(طرح)

## Humidification and Water Cooling

humidification : increase the amount of vapour in a gas stream.
dehumidification : reduce the amount of vapour present in the gas stream.

in humidification the vapour increase by passing the gas over a liquid which then evaporates into the gas stream.
the system is air-water.

Humidity $\mathcal{H}$ : mass of vapour associated with unit mass of dry air (kg water/kg dry air)

Humidity of saturated gas ($\mathcal{H}_0$) ; humidity of gas when it is saturated with vapour at a given temp. (saturation humidity.)

percent humidity : $\mathcal{H}/\mathcal{H}_0 \times 100$ .

Humid heat (S) : heat required to raise the temperature of unit mass of dry gas (air) and its associated vapour (water) by 1 °C.

$$S = C_a + \mathcal{H} C_w$$

where $C_a$ = specific heat capacity of gas
$C_w$ = specific heat capacity of the vapour

for air water system

$$S = 1.0 + 1.9\,\mathcal{H} \quad KJ/kg\,K$$

Humid volume : volume occupied by unit mass of dry gas and its associated vapour

Saturation volume : humid volume of saturated gas.

Dew point : temperature at which the gas is saturated with vapour. If a gas is cooled, the dew point is the temperature at which condensation will first occur.

% relative humidity = $\frac{\text{Partial pressure of water vapour in gas}}{\text{Partial pressure of water vapour in saturated gas}}$

$= \frac{P_w}{P_{w_s}} \times 100$

relationship between partial pressure of the vapour ($P_w$) and humidity of gas ($\mathcal{H}$):

consider 1 unit volume of gas (assume ideal gas)

mass of water vapour $= P_w M_w / RT$ $\quad PV = \frac{m}{M} RT$

mass of air $= P_A M_A / RT$

$$\mathcal{H} = \frac{P_w M_w}{P_A M_A} = \frac{P_w}{P_A} \times \frac{18}{29}$$

$\mathcal{H} = \frac{\text{kg water}}{\text{kg dry air}}$

$P_v = \frac{m}{M} RT$

$P_A + P_w = P_T$

air water

$$\mathcal{H} = \frac{P_w}{P_T - P_w} \times \frac{18}{29} \quad \text{——— (1)}$$

$P_w << P_T$

$$\mathcal{H} = \frac{P_w}{P_T} \frac{18}{29}$$

where $P$ = total pressure.

$$\mathcal{H}_s = \frac{P_{w_s}}{P_T - P_{w_s}} \frac{M_w}{M_A} \quad \text{——— (2)}$$

% humidity $= \frac{\mathcal{H}}{\mathcal{H}_s} \times 100$

from (1) & (2)

% humidity $= \frac{P_T - P_{w_s}}{P_T - P_w} \cdot \left( \frac{P_w}{P_{w_s}} \times 100 \right)$ → Percent humidity

% humidity $= \frac{P_T - P_{w_s}}{P_T - P_w} \cdot Z$ ← % relative humidity, ——— (3)

% humidity = % relative humidity when $\frac{P - P_{w_s}}{P - P_w} \approx 1$

↓

when partial pressure of wat vapour is a small proportion o the total pressur or when the

ex: 11.2 Page 963

Ex: In a vessel at 101.3 kN/m² and 300K, the percentage relative humidity of the water vapour in the air is 25. If the partial pressure pressure of water vapour when air is saturated with vapour at 300K is 3.6 kN/m², Calculate ?

a) the partial pressure of the water vapour in the vessel
b) the specific volumes of the air and water vapour.
c) the humidity of the air and humid volume
d) the percentage humidity.

Solution: a) % relative humidity = $Z = \frac{P_w}{P_{w'}} \times 100$

$P_w = 3.6$ kN/m².

$\frac{25}{100} = \frac{P_w}{3.6} \times 100$

$P_w = \frac{25}{100} \times 3.6$

$= 0.9$ kN/m²

b) for 1 m³ of air

S.P volume = $\frac{1}{\text{density}} = \frac{1}{\rho}$

mass of water vapour = $P_w M_w / RT$

$= 0.9 \times 18 / (8.314 \times 300)$

$= 0.0064$ kg

mass air = Pair Mw

$P_A = (P_{total} - P_{water})$

mass of air = $\frac{(101.3 - 0.9) \times 29}{(8.314 \times 300)}$

$= 1.167$ kg

specific volume of water vapour = $\frac{1}{0.0064} = 156.3 \frac{m^3}{kg}$

specific volume of air = $\frac{1}{1.1673} = 0.857 \frac{m^3}{kg}$

c) $\chi = \frac{0.0064}{1.1673} = 0.00548 \frac{\text{kg water}}{\text{kg dry air}}$

or $\chi = \frac{P_w}{P_1} \times \frac{18}{29}$

$= \frac{0.9 \times 18}{101.3 \times 29} = 0.00551 \frac{\text{kg water}}{\text{kg dry air}}$

or $\chi = \frac{0.9 \times 1}{(101.3 - 0}$

$= 0.0$

in 1 kg air vapour mixture, there is $\frac{0.0055}{1.0055} = 0.0054$ kg w

and $(1 - 0.0054) = 0.9946$ kg air

Humid volume = $(0.9946 \text{ kg air} \times 0.857 \frac{m^3}{\text{kg air}}) + (0.0054 \text{ kg water}$

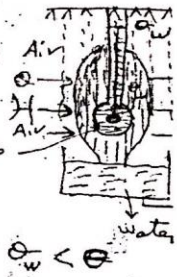
## Wet-bulb Temperature ($\Theta_w$)

when a stream of unsaturated gas is passed over surface of a liquid, the humidity of the gas is increased to evaporation of the liquid. The temperature of the liqu. falls below that of the gas and heat is transferred fro the gas to the liquid. At equilibrium the rate of hea transfer from the gas just balances that required to vapo the liquid is said to be at the wet-bulb temperature.

The rate of heat at which this temp. is reached depends the initial temperature and 2) the rate of flow of gas past the surface

heat transfer from gas to liquid $Q_w = h A (\Theta - \Theta_w)$ —(4)

gas temp. liquid temp.




$Q$ = heat flow

$h$ = heat transfer coefficient $W/m^2 K$

$A$ = area for transfer $m^2$

$\Theta$ = temp of the gas phase $K$

$\Theta_w$ = temp of the liquid phase $K$ (wet bulb tem.) $\Theta_w < \Theta$

The liquid evaporating into the gas is transferred $H < H_s$

by diffusion from the interface to the gas stream as a re of a concentration difference $(C_s - C)$

$$W = K_G A (C_s - C) \quad \text{——(5)}$$

$W$ = rate of evaporation $kg/s$

$K_G$ = mass transfer coefficient $kmol/(kmole/m^3)m^2 s$ $(m/s)$.

$A$ = area for transfer $m^2$

$C_s$ = the concentration of water vapour at the surface (saturated conc.) $(kg/m^3)$.

