1-3 Newton’s Law of Viscosity and Momentum Transfer

Consider the top plate to be subject to a force in the x direction that causes it to move with a velocity u_1, and the lower plate is stationary ($u_o = 0$) figure (1-1).

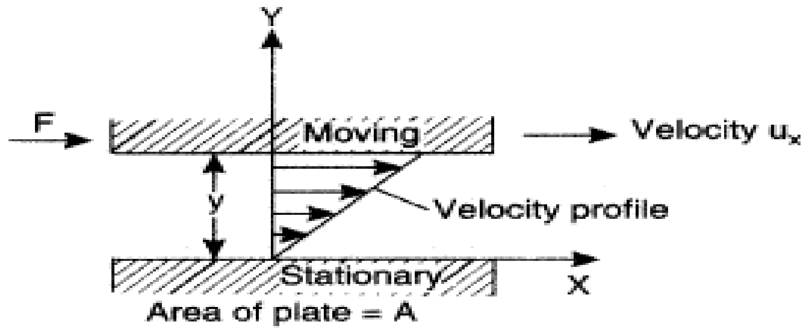


Figure (1-1): The behavior of a fluid in laminar flow between two parallel plates

The fluid in contact with the upper plate sticks to the plate surface and moves with it at the same velocity, and the shear stress t acting on this fluid layer is

$$\tau = \frac{F}{A}$$

In steady laminar flow, the fluid velocity between the plates varies linearly between 0 and *u*, and thus the *velocity profile* and the *velocity gradient* are

$$\tau \propto \frac{u_x}{y} \propto \frac{du}{dy}$$

In one-dimensional Newton’s law of viscosity can be written in terms of the shear stress instead of the momentum flux by the linear relationship

$$\boldsymbol{shear\ stress} \qquad \boldsymbol{\tau_{yx} = \mu \frac{du}{dy}} \qquad \boldsymbol{\frac{N}{m^2}} \qquad \ldots.(\mathbf{1-2})$$

Fluids for which the rate of deformation is proportional to the shear stress are called ***Newtonian fluids***.

A plot of shear stress versus the rate of deformation (velocity gradient) for a Newtonian fluid is a straight line whose slope is the viscosity of the fluid, as shown in Figure (1-2). Note: that viscosity is independent of the rate of deformation

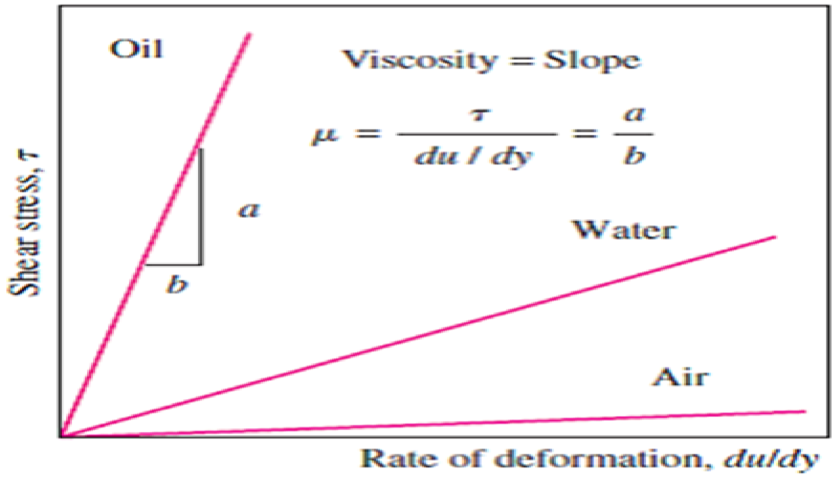


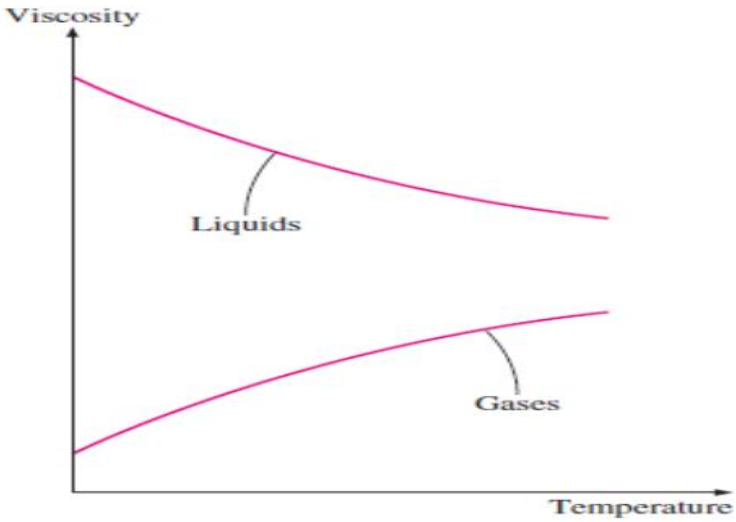
Figure (1-2): The rate of deformation (velocity gradient)

The ***<u>shear force</u>*** acting on a Newtonian fluid layer (or, by Newton's third law, the force acting on the plate) is

$$\textbf{shear force} \qquad \mathbf{F = \tau A = \mu A \frac{du}{dy}} \qquad \mathbf{(N)} \qquad \mathbf{\ldots(1-3)}$$

The viscosity of liquids decreases with temperature, whereas the viscosity of gases increases with temperature. This is because in a liquid the molecules possess more energy at higher temperatures, and they can oppose the large cohesive intermolecular forces more strongly. As a result, the energized liquid molecules can move more freely.

