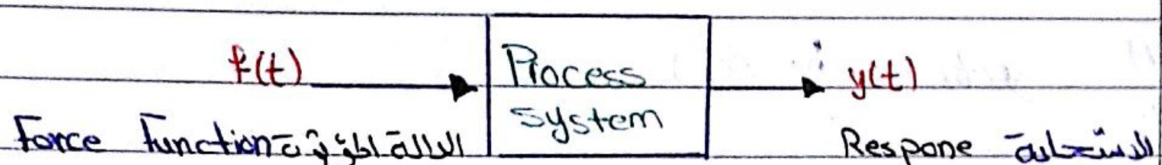
Transfer Function

$G(s) = \dfrac{y(s)}{f(s)}$; $\dfrac{dy}{dt}$ State Variable



Transfer Function: Is algerbic expersion for the dynamic relation between the Input " Force Function " and the Output oF the process " Respone " model and both expersion is " S " domain y(s) and f(s).

دالة جبرية بين حالة ديناميكية تربط بين الداخل والخارج

$$\boxed{G(s) = \frac{\text{Output Variable in S domain}}{\text{Input Variable in S domain}}}$$

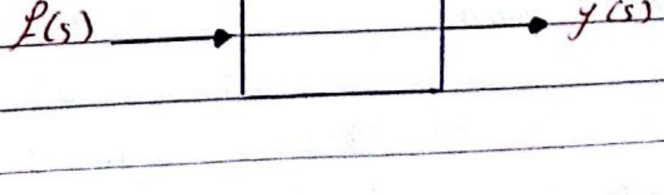
$G(s) = K$ this mean $\dfrac{\text{laplace transform oF respone Variable}}{\text{laplace transform oF Force Variable}}$.

Block diagram: block diagram is graphical representation oF their Function and their interaction.

zero جذور الاصفار

$$G(s) = \frac{(s+Z_1)(S+Z_2)+\cdots+(S+Z_n)}{(s+P_1)(s+P_2)+\cdots+(S+P_n)}$$

Pole الأقطاب



① Tranfer function of a first Order System.

$$a_1 \frac{dy}{dt} + a_0 y(t) = b_0 f(t)$$

$$\left(\frac{a_1}{a_0}\right) \frac{dy}{dt} + y(t) = \frac{b_0}{a_0} f(t)$$

$$\frac{a_1}{a_0} = \tau_p \quad ; \quad \frac{b_0}{a_0} = k_p$$

$$\boxed{\tau_p \frac{dy}{dt} + y(t) = k_p f(t)}$$ Model of First Order System In time domain

* Assuming Zero Initial Condition; Taking laplace transform

$$\tau_p [s y(s) - y(0)] + y(s) = k_p f(s)$$

$$\tau_p s y(s) + y(s) = k_p f(s)$$

$$y(s)[\tau_p s + 1] = k_p f(s)$$

$$\boxed{G(s) = \frac{y(s)}{f(s)} = \frac{k_p \cancel{f(s)}}{(\tau_p s + 1)\cancel{f(s)}} = \frac{k_p}{\tau_p s + 1}}$$ Transfer Function of First Order In laplace.

τ_p = Time Constant Process "Time".

k_p = Steady state gain.

$$= \frac{\text{Unit Respone Variable}}{\text{Unit Force Variable}}$$ نسبة الحالة المستقرة

Example $50 \xrightarrow{+5} 55$
$30 \xrightarrow{+5} 35$ } $\frac{5}{5} = 1$

$50 \xrightarrow{-5} 45$
$25 \xrightarrow{-2} 23$ } $\frac{-5}{-2} = \frac{5}{2}$

K_p : give that ; $+ve$, $-ve$, >1
<1

{at steady state ($\tau_p = 0$) ; ($G(s) = K_p$)}

② Tranfer Function For Second Order System.

$$a_2 \frac{d^2y}{dt^2} + a_1 \frac{dy}{dt} + a_0 y(t) = b_0 f(t)$$

$$\left(\frac{a_2}{a_0}\right) \frac{d^2y}{dt^2} + \left(\frac{a_1}{a_0}\right) \frac{dy}{dt} + y(t) = \left(\frac{b_0}{a_0}\right) f(t)$$

let $\tau_p^2 = \frac{a_2}{a_0}$; $\tau_p \zeta = \frac{a_1}{a_0}$; $k_p = \frac{b_0}{a_0}$

$$\boxed{\tau_p^2 \frac{d^2y}{dt^2} + \zeta \tau_p \frac{dy}{dt} + y(t) = k_p f(t)}$$ Model of Second Order System " time ".

Asuming for Initial Condition; Taking laplace tranForm.

$$\tau_p^2 \left[s^2 y(s) - s\overset{0}{y(0)} - \overset{0}{y'(0)} \right] + 2\zeta \tau_p \left[s y(s) - \overset{0}{y(0)} \right] + y(s) = k_p f(s)$$

$$\tau_p^2 S^2 y(s) + 2\zeta \tau_p S\, y(s) + y(s) = k_p f(s)$$

$$y(s)\left[\tau_p^2 S^2 + 2\zeta \tau_p S + 1\right] = k_p f(s).$$

$$\boxed{G(s) = \frac{y(s)}{f(s)} = \frac{k_p}{\tau_p^2 S^2 + 2\zeta \tau_p S + 1}}$$ Tranfer Function of Second Order.

τ_p = Time Constant "Time".
k_p = steady state gain.
ζ = Damping Factor.

(3) Tranfer Function of Zero Order System

$a_0 y(t) = b_0 f(t)$
$a_0 y(s) = b_0 f(s)$

$$\boxed{G(s) = \frac{y(s)}{f(s)} = \frac{b_0}{a_0} = k_p}$$ Transfer Fuction On Zero Order Laplace.

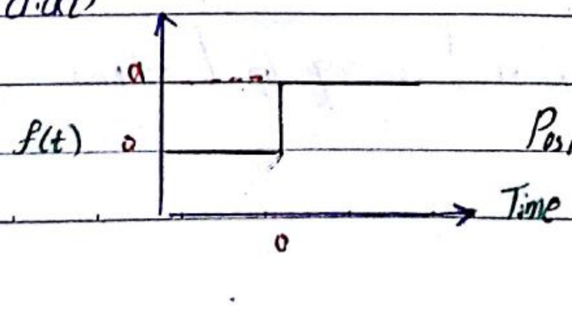
(*) Force function

(1) Step function.

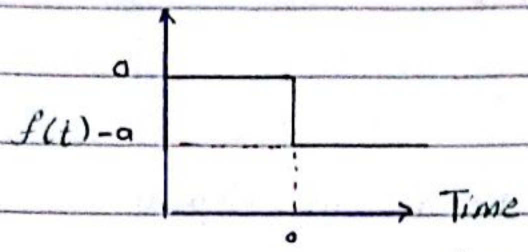
$$f(t) = \begin{cases} 0 & t < 0 \\ a & t \geq 0 \end{cases}$$

$f(t) = a$; $f(t) = a.u(t)$

$f(s) = U(s)$; $\frac{+a}{S}$



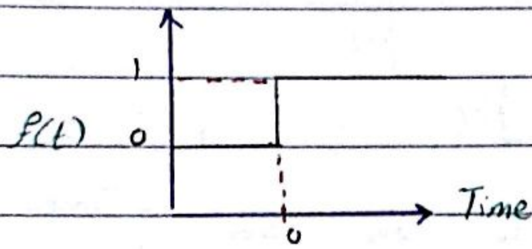
Positive step change



Negative step change:

$$f(t) = \begin{cases} 0 & t < 0 \\ -a & t \geq 0 \end{cases} \quad ; \quad f(t) = -a.u(t) \quad ; \quad f(t) = -a.$$

$$f(s) = u(s) = \frac{-a}{s}$$



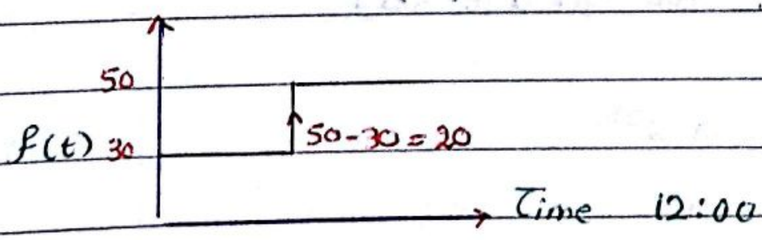
Unit step

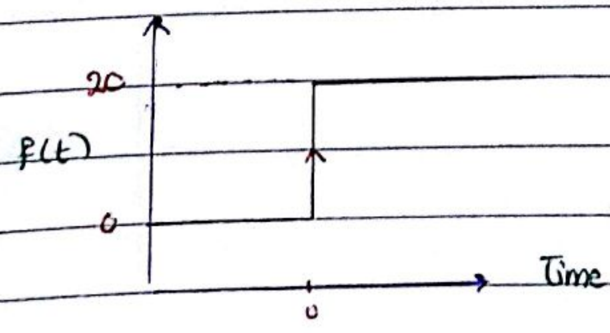
$$f(t) = u(t) = 1 \quad ; \quad f(s) = u(s) = \frac{1}{s}$$

Example:

$T_1 = 30°C$; $T_2 = 50°C$

time = 12:00





$$f(t) = T(t) \begin{cases} 0 & t < 0 \\ 20 & t \geq 0 \end{cases}$$

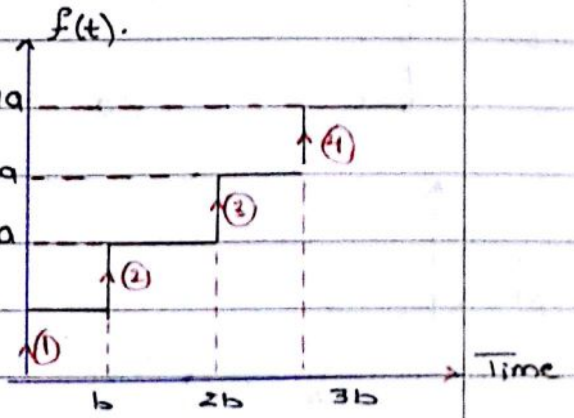
$$\therefore f(t) = 20 \quad ; \quad f(s) = \frac{20}{s}$$

$a =$ Magnitude of step function ; height.