$C$ = the concentration in the gas stream $(kg/m^3)$.

$$P_w V = \frac{m_w}{M_w} R T$$

$$C = \frac{m_w}{V} = \frac{P_w M_w}{RT}$$

$$W = K_G A \left(\frac{M_w}{RT}\right)(P_{w_s} - P_w) \quad \text{——(6)}$$

$P_w$ = Partial pressure of vapour H/M in gas

$P_{w_s}$ = Partial pressure of

if $P_w << P_T$

$P_{w_0} << P_T$

$P_T - P_w = P_A$

$P_T - P_{w_0} = P_A$

$\mathcal{H}_0 - \mathcal{H} = (P_{w_0} - P_w)\frac{1}{P_A}\frac{M_w}{M_A}$

$P_{w_0} - P_w = (\mathcal{H}_0 - \mathcal{H})\frac{P_A M_A}{M_w}$

$W = \left[k_G A (\mathcal{H}_0 - \mathcal{H})\frac{M_w}{RT}\right]\frac{P_A M_A}{M_w}$

where $P_A$ = mean partial pressure of the gas.

$\mathcal{H} = \frac{P_w}{P - P_w}\frac{M_w}{M_A}$ — (1)

$\mathcal{H}_0 = \frac{P_{w_0}}{P - P_{w_0}}\frac{M_w}{M_A}$ — (2)

$PV = \frac{wt}{M_{wt}}RT$

$P \cdot M_{wt} = \frac{wt}{V}RT$

$\rho = \frac{P \cdot M_{wt}}{RT}$

$W = k_G A \rho_A (\mathcal{H}_0 - \mathcal{H})$

$\mathcal{H}_0$ = humidity of saturated gas

$\mathcal{H}_w$ = humidity of gas saturated at wet bulb temp.

$W = k_G A \rho_A (\mathcal{H}_w - \mathcal{H})$ — (8)

where $\rho_A$ = is the density of the gas at partial pressure

The heat transfer required to maintain the rate of evaporation

$Q = W\lambda$

where $\lambda$ = latent heat of vaporization of the liquid J/kg

$Q = k_G A \rho_A (\mathcal{H}_w - \mathcal{H})\lambda$ — (9)

from eq. (4) & (9)

$Q_w = h A (\theta - \theta_w)$

$h A (\theta - \theta_w) = k_G A \rho_A (\mathcal{H}_w - \mathcal{H})\lambda$

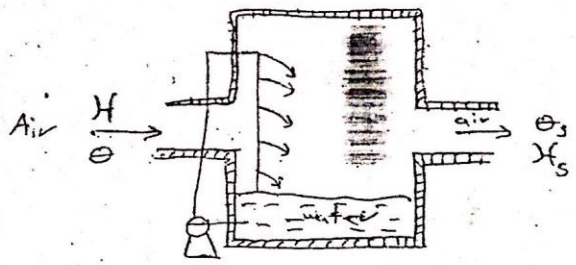
$$\mathcal{H}_w - \mathcal{H}_{air} = \frac{h}{k_G \rho_A \lambda}(\theta - \theta_w)$$ — (10)

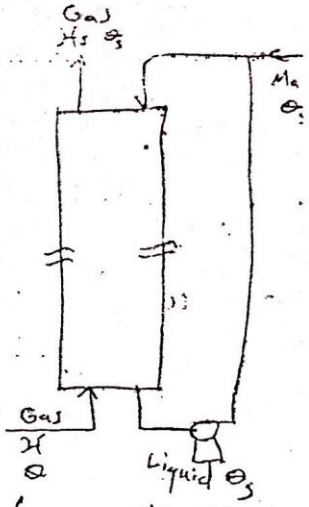
$\mathcal{H} - \mathcal{H}_w = -\left(\frac{h}{k_G \rho_A \lambda}\right)(\theta - \theta_w)$

for air-water system $\frac{h}{k_G \rho_A} \cong 1.0 \frac{KJ}{Kg \cdot K}$

$= 1.5 - 2$ KJ

## Adiabatic Saturation Temperature ($\theta_s$)

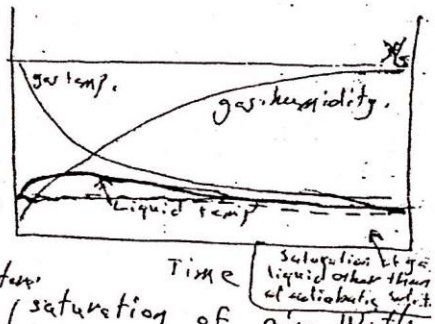



$H_s$ = humidity of gas saturated at adiabatic saturation temperature.

If the gas is passed over the liquid at such a rate that the time of contact is sufficient for equilibrium.

In a thermal insulation system, the total sensible heat falls by an amount equal to the latent heat of the liquid evaporated. The temperature of the liquid gradually approaches an equilibrium value which is known as the adiabatic saturation temperature.




Basis = unit mass of dry gas.

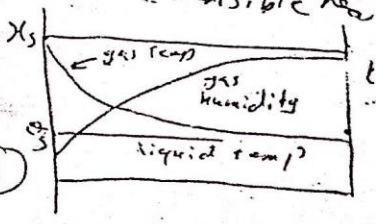
Heat balance over the column:

Heat of vaporization of the liquid come from the sensible hea in the gas.

$$(\theta - \theta_s)S = (H_s - H)\lambda$$

($H_s - H$: liquid evaporated)

$$(H - H_s) = -\left(\frac{S}{\lambda}\right)(\theta - \theta_s) \quad \text{---- (11)}$$




Where $S$ = humid heat of the gas

$\lambda$ = latent heat of vaporization at $\theta_s$.

1- $\mathcal{H}$ for various values of the % relative humidity.

For saturated gas $\mathcal{H}_0 = \left[\frac{P_{w_0}}{P - P_{w_0}}\left(\frac{M_w}{M_a}\right)\right]$ — (2)

percent humidity $= \frac{P - P_{w_0}}{P - P_w} \times \frac{P_w}{P_0} \times 100$ — 13

$\% \mathcal{H} = \frac{P - P_{w_0}}{P - P_w} \times$ % relative humidity $= Z$ ( % relative humidity

$\frac{\mathcal{H}}{\mathcal{H}_0} \times 100 = \frac{P - P_{w_0}}{P - P_w} \times Z$

$\mathcal{H} = \mathcal{H}_0 \left(\frac{Z}{100}\right)\left(\frac{P - P_{w_0}}{P - P_w}\right)$ — (12)

2- specific volume of dry gas: this is a linear functi of temperature.

