Height of column based on conditions in gas film

let:

G_m = moles of inert gas / (unit time) (unit C.S.A) of tower)

L_m = moles of solvent (solute free) / (unit time) (unit C.S.A of tower)

Y = moles of solute of gas A / moles of inert gas B in gas phase $\left[\frac{A}{B}\right]$

X = moles of solute of gas A / moles of solvent (solute free) $\left[\frac{A}{L_m}\right]$

over a small height dZ, the mols of gas leaving the gas phase will equal the moles taken up by the liquid.

$$\underset{\frac{A}{B}}{dY}\ \underset{B}{G_m}\ \underset{\frac{(C.S.A)}{m^2}}{A} = \underset{\frac{A}{L_m}}{dX}\ L_m\ \underset{m^2}{A} \quad \text{—— (15)}$$

$$\frac{kmol}{m^2 \cdot s} \times m^2 = \frac{kmol}{s}$$

gas leaving the gas phase = moles taken up by the liquid

gas diffusion

$$G_m \cancel{A}\, dY = K_G a \left[P_G - P_i\right] \cancel{A}\, dZ \quad \text{—— (16)}$$

but $P_G = y\, P_T$ —— (17)

y = mole fraction $\frac{A}{A+B}$

Y = mole ratio $= \frac{A}{B}$

$$y = \frac{A}{A+B} = \frac{\frac{A}{B}}{\frac{A}{B}+\frac{B}{B}} = \frac{Y}{Y+1}$$

substitute in eq. 17:

$$P_G = \frac{Y}{Y+1}\, P_T$$

$$G_m\, dY = K_G a \left[\frac{Y}{Y+1} P_T - \frac{Y_i}{Y_i+1} P_T\right] dZ$$

$$= K_G a P_T \left[\frac{Y}{Y+1} - \frac{Y_i}{Y_i+1}\right] dZ$$

$$G_m\, dY = K_G a P_T \left[\frac{Y(1+Y_i) - Y_i(1+Y)}{(1+Y)(1+Y_i)}\right] dZ$$

$$G_m\, dY = K_G a P_T \left[\frac{Y - Y_i}{(1+Y)(1+Y_i)}\right] dZ$$

$$\int_0^Z dZ = \frac{G_m}{K_G a P_T} \int_{Y_2}^{Y_1} \frac{(1+Y)(1+Y_i)}{Y - Y_i}\, dY \quad \text{—— (18)}$$

For weak (dilute solution) ($\leq 5\%$) mixture can be written as:

$$\boxed{Z = \frac{G_m}{K_G a P_T} \int_{Y_2}^{Y_1} \frac{dY}{Y - Y_i}} \quad —— (19)$$

Z = height of packing.

Height of column Based on condition in liquid film

$$A L_m\, dx = k_L a (C_i - C_L) A\, dZ \quad —— (20)$$

where the concentration C are in terms of moles of solute per unit volume of liquid.

$$C_T = \frac{\text{moles of solute + solvent}}{\text{volume of liquid}}$$

$$X = \frac{\text{mols of solute}}{\text{mols of solvent}} = \frac{C}{C_T - C}$$

$$\therefore \boxed{C = \frac{X}{1+X} C_T}$$

$$X = \frac{C}{C_T - C}$$

$$X C_T - X C = C$$

$$X C_T = C + X C$$

$$X C_T = C(1+X)$$

$$C = \frac{X}{1+X} C_T$$

$$L_m\, dX = k_L a\, C_T \left[\frac{X_i}{1+X_i} - \frac{X}{1+X}\right] dZ$$

$$\boxed{\int_0^Z dZ = Z = \frac{L_m}{k_L a C_T} \int_{X_2}^{X_1} \frac{(1+X_i)(1+X)\, dX}{X_i - X}} \quad 21)$$

and for dilute solution this gives

$$\boxed{Z = \frac{L_m}{k_L a C_T} \int_{X_2}^{X_1} \frac{dX}{X_i - X}} \quad —— (22)$$

where C_T and k_L have taken as constant over the column.