2) Stair Function دالـة الـدرج

This is asceires step Function as shown in Figure and Can be express as:

$$f(t) = \begin{cases} 0 & t < 0 \\ a & 0 \leq t < b \\ 2a & b \leq t < 2b \\ 3a & 2b \leq t < 3b \\ 4a & t \geq 3b \end{cases}$$

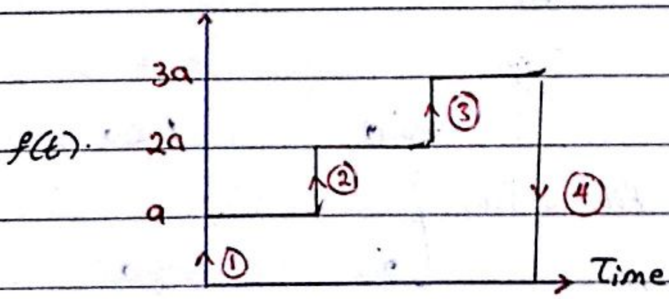


$$f(t) = \underset{1}{a.u(t)} + \underset{2}{a.u(t-b)} + \underset{3}{a.u(t-2b)} + \underset{4}{a.u(t-3b)}.$$

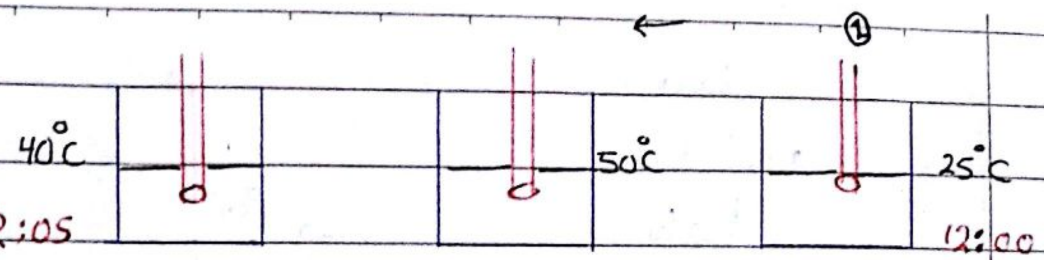
$$\boxed{f(s) = \frac{a}{s} + \frac{a}{s} e^{-bs} + \frac{a}{s} \cdot e^{-2b} + \frac{a}{s} e^{-3bs}}$$

$$f(t) = a.u(t) + a.u(t-b) + a.u(t-2b) - 3a.u(t-3b)$$

$$f(s) = \frac{a}{s} + \frac{a}{s} e^{-bs} + \frac{a}{s} \cdot e^{-2b} - 3\frac{a}{s} e^{-3bs}.$$



Example:

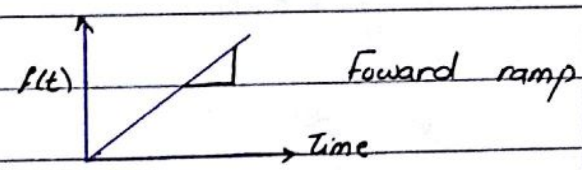


$f(s) = \frac{25}{s} - \frac{10}{s} e^{-5s}$; $f(t) = 25 - 10U(t-5)$

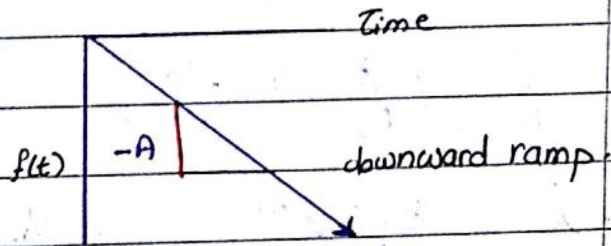
③ Ramp Function دالة المنحدر

The ramp function is a Uniform change with time.

∴ $f(t) = At$; $f(s) = \frac{A}{s^2}$



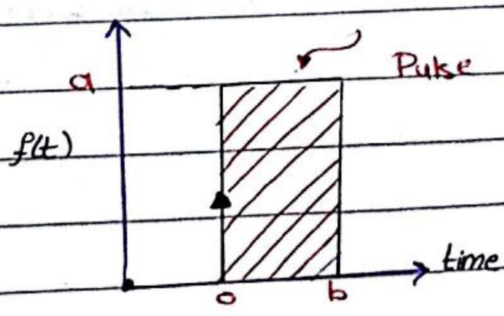
∴ $f(t) = -At$; $f(s) = \frac{-A}{s^2}$



④ Rectangular pulse النبضة المستطيلة

Is positive step Function change at period of the time and then negative step change.

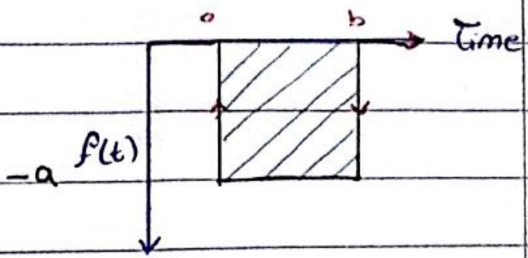
$$f(t) = \begin{cases} 0 & t < 0 \\ a & a \leq t \leq b \\ 0 & t \geq b \end{cases}$$



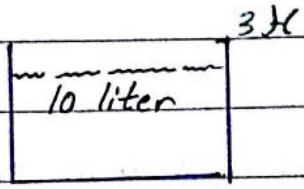
$f(t) = a.u(t) - a\,u(t-b)$

$f(s) = \frac{a}{s} - \frac{a}{s} e^{-bs}$

$$f(t) = \begin{cases} 0 & t<0 \\ -a & a \leq t \leq b \\ 0 & t \geq b \end{cases}$$



⑤ Impluse "$\delta(t)$"   النبضة الآنية



1 liter = Area of Impluse

الارتفاع

$$f(t) = \frac{k}{b} u(t) - \frac{k}{b} u(t-b).$$

$$\boxed{f(s) = \frac{k}{bs} - \frac{k}{b} \frac{e^{-bs}}{s}}$$

$$\mathcal{L}\,\delta(t) = \lim_{b \to 0} \left(\frac{k}{bs} (1 - e^{-bs}) \right).$$

$$= \lim_{b \to 0} \frac{k \cdot b\, e^{-bs}}{b} \quad \text{by L'Hoptial Rule}$$

$$\delta(s) = k$$

	f(t)	f(s)
Unit step	1	1/s
Unit Implus	$\delta(t)$	1

رسمة ال Impluse Function

Example: A System have the Following transfer Function

$$G_{(s)} = \frac{\theta_{o(s)}}{\theta_{i(s)}} = \frac{3}{S} \quad \frac{\text{Out put}}{\text{In put}}$$

a/ Find the differenatial equation?

b/ Find the response (θ_o) in time (τ) domain and sketch it For the Following Input (θ_i) Functions:

① Exponential Function $\frac{1}{5}e^{-t/s}$ start after $t=10$?

② Cosine Function start at $t=4$, $a=4$ and Frequency time of (π)

Sol:

a/ $$\frac{\theta_{o(s)}}{\theta_{i(s)}} = \frac{3}{S}$$

دائماً يكون معامل θ_i = 1 لذلك نقسم $(\frac{1}{3})$

$$S\theta_{o(s)} = 3\theta_{i(s)} \Rightarrow \frac{1}{3} S\theta_{o(s)} = \theta_{i(s)}$$

مشتقة أولى

$$\frac{1}{3}\frac{d\theta_{o(t)}}{dt} = \theta_{i(t)} \Leftarrow \frac{1}{3}S\theta_{o(s)} - \theta_{(0)} = 0$$

في حالة التحويل إلى Transfer Function

b/ ① $$G_{(s)} = \frac{\theta_{o(s)}}{\theta_{i(s)}} = \frac{3}{S} \quad ; \quad \theta_{i(t)} = \frac{1}{5}e^{-t/5}$$

$$\theta_{i(s)} = \left(\frac{1}{5(s+1}\right)e^{-10s}$$

$$\theta_{o(s)} = G(s) \cdot \theta_{i(s)}$$

$$\theta_{o(t)} = \mathcal{L}^{-1}\, G(s) \cdot \theta_{i(s)}$$

$$\theta_{o(t)} = \mathcal{L}^{-1}\,\theta_{o(s)} = \mathcal{L}^{-1}\frac{3}{S}\left(\frac{1}{5s+1}\right)e^{-10s}$$

$$\frac{3}{5}\mathcal{L}^{-1}\frac{1}{s}\left(\frac{1}{s+\frac{1}{5}}\right)e^{-10s}$$

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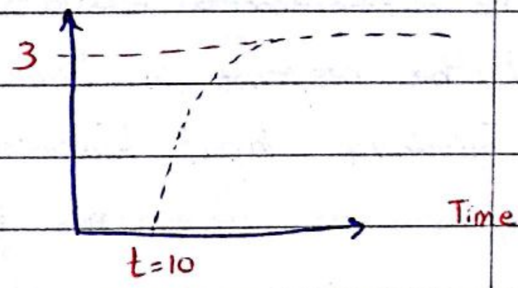
$$= \frac{3}{5}\mathcal{L}^{-1}\frac{1}{s(s+\frac{1}{5})}e^{-10s} = \frac{3}{5}\mathcal{L}^{-1}\frac{5}{s(5s+1)}e^{-10s}$$

$$= \frac{3}{5}\left[\frac{1}{\frac{1}{5}}\left(1-e^{-\frac{1}{5}(t-10)}\right)\right]$$



$$\theta_o(t) = 3\left(i - e^{-(t-10)/5}\right).$$

t	$\theta_{(o)}t$
10	-
11	-
12	-

b/2 $\theta_{i(t)} = a\cos w(t-td)$

$$w = \frac{2\pi}{time} = \frac{2\pi}{\pi} = 2 \quad ; \quad W = 2\pi f.$$

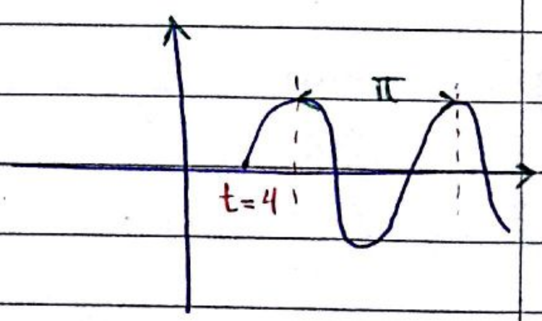
$$\theta_{i(t)} = 4\cos 2(t-4)$$

$$\theta_i(t) = 4\cos(2t-8)$$

Time	$\theta_{(o)}t$
4	-
5	-
6	-

$$\theta_{i(s)} = \frac{4s}{s^2+4}e^{-4s}$$

$$\theta_o(t) = \mathcal{L}^{-1}\left(\frac{3}{s}\right)\left(\frac{4s}{s^2+4}\right)e^{-4s}$$



$$\theta_o(t) = 3\sin 2(t-4)$$

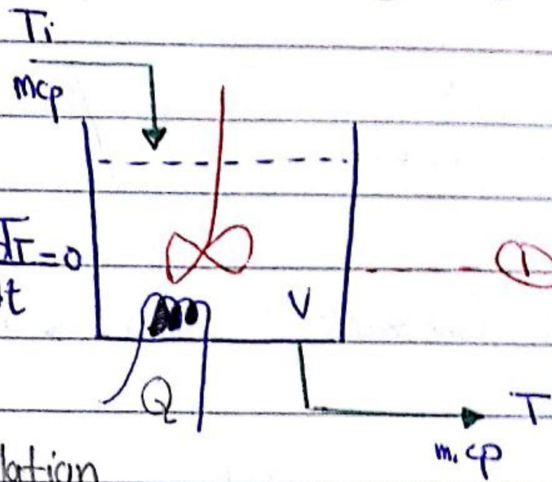
$$\theta_o(t) = 3\sin(2t-8)$$

Example: Continuous stirred tank heater ; متغيرات الفروق

Heat Balance :

S.S (steady state) ⊖

$$\bar{Q} + mc_p \bar{T_i} - mc_p\bar{T} = \rho c_p V \frac{d\bar{T}}{dt} = 0 \quad \text{--- } (1)$$



UnSteady state (t)

In - Out = Accumulation

$$Q'(t) + mc_p T_i'(t) - mc_p T'(t) = \rho V c_p \frac{dT'(t)}{dt} \quad \text{------ } (2)$$

$$kJ + kg \cdot \frac{kJ}{kg \cdot C} \cdot C = \frac{kg}{m^3} \cdot m^3 \, \frac{kJ}{kg \cdot C} \, \frac{C}{}$$

2-1 gives

* لازم ان يكون كل المتغيرات

$$mc_p(T_i'(t) - \bar{T_i}) + (Q'(t) - \bar{Q}') - mc_p(T'(t) - \bar{T}) =$$

$$\rho V c_p \frac{d}{dt}(T'(t) - \bar{T})$$

$$\boxed{mc_p T_i(t) + Q(t) - mc_p T(t) = \rho V c_p \frac{dT(t)}{dt}}$$

← Input معامل * نقسم على mcp

$$T_i(t) + \frac{Q(t)}{mc_p} - T(t) = \underbrace{\left(\frac{\rho V}{m}\right)}_{\tau} \frac{dT(t)}{dt}$$

$\tau = \rho V / m$

$k = \frac{1}{mc_p}$

$$\therefore \tau \frac{dT}{dt} + T(t) = T_i(t) + \underbrace{\left(\frac{1}{mc_p}\right)}_{k} Q(t)$$