3- the saturated volume, increase more rapidly with tempe

4- the latent heat of vaporisation and humid heat are plotted

## Enthalpy-Humidity Chart.

if $H$ = enthalpy of the humid gas per unit mass of dry gas $\bar{J}/kg$

$H_a$ = enthalpy of the dry gas per unit mass $\bar{J}/kg$ (air tahlili)

$H_w$ = enthalpy of the vapour per unit mass $\bar{J}/kg$

$C_a$ = specific heat of the gas at constant pressure $\bar{J}/kg$

$C_w$ = specific heat of the vapour at constant pressure $\bar{J}/k$

$\theta$ = temp. of humid gas K

$\theta_0$ = reference temperature K (melting point of the wate

$\lambda$ = latent heat of vaporisation of the liquid at $\theta_0$ $\bar{J}/kg$

$\mathcal{H}$ = humidity of the gas. kg/kg

for an unsaturated gas

$H = H_a + H_w \mathcal{H}$ — 13

↑ T , kJ water

$$(H - H_w) = -\frac{h}{k_g \rho_a} (\theta - \theta_w) \quad \text{---(10)}$$

$$(H - H_s) = -\frac{s}{q} (\theta - \theta_s) \quad \text{--- eq (11)} \quad (6)$$

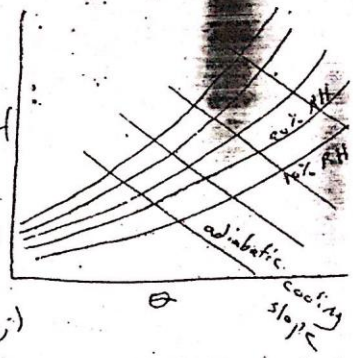


A curve of humidity versus temperature for gases with ag adiabatic saturation temperature is known as an adiabatic cooling

Eq. (11) indicate approximately linear relationship between $H$ and temp ($\theta$)

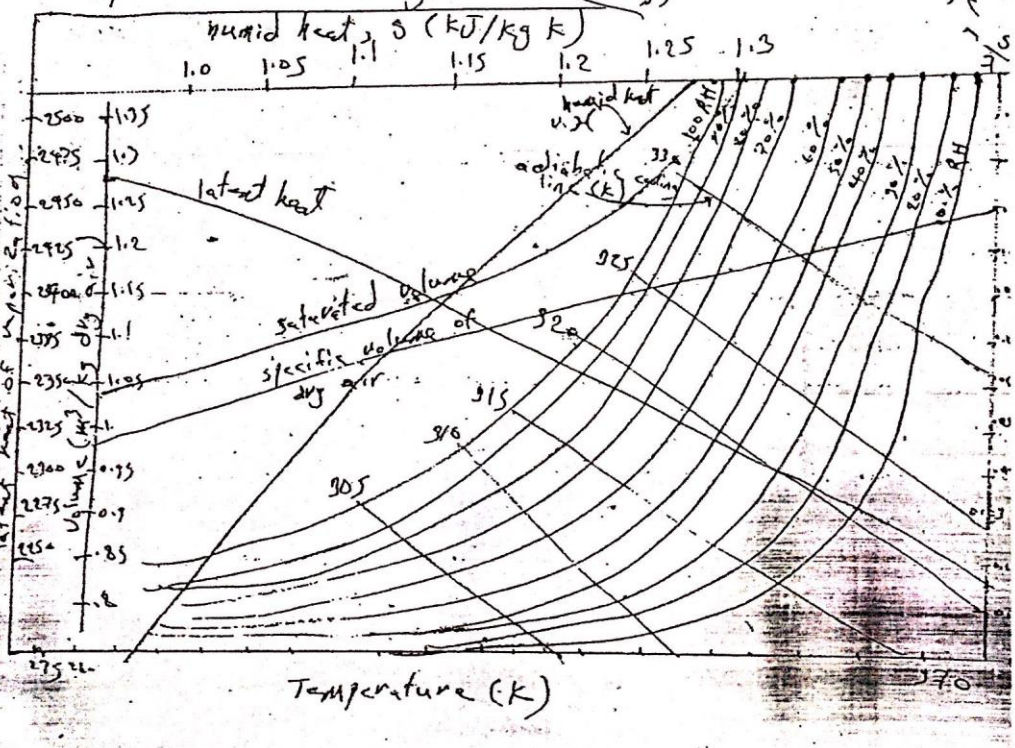
comparing eq (10) & (11) seen that

$\theta_s = \theta_w$ when $s = \frac{h}{k_g \rho_a}$




This is the case for most water vapour system (i.e for water-air system $\theta_s = \theta_w$)

Temperature - Humidity chart (مخطط درجة الحرارة والرطوبة)




Humidity-temperature diagram for air-water system at atmospheric pressure

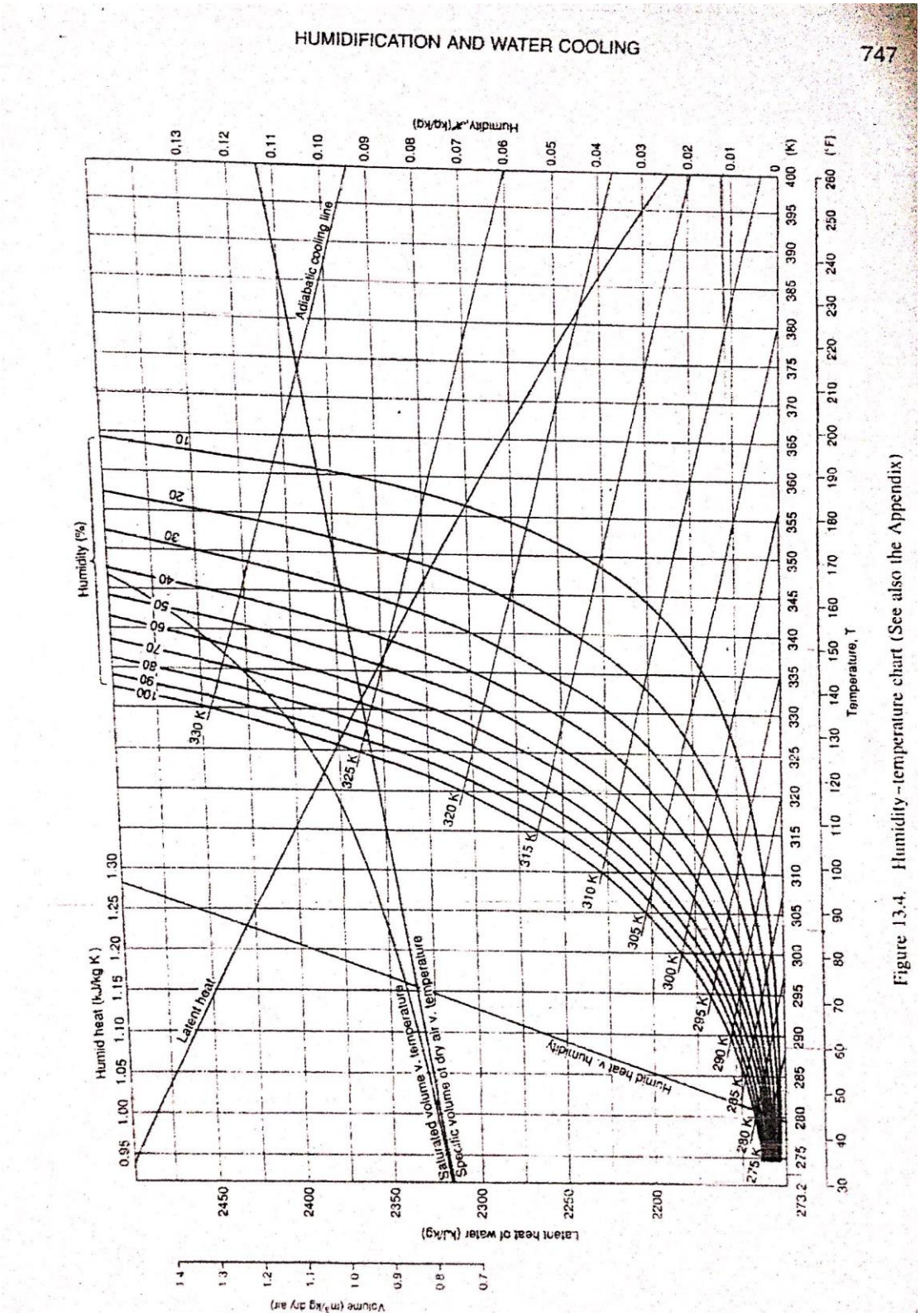



Figure 13.4. Humidity–temperature chart (See also the Appendix)