Height based on overall coefficient

If the driving force based on the gas concentration is written as $(Y - Y_e)$ and the overall gas transfer coefficient is KOG than the hight of the tower for dilute concentration becomes:

$$\boxed{Z = \frac{G_m}{K_{OG}\, a\, P_T} \int_{Y_2}^{Y_1} \frac{dY}{Y - Y_e}} \quad \text{—— (23)}$$

or in term of the liquar concentration as:

$$\boxed{Z = \frac{L_m}{K_{OL}\, a\, C_T} \int_{X_2}^{X_1} \frac{dX}{X_e - X}} \quad \text{—— (24)}$$

For dilute concentration

$y = 0.01$ $\qquad y = \frac{Y}{Y+1}$

$0.01 = \frac{Y}{Y+1} \Rightarrow Y = 0.01Y + 0.01$

$Y - 0.01Y = 0.01$

$0.99Y = 0.01 \Longrightarrow Y = \frac{0.01}{0.99} = 0.0101$

$Y \approx y$.

As the mol fraction is approximately equal to the molar ratio at dilute concentration then the gas film is

$$\boxed{Z = \frac{G_m}{K_{OG}\, a\, P_T} \int_{y_2}^{y_1} \frac{dy}{y - y_e}} \quad \text{—— (25)}$$

and liquid film

$$\boxed{Z = \frac{L_m}{K_{OL}\, a\, P_T} \int_{x_2}^{x_1} \frac{dx}{x_e - x}} \quad \text{—— (26)}$$

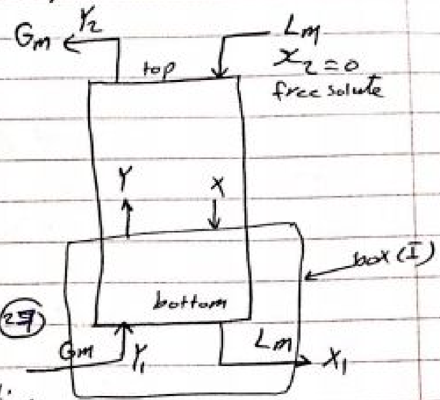
$Z = HOG \times NOG$

HOG = high of transfer unit $= \frac{G_m}{K_{OG}\, a\, P_T} = m$

NOG = Number of transfer unit = $\int \frac{dy}{y - y_e}$

بدون وحدات

The operating line :



box I M.B

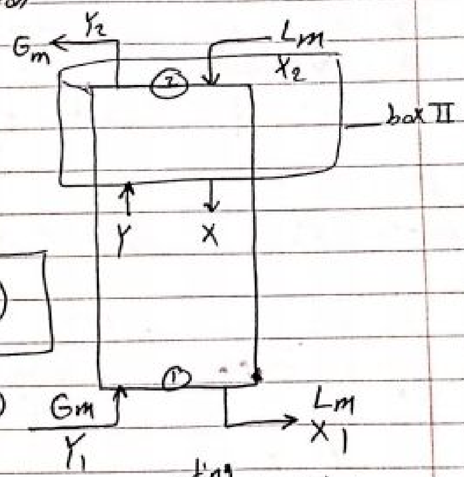
$$G_m Y_1 + L_m X = G_m Y + L_m X_1$$

$$G_m (Y_1 - Y) = L_m (X_1 - X)$$

$$Y_1 - Y = \frac{L_m}{G_m} (X_1 - X)$$

$$\boxed{Y = \frac{L_m}{G_m} X + \left(Y_1 - \frac{L_m}{G_m} X_1\right)} \quad (27)$$