In نتيجة , Output

$$\tau s T(s) + T(s) = T_i(s) + kQ(s)$$

$$T(s)[\tau s + 1] = T_i(s) + kQ(s)$$

$$T(s) = \frac{1}{\tau s + 1} T_i(s) + \frac{k}{\tau s + 1} Q(s)$$

Now: if Q is Constant $Q(s) = 0$ Input (بين الاثنتين أخد

وبقى $\xrightarrow{T_i}$

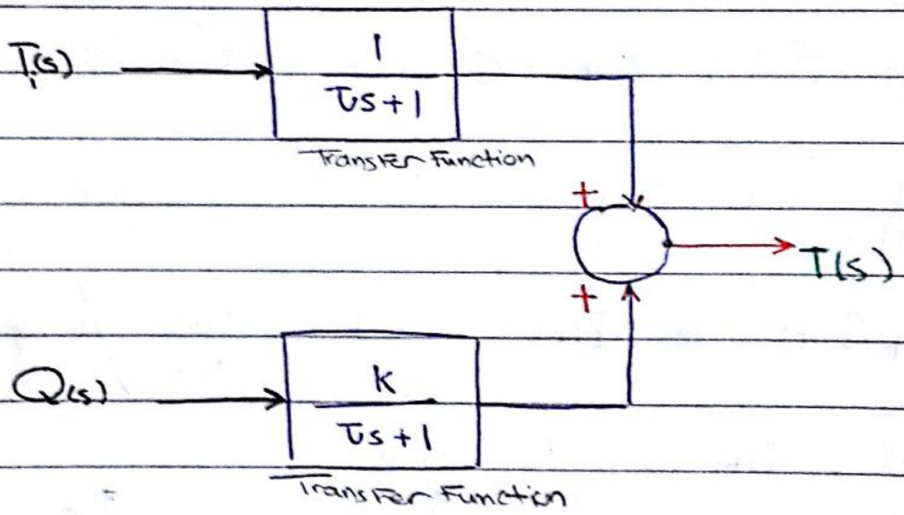
$\xrightarrow{Q}$

$$\boxed{\frac{T(s)}{T_i(s)} = \frac{1}{\tau s + 1} = G(s)}$$

If T_i is Constant $T_i(s) \to 0$

$$\frac{T(s)}{Q(s)} = \frac{K}{\tau s + 1} = G(s)$$

Block diagram



* معادلة لاخطية الدرس #1

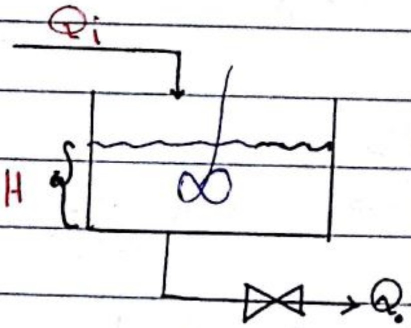
تحويل الدالة اللاخطية إلى خطية * لا يمكن تحويل S.S و Un الا إذا كانت خطية

ان الطريقة المناسبة لتحويل الدالة اللاخطية إلى خطية وذلك باستخدام مفكوك Taylors

$$\boxed{f(x) = f(x_o) + \frac{df}{x}\Big|_{x_o}(x - x_o) + \frac{1}{2!}\frac{d^2f}{dx^2}\Big|_{x_o}(x - x_o)^2}$$

$Q_o = k\sqrt{H}$ معادلة لاخطية

$$Q = k\sqrt{H_o} + \frac{k}{2\sqrt{H_o}}(H - H_o)$$



Example: Mass Balance.

* استجابة المنظومة Response: تغيرات (H) وليس Q_o دائماً

$$\rho\bar{Q}_i - \rho\bar{Q}_o = \rho A\frac{d\bar{H}}{dt} = 0 \quad S.S \quad \text{---} \quad (1)$$

$$\rho Q_i'(t) - \rho Q_o'(t) = \rho A\frac{dH'_{(t)}}{dt} \quad Un\ S.S$$

in - Out = Accumlation.

$Q_{o(t)} = kH^{1/2}$ Un S.S لاخطية At Steady state $\bar{Q}_o = k\sqrt{\bar{H}}$

↓ Unsteady state

$$Q_o'(t) = k\bar{H}^{1/2} + \frac{k}{2\bar{H}^{1/2}}(H'(t) - \bar{H})$$ خطية

$$\rho Q_i'(t) - \rho k\bar{H}^{1/2} + \frac{k}{2\bar{H}^{1/2}}(H'(t) - \bar{H}) = \rho A\frac{dH'_{(t)}}{dt} \quad \text{-----} \quad (2)$$ معادلة خطية

Subtract

طرح 2-1 ⇒

$$\underset{Un\,s.s}{Q_i'(t)} - \underset{s.s}{\bar{Q}_i} - \left[k\bar{H}^{1/2} + \frac{k}{2\bar{H}^{1/2}}H'(t) - k\bar{H}^{1/2}\right] = A\frac{d(H'_{(t)} - \bar{H})}{dt}$$

$$Q_i(t) - \frac{k}{2H^{-1/2}} H(t) = A\frac{dH(t)}{dt}$$

$$A\frac{dH(t)}{dt} + \frac{k}{2H^{-1/2}} H(t) = Q_i(t)$$

$$A\left(sH(s) - H(0)\right) + \frac{k}{2H^{-1/2}} H(s) = Q_i(s)$$

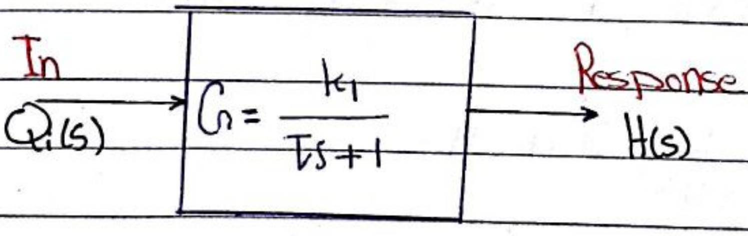
$$H(s)\left(As + \frac{k}{2H^{-1/2}}\right) = Q_i(s).$$

$$G(s) = \frac{H(s)}{Q(s)} = \frac{1}{As + \frac{k}{2\sqrt{H}}} = \left(\frac{2\sqrt{H}/K}{\frac{A2\sqrt{H}s}{k} + 1}\right)$$

Or $G(s) = \dfrac{k_1}{\tau s + 1}$; $k_1 = \dfrac{2\sqrt{H}}{k}$

$$\tau = \frac{2A\sqrt{H}}{k}$$

Block diagram



H.W/ for the above example if the following data are given $A = 1\,m^2$, $\bar{H} = 1.44\,m$, $\bar{Q}_i = \bar{Q}_o = 2.4 \times 10^{-3}\ m^3/s$ and a step change in Inlet Flow rate = $0.6 \times 10^{-3}\ m^3/s$ find the response and sketch it

ans/ $H'(t) = 2.16\ m$ (Final Value).

تحويل الدالة اللاخطية التي تعتمد على متغيرين أو أكثر إلى خطية

$$f(x,y) = f(x_0,y_0) + \left.\frac{df}{dx}\right|_{x_0,y_0}(x-x_0) + \left.\frac{df}{dy}\right|_{x_0,y_0}(y-y_0)$$

Example: Change the Following function to linear

$f(x,y) = x.y \qquad y_0 = 5 \quad , \quad x_0 = 2$

$$\therefore f(x,y) = x_0 . y_0 + y_0(x - x_0) + x_0(y - y_0)$$

$$= x_0 . y_0 + y_0 x - y_0 x_0 + x_0 y - x_0 y_0$$

$$\boxed{f(x,y) = 5x + 2y - 10}$$

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Example: A Well_stirred tank Contain (M) kg of a liquid with Specific heat Capacity of (J/kg.°C). The liquid is heated electrically with Constant rate of (Q). At Steady state the liquid inlet at a rate of m kg/h a temp of θ_i and Outlet at θ_o. Find the response to step change in inlet Flow rate of 20% [redacted] / [redacted] ? From steady state ?

∴ In step change ع المتغير المؤثر

Sol/ S. State.

$* \; m(t) c_p \theta_i$ ذ In نثبت متغير ونتحكم بالآخر

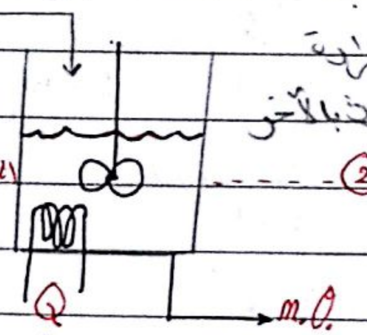
$$\bar{Q} + \bar{m} c_p \bar{\theta}_i - m c_p \bar{\theta}_o = M c_p \frac{d\bar{\theta}_o}{dt} = 0 \quad \text{--------} \quad (1)$$

(*) نثبت (m) وذلك باعتبار Q ثابت

يوجد تغير في الحرارة

لكن التغير يحدث بالآخر

Unsteady state



Taylor Series at (3).

$$\acute{Q} + \acute{m}(t) c_p \bar{\theta}_i - \acute{m}(t) c_p \acute{\theta}_o(t) = M c_p \frac{d\theta_o(t)}{dt} \quad \text{-----} \quad (2)$$

$$\acute{m}(t)\acute{\theta}_o(t) = \bar{m}\bar{\theta}_o + \bar{\theta}_o(\acute{m}(t) - \bar{m}) +$$

Taylor Series تطبيق

$$\bar{m}(\acute{\theta}_o(t) - \bar{\theta}_o) \quad \text{---} \quad (3)$$

$$\therefore \; \acute{m}(t) c_p \theta_i - \bar{m} c_p \bar{\theta}_i - \left(\bar{m} c_p \bar{\theta}_o + c_p \bar{\theta}_o(\acute{m}(t) - \bar{m}) + \right.$$

2-1 طرح

$$\left. \bar{m} c_p \acute{\theta}_o(t) - \bar{\theta}_o \right) + \bar{m} c_p \bar{\theta}_o = M c_p \frac{d\theta(t)}{dt}$$

$$m(t) c_p \theta_i - c_p \bar{\theta}_o m(t) - c_p \bar{m}\, \theta_o(t) = M c_p \frac{d\theta_o(t)}{dt}$$

$$(c_p \bar{\theta}_i - c_p \bar{\theta}_o) m(t) - c_p \bar{m}\, \theta_o(t) = M c_p \frac{d\theta_o(t)}{dt}$$

$$\left(\frac{\bar{\theta}_i - \bar{\theta}_o}{\mu}\right) m(t) - \frac{\bar{m}}{\mu} \theta_o(t) = \frac{d\theta_o(t)}{dt}$$

$$\frac{\mu}{\bar{m}} \cdot \frac{d\theta_o(t)}{dt} + \theta_o(t) = \left(\frac{\bar{\theta}_i - \bar{\theta}_o}{\bar{m}}\right) m(t)$$

(2) $$\tau \frac{d\theta_o(t)}{dt} + \theta_o(t) = k\, m(t)$$

$$\tau s \theta_o(s) + \theta_o(s) = k\, m(s)$$

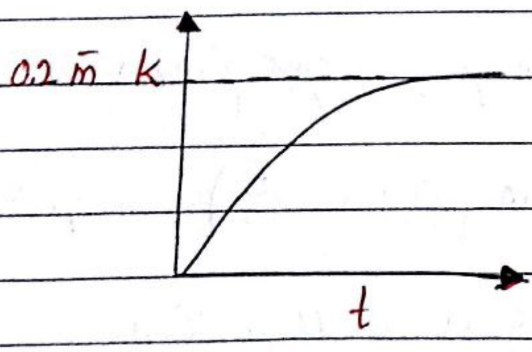
$$\theta_o(s)\left[\tau s + 1\right] = k\, m(s) \qquad k = \frac{\bar{\theta}_i - \bar{\theta}_o}{\bar{m}}$$

$$G(s) = \frac{\theta_o(s)}{m(s)} = \frac{k}{\tau s + 1} \qquad \tau = \frac{m}{\bar{m}}$$

$$\theta_o(s) = \frac{k}{\tau s + 1} m(s) \quad ; \quad m(s) = 0.2\,\bar{m}$$

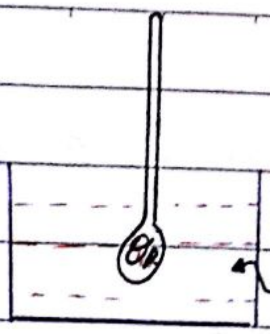
Steady state.