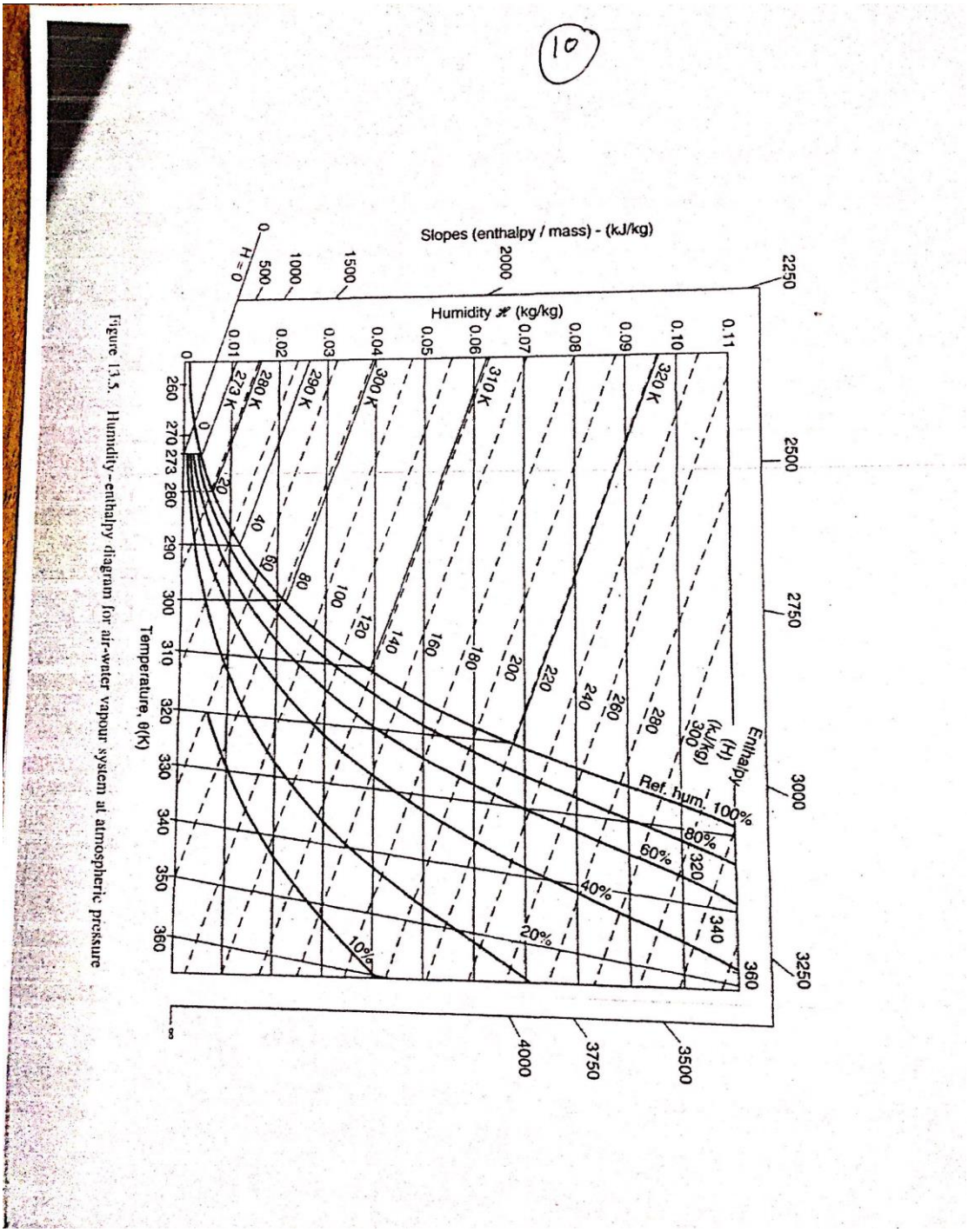
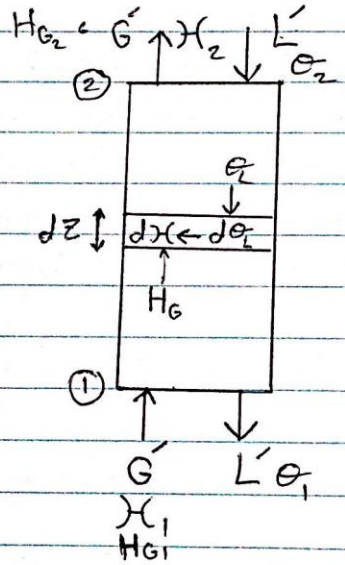



Figure 13.5. Humidity-enthalpy diagram for air-water vapour system at atmospheric pressure

## Cooling Tower أبراج التبريد

- Air and water are brought into counter-current contact in a cooling tower. The water flows down over a series of wooden slats (ألواح خشبية) which give a large interfacial area and promote turbelnt in the liquid.
- The air is humidified, while the water is cooled.
- The atmospheric conditions and the temperature and quantity of water is a very important effect on the operation of the cooling tower.




Let G' = mass rate of flow of air per unit cross section area ($kg/m^2s$) is constant through the whole hight of the tower (dry air only)

L' = Mass rate of flow of liquid per unit area ($kg/m^2s$) can be taken as constant because the water is normally evaporated (1-5 %)

$\mathcal{H}$ = humidity, $\theta$ = temperature K

H = enthalpy (kJ/kg)

