Modeling

Once the variables are selected and the dimensional analysis is performed, the experimenter seeks to achieve similarity between the model tested and the prototype to be designed. With sufficient testing, the model data will reveal the desired dimensionless function between variable.

Suppose one knew that the force F on a particular body immersed in a stream of fluid depended only on the body length L, stream velocity u, fluid density ρ, and fluid viscosity μ, that is

$$F = f\,(L, u, \rho, \mu) \qquad (2.1)$$

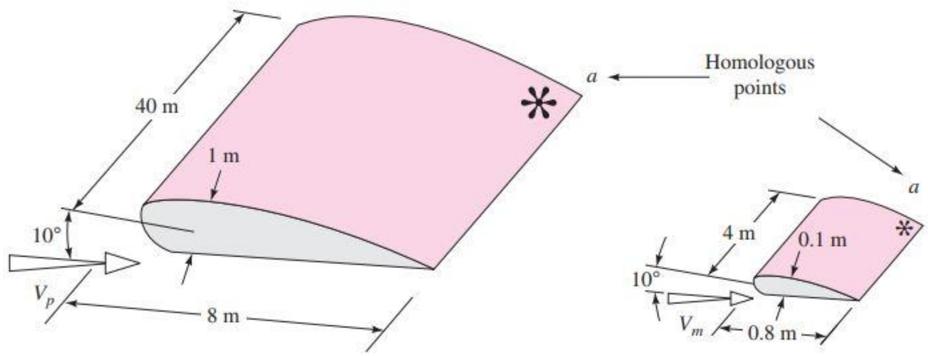
To find the effect of body length in Eq. (5.1), we have to run the experiment for 10 lengths L. For each L we need 10 values of u, 10 values of ρ, and 10 values of μ, making a grand total of 10^4, or 10,000, experiments. At $50 per experiment well, you see what we are getting into. However, with dimensional analysis, we can immediately reduce Eq. (2.1) to the equivalent form

$$\frac{F}{l^2 u^2 \rho} = f\left[\frac{\rho\, u\, l}{\mu}\right] \qquad (2.2)$$

$$C_F = f[Re] \qquad (2.3)$$

Although its purpose is to reduce variables and group them in dimensionless form, dimensional analysis has several side benefits. The first is enormous savings in time and money.

A second benefit is that dimensional analysis provides scaling laws which can convert data from a cheap, small model to design information for an expensive, large prototype. We do not build a million-dollar airplane and see whether it has enough lift force. We measure the lift on a small model and use a scaling law to predict the lift on the full-scale prototype airplane. There are rules we shall explain for finding scaling laws. When the scaling law is valid, we say that a condition of similarity exists between the model and the prototype.



(a) (b)

Similarity is achieved if the Reynolds number is the same for the model and prototype because the function g then requires the force coefficient to be the same also:

$$if\ \ Re_m = Re_p \quad then \qquad C_{fm} = C_{fp} \tag{2.4}$$

Example (2.7)

A body length L=1 mm immersed in a stream of fluid. We want to know the drag force on the body when it moves slowly in fresh water. A scale model 100 times larger is made and tested in glycerin at 30 cm/s. The measured drag on the model is 1.3 N. For similar conditions, what are the velocity and drag of the body in water?

Water (prototype): $\mu = 0.001$ kg/ (m.s), $\rho = 998$ kg/m^3

Glycerin (model): $\mu = 1.5$ kg/ (m.s), $\rho = 1263$ kg/m^3

The length scales are $L_m = 100$ mm and $L_p = 1$ mm. We are given enough model data to compute the Reynolds number and force coefficient

$$Re_m = \frac{\rho_m \, u_m \, l_m}{\mu_m} = \frac{1263 \frac{kg}{m3} * 0.3 \frac{m}{s} * 0.1 \, m}{1.5 \frac{kg}{m.s}} = 25.3$$

$$Re_m = Re_p = 25.3 = \frac{998 \frac{kg}{m3} * u * 0.001 \, m}{0.001 \frac{kg}{m.s}} \quad ---- \qquad u_p = 0.025 \frac{m}{s}$$

$$C_{fm} = \frac{F_m}{{l_m}^2 {u_m}^2 \rho_m} = \frac{1.3N}{1263 \frac{kg}{m3} * \left(\frac{0.3m}{s}\right)^2 * (0.1m)^2} = 1.14$$

$$C_{fp} = C_{fm} = 1.14 = \frac{F_p}{{l_p}^2 {u_p}^2 \rho_p} = \frac{F_p}{998 \frac{kg}{m3} * \left(\frac{0.025m}{s}\right)^2 * (0.001m)^2}$$

$$F_p = 7.13 * 10^{-7} N$$