it is a straight line equation of slope $\frac{L_m}{G_m}$

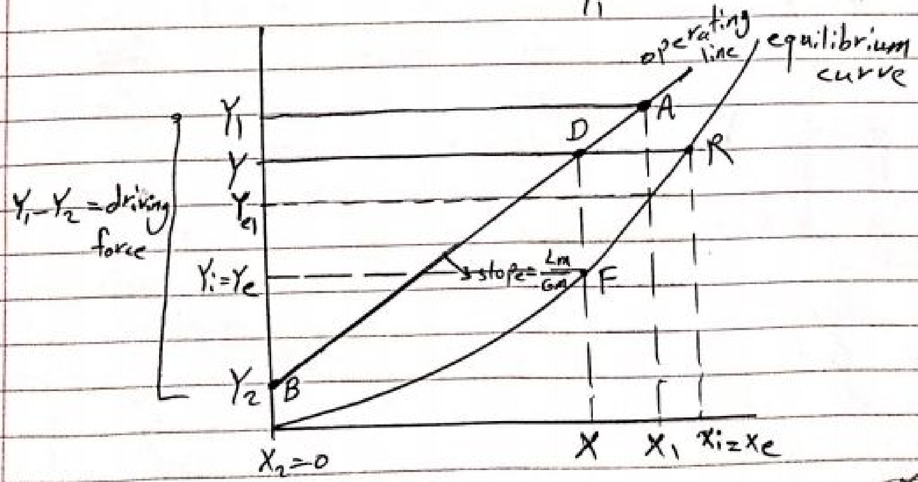


box II . M.B

$$G_m Y + L_m X_2 = G_m Y_2 + L_m X$$

$$Y_2 - Y = \frac{L_m}{G_m} (X_2 - X)$$

$$\boxed{Y = \frac{L_m}{G_m} X + \left(Y_2 - \frac{L_m}{G_m} X_2\right)} \quad (28)$$



This is an operating line (straight line) of slope $\frac{L_m}{G_m}$ and represent the condition at any point in the column.

AB = operating line

A = concentration at the bottom.

B = concentration at the top.

D (X, Y) = the condition of the bulk of the liquid and gas at any point in the column

IF the gas film is controlling the process:

$$Y_e = Y_i \text{ (point F)}$$

The driving force = DF = $Y - Y_i$

Therefore it is possible to evaluate the expression by selecting value of (Y) reading from the diagram the corresponding value of Y_i and then calculate $\frac{1}{Y - Y_i}$

- IF the liquid film controls the process

$$X_i = X_e$$

the driving force $X_i - X = DR$

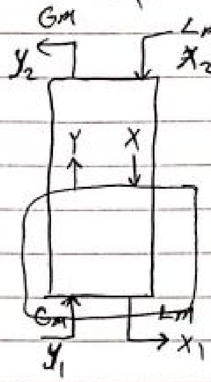
and $\int_{X_2}^{X_1} \frac{dX}{X_i - X}$ calculated

- Special case when equilibrium curve is a straight line and dilute solution:

For dilute concentration over a small height dZ the absorption is:

$$N_A A a \, dZ = G_m A \, dY = K_{OG} a P_T (y - y_e) \, dZ \quad (26)$$

$$y_e = mX + c \quad (27)$$ equilibriume line يكون خط مستقيم



$$y_{e1} = mX_1 + c \Rightarrow y_{e1} - mX_1 = c$$

$$y_{e2} = mX_2 + c \Rightarrow y_{e2} - mX_2 = c$$

$$\therefore y_{e1} - mX_1 = y_{e2} - mX_2$$

$$m = \frac{y_{e1} - y_{e2}}{X_1 - X_2} \quad (28)$$

Overall M.B:

$$G_m y_1 + L_m X_2 = G_m y_2 + L_m X_1$$

$$G_m (y_1 - y_2) = L_m (X_1 - X_2)$$

$$\frac{G_m}{L_m} = \frac{X_1 - X_2}{y_1 - y_2}$$

Material Balance over the lower portion of the column

$$L_m (X - X_1) = G_m (y - y_1)$$

$$X = X_1 + \frac{G_m}{L_m} (y - y_1) \quad (29)$$

from eq. (22)

$$Z = \frac{G_m}{K_{OG} a P_T} \int_{y_2}^{y_1} \frac{dy}{y - y_e} \qquad (y_e = mX + c)$$

$$\therefore Z = \frac{G_m}{K_{OG} a P_T} \int_{y_2}^{y_1} \frac{dy}{y - mX - c}$$

$$Z = H_{OG} \times N_{OG}$$

from eq. (29)

$$NOG=\int_{y_2}^{y_1}\frac{dy}{y-m\left[x_1+\frac{G_m}{L_m}(y-y_1)\right]-c}$$

$$NOG=\int_{y_2}^{y_1}\frac{dy}{y\left(1-m\frac{G_m}{L_m}\right)-m\left(x_1-\frac{G_m}{L_m}y_1\right)}\times\frac{1-m\frac{G_m}{L_m}}{1-m\frac{G_m}{L_m}}$$

$$NOG=\underbrace{\frac{1}{1-m\frac{G_m}{L_m}}}_{E}\,Ln\underbrace{\frac{y_1\left(1-m\frac{G_m}{L_m}\right)-m\left(x_1-\frac{G_m}{L_m}y_1\right)}{y_2\left(1-m\frac{G_m}{L_m}\right)-m\left(x_1-\frac{G_m}{L_m}y_1\right)}}_{=F}$$