For an increment height of column, $dz$

① Water balance

$$dL' = G' dx$$

② Enthalpy balance

$$L' dH_L = G' dH_G$$

but $H_G = s(\theta_G - \theta_0) + \lambda x$

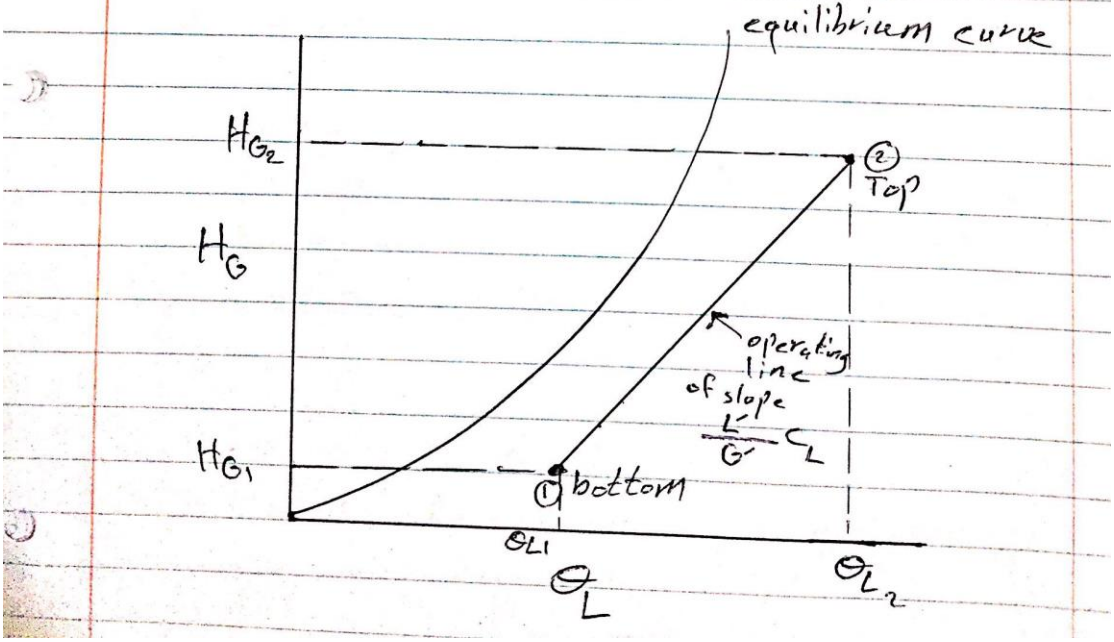
$$H_L = C_L(\theta_L - \theta_0)$$

$$G'(H_{G_2} - H_{G_1}) = L' C_L (\theta_{L_2} - \theta_{L_1})$$

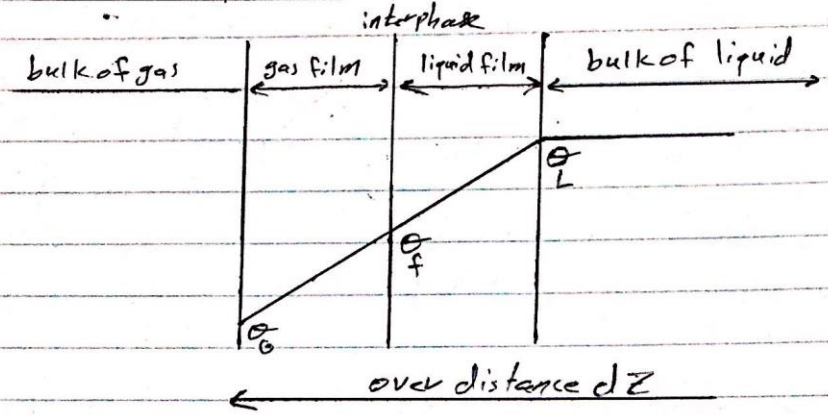
$$H_G - H_{G_1} = \frac{L' C_L}{G'}(\theta_L - \theta_{L_1})$$

$$\boxed{H_G = \frac{L'}{G'} C_L \theta_L + \left(H_{G_1} - \frac{L'}{G'} C_L \theta_{L_1}\right)}$$

operating line equation of slope $\frac{L'}{G'} C_L$



(3) Heat transfer from the body of the liquid to the interphase.




$$h_L\, a\, dz\, (\theta_L - \theta_f) = L' C_L\, d\theta_L$$

$$\frac{W}{m^2 K} \cdot \frac{m^2}{m^3} \cdot m \cdot K = \frac{kg}{m^2 s} \cdot \frac{J}{kg\,K} \cdot K$$

$$W = \frac{J}{s} = W \checkmark$$

where $h_L$ = heat transfer coefficient in the liquid phase ($W/m^2K$)

$a$ = interfacial area per unit volume of column

$$\boxed{\frac{d\theta_L}{\theta_L - \theta_f} = \frac{h_L\, a\, dz}{L' C_L}} \quad \text{—— (1)}$$

(4) Heat transfer from the interphase to the bulk of the gas.

$$h_G\, a\, dz\, (\theta_f - \theta_G) = G' S\, d\theta_G$$

$h_G$ = heat transfer coefficient in the gas phase ($\frac{W}{m^2 s}$)

$$\boxed{\frac{d\theta_G}{\theta_f - \theta_G} = \frac{h_G\, a\, dz}{G' S}} \quad \text{—— (2)}$$

(5) Mass transfer from the interphase to the gas

$$K_G P a dz (\mathcal{H}_f - \mathcal{H}) = \acute{G}\, d\mathcal{H}$$

where $K_G$ = mass transfer coefficient for the gas $(\frac{m}{s})$

$P$ = mean density of air $\frac{kg}{m^3}$

$\mathcal{H}_f$ = humidity at interphase

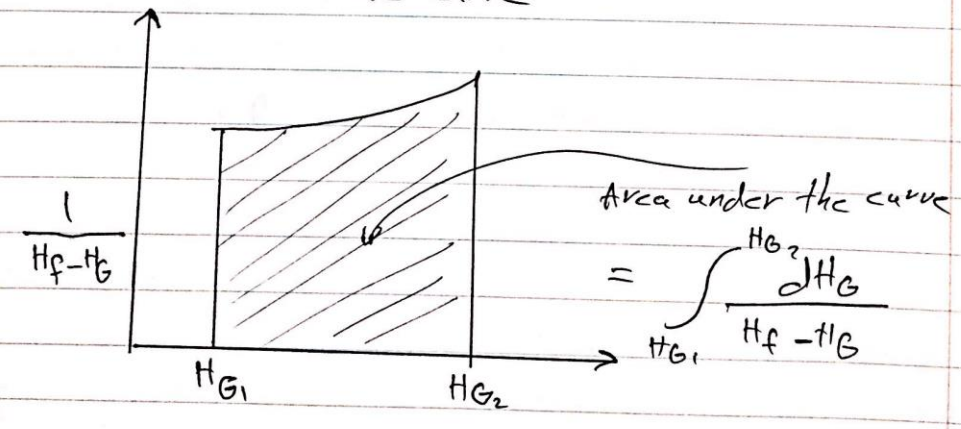
$$\frac{d\mathcal{H}}{\mathcal{H}_f - \mathcal{H}} = \frac{K_G P a}{\acute{G}}\, dz \quad \text{(3)}$$

