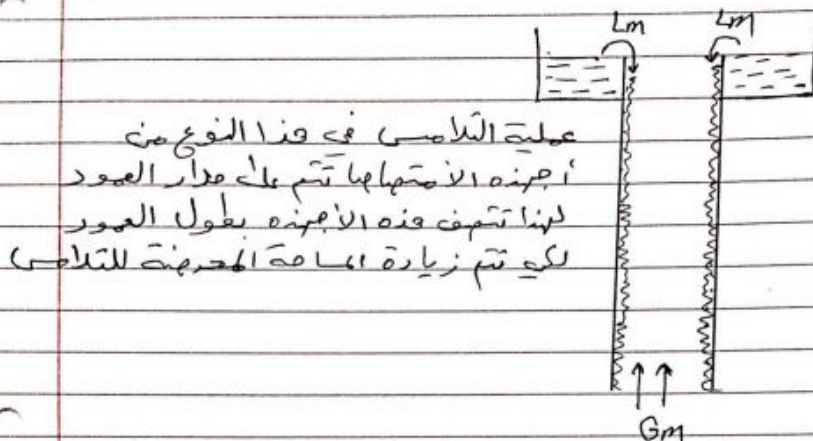
$$NOG = 4.64$$

$$Z = NOG HOG$$

$$= 4.64 \times 0.6$$

$$Z = 2.78 m$$

(12)

Wetted-Wall column Absorber:

The general equation for mass transfer in a wetted wall column is:

$$\frac{h_0 d}{D_v} \frac{P_B L M}{P_T} = \beta Re^{0.83} Sc^{0.44} \quad (1)$$

where: Re : Reynold No. $= \left(\frac{\rho u d}{\mu} \right)_{\text{gas}}$

Sc : Schmidt No. $= \left(\frac{\mu}{\rho D} \right)_{\text{gas}}$

h_0 : Mass transfer coefficient $= \left(\frac{D_v}{Z_0} \right)$

D_v : Vapor diffusivity $\frac{m^2}{\text{sec}}$

Z_0 : Thickness of gas film

d : Column diameter (m)

P_{BLM} : log mean of (P_B)

P_B : Partial pressure of B

P_T : Total pressure.

$$K_G = \left(\frac{h_0}{RT} \right) \left(\frac{P_{BLM}}{P_T} \right)$$

For wetted wall column the mass transfer coefficient for gas film h_0

$$\frac{h_0}{\mu} = 0.04 \left(\frac{\rho u d}{\mu} \right)^{-0.25} \left(\frac{\mu}{\rho u d} \right)^{-0.5} \left(\frac{P_T}{P_{BLM}} \right) \quad (2)$$

When for Packed tower:

$$\frac{h_0}{\mu} \frac{P_{BLM}}{P_T} \left[\frac{\mu}{\rho u d} \right]^{0.56} = \beta Re^{-0.17} = j_d = j_g \quad (3)$$

(13)