$$\theta_o(s) = 0.2\,\bar{m}\left(\frac{k}{s(\tau s + 1)}\right) = 0.2\,\bar{m}\,k\left(1 - e^{-t/\tau}\right)$$



"First Order System" أس الـ $s = 1$

Heat Balance:



θ_i ← حرارة المحيط

لا يوجد $= \theta_o$

In = Out + ACC

In = ACC

at steady state

$$hA(\underbrace{\bar{\theta}_i - \bar{\theta}_R}_{\Delta T}) = Mcp \frac{d\bar{\theta}_R}{dt} = 0 \quad \text{------} \quad (1)$$

Unsteady state

$$hA(\acute{\theta}_i - \acute{\theta}_R) = Mcp \frac{d\acute{\theta}_R}{dt} \quad \text{------} \quad (2)$$

Subtract 2 - 1

$$hA(\acute{\theta}_i - \acute{\theta}_R - \bar{\theta}_i + \bar{\theta}_R) = Mcp \frac{d(\acute{\theta}_R - \bar{\theta}_R)}{dt}$$

$$hA\left(\theta_i(t) - \theta_R(t)\right) = Mcp \frac{d\theta_R(t)}{dt}$$

$$\theta_i(t) - \theta_R(t) = \frac{Mcp}{hA} \cdot \frac{d\theta_R(t)}{dt}$$ المعامل عند المشتقة دائماً = 1

let $\frac{Mcp}{hA} = \tau$

$$\tau \frac{d\theta_R(t)}{dt} - \theta_i(t) = \theta_R(t)$$

Taking Laplace Trans Form T.F عند $\theta_R(0) = 0$ شرط

$$\tau\left(s\,\theta_R(s) - \cancel{\theta_R(0)}^{\,0}\right) - \theta_R(s) = \theta_i(s)$$ K For Thermometer = 1

$(\tau s+1)\,\theta_R(s) = \theta_i(s)$

$$\therefore G(s) = \frac{\theta_R(s)}{\theta_i(s)} = \frac{1}{\tau s+1}$$

* Transform function of 1^{st} Order System

$$G(s) = \frac{y(s)}{X(s)} = \frac{k}{\tau s+1} \leftarrow \text{General Form}$$

k : Steady state gain
τ : Time Constant

Notice that steady state gain for the thermometer = 1

* لماذا تصنف أجهزة First Order System

أولاً : لها ثابت زمن = 1 (τ)

ثانياً : التعبير الجبري لتمام دالة التحويل من الدرجة الأولى ($\tau S+1$)

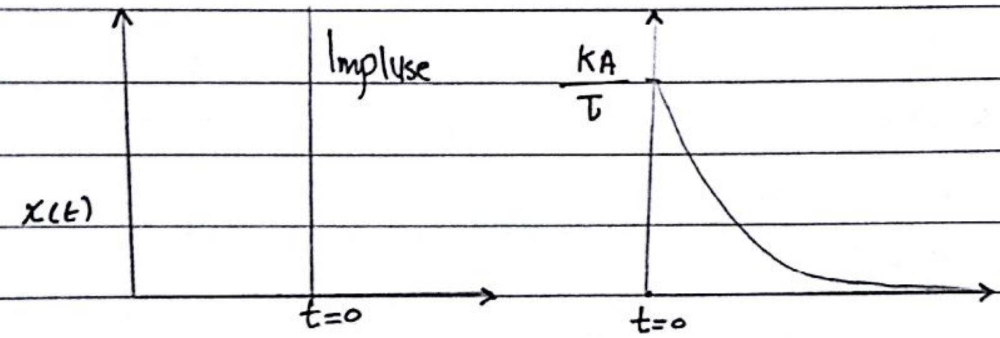
ثالثاً : نسبة الحالة المستقرة = K وكحالة خاصة للحرارة فإن

$K = 1$

Response of 1^{st} Order element to Impluse force

$$G(s) = \frac{y(s)}{x(s)} = \frac{k}{\tau s+1} \quad ; \quad x(s) = A \qquad \text{Impluse}$$

$$y(s) = kA\left[\frac{1}{\tau s+1}\right] \Rightarrow y(t) = \frac{kA}{\tau} e^{-t/\tau}$$

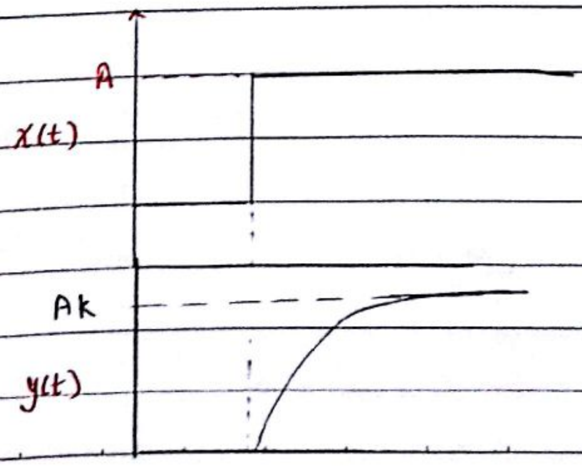


Response oF 1^{st} Order System to step force.

$$G(s) = \frac{y(s)}{x(s)} = \frac{k}{\tau s+1} \qquad x(s) = \frac{A}{s}$$

$$y(s) = kA\left(\frac{1}{s(\tau s+1)}\right) \quad ; \quad \frac{A}{s}\left[\frac{k}{\tau s+1}\right]$$

$$y(t) = kA\left(1 - e^{-t/\tau}\right)$$



Response of 1^{st} Order to pulse change. $<$ step$^+$, step$^-$

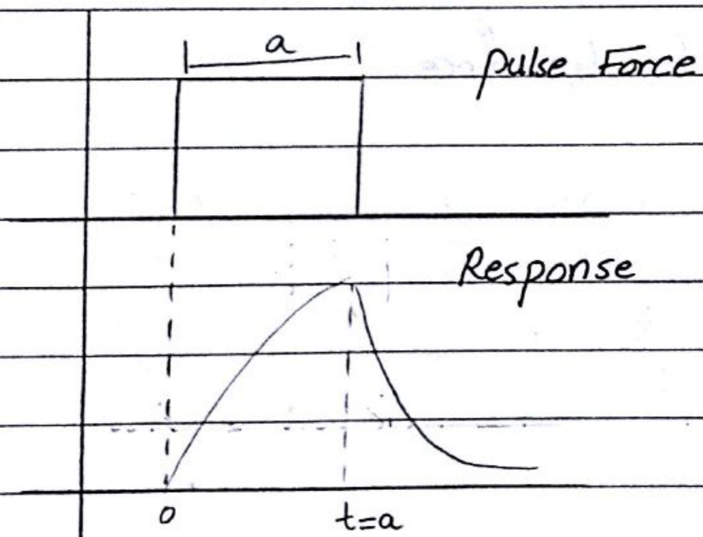
$$G(s) = \frac{k}{\tau s + 1} = \frac{y(s)}{x(s)} \quad , \quad x(s) = \frac{A}{s}\left(1 - e^{-as}\right)$$

$$y(s) = \frac{kA}{s} \frac{(1 - e^{-as})}{(\tau s + 1)}$$

$$y(s) = \frac{kA}{s(\tau s+1)} - \frac{kA\, e^{-as}}{s(\tau s+1)}$$

$$y(s) = \frac{kA}{s(\tau s+1)} - \frac{kA}{s(\tau s+1)} e^{-as}$$

$$y(t) = kA\left(1 - e^{-t/\tau}\right) - kA\left(1 - e^{\frac{-(t-a)}{\tau}}\right).$$



Response of 1^{st} Order System to Sine Function Force

$$G_{(s)} = \frac{\theta_{o(s)}}{\theta_{i(s)}} = \frac{k}{\tau s + 1}$$

$$\theta_{i(t)} = a \sin \omega t \quad ; \quad \theta_{i(s)} = \frac{a\omega}{s^2 + \omega^2}$$

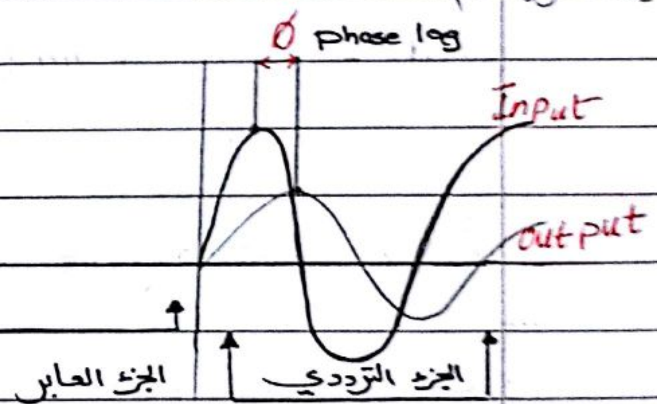
$$\theta_{o(s)} = \left(\frac{k}{\tau s + 1}\right)\left(\frac{a\omega}{s^2 + \omega^2}\right)$$

$$\theta_{o(s)} = ka\omega\left(\frac{1}{(\tau s + 1)(s^2 + \omega^2)}\right) \quad \frac{A}{\tau s + 1} + \frac{B}{s^2 + \omega^2}$$ استخدام التجزئة

$$\theta_{o(t)} = \frac{ak\omega\tau}{\tau^2\omega^2 + 1} e^{-t/\tau} + \frac{ak}{\sqrt{\tau^2\omega^2 + 1}} \sin(\omega t + \phi)$$

exponential — Oscillatory

بين التأخر والتقدم



Or $$\theta_{o(t)} = \frac{ak}{\sqrt{\tau^2\omega^2 + 1}} \sin(\omega t + \phi), \quad t \to \infty$$

Amplitude Ratio (AR)

$$\frac{\text{سعة الاستجابة}}{\text{سعة الدالة المؤثرة}} = \text{نسبة السعة}$$

$$AR = \frac{\frac{ak}{\sqrt{\tau^2\omega^2 + 1}}}{a} = \frac{k}{\sqrt{\tau^2\omega^2 + 1}}$$

$\omega = 2\pi f$, f = frequency

ω = Angular frequency.

ϕ phase lag in radian

frequency time $= \frac{1}{f}$

$t = \frac{|\phi|}{\omega}$ phase lag in time

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$\phi * \frac{180}{\pi}$ phase lag in degree.

Example: A thermometer have a Constant time of 6 sec is placed in a liquid of 100°C for a long time if the Input Force is a Sine wave of amplitude $a = 2°C$ and a frequency of $\frac{1}{6\pi}$ Hz. Find the Respones in time domain and sketch it.