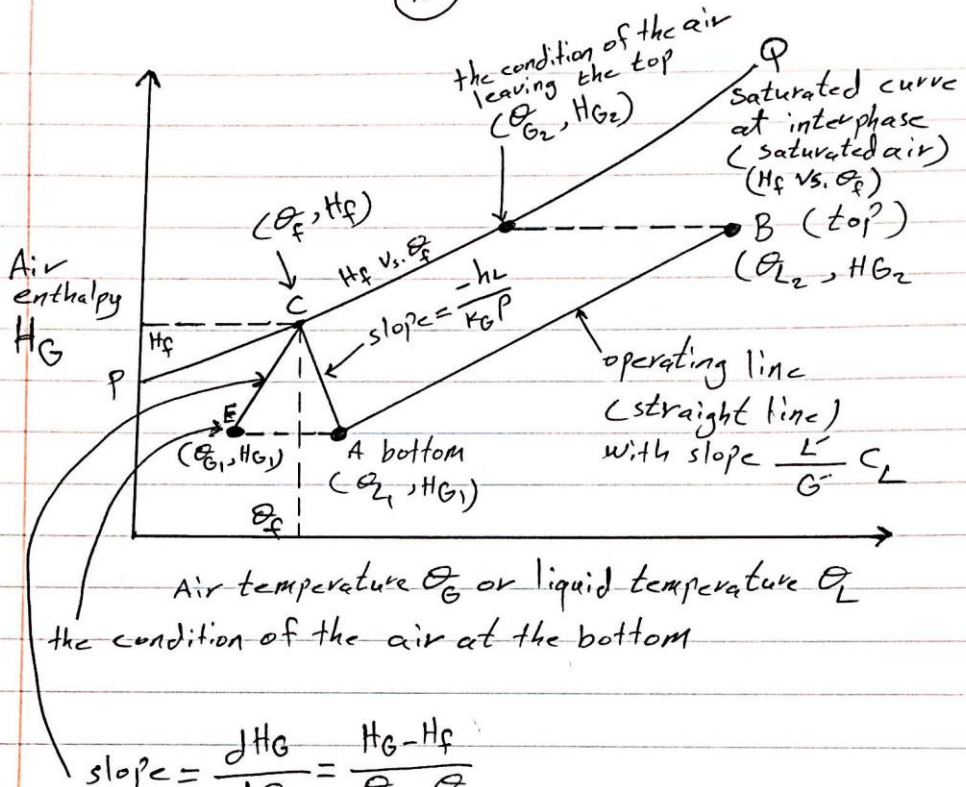
وبإشتقاق المعادلات (1) (2) و (3) نحصل على المعادلة النهائية

المعادلة النهائية لحساب ارتفاع برج التبريد

$$Z = \frac{\acute{G}}{K_G P a} \int_{H_{G_1}}^{H_{G_2}} \frac{dH_G}{H_f - H_G}$$

وحل التكامل بواسطة Area under the curve





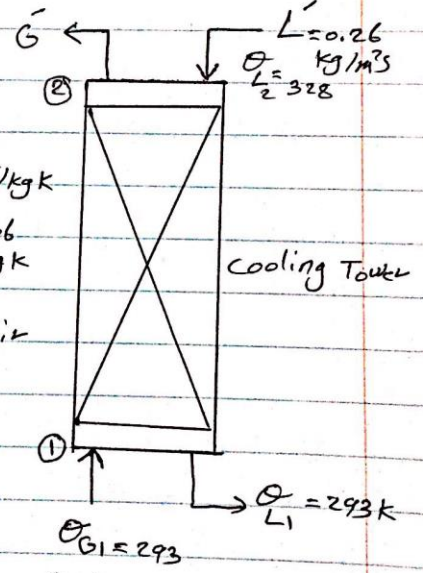

$$\text{slope} = \frac{dH_G}{d\Theta_G} = \frac{H_G - H_f}{\Theta_G - \Theta_f}$$

* PQ enthalpy – temperature curve for saturated air (from the humidity chart). Above the line PQ super saturated. Below the line PQ unsaturated air, it is assumed that the air in contact with the liquid surface is saturated with water vapor,
* This curve represent the relation between air-enthalpy $H_f$ and temperature $\Theta_f$ at the interphase.
* The line for the bottom of the column passes through the point A $(\Theta_{L_1}, H_{G_1})$ and cut the enthalpy-temp. curve for saturated air at point C (which represent the condition at inter face) $(\Theta_f, H_f)$

Volume 1 ch5

(1) Example: (13.9 مثال page 775 الطبعة الرابعة)

Water is to be cooled from 328 K to 293 K by means of a counter-current air stream entering at 293 K with a relative humidity of 20% in a cooling tower. The flow of air is 0.68 $m^3/m^2 s$ and the water through is 0.26 $kg/m^2 s$. The whole of the resistance to heat and mass transfer may be assumed to be in the gas phase and $K_G a$ may be taken as 0.2 $(m/s)(m^2/m^3)$. what is the required height of packing and the condition of the exit air stream?




Given:

$\lambda$ of water at 273 K = 2495 kJ/kg

specific heat of air ($C_a$) = 1.003 kJ/kg K

specific heat of water vapor ($C_w$) = 2.006 kJ/kg K

specific heat of water ($C_L$) = 4.18 J/kg K at 293

solution: The enthalpy of the inlet air stream

$$H_{G1} = H_a + H_w \lambda$$

$$H_a = C_a (\theta - \theta_0)$$

$$H_w = C_w (\theta - \theta_0) + \lambda$$

From Fig. (13.4) Page (9)

at $\theta = 293$ K, $RH = 20\%$ $\Big\}$ $\lambda = 0.003$ kg/kg

$$H_{G1} = 1.003 (293 - 273) + 0.003 [2.006 (293 - 273) + 2495]$$

$$H_{G1} = 27.67 \text{ kJ/kg}$$

$$0.003 \frac{\text{kg water}}{\text{kg dry air}} \times \frac{\text{kmole water}}{18 \text{ kg water}} \times \frac{29 \text{ kg dry air}}{\text{kmole dry air}} = 0.005 \frac{\text{kmol water}}{\text{kmol dry air}}$$

The flow of dry air = 0.68 − 0.005

= 0.677 $m^3/m^2s$

Density of air at 293 K = $\frac{273}{293} \times \frac{29}{22.4}$ = 1.206 $\frac{kg}{m3}$

mass flow of dry air = 1.206 $\frac{kg}{m3} \times 0.677 \frac{m^3}{m^2s}$

= 0.817 $kg/m^2s$

slope of operating line = $\frac{L'}{G'} \times cL$

= $\frac{0.26}{0.817} \times 4.18$

= 1.33

* The coordinate of the bottom of the operating line are: $\theta_{L_1}$ = 293 K, $H_{G_1}$ = 27.67 kJ/kg

Hence on an enthalpy–temperature diagram, the operating line of slope 1.33 is drawn through the point (293, 27.67) ($\theta_{L_1}$, $H_{G_1}$).

The top point of the operating line is given by $\theta_{L_2}$ = 328 K and $H_{G_2}$ is found to be 76.5 kJ/kg.