Plate Tower Absorption: أعمدة الامتصاص ذات ألواح

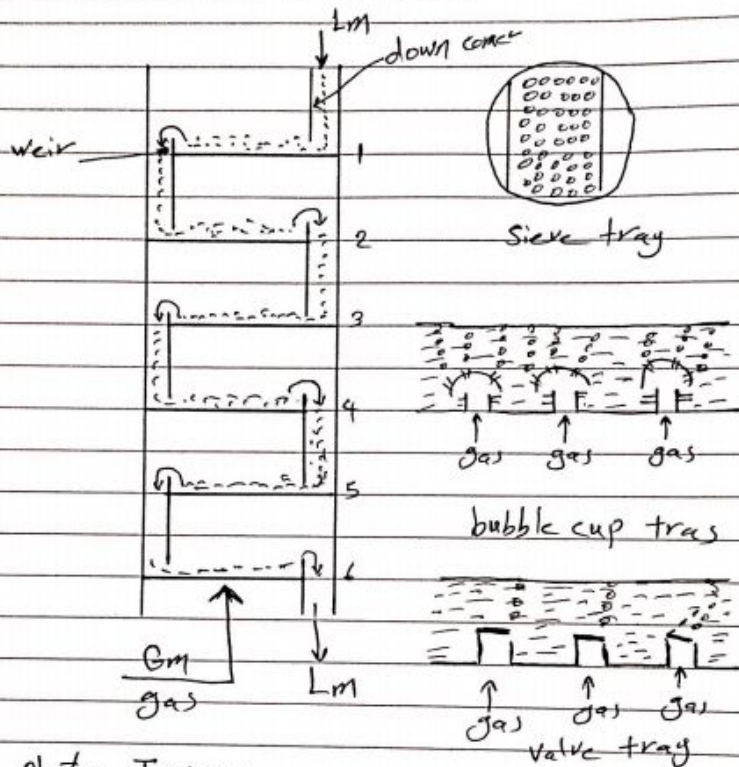


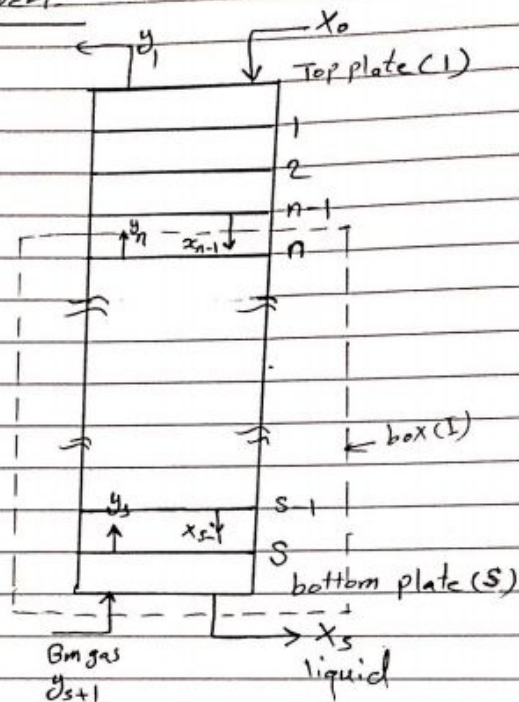
plate Tower:

Bubble-cup, sieve and valve tray are some times used for gas absorption particularly when the load is more than can be handled in a packed tower of about (1m) (when the liquid rate is sufficient to flood in a packed tower). plate tower consist of vertical cylindrical shell with number of plate.

when diameter $< 2m$ packed column are more often used,

في هذا النوع من الأبرشة يتم عملية الامتصاص على ألواح
فتم تعمل هذه الألواح على تقوية مساحة تلامس كمية بين الغاز
والسائل من طريق تكوين فقاعات للغاز العائد للأعلى مع السائل النازل
للأسفل وفي أبرشة صممة تعمل لكيارات السطح العالية للغاز أو السائل

plate Absorption Tower (14)



let:

- L_m = is the molar rate of liquid per (unit area, unit time)
- G_m = is the molar rate of gas per (unit area, unit time)
- n = refers to the plate number from the top downward.
- X = mole fraction of the absorbed component in the liquid.
- y = mole fraction of the absorbed component in the gas.
- S = Total number of plate in the column.

plate Absorption Tower:

For dilute solution $\Rightarrow y = Y$ & $x = X$

The gas leaving of composition y_n in equilibrium with the liquid of composition x_n .