$$NOG=E\,Ln\,F$$

$$E=\frac{1}{1-\underbrace{\frac{y_{e1}-y_{e2}}{x_1-x_2}}_{m}\cdot\underbrace{\frac{x_1-x_2}{y_1-y_2}}_{\frac{G_m}{L_m}}}=\frac{1}{\frac{y_1-y_2-y_{e1}+y_{e2}}{y_1-y_2}}$$

$$E=\frac{y_1-y_2}{(y_1-y_{e1})-(y_2-y_{e2})}$$

$$F=\frac{y_1\left[1-\frac{y_{e1}-y_{e2}}{\cancel{x_1-x_2}}\cdot\frac{\cancel{x_1-x_2}}{y_1-y_2}\right]-\frac{y_{e1}-y_{e2}}{x_1-x_2}x_1+\left[\frac{y_{e1}-y_{e2}}{\cancel{x_1-x_2}}\times\frac{\cancel{x_1-x_2}}{y_1-y_2}\right]y_1}{y_2\left[1-\frac{y_{e1}-y_{e2}}{\cancel{x_1-x_2}}\cdot\frac{\cancel{x_1-x_2}}{y_1-y_2}\right]+\left[\frac{y_{e1}-y_{e2}}{y_1-y_2}\right]y_1-\frac{y_{e1}-y_{e2}}{x_1-x_2}x_1}$$

$= y_{e1}$

$$F=\frac{y_1\left[1-\frac{y_{e1}-y_{e2}}{y_1-y_2}\right]+\left[\frac{y_{e1}-y_{e2}}{y_1-y_2}\right]y_1-\overbrace{\frac{y_{e1}-y_{e2}}{x_1-x_2}}^{=y_{e1}}x_1}{y_2\left[1-\frac{y_{e1}-y_{e2}}{y_1-y_2}\right]+\left(\frac{y_{e1}-y_{e2}}{y_1-y_2}\right)y_1-\underbrace{\frac{y_{e1}-y_{e2}}{x_1-x_2}}_{=y_{e1}=mx_1}x_1}$$

$$F=\frac{y_1-\cancel{\left(\frac{y_{e1}-y_{e2}}{y_1-y_2}\right)y_1}+\cancel{\left(\frac{y_{e1}-y_{e2}}{y_1-y_2}\right)y_1}-y_{e1}}{y_2-\left(\frac{y_{e1}-y_{e2}}{y_1-y_2}\right)y_2+\left(\frac{y_{e1}-y_{e2}}{y_1-y_2}\right)y_1-y_{e1}}$$

$$F=\frac{y_1-y_{e1}}{y_2+\cancel{(y_1-y_2)}\left(\frac{y_{e1}-y_{e2}}{\cancel{y_1-y_2}}\right)-y_{e1}}$$

$$\boxed{F=\frac{y_1-y_{e1}}{y_2+\cancel{y_{e1}}-y_{e2}-\cancel{y_{e1}}}=\frac{y_1-y_{e1}}{y_2-y_{e2}}}\qquad(30)$$

$$\therefore N_{OG} = \frac{y_1 - y_2}{(y-y_e)_1 - (y-y_e)_2} \ln \frac{(y-y_e)_1}{(y-y_e)_2} \quad (9)$$

E F

$$\therefore \boxed{N_{OG} = \frac{y_1 - y_2}{(y-y_e)_{LM}}} \quad (34)$$

where $$\boxed{(y-y_e)_{LM} = \frac{(y-y_e)_1 - (y-y_e)_2}{\ln \frac{(y-y_e)_1}{(y-y_e)_2}}}$$ when equilibrium curve is straight line (35)

$(y-y_e)_1$ = bottom driving force
$(y-y_e)_2$ = top driving force

$$\boxed{Z = \underbrace{\frac{G_m}{K_{OG}\, a\, P_T}}_{H_{OG}} \cdot \underbrace{\frac{y_1 - y_2}{(y-y_e)_{LM}}}_{N_{OG}}} \quad (36)$$

in term of mol. fraction

$$\boxed{a A Z N_A = G_m (y_1 - y_2) A = K_{OG}\, a\, A\, P_T\, Z\, (y-y_e)_{LM}} \quad (37)$$

$$\boxed{a A Z N_A = G_m (Y_1 - Y_2) A = K_{OG}\, a\, A\, P_T\, Z\, (Y-Y_e)_{LM}} \quad (38)$$

interm of mol ratio