From fig. 13.4 & Fig 13.5 (Page (9)(10))

The curve representing the enthalpy of saturated air as a function of temperature, is obtained and drawn. Also this plot may be calculated from this equation:

$H_f = c_a(\theta_f - 273) + [c_w(\theta_f - 273) + \lambda]H_0$ kJ/kg

وذلك بفرض $\theta_f$ ثم نوجد $H_0$ من الجدول الموجود في (9)(10)

The curve represent the relation between enthalpy and temperature at interface that is $H_f$ as a function of $\theta_f$.

between limits, $H_{G_1}$ = 27.7 kJ/kg

$H_{G_2}$ = 76.5 kJ/kg

various values of $H_G$ between these limit are selected and the value of $\theta$ obtained from the operating line.

Assuming that the resistance to mass transfer lies entirely within the gas phase, the lines connecting $\theta_L$ and $\theta_f$ are parallel to the enthalpy axis.

$$\text{slope} = -\frac{h_L}{k_G P}$$

The corresponding value of $H_f$ is obtained from the curve of saturated air.

| $H_G$ | $\theta = \theta_f$ | $H_f$ | $(H_f - H_G)$ | $1/(H_f - H_G)$ |
|---|---|---|---|---|
| 27.7 | 293 | 57.7 | 30 | 0.033 |
| 30 | 294.5 | 65 | 35 | 0.0285 |
| 40 | 302 | 98 | 58 | 0.0172 |
| 50 | 309 | 137 | 87 | 0.0114 |
| 60 | 316 | 190 | 130 | 0.0076 |
| 70 | 323 | 265 | 195 | 0.0051 |
| 76.5 | 328 | 355 | 279 | 0.0035 |

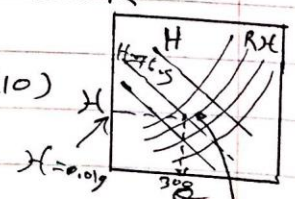
A plot of $1/(H_f - H_G)$ and $H_G$ Area under the curve $\Rightarrow A = 0.648$

$$Z = \int_{H_{G1}}^{H_{G2}} \frac{dH_G}{H_f - H_G} \times \frac{G}{k_G a P}$$

$$= 0.648 \times \frac{0.817}{0.2 \times 1.206} = 2.2 \text{ m}$$

$\theta_{G_2}$ corresponding to $H_G = 76.5 \frac{kJ}{kg} = 300\ K$

$H_{G_2} = 0.019$
$RH = 83\%$
} from fig 13.5 page (10)



It is seen that the liquid film coefficient is generally considerably higher than the gas film coefficient, but that it is not always safe to ignore the resistance to transfer in the liquid phase.

LOWE and CHRISTIE[17] used a 1.3 m square experimental column fitted with a number of different types of packing and measured heat and mass transfer coefficients and pressure drops. They showed that in most cases:

$$h_D a \propto L'^{1-n} G'^{n} \tag{13.58}$$

The index $n$ was found to vary from about 0.4 to 0.8 according to the type of packing. It will be noted that when $n \approx 0.75$, there is close agreement with the results given by equation 13.57.

The heat-transfer coefficient for the liquid is often large compared with that for the gas phase. As a first approximation, therefore, it can be assumed that the whole of the resistance to heat transfer lies within the gas phase and that the temperature at the water-air interface is equal to the temperature of the bulk of the liquid. Thus, everywhere in the tower, $\theta_f = \theta_L$. This simplifies the calculations, since the lines $AC$, $HJ$, and so on, have a slope of $-\infty$, that is, they become parallel to the enthalpy axis.

Some workers have attempted to base the design of humidifiers on the overall heat transfer coefficient between the liquid and gas phases. This treatment is not satisfactory since the quantities of heat transferred through the liquid and through the gas are not the same, as some of the heat is utilised in effecting evaporation at the interface. In fact, at the bottom of a tall tower, the transfer of heat in both the liquid and the gas phases may be towards the interface, as already indicated. A further objection to the use of overall coefficients is that the Lewis relation may be applied only to the heat and mass transfer coefficients in the gas phase.

In the design of commercial units, nomographs[18,19] are available which give a performance characteristic $(KaV/L')$, where $K$ is a mass transfer coefficient (kg water/m$^2$s) and $V$ is the active cooling volume (m$^3$/m$^2$ plan area), as a function of $\theta$, $\theta_w$ and $(L'/G')$. For a given duty $(KaV/L')$ is calculated from:

$$\frac{KaV}{C_L L'} = \int_{\theta_1}^{\theta_2} \frac{\mathrm{d}\theta}{(H_f - H_G)} \tag{13.59}$$

and then a suitable tower with this value of $(KaV/L')$ is sought from performance curves[20,21]. In normal applications the performance characteristic varies between 0.5–2.5.

**Example 13.9**

Water is to be cooled from 328 to 293 K by means of a countercurrent air stream entering at 293 K with a relative humidity of 20 per cent. The flow of air is 0.68 m$^3$/m$^2$s and the water throughput is 0.26 kg/m$^2$s. The whole of the resistance to heat and mass transfer may be assumed to be in the gas phase and the product, $(h_D a)$, may be taken as 0.2 (m/s)(m$^2$/m$^3$), that is 0.2 s$^{-1}$.

What is the required height of packing and the condition of the exit air stream?

**Solution**

Assuming the latent heat of water at 273 K = 2495 kJ/kg
specific heat of air = 1.003 kJ/kg K
and specific heat of water vapour = 2.006 kJ/kg K

(20)



the enthalpy of the inlet air stream,

$$H_{G1} = 1.003(293 - 273) + \mathscr{H}'[2495 + 2.006(293 - 273)]$$

From Figure 13.4:

at $\theta = 293$ K and 20 per cent RH, $\mathscr{H}' = 0.003$ kg/kg, and hence

$$H_{G1} = (1.003 \times 20) + 0.003[2495 + (2.006 \times 20)]$$

$$= 27.67 \text{ kJ/kg}$$

In the inlet air, water vapour = 0.003 kg/kg dry air

or:
$$\frac{(0.003/18)}{(1/29)} = 0.005 \text{ kmol/kmol dry air}$$

Thus flow of dry air $= (1 - 0.005)0.68 = 0.677 \text{ m}^3/\text{m}^2\text{s}$

Density of air at 293 K $= \left(\frac{29}{22.4}\right)\left(\frac{273}{293}\right) = 1.206 \text{ kg/m}^3$

and mass flow of dry air $= (1.206 \times 0.677) = 0.817 \text{ kg/m}^2\text{s}$

Slope of operating line: $(L'C_L/G') = \dfrac{(0.26 \times 4.18)}{0.817} = 1.33$

The coordinates of the bottom of the operating line are:

$$\theta_{L1} = 293 \text{ K}, \quad H_{G1} = 27.67 \text{ kJ/kg}$$

Hence on an enthalpy–temperature diagram, the operating line of slope 1.33 is drawn through the point $(293, 27.67) \equiv (\theta_{L1}, H_{G1})$.

The top point of the operating line is given by $\theta_{L2} = 328$ K, and $H_{G2}$ is found to be 76.5 kJ/kg (Figure 13.19).

From Figures 13.4 and 13.5 the curve representing the enthalpy of saturated air as a function of temperature is obtained and drawn in. Alternatively, this plot may be calculated from:

$$H_F = C_a(\theta_f - 273) + \mathscr{H}'_0[C_w(\theta_f - 273) + \lambda] \text{ kJ/kg}$$

The curve represents the relation between enthalpy and temperature at the interface, that is $H_f$ as a function of $\theta_f$.