Material Balance for absorber component (box I)

from bottom to a plane above plane (n) will give:

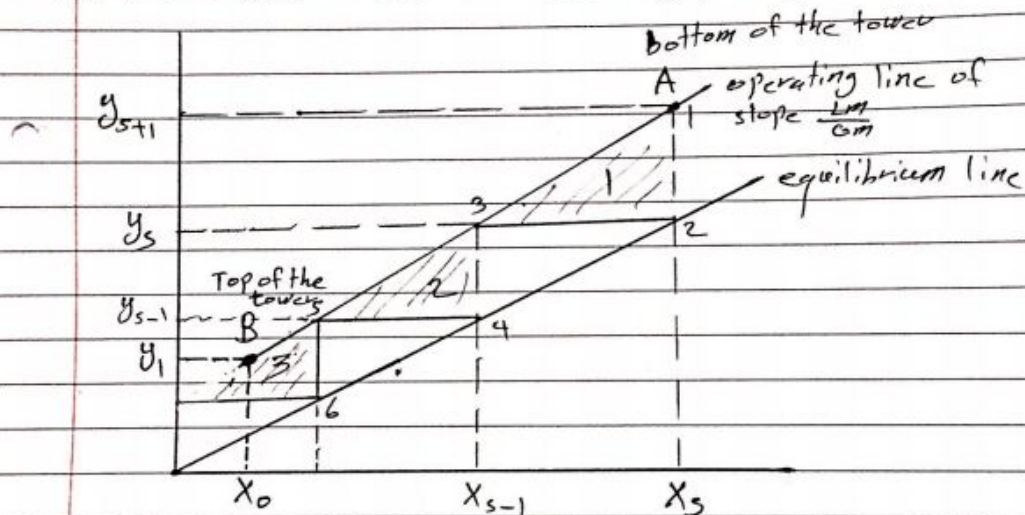
$$G_m y_{S+1} + L_m x_{n-1} = G_m y_n + L_m x_S$$

(15)

$$y_n = \frac{L_m}{G_m} x_{n-1} + y_{s+1} - \frac{L_m}{G_m} x_s$$

operating line of slope $\frac{L_m}{G_m}$

أقل تركيز لل (solute) في الغاز يكون في Top x_0
 أقل تركيز لل (solute) في السائل يكون في Top y_1
 أعلى تركيز لل (solute) في الغاز يكون عند دخول الغاز لل absorber y_{s+1}
 أعلى تركيز لل (solute) في السائل يكون عند خروج السائل من absorber x_s



- بالنسبة لل equilibrium
 لا يوجد فرق في رقم ال stage
 بين (x_1, y_1) و (x_2, y_2)
 (x_3, y_3) تكون في نفس ال stage
 at equilibrium

$\uparrow y_1$	$\downarrow x_0$	0
$\uparrow y_2$	$\downarrow x_1$	1
$\uparrow y_3$	$\downarrow x_2$	2
	$\downarrow x_3$	3

- بالنسبة لل operating
 يوجد فرق في رقم ال stage
 بين (x_0, y_1) و (x_1, y_2)
 و (x_2, y_3)