$K = 1$

sol/ $\omega = 2\pi f = \frac{2\pi}{6\pi} = \frac{1}{3}$ rad/sec ; $\phi = \tan^{-1}(-\omega\tau) = -1.11$ rad

, $\tau = 6$sec , $a = 2$

$$\theta_o(t) = \frac{a k}{\sqrt{\tau^2 \omega^2 + 1}} \sin(\omega t + \phi)$$

$\phi = \tan^{-1}(-\omega t)$ or $\phi = \tan^{-1}(\omega t)$ ⊖

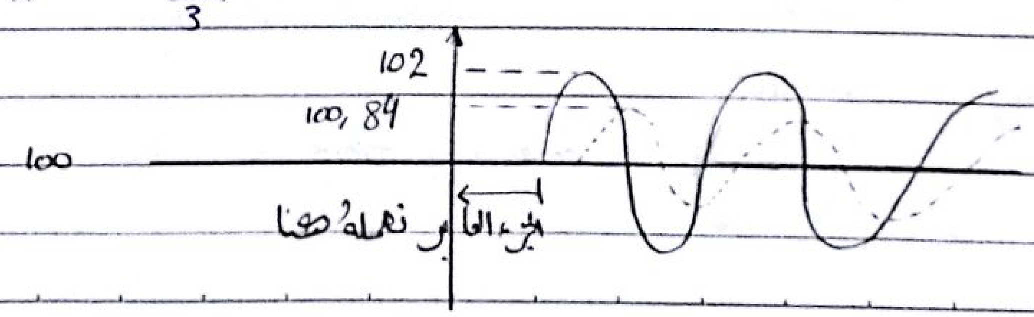
$$\theta_o(t) = \frac{1}{\sqrt{6^2 \left(\frac{1}{3}\right)^2 + 1}} \sin\left(\frac{1}{3} t - 1.11\right)$$

التعويض Rad

$$\theta_o(t) = 0.89 \sin\left(\frac{1}{3} t - 1.11\right)$$

$\phi = -1.11$ rad ; $\phi = \tan^{-1}(-\omega t)$ Phase lag

$$\frac{|\phi|}{\omega} = \frac{1.11}{\frac{1}{3}} = 3.33 \text{ Sec in time}$$



(a)

Example: A Sine Wave of frequency $\left(\frac{1}{\pi}\ Hz\right)$ and amplitude of 5 is affect On system whose T.F is

$$G(s) = \frac{y(s)}{X(s)} = \frac{1}{s+2} = \frac{k}{\tau s+1}$$ معادلة درجة أولى للبدال تكون هكذا

Show that $y(t) = \frac{5}{4} e^{-2t} + \frac{5}{2\sqrt{2}} \sin(114^\circ t - 45^\circ)$

الجزء العابر لم يُهمل هنا

Sol

$$G(s) = \frac{1}{s+2} = \frac{\frac{1}{2}}{\frac{1}{2}s+1} = \frac{k}{\tau s+1} \qquad k = \frac{1}{2} \; ; \; \tau = \frac{1}{2}$$

$$y(t) = \frac{ka\omega\tau}{\tau^2\omega^2+1} e^{-t/\tau} + \frac{ak}{\sqrt{\tau^2\omega^2+1}} \sin(\omega t + \phi)$$

$$\phi = \tan^{-1}(-\omega t) \quad , \quad \omega = 2 \quad , \quad a = 5$$

$$y(t) = \frac{5(\frac{1}{2})(2)(\frac{1}{2}) e^{-t/\frac{1}{2}}}{(\frac{1}{2}+2)+1} + \frac{5 * \frac{1}{2}}{\sqrt{(\frac{1}{2}+2)^2+1}} \sin(2t + \tan^{-1}(2 * \frac{1}{2}))$$

$$y(t) = \frac{5}{4} e^{-2t} + \frac{5}{2\sqrt{2}} \sin(2t - 0.7854)$$

$$y(t) = \frac{5}{4} e^{-2t} + \frac{5}{2\sqrt{2}} \sin(114^\circ t - 45^\circ)$$

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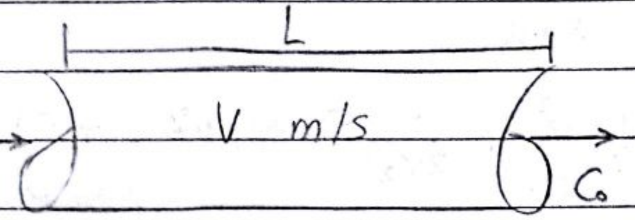
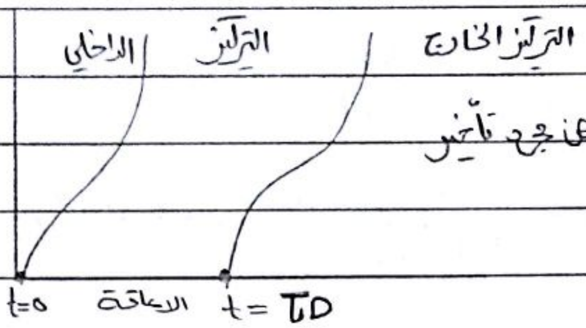
Pure time delay

أجهزة التفوق الزمني للفحص

هي تلك الأجهزة التي لا يطرأ منها أي تغير على مقدار أو شكل الإشارة الداخلة بل يكون تأثير الجهاز عن إعاقة زمنية محضة





نفس الشكل ونفس الهيئة ولكن مع تأخير

T.F of pure ~~time~~ delay

$$G_{(s)} = \frac{y(s)}{f(s)} = e^{-\tau_D s} \rightarrow \tau_D$$ الإعاقة والتأخير

$$\tau_D = \frac{L}{V}$$

Example: An Insulated tube transfer a liquid with Velocity of 0.5 m/s (V). At time = 0, it's temp is exposed to Sine Change $Q_i(t) = 10 \sin 2t$ find the response of a thermometer placed at a distance 20 m (L) away from the Inlet?

$$G_{(s)} = \frac{Q_{o(s)}}{Q_{i(s)}} = e^{-\tau_D s}$$

$$Q_{o(s)} = Q_i(s)\, e^{-\tau_D s} \qquad \frac{Q_{o(s)}}{Q_i} = e^{-\tau_D s}$$

$$Q_o(t) = Q_i(s)\, e^{-\tau_D s}$$

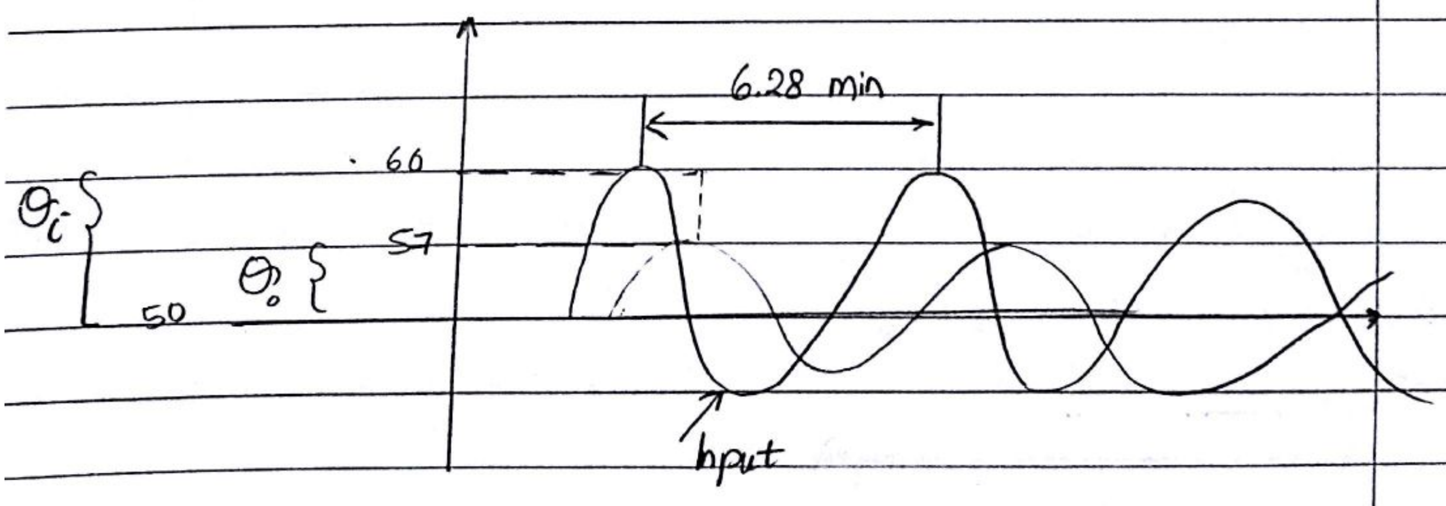
$$Q_{o(t)} = \mathcal{L}^{-1}\, Q_{i(s)}\, e^{-\tau_D s}$$

$$Q_o(t) = \mathcal{L}^{-1} \frac{a\omega}{s^2+\omega^2}\, e^{-\tau_D s} \qquad \tau_D = \frac{L}{V} = 40$$

$$Q_{(o)}t = \mathcal{L}^{-1} \frac{10(2)}{s^2+4}\, e^{-40 s}$$

$$Q_{o(t)} = 10 \sin 2(t-40)$$

H.w/ A first Order System is exposed to a sinewave and the sketch belows show both the Input and the OutPut response. Find T.F of the System and the Input and Output Force ?



كيفية تعيين ثابت زمن جهاز مرتبة أولى عمليا؟

يتم تعيين ثابت جهاز مرتبة أولى وذلك بتعريض الجهاز
الى تغير درجي تغير مناسب ومن خلال اخذ عدة قراءات
لإستجابة مع الزمن نتمكن من تعيين ثابت زمن الجهاز وكالتالي :

① Initial Slope

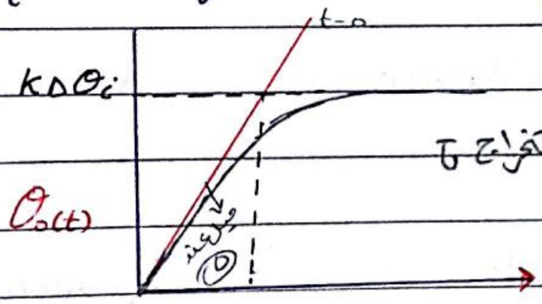
طريقة الميل الابتدائي

$\theta_{o(s)} = G_{(s)} \cdot \theta_{i(s)}$; $\theta_{o(s)} = \dfrac{k}{\tau s+1} \cdot \dfrac{\theta_{i(s)}}{s}$

$= k\Delta\theta_i \left(\dfrac{1}{s(\tau s+1)}\right)$

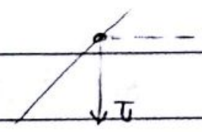
$\theta_{o(t)} = k\Delta\theta_i \left(1 - e^{-t/\tau}\right)$ step chang $\leftarrow_{t_0}$ 1^{st} استجابة جهاز من

$\dfrac{d\theta_{o(t)}}{dt} = \dfrac{k\Delta\theta_i}{\tau} e^{-t/\tau}$; $\left.\dfrac{d\theta_{o(t)}}{dt}\right|_{t=0} = \dfrac{k\Delta\theta_i}{\tau}$



Slope $= \dfrac{k\Delta\theta_i}{\tau} = \Delta y / \Delta x$

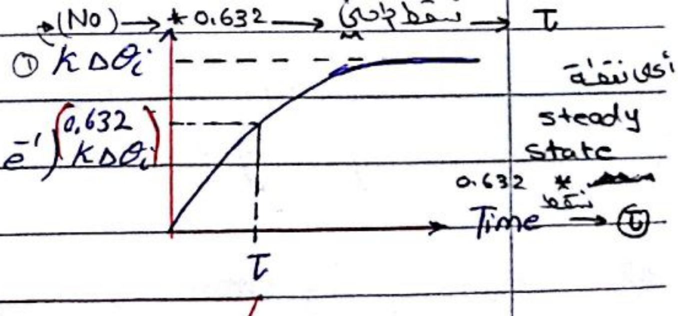
ناخذ ميله مع المنحنى ونسقطها لإستخراج τ



② 0.632 from the response

$\theta_{o(t)} = k\Delta\theta_i \left(1 - e^{-t/\tau}\right)$

if $t = \tau \Rightarrow \theta_{o(t)} = k\Delta\theta_i (1 - e^{-1})$



$\theta_{o(t)} = 0.632\ k\Delta\theta_i$

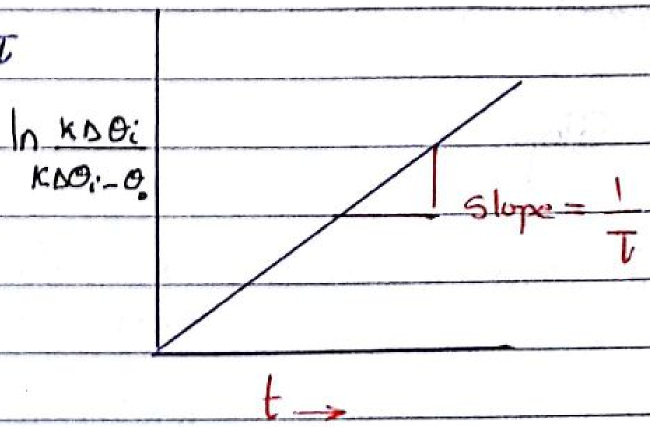
at $t = \tau$, $\boxed{\theta_{o(t)} = 0.632\ k\Delta\theta_i}$

