3-2General Equation For Variation Of Pressure In A Static Fluid

Consider the cylindrical element of fluid in the figure (3-2), inclined at an angle θ to the vertical, lengthδs, cross-sectional area A in a static fluid of mass density ρ . The pressure at the end with height *z* is *p* and at the end of height z+δs is p+δp

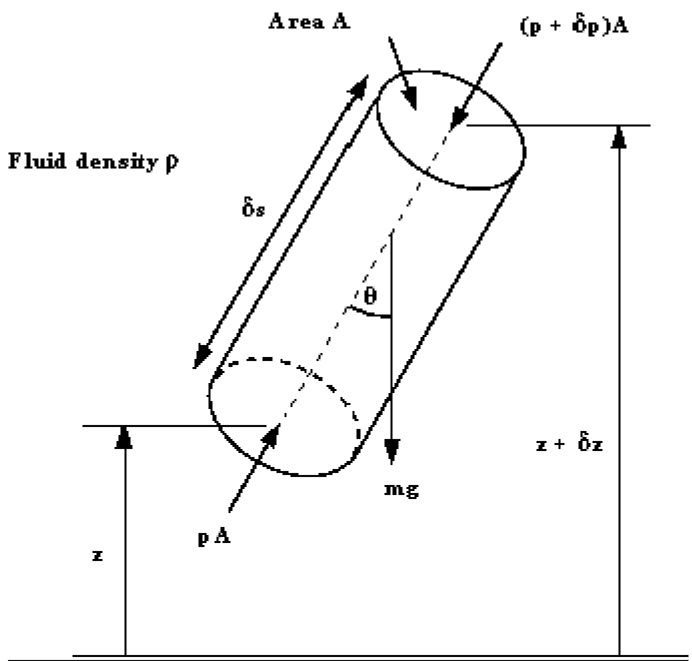


Figure (3-2): A cylindrical element of fluid at an arbitrary orientation

The forces acting on the element are

pA acting at right − angles to the end of the face at z

$(\mathrm{p}+\delta p)\mathrm{A}$ acting at right − angles to the end of the face at zδs

$\mathrm{mg}=\rho g A \delta s \cos \theta$

Resolving the forces in the direction along the central axis gives

$$\mathrm{pA}-(\mathrm{p}+\delta p)\mathrm{A}-\rho g A \delta s \cos \theta=0$$

$$\delta p=-\rho g \delta s \cos \theta$$

$$\frac{\delta p}{\delta s}=-\rho g \cos \theta$$

Or in the differential form

$$\frac{dp}{ds} = -\rho g\ cos\ \ \theta$$

If θ = 90 then s is in the x or y directions, (i.e. horizontal),so

$$\left[\frac{dP}{ds}\right]_{\theta=90} = \frac{dP}{dx} = \frac{dP}{dy} = 0$$

Confirming that pressure on any horizontal plane is zero.

If θ = 0 then s is in the z directions, (i.e. vertical),

$$\left[\frac{dP}{ds}\right]_{\theta=0} = \frac{dP}{dz} = -\rho g$$

Confirming the result

$$\frac{P_2 - P_1}{z_2 - z_1} = -\rho g$$

$$\boxed{P_2 - P_1 = -\rho g(z_2 - z_1)}$$

3.3 Pressure in a Fluid

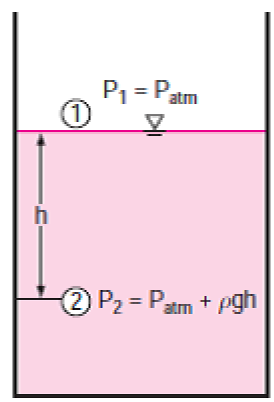
In a static fluid of constant density we have the relationship

$$\frac{dP}{dz} = -\rho g$$

This can be integrated to give

$$P = -\rho gz + constant$$

In a liquid with a free surface the pressure at any depth z measured from the free surface so that z = -h (see the figure below)



This gives the pressure

$$P = \rho g h + constant$$

At the surface of fluids we are normally concerned with, the pressure is the atmospheric pressure. So

$$\boxed{P = \rho g h + P_{atm}}$$

Note: The shape of the vessel does not affect the pressure at the bottom.

3.4 Absolute and Relative Pressure

The term pressure is sometimes associated with different terms such as *atmospheric*, *gauge*, *absolute*, and *vacuum*. The meanings of these terms have to be understood well before solving problems in hydraulic and fluid mechanics.

1- Atmospheric Pressure

It is the pressure exerted by atmospheric air on the earth due to its weight. This pressure is change as the density of air varies according to the altitudes. Also it may vary because of the temperature and humidity of air.

1 atm = 1.01325 bar = 101.325 kPa = 10.328 m H2o = 760 torr (mm Hg) = 14.7 psi

2- Gauge Pressure or Positive Pressure

It is the pressure recorded by an instrument. This is always above atmospheric.

3- Vacuum Pressure or Negative Pressure

This pressure is caused either artificially or by flow conditions. The pressure intensity will be less than the atmospheric pressure whenever vacuum is formed.

4- Absolute Pressure

Absolute pressure is the algebraic sum of atmospheric pressure and gauge pressure.

$$P_{abs} = \rho g h + P_{atm}$$

Absolute Pressure = Gauge Pressure + Atmospheric Pressure

Absolute Pressure = Atmospheric Pressure – Vacuum Pressure

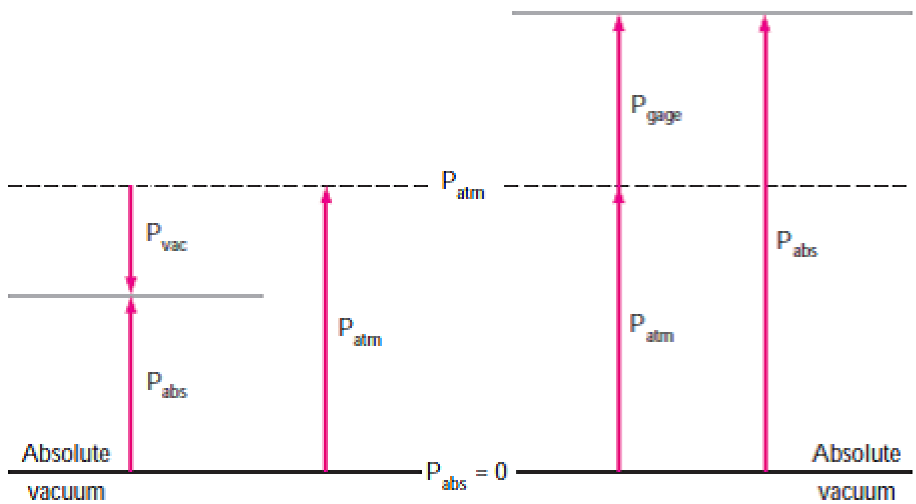


Figure (3-3): absolute, gage and vacuum pressures.