Fluid Static and Its Applications

3- Introduction

Static fluids: means that the fluids are at rest.

3.1 Pascal's Law for Pressure at a Point

By considering a small element of fluid in the form of a triangular prism which contains a point P, we can establish a relationship between the three pressures p_x in the x direction, p_y in the y direction and p_s in the direction normal to the sloping face as shown in figure (3-1).

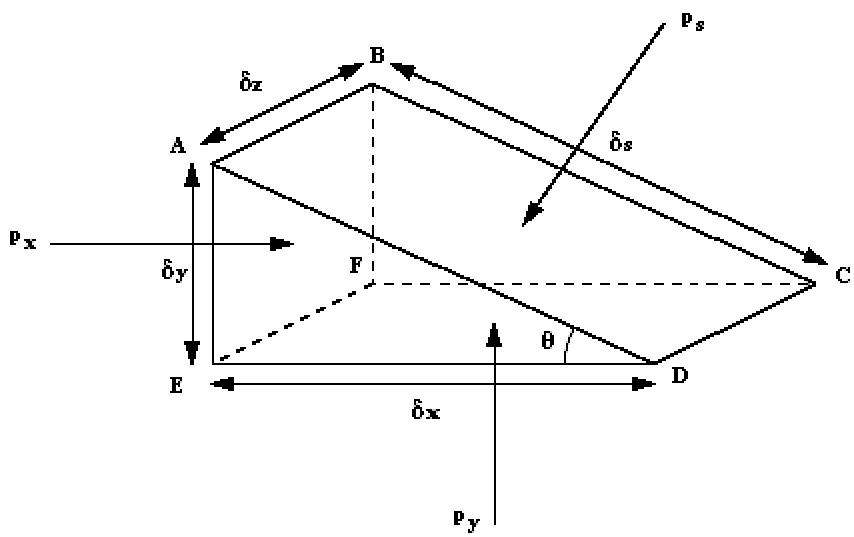

Figure (3-1) Triangular prismatic element of fluid

The fluid is a rest, so we know there are no shearing forces, and we know that all force are acting at right angles to the surfaces .i.e.

p_s acts perpendicular to surface ABCD,
p_x acts perpendicular to surface ABFE and
p_y acts perpendicular to surface FECD.

And, as the fluid is at rest, in equilibrium, the sum of the forces in any direction is zero. Summing forces in the x-direction:

Force due to *px* ,

$$F_{X_X} = P_X * Area_{ABFD} = P_X\ \delta y \delta z$$

Component of force in the x-direction due to *ps* ,

$$F_{X_S} = -P_S\ sin\theta * Area_{ABCD}$$

$$F_{X_S} = -P_S\ \frac{\delta y}{\delta s} * \delta z \delta s\ = -P_S \delta y \delta z$$

$where \quad sin\theta = \frac{\delta y}{\delta s}$

Component of force in x-direction due to p_y ,

$$F_{X_y} = 0$$

To be at rest (in equilibrium)

$$F_{X_x} + F_{X_y} + F_{X_S} = 0$$

$$P_X\ \delta y \delta z + 0 + (-P_S \delta y \delta z) = 0$$

$$P_X\ = P_S$$

Similarly, summing forces in the y direction Force due to p_y ,

$$F_{y_y} = P_y * Area_{ABCD} = P_y\ \delta x \delta z$$

Component of force in the y-direction due to *ps* ,

$$F_{y_S} = -P_S\ cos\theta * Area_{ABCD}$$

$$F_{y_S} = -P_S\ \frac{\delta x}{\delta s} * \delta z \delta s\ = -P_S \delta x \delta z$$

$where \quad cos\theta = \frac{\delta x}{\delta s}$

Component of force in x-direction due to p_y ,

$$F_{y_x} = 0$$

Force due to gravity,

Weight = - specific weight * volume of element

$$w = -\rho g * \frac{1}{2}\ \delta x \delta z \delta y$$

To be at rest (in equilibrium)

$$F_{y_y} + F_{y_x} + w + F_{y_S} = 0$$

$$P_y\ \delta x \delta z + 0 + \left(-\rho g * \frac{1}{2}\ \delta x \delta z \delta y\right) + (-P_S \delta x \delta z) = 0$$

The element is small i.e. $\delta x \delta z\ and \delta y$ are small, and so $\delta x \delta z \delta y$ is very small and considered negligible, hence

$$P_y\ = P_S$$

Thus

$$P_y\ = P_x = P_S$$

Considering the prismatic element again, p_s is the pressure on a plane at any angle θ, the x, y and z directions could be any orientation. The element is so small that it can be considered a point so the derived expression $p_x = p_y = p_s$. Indicates that pressure at any point is the same in all directions

<u>Pressure at any point is the same in all directions.</u>
<u>This is known as **Pascal's Law** and applies to fluids at rest</u>