↓ steady state

(3) طريقة الرسم النصف لوغاريتمي

$$\theta_{(t)} = k\Delta\theta_i\left(1 - e^{-t/\tau}\right)$$

$$= k\Delta\theta_i - k\Delta\theta_i e^{-t/\tau}$$

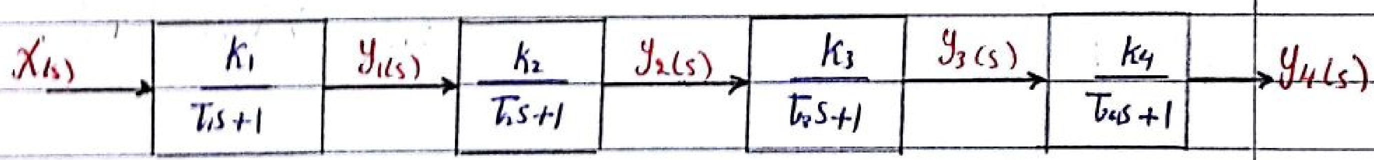


$$\frac{k\Delta\theta_i - \theta_{o(t)}}{k\Delta\theta_i} = e^{-t/\tau}$$

$$-\ln \frac{k\Delta\theta_i - \theta_{o(t)}}{k\Delta\theta_i} = t/\tau$$

$$\ln \frac{k\Delta\theta_i}{k\Delta\theta_i - \theta_{(t)}} = t/\tau$$

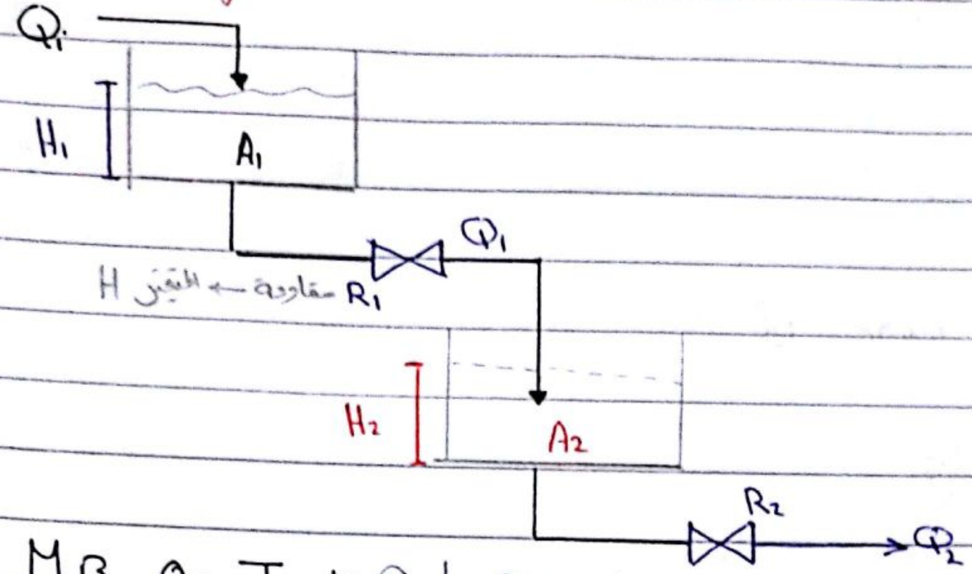
Response of first Order System in Series أس الأس = ١



1- Non interacting System
2- Interacting System.

Non-interacting system: أي دالة تحويل اي جهاز لا يحتوي على متغير الأجهزة السابقة

For Example



M.B On Tank (1) In − Out = Acc.

$$\rho Q_i(t) - \rho Q_1(t) = A\rho \frac{dH_1(t)}{dt}$$

if the Flow is laminar (linear Relationship Q of H_1)

$$\left(Q_1 = \frac{H_1(t)}{R_1} \right) \quad \therefore Q_i(t) - \frac{H_1(t)}{R_1} = A_1 \frac{dH_1(t)}{dt}$$

(Value + R وجود / $Q_1 = \frac{H_1(t)}{R}$ يعني) إذا وجد

$$T_1 \frac{dH_1(t)}{dt} + H_1(t) = R_1 Q_i(t) \quad ; \quad T = A_1 R_1$$

Flow laminar) علم

$$H_1(s)\left(T_1(s)+1\right) = R_1 Q_i(s)$$

$$\frac{H_1(s)}{Q_i(s)} = \frac{R_1}{T_1 S + 1} \quad (1) \quad \frac{\text{Respone}}{\text{Out}}$$

if a step change occurs in the Inlet Flow Rate

$$Q_i(s) = \frac{\Delta Q_i}{S}$$

Transfer Function

$$H_1(s) = \frac{\Delta Q_i}{s}\left(\frac{R_1}{\tau_1 s + 1}\right)$$

$$H_1(t) = \underbrace{R_1 \Delta Q_1}_{\text{قيمة ع الاتزان}} \left(1 - e^{-t/\tau}\right)$$ First Order Step Change

Mass Balance a take ②

$$Q_1(t) - Q_2(t) = A_2 \frac{dH_2}{dt}$$

$$\tau_2 \frac{dH_2(t)}{dt} + H_2(t) = R_2 Q_1(t)$$

$$(\tau_2 s + 1)(H_2(s)) = R_2 Q_1(s)$$

$$\frac{H_2(s)}{Q_1(s)} = \frac{R_2}{\tau_2 s + 1} \quad \text{-----} \quad ②$$

but $Q_1(t) = \dfrac{H_1(t)}{R_1}$

$$\frac{H_2(s)}{H_1(s)} = \frac{R_2}{R_1}\left(\frac{1}{\tau_2 s + 1}\right)$$

$$\frac{H_1(s)}{Q_i(s)} = R_1\left(\frac{1}{\tau_1 s + 1}\right)$$

$$\therefore \frac{H_2(s)}{Q_i(s)} = \frac{R_2}{(\tau_1 s + 1)(\tau_2 s + 1)}$$

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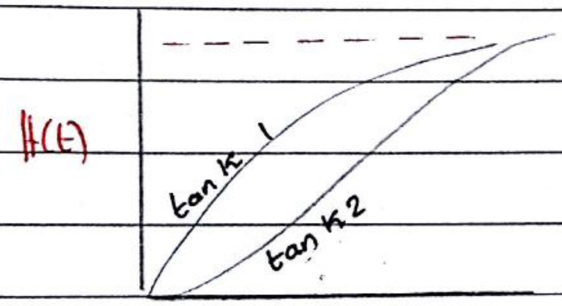
$$H_2(s) = \frac{R_2}{(\tau_1 s+1)(\tau_2 s+1)} Q_i(s)$$

if a step change Occurs in inlet flow rate $Q_i(s) = \frac{\Delta Q_i}{S}$

$$H_2(t) = \Delta Q_i R_2 \left(1 - \frac{\tau_1 e^{-t/\tau_1} - \tau_2 e^{-t/\tau_2}}{\tau_1 - \tau_2} \right)$$



Block diagram

$$Q_i(s) \rightarrow \boxed{\frac{R_1}{\tau_1 S+1}} \xrightarrow{H_1(s)} \boxed{\frac{R_2/R_1}{\tau_2 S+1}} \rightarrow H_2(s)$$

Now if $A_1 = A_2$ & $R_1 = R_2 = R$, $\tau_1 = \tau_2 = \tau$

$$\frac{H_2(S)}{Q_i(s)} = \frac{R_2}{(\tau S+1)^2}$$

if $Q_i(s) = \frac{\Delta \theta_i}{S}$

$$H_2(t) = \mathcal{L}^{-1} \frac{\Delta Q_i R_2}{S(\tau S+1)^2}$$

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$$H_2(t) = \Delta Q_i R_2 \left(1 - e^{-t/\tau} - \frac{t}{\tau} e^{-t/\tau}\right)$$

In general

$$\frac{Y_n(s)}{X(s)} = \frac{K_1 K_2 K_3}{(\tau_1 s+1)(\tau_2 s+1)(\tau_3 s+1)\ldots}$$

if $K = K_1 = K_2 = K_3$ ...

$\tau = \tau_1 = \tau_2 = \tau_3$ ...

$$\frac{Y_n(s)}{X(s)} = \frac{K^n}{(\tau s+1)^n}$$ In Series → $(K = \tau)$ n فقط نرفع لـ.

This Can be represented by:

$$\frac{y(s)}{X(s)} = \frac{K e^{-\tau_D s}}{\tau s+1}$$; $\tau_D = n\tau \xleftarrow{\text{فقط}} \tau = \frac{1}{V}$ (Tube) في

$$\frac{y(s)}{X(s)} = \frac{K^n}{(\tau s+1)^n} = \frac{K e^{-\tau_D s}}{\tau s+1}$$

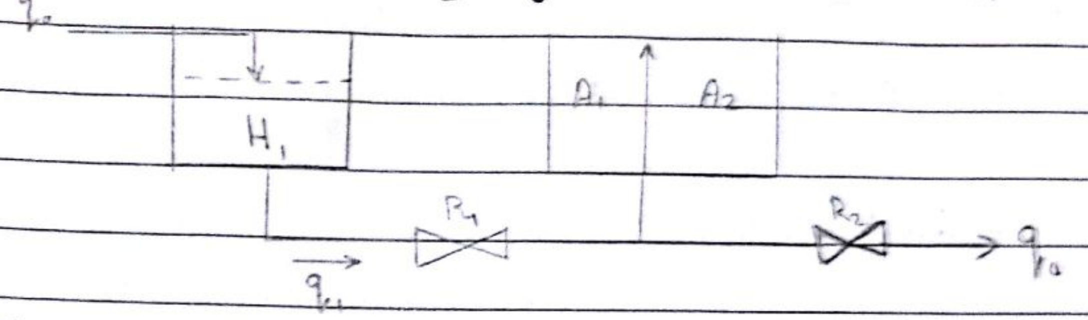
$$\boxed{\frac{y(s)}{X(s)} = \frac{K e^{-n\tau s}}{\tau s+1}}$$

هذا القانون فقط عند * وجود Series + $\tau = K$

$$\frac{y_n(s)}{X(s)} = \frac{K_1 K_2 K_3 \ldots}{(\tau_1 s+1)(\tau_2 s+1)(\tau_3 s+1)\ldots}$$ في حالة عدم $\tau \neq K$ *

Example: Interacting System.