EX: 12:17

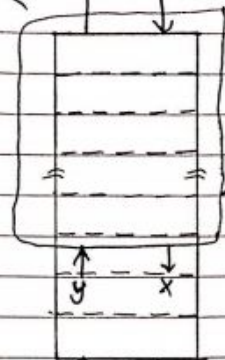
16

$$y_2 = 0.000$$

Fresh water

$$x_2 = 0$$

$$L_m = 0.65 \frac{\text{kg}}{\text{m}^2 \cdot \text{s}}$$



$$y_1 = 0.005$$

(dilute solution)
 $G_m = 0.4 \frac{\text{kg}}{\text{m}^2 \cdot \text{s}}$

$$L_m = \frac{0.65}{18} = 0.036 \frac{\text{kmol}}{\text{m}^2 \cdot \text{s}}$$

$$G_m = \frac{0.4}{29} = 0.0138 \frac{\text{kmol}}{\text{m}^2 \cdot \text{s}}$$

لما بـ حساب المادتين نجد أن معادلة operating line

box I: Material Balance

$$G_m (y - y_2) = L_m (x - x_2)$$

$$0.0138 (y - 0.000) = 0.036 x$$

$$y = 2.61 x + 0.000 \rightarrow \text{operating line equation}$$

ثم نرسم معادلة الـ operating line ومعادلة الـ equilibrium line

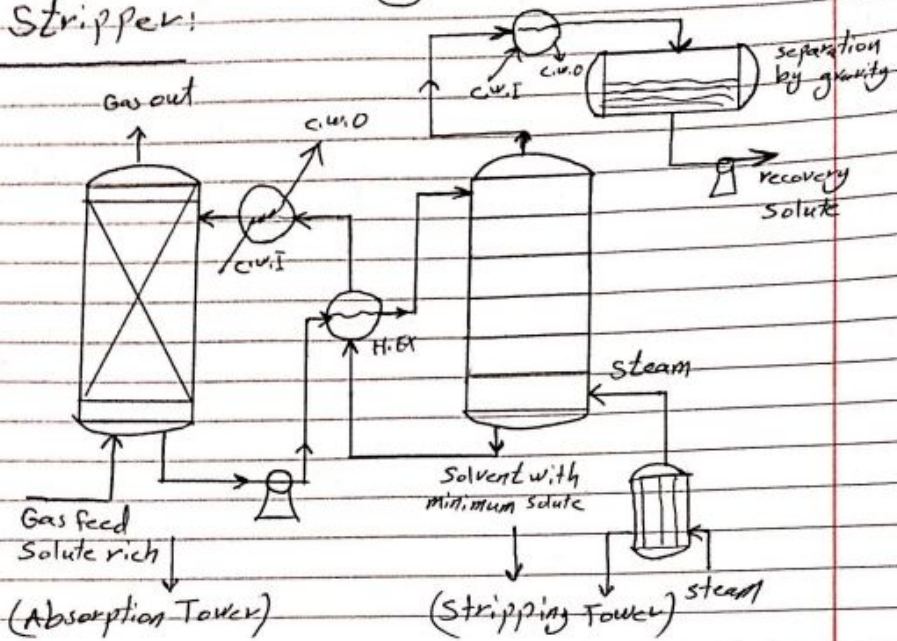
equilibrium line $y = x$ (45° خط)

ويوجد أن نرسم على المخطط من $y_1 = 0.005$ إلى $y_2 = 0.000$

يتم كتابة السؤال، ورسمة H.W.
من قبل الطلبة.

(17)

Stripper:



Stripper:

Stripping involves the removal of one or more volatile components from a liquid by contacting with an inert gas such as steam in mass transfer occur from liquid to gas ($L \rightarrow G$)

plate tower absorption

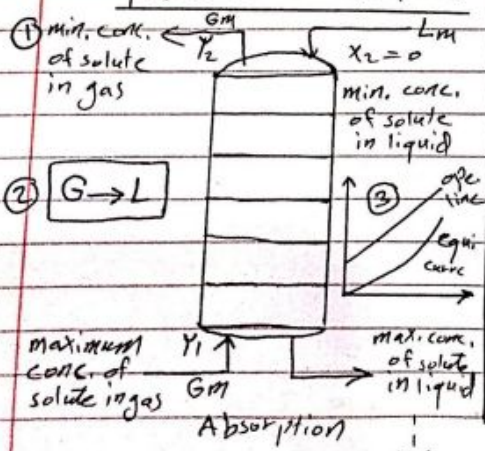
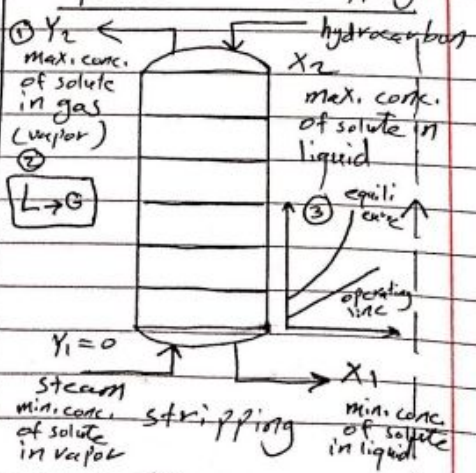
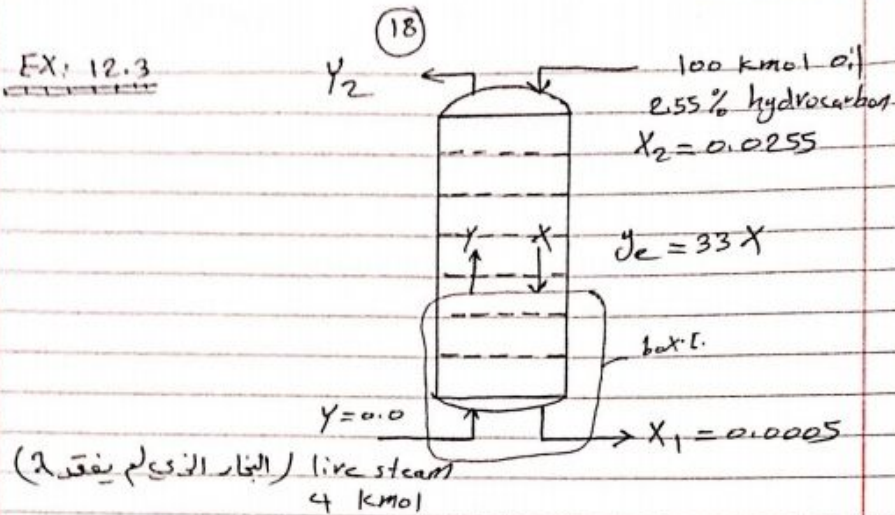