It now remains to evaluate the integral $\int dH_G/(H_f - H_G)$ between the limits, $H_{G1} = 27.7$ kJ/kg and $H_{G2} = 76.5$ kJ/kg. Various values of $H_G$ between these limits are selected and the value of $\theta$ obtained from the operating line. At this value of $\theta_1$, now $\theta_f$, the corresponding value of $H_f$ is obtained from the curve for saturated air. The working is as follows:

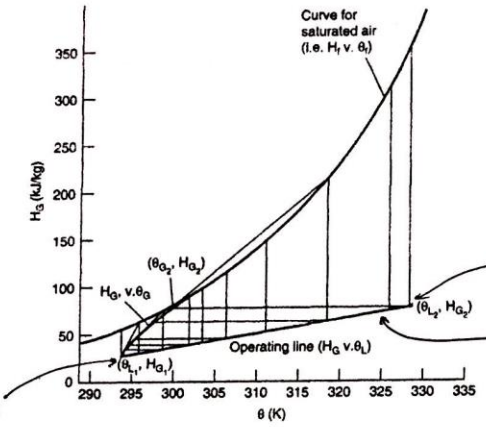
| $H_G$ | $\theta = \theta_f$ | $H_f$ | $(H_f - H_G)$ | $1/(H_f - H_G)$ |
|---|---|---|---|---|
| 27.7 | 293 | 57.7 | 30 | 0.0330 |
| 30 | 294.5 | 65 | 35 | 0.0285 |
| 40 | 302 | 98 | 58 | 0.0172 |
| 50 | 309 | 137 | 87 | 0.0114 |
| 60 | 316 | 190 | 130 | 0.0076 |
| 70 | 323 | 265 | 195 | 0.0051 |
| 76.5 | 328 | 355 | 279 | 0.0035 |

A plot of $1/(H_f - H_G)$ and $H_G$ is now made as shown in Figure 13.20 from which the area under the curve = 0.65. This value may be checked using the approximate solution of CAREY and WILLIAMSON[14]. At the bottom of the column:

$$H_{G1} = 27.7 \text{ kJ/kg}, \quad H_{f1} = 57.7 \text{ kJ/kg} \quad \therefore \Delta H_1 = 30 \text{ kJ/kg}$$

(21)



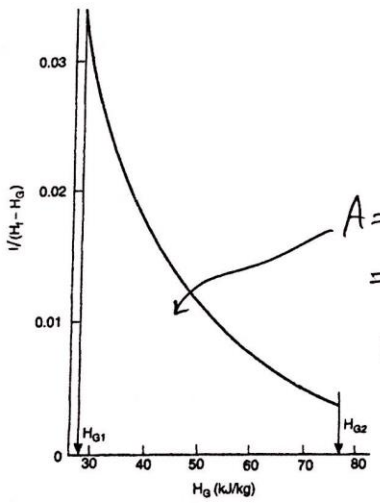



$(\theta_{L_2}, H_{G_2})$
$(328, 76.6)$

slope of operating line $= \frac{L'}{G'} \times c_l$
$= 1.33$

$(\theta_{L_1}, H_{G_1})$
$(293, 27.67)$

Figure 13.19. Calculation of the height of a water-cooling tower




A = Area under the curve

$$= \int_{H_{G_1}}^{H_{G_2}} \frac{dH_G}{H_f - H_G} = 0.648$$

Figure 13.20. Evaluation of the integral of $dH_G/(H_f - H_G)$

*At the top of the column:*

$$H_{G2} = 76.5 \text{ kJ/kg}, \quad H_{f2} = 355 \text{ kJ/kg} \quad \therefore \Delta H_2 = 279 \text{ kJ/kg}$$

At the mean water temperature of $0.5(328 + 293) = 310.5$ K:

$$H_{Gm} = 52 \text{ kJ/kg}, \quad H_f = 145 \text{ kJ/kg} \quad \therefore \Delta H_m = 93 \text{ kJ/kg}$$

$$\frac{\Delta H_m}{\Delta H_1} = 3.10, \quad \frac{\Delta H_m}{\Delta H_2} = 0.333,$$

and from Figure 13.16: $f = 0.79$

*Thus:*
$$\frac{(H_{G2} - H_{G1})}{f\Delta H_m} = \frac{(76.5 - 27.7)}{(0.79 \times 93)} = 0.66$$

which agrees well with the value (0.65) obtained by graphical integration.

Thus, in equation 13.53:

$$\text{height of packing, } z = \int_{H_{G1}}^{H_{G2}} \frac{dH_G}{(H_f - H_G)} \frac{G'}{h_D a \rho}$$

$$= \frac{(0.65 \times 0.817)}{(0.2 \times 1.206)}$$

$$= \underline{\underline{2.20 \text{ m}}}$$

Assuming that the resistance to mass transfer lies entirely within the gas phase, the lines connecting $\theta_L$ and $\theta_f$ are parallel with the enthalpy axis.

In Figure 13.18 a plot of $H_G$ and $\theta_G$ is obtained using the construction given in Section 13.6.4 and shown in Figure 13.15. From this curve, the value of $\theta_{G2}$ corresponding to $H_{G2} = 76.5$ kJ/kg is 300 K. From Figure 13.5, under these conditions, the exit air has a humidity of 0.019 kg/kg which from Figure 13.4 corresponds to a relative humidity of 83 per cent.

### 13.6.7. Humidifying towers

If the main function of the tower is to produce a stream of humidified air, the final temperature of the liquid will not be specified, and the humidity of the gas leaving the top of the tower will be given instead. It is therefore not possible to fix any point on the operating line, though its slope can be calculated from the liquid and gas rates. In designing a humidifier, therefore, it is necessary to calculate the temperature and enthalpy, and hence the humidity, of the gas leaving the tower for a number of assumed water outlet temperatures and thereby determine the outlet water temperature resulting in the air leaving the tower with the required humidity. The operating line for this water-outlet temperature is then used in the calculation of the height of the tower required to effect this degree of humidification. The calculation of the dimensions of a humidifier is therefore rather more tedious than that for the water-cooling tower.

In a humidifier in which the make-up liquid is only a small proportion of the total liquid circulating, its temperature approaches the adiabatic saturation temperature $\theta_s$, and remains constant, so that there is no temperature gradient in the liquid. The gas in contact with the liquid surface is approximately saturated and has a humidity $\mathscr{H}_s$.

*Thus:*
$$d\theta_L = 0$$

and:
$$\theta_{L1} = \theta_{L2} = \theta_L = \theta_f = \theta_s$$

Hence:
$$-G's\, d\theta_G = h_G a\, dz(\theta_G - \theta_s) \quad \text{(from equation 13.42)}$$

and:
$$-G'\, d\mathscr{H} = h_D \rho a\, dz(\mathscr{H} - \mathscr{H}_s) \quad \text{(from equation 13.44)}$$