$Q_1 = \frac{H_1}{R_1}$, $Q_2 = \frac{H_2}{R_2}$



find $G_{(s)} = \frac{H_2(s)}{q_o(s)}$

Sol M.B On tank ①

$$q_i - q_o = A_1 \frac{dH_1}{dt}$$ In - Out = Acc ; $q_1 = \frac{H_1 - H_2}{R_1}$ Inter acting

$$A \frac{dH_1}{dt} + \frac{H_1 - H_2}{R_1} = q_i$$

$$R_1 A_1 \frac{dH_1}{dt} + H_1 = R_1 q_i + H_2$$

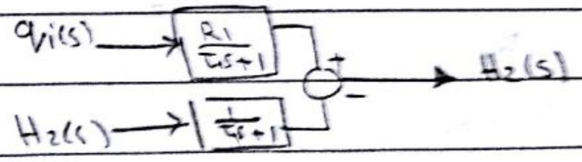
$$\tau_1 \frac{dH_1}{dt} + H_1 = R_1 q_i + H_2$$

$$(\tau_1 S + 1) H_1(s) = R_1 q_i(s) + H_2 S \quad \text{——} \quad ①$$

Transfer Function

$$H_1(s) = \frac{R_1}{\tau_1 S + 1} q_i(s) + \frac{1}{\tau S + 1} H_2(s)$$

M.B On tank ②



$$q_1 - q_o = A_2 \frac{dH_2}{dt}$$

$$A_2 \frac{dH_2}{dt} + \frac{H_2}{R_2} = \frac{H_1 - H_2}{R_2}$$

$$A_2 \frac{dH_2}{dt} + \frac{H_2}{R_2} + \frac{H_2}{R_1} = \frac{H_1}{R_1}$$

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$$A_2 \frac{dH_2}{dt} + H_2\left(\frac{R_2 + R_1}{R_1 R_2}\right) = \frac{H_1}{R_1}$$

$$(\tau^* s + 1)\, H_2(s) = \frac{R_2}{R_1 + R_2} H_1(s) \quad ②$$

$$(\tau^* s + 1)\, H_2(s) = k_1 H_1(s)$$

$$\frac{(\tau_1 s + 1)(\tau^* s + 1)}{k_1} H_2(s) = R_1 q_i(s) + H_2(s)$$

$$(\tau_1 s + 1)(\tau^* s + 1) H_2(s) - k_1 H_2(s) = R_1 k_1 q_i(s)$$

$$\left(\tau_1 \tau^* s^2 + (\tau_1 + \tau^*) + s + 1 - k_1\right) H_2(s) = R_1 k_1 q_i(s)$$

$$\left(\frac{\tau_1 \tau^*}{1-k_1} s^2 + \frac{\tau_1 + \tau^*}{1-k_1} s + 1\right) H_2(s) = \frac{R_1 k_1}{1-k_1} q_i(s)$$

$$1 - k_1 = 1 - \frac{R_2}{R_1 + R_2} = \frac{R_1}{R_1 + R_2}$$

$$\frac{\tau^*}{1-k_1} = \frac{A_2 R_2 R_1 / (R_1 + R_2)}{\frac{R_1}{R_1 + R_2}} = A_2 R_2 = \tau_2$$

$$\frac{\tau_1 + \tau^*}{1-k_1} = \frac{A_1 R_1 + A_2 R_1 R_2 / R_1 + R_2}{\frac{R_1}{R_1 + R_2}} = \frac{AR_1(R_1 + R_2) + A_2 R_1 R_2}{R_1}$$

$$= A_1 R_1 + A_1 R_2 + A_2 R_2$$

$$= \tau_1 + \tau' + \tau_2$$

$$\frac{R_1 K_1}{1 - K_1} = \frac{\frac{R_2}{R_1 R_2 + R_2}}{\frac{R_1}{R_1 + R_2}} = R_2$$

$$\therefore \frac{H_2(s)}{q_i(s)} = \frac{R_2}{\tau_1 \tau_2 S^2 + (\tau_1 + \tau' + \tau_2) S}$$

$\tau_1 = A_1 R_1$; $\tau_2 = A_2 R_2$

$\tau' = A_1 R_2$

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Second Order System

As an example for second Order System we Can Say " Manumeter".

after Making a Force balance, we get:

$$G(s) = \frac{y(s)}{x(s)} = \frac{K}{\tau^2 s^2 + 2 S \tau s + 1}$$

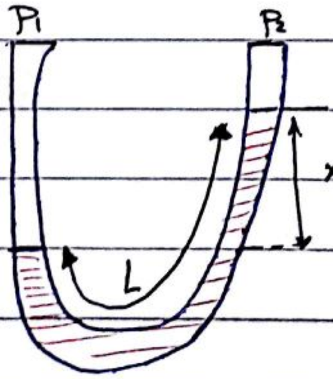
where

T : Natural period of Oscillation

$$T^2 = \frac{1}{Wa^2}$$ where W_a = natural frequency.

S = damping Factor (coefficent).

We Can Solve as Charactristic eq. like $(\tau^2 s^2 + 2 S \tau s + 1 = 0)$



$$S = \frac{-2S\tau \mp \sqrt{4S^2\tau^2 - 4\tau^2}}{2\tau^2}$$

$$= \frac{2S\tau}{2\tau^2} \mp \frac{\sqrt{S^2 - 1}}{\tau}$$

$$S = \frac{-S}{\tau} \mp \frac{\sqrt{S^2 - 1}}{\tau}$$

Notice that $\sqrt{S^2 - 1}$ detrime the nature of response and as follows:

① if $\delta > 1$ Unequal roots $(-ve)$ real roots

$\therefore y = A e^{-m_1 t} + B e^{-m_2 t}$ المعادلة

Over depend System اسم الاستجابة

*not Oscillatory (ميزاتها)

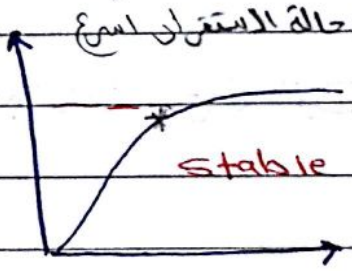
*stable System.

② $\delta = 1$ equal roots $(-ve)$: $m_1 = m_2$

ووصولها لحالة الاستقرار أسرع

$\therefore y = A e^{-mt} + Bt e^{-mt}$

$= e^{-mt}(A + Bt)$.



Critically damped response. اسم الاستجابة

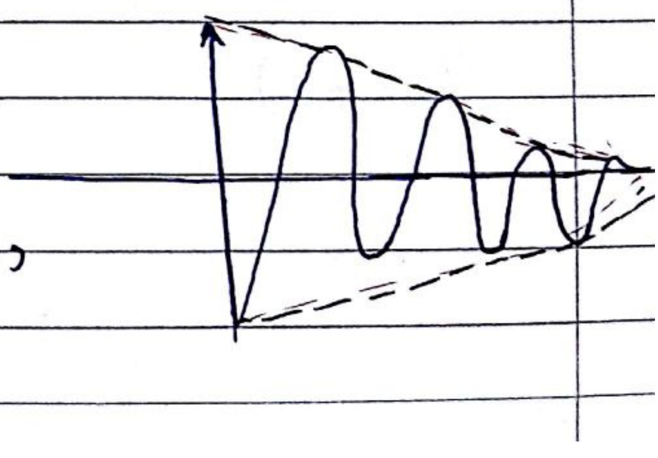
* stable and the reaching to the desired Value Faster than over damped. ميزاتها

③ $0 < \delta < 1 \Rightarrow$ Complex $(-ve)$ real part

الاسم

S_1 and $S_2 = -\alpha \mp i\beta$

$y = e^{-\alpha t} \sin \beta t$



Under damped response, stable, Oscillation.

* إنّ القيمة المثالية لمعامل التضاؤل للوصول الى حالة الاستقرار تساوي =

$\zeta_{\text{المثالي}} = 0.214$

واغلب الاجهزة في مجال عملها تكون من النوع الثالث (Under damped).

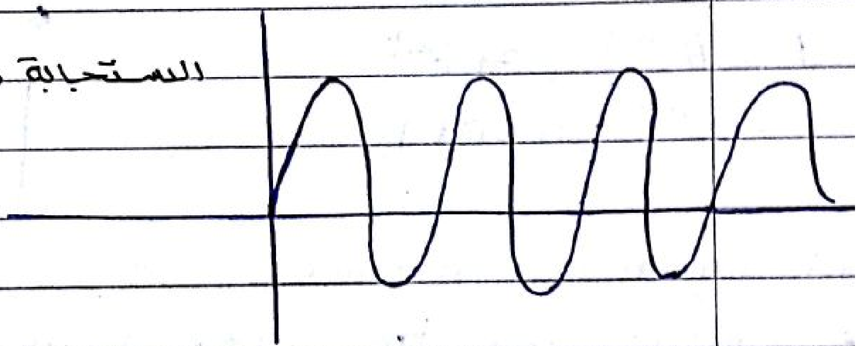
(4) $\delta = 0$ pure Imaginary.

S_1 and $S_2 = \mp i\beta$

$y = A \sin \beta t$

Critically Stable or Free Oscillators. التذبذب بسعة ثابتة

الاستجابة ذات سعة ثابت

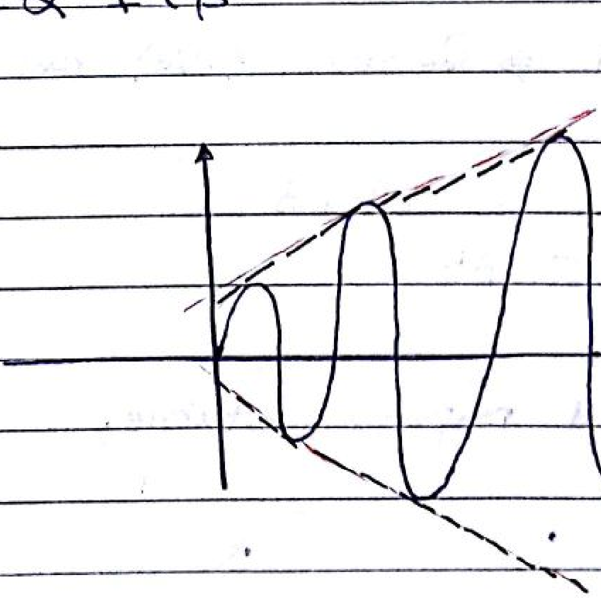


(5) $\delta < 0$ Complex (+ve real part)

S_1 and $S_2 = +\alpha \mp i\beta$

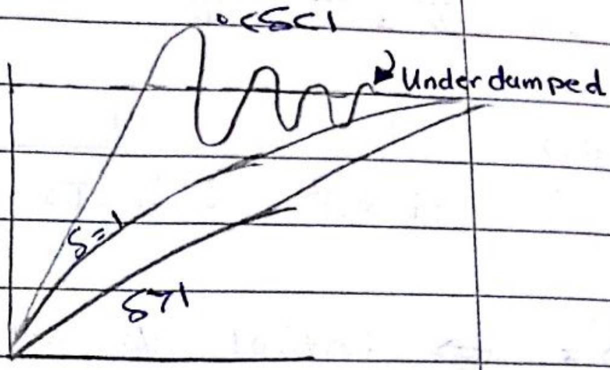
$y = e^{mt} \sin \beta t$

(Unstable)



In general for Stable System

Optimum $\delta = 0.214$



Tow first Order System In Series

$$X(s) \rightarrow \boxed{\frac{k_1}{\tau_1 s+1}} \rightarrow \boxed{\frac{k_2}{\tau_2 s+1}} \rightarrow y(s)$$

Transfer function : $\frac{y(s)}{X(s)} = \frac{k_1 k_2}{(\tau_1 s+1)(\tau_2 s+1)} = \frac{k}{\tau_1 \tau_2 s^2 + (\tau_1+\tau_2)s + 1}$ المعادلة الرئيسية لـ 2nd

$\tau_1 + \tau_2 = 2\delta$

1. $\tau^2 = \tau_1 \tau_2 \Rightarrow \tau = \sqrt{\tau_1 \tau_2}$

2. $2\delta\tau = \tau_1 + \tau_2 \quad \therefore \tau_1 \neq \tau_2$

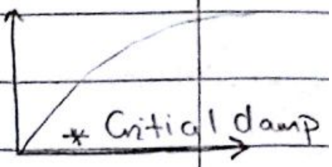
(first Order In Series Only)

$$\delta = \frac{\tau_1+\tau_2}{2\tau} = \frac{\tau_1+\tau_2^2}{2\sqrt{\tau_1 \tau_2}} > 1$$ always larger than 1.

$\therefore$ Over damped System.

If $\tau_1 = \tau_2 = \tau$

$$\frac{y(s)}{x(s)} = \frac{1}{(\tau s+1)^2} = \frac{1}{\tau^2 s^2 + 2\tau s + 1} = \frac{1}{\tau^2 s^2 + 2\delta\tau s + 1}$$



$\delta = 1 \Rightarrow$ Critical damped, no Oscillation Occurs

* Response of 2^{nd} Order System (Under damped to step Change)

If the Input force is a step change.