plate tower stripping



EX: 12.3



لعداد 18 والنفذ معادلة line steam
والمعادلة

box I. Material Balance

$$L_m(X - X_1) = G_m(Y - Y_1)$$

$$Y G_m = L_m X - L_m X_1$$

$$Y = \frac{L_m}{G_m} X - \frac{L_m}{G_m} X_1$$

$$Y = \frac{100}{4} X - 25 \times (0.0005)$$

$$Y = 25X - 0.0125 \text{ operating line equation.}$$

$$Y_e = 33X$$

$$\frac{Y_e}{1 + Y_e} = \frac{33X}{1 + X}$$

$$Y_e = \frac{33X}{1 - 32X} \text{ equilibrium curve}$$

فرصة
equilib.
curve

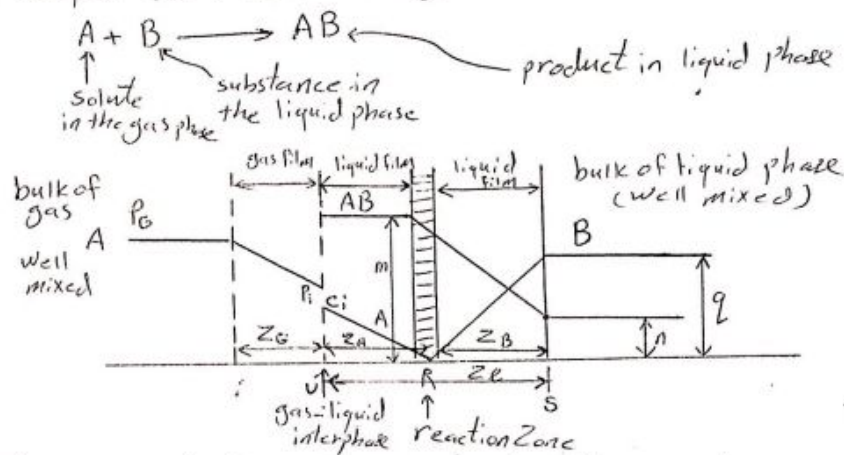
$X_1 = 0.005$	$Y_1 = 0.17$
$X_2 = 0.01$	$Y_2 = 0.48$
$X_3 = 0.015$	$Y_3 = 0.75$

H.W. complete the drawing
and find number of
stage.