In a gas, on the other hand, the intermolecular forces are negligible, and the gas molecules at high temperatures move randomly at higher velocities. This results in more molecular collisions per unit volume per unit time and therefore result greater resistance to flow.



Viscometer

Consider a fluid layer of thickness L within a small gap between two concentric cylinders, as shown in figure (1-3) below such as the thin layer of oil in a journal bearing.

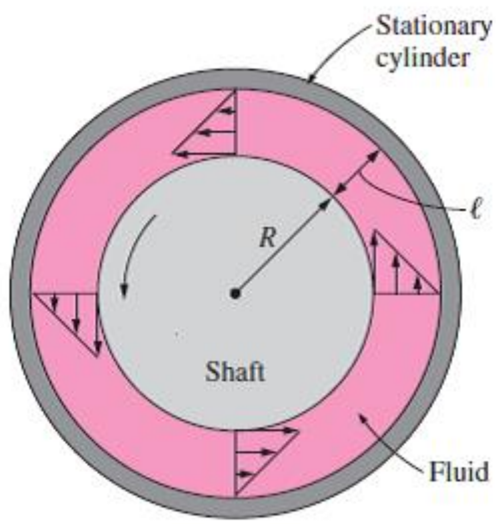


Figure (1-3): Cylindrical moving

The gap between the cylinders can be modeled as two parallel flat plates separated by a fluid.

$$torque \qquad T = FR$$

$$tangential\ velocity \qquad v = \omega R$$

$$angular\ velocity \qquad \omega = 2\pi \dot{n}$$

The wetted surface area of the inner cylinder to be $A = 2\pi RL$
Torque can be expressed as

$$torque \qquad T = FR = \mu \frac{2\pi R^3 \omega L}{y} = \mu \frac{4\pi^2 R^3 \dot{n} L}{y} \qquad \dots.(1-3)$$

Where L is the length of the cylinder and n is the number of revolutions per unit time, Equatio1-3 can be used to calculate the viscosity of a fluid by measuring torque at a specified angular velocity. Therefore, two concentric cylinders can be used as a ***viscometer***, a device that measures viscosity.

1-4 Newtonian and non-Newtonian fluids

Plot the relationship between shear stress and rate of deformation as shown in figure (1-4). The slope of the curve on the (ι) versus du/dy chart is referred to as the *apparent viscosity* of the fluid.

- For Newtonian fluids, the relationship between shear stress and rate of deformation is linear.
- For non-Newtonian fluids, the relationship between shear stress and rate of deformation is not linear, and are divided into

a) Fluids for which the apparent viscosity increases with the rate of deformation (such as solutions with suspended starch or sand) are referred to as ***dilatant .***

b) The fluid becoming less viscous as it is sheared harder (such as some paints, polymer solutions, and fluids with suspended particles) are referred to as ***pseudoplastic*** or shear thinning fluids.

c) Some materials such as toothpaste can resist a finite shear stress and thus behave as a solid, but deform continuously when the shear stress exceeds the yield stress and thus behave as a fluid. Such materials are referred to as ***Bingham plastics***

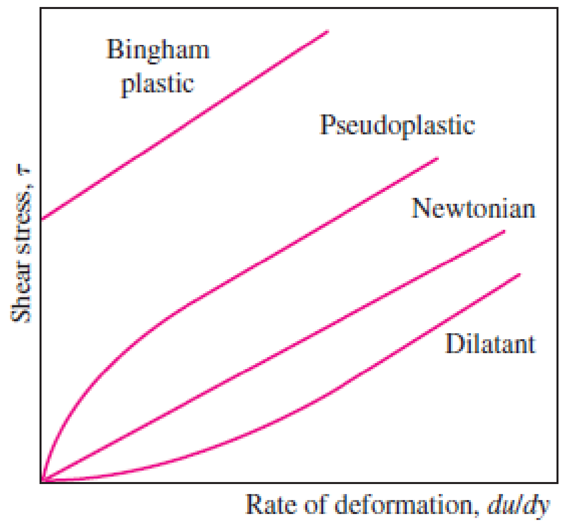


Figure (1-4): Plot the relationship between shear stress and rate of deformation

Example (1.1)

One liter of certain oil weighs 0.8 kg calculates the specific weight, density, specific volume, and specific gravity of it.

Solution:

- Specific weight

$$\gamma = \rho g \quad = \quad \frac{0.8\,kg}{1 * 10^{-3} m^3} * 9.81 \frac{m}{sec^2} \;=\; 7848\ \frac{N}{m^3}$$

- Density

$$\rho = \frac{m}{V} \;=\; \frac{0.8\ \mathrm{kg}}{1 * 10^{-3}\mathrm{m}^3} \;=\; 800\ \frac{\mathrm{kg}}{\mathrm{m}^3}$$

- Specific volume

$$\boldsymbol{v} = \frac{1}{\rho} = \frac{1 * 10^{-3}\mathrm{m}^3}{0.8\ \mathrm{kg}} \;=\; 1.25 * 10^{-3}\ \frac{\mathrm{m}^3}{\mathrm{kg}}$$

- Specific gravity

$$\boldsymbol{SG_{liquid}} = \frac{\rho_{liq}}{\rho_{water}} = \frac{800\ \frac{\mathrm{kg}}{\mathrm{m}^3}}{1000\ (\frac{kg}{m^3})} \;=\; 0.8$$