Absorption Associated with chemical reaction (17)

The gas on absorption react chemically with a component of the liquid phase for example the absorption of carbon dioxide by caustic soda $(\text{CO}_2) + \text{NaOH}(\text{L}) \rightarrow \text{Na}_2\text{CO}_3 + \text{H}_2\text{O}$

In such process the condition in the gas phase are similar to those already discussed, but in the liquid phase there is a liquid film followed by a reaction zone.



- ① - The gas A diffuses through part of the liquid film before meeting B
 - ② - The concentration of B at the interphase falls, this results in diffusion of B from the bulk of liquid phase to the interphase.
 - ③ - The new product AB formed, diffuses towards the main body of the liquid.
 - ④ - The driving force of A which is diffusing through the gas film $= P_G - P_i$
 - ⑤ - The driving force of A which is diffusing to the reaction zone $= C_i - 0$ (in the liquid phase)
 - ⑥ - The component B diffuses from the main body of the liquid to reaction zone under driving force $C_B - C_i$.
 - ⑦ - The product AB diffuses back to the main bulk of the liquid under a driving force $m - n$.
- For transfer in the gas phase $N_A = K_G (P_G - P_i)$
 and in the liquid phase $N_A = K_L (C_i - 0)$

for irreversible chemical reaction at steady state (20)

$$\propto N_A = N_B \quad , \quad Z_L = Z_A + Z_B$$

$$N_A = \frac{D_A}{Z_A} (C_{A_i} - 0) \Rightarrow Z_A = \frac{D_A}{N_A} C_{A_i} \quad \text{--- (1)}$$

$$N_B = \frac{D_B}{Z_B} (C_{B_i} - 0) \Rightarrow Z_B = \frac{D_B}{N_B} C_{B_i} \quad \text{--- (2)}$$

$$Z_L = \frac{D_A C_{A_i}}{N_A} + \frac{D_B C_{B_i}}{N_B}$$

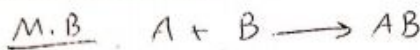
$$Z_L = \frac{D_A C_{A_i}}{N_A} + \frac{D_B C_{B_i}}{\alpha N_A} = \frac{1}{N_A} \left[D_A C_{A_i} + \frac{D_B C_{B_i}}{\alpha} \right]$$

$$Z_L = \frac{D_A C_{A_i}}{N_A} \left[1 + \frac{D_B C_{B_i}}{\alpha D_A C_{A_i}} \right] \quad \text{--- (3)} \Rightarrow \frac{N_A}{C_{A_i}} = \frac{D_A}{Z_L} \left[1 + \frac{D_B C_{B_i}}{\alpha D_A C_{A_i}} \right]$$

$$\text{Define } K_L \Rightarrow N_A = K_L (C_{A_i} - 0) \Rightarrow K_L = \frac{N_A}{C_{A_i}}$$

$$\therefore K_L = \frac{D_A}{Z_L} \left[1 + \frac{D_B C_{B_i}}{\alpha D_A C_{A_i}} \right] \quad \text{--- (4)}$$

equation of operating line



$$G(y_{A \text{ bottom}} - y_A) = L(c_{B \text{ bottom}} - c_B)$$

$$\text{where } y_A = \frac{p_A}{P}$$

$$y_{A \text{ bottom}} - y_A = \frac{L}{G} (c_{B \text{ bottom}} - c_B)$$

$$-y_A = \frac{L}{G} (c_{B \text{ bottom}} - c_B) - y_{A \text{ bottom}}$$

$$y_A = \frac{L}{G} c_B - \frac{L}{G} c_{B \text{ bottom}} + y_{A \text{ bottom}}$$

$$\boxed{y_A = \frac{L}{G} c_B + (y_{A \text{ bottom}} - \frac{L}{G} c_{B \text{ bottom}})} \quad \text{equation of operating line} \quad \text{--- (5)}$$

$$Z = \frac{G}{K_O G a P_T} \int_{p_{A \text{ top}}}^{p_{A \text{ bottom}}} \frac{d p_A}{p_A + \frac{H D_B C_B}{D_A}}$$