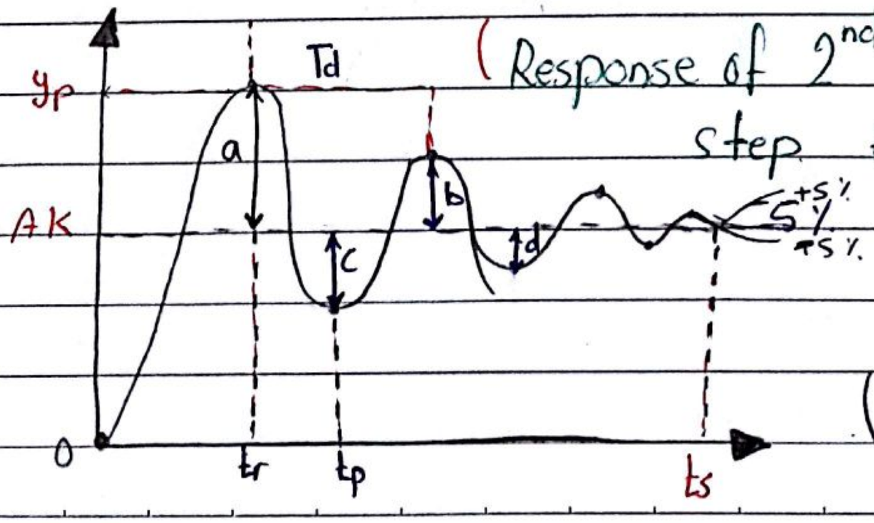
$$y(s) = \frac{K}{s(\tau^2 s^2 + 2\delta\tau s + 1)}$$

Solving by Convalution Laplace طريقة *

$$y(t) = KA\left(1 - \frac{e^{-\delta t/\tau}}{\sqrt{1-\delta^2}} \sin\left(\frac{\sqrt{1-\delta^2}}{\tau} t + \phi\right)\right)$$

$$\phi = \tan^{-1}\left(\frac{\sqrt{1-\delta^2}}{\delta}\right).$$

* Force peak التوقيتين *
Response peak 9



(Response of 2^{nd} Order System to step Function)

Oscillation تلاشي

(steady state) وتنتهي

ω: Angular frequency

$$\omega = \frac{\sqrt{1-\delta^2}}{T} = \omega_n \sqrt{1-\delta^2}$$

Cycle Period : T_d

$$T_d = \frac{2\pi}{\omega} = \frac{2\pi}{\omega_n\sqrt{1-\delta^2}}$$

* وقت بين peak إلى peak

Time of first Peak : t_p

$$t_p = \frac{\pi \tau}{\sqrt{1-\delta^2}}$$

* أول قمة تظهر في الاستجابة

Overshoot

* هو الفرق بين القمة الأولى والاستجابة النهائية
* هو أعلى انحراف عن المواصفات أو القيم المرغوبة (أعلى قمة)

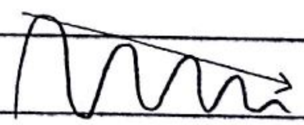
$$\text{Over shoot} = Ak\, e^{-\frac{\pi\delta}{\sqrt{1-\delta^2}}}$$

Ak % بدون ×

$$\underline{\underline{\%}}\ \text{Over shoot} = e^{-\frac{\pi\delta}{\sqrt{1-\delta^2}}} * 100$$

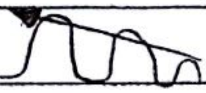
Subsidence Ratio (SR) — نسبة التناقص

$$SR = \frac{a}{b} = \frac{c}{d} = e^{+\frac{2\pi\delta}{\sqrt{1-\delta^2}}}$$



Decay Ratio (DR) — نسبة التخامد

$$\frac{1}{SR} = \frac{b}{a} = \frac{d}{c} = e^{-\frac{2\pi\delta}{\sqrt{1-\delta^2}}} = (D_R)$$



Optimum DR $= \frac{1}{4}$ (المثالي)

Optimum SR $= 4$

$\delta = 0.214$

Rise time t_r : This is the time required for the response to reach the first it's Ultimate Value and it's equation:

* الزمن اللازم للوصول لأول قمة مرة واحدة

$$t_R = \frac{T(\pi - \phi)}{\sqrt{1-\delta^2}}$$

$$= \frac{T}{\sqrt{1-\delta^2}} \left(\pi - \tan^{-1}\frac{\sqrt{1-\delta^2}}{\delta}\right).$$

Steady time : t_s الزمن للوصول الى حالة الاستقرار النهائية والتي تحصل عندما تقل الاستجابة النهائية الى $\pm 5\ \%$ من القيمة المرغوبة

* استخراجها من الرسم عملياً *

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Ex: a/ find SR for the function :

$$f(t) = a\,e^{-4t}\,\sin 0.5t$$

b/ find δ where $SR = 4$

Sol/ a =) $e^{-4t} = e^{-\frac{\delta}{T}t}$

$$4 = \frac{\delta}{T} \quad \text{---------} \quad (1)$$

$$\sin 0.5t = \sin \frac{\sqrt{1-\delta^2}}{T}\,t$$

$$\frac{\sqrt{1-\delta^2}}{T} = 0.5 \quad \text{---------} \quad (2)$$

* from (1) and (2) *

$$\frac{\sqrt{1-\delta^2}}{\delta} = \frac{0.4}{5} \quad ; \quad 1-\delta^2 = 0.015625\,\delta^2$$

$$\delta^2 = 0.9922$$

$$SR = e^{2\pi\delta/\sqrt{1-\delta^2}} = e^{2\pi(8)} = e^{16\pi}$$

b/ If $SR = 4$

$$4 = e^{\frac{2\pi\delta}{\sqrt{1-\delta^2}}}$$

$$1.386 = 2\pi \cdot \frac{\delta}{\sqrt{1-\delta^2}}$$

$$0.22 = \frac{8}{\sqrt{1-\delta^2}} \Rightarrow {}^{*}\ \delta = 0.214\ {}^{*}$$

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Response of 2^{nd} Order System to Impluse:

$$y(t) = \frac{K}{T^2S^2 + 2\delta TS + 1} X(s)$$

$$X(s) = A$$

$$y(s) = KA\left(\frac{1}{T^2S^2 + 2\delta TS + 1}\right)$$

$$\boxed{y(t) = \frac{KA}{\sqrt{1-\delta^2}\,T} e^{-\frac{\delta}{T}t} \sin \frac{\sqrt{1-\delta^2}}{T} t}$$ حفظ

Response of 2^{nd} Order System for Sinewave.

$$y(s) = \frac{k}{T^2S^2 + 2\delta TS + 1} X(s)$$

$$X(s) = \frac{a\omega}{S^2 + \omega^2} \quad , \quad x(t) = a \sin \omega t$$

$$y(t) = \frac{Ka}{\sqrt{(1-\omega^2T^2) + (2\delta\omega T)^2}} \sin(\omega t + \phi)$$ (حفظ)

$$\phi = \tan^{-1} \frac{2\delta\omega T}{1-\omega^2 T^2}$$

Ex: The dynamic of particular process are represented by a Single time Constant element in Seires with a Second Order (Oscillatory) element. The first Order time Constant is Unity, while the second Order time Constant and damping factor both have a value of 0.2425 min. The steady state gain factor of the process is 0.941. Show that the Overall T.F Can be Written As:

$$G_{(s)} = \frac{16}{(s+1)\left[(s+1)^2+16\right]}$$

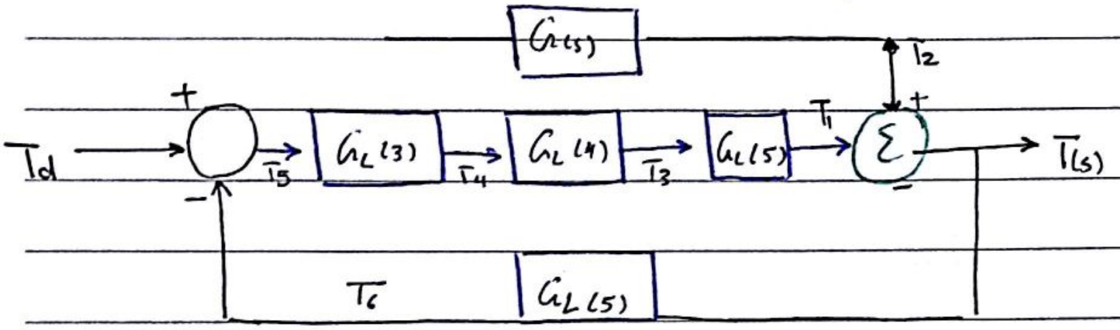
If a Unit Impluse disturbance enter the System, determine the response as a function of time and sketch it.

Sol : Answer $y(t) = (1-\cos 4t)\, e^{-t}$

Closed System

Closed System divided into:

① General closed loop system, if he didn't give Us any closed loop we Can Consider it general Closed system.



" General Closed loop "

Derivation of T.F of general loop:

$T_{(s)} = T_2 + T_1$

$T_{(s)} = m(s) . G_L(s) + T_3 \, G_p(s)$

$T_{(s)} = m(s) . G_L(s) + T_4 \, G_{v(s)} . G_p(s)$

$T_{(s)} = m(s) . G_L(s) + T_5 \, G_v . G_c \, G_p(s)$

$T_{(s)} = m(s) . G_L(s) + (T_d - T_6) \, G_c . G_p . G_v .$

$T(s) = m(s) \cdot G_L(s) + T_d\, G_c\, G_p\, G_v - T(s) \cdot G_m\, G_c \cdot G_v \cdot G_p$

$T(s)\left(1 + G_m(s) \cdot G_c(s) \cdot G_v(s) \cdot G_p(s)\right) = m(s) \cdot G_L(s) + T_d \cdot G_c(s) \cdot G_v(s) \cdot G_p(s)$

$\bar{G}(s) = G_c(s) \cdot G_v(s) \cdot G_p(s)$

$$T(s) = \frac{G_L(s)}{1 + G_m(s) \cdot \bar{G}(s)}\, m(s) + \frac{\bar{G}(s)}{1 + G_m \cdot \bar{G}(s)}\, T_d$$

Transfer Function of general closed loop can divided into:

① Servo System

In which the desired value changing and the load value is constant.

$$G(s) = \frac{T(s)}{T_d(s)} = \frac{\bar{G}(s)}{1 + G_m(s) + \bar{G}(s)}$$

② Regulator System

In which the desired value is constant and the load value changing.

$$G(s) = \frac{T(s)}{m(s)} = \frac{G_L(s)}{1 + G_m(s) \cdot \bar{G}(s)}$$

* يغير load ✓

Ex: Regulator loop with the Following elements:

$k=3$, $\tau=10$ ⟵ $G_p = \dfrac{3}{10S+1}$, $G_{L(s)} = \dfrac{1}{10S+1}$

$G_{m(s)} = 1$, $G_{v(s)} = 1.5$, $G_{c(s)} = 2$

Determine the System response for a Unit step in load.

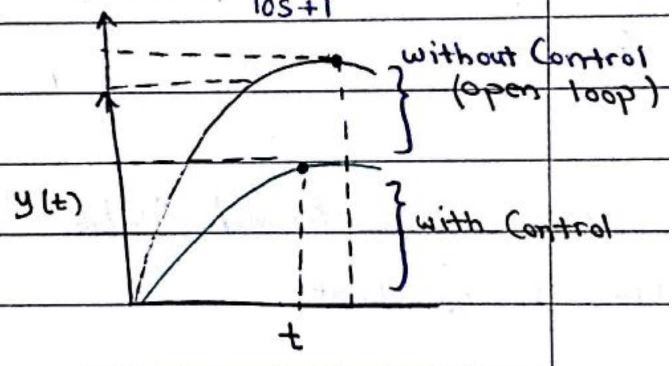
$\frac{1}{S}$

Sol:

$$T(s) = F(s) \cdot G(s)$$

$$= \frac{1}{S} \cdot \frac{G_{L(s)}}{1 + G_{m(s)} \cdot G(s)} = \frac{1}{S} * \frac{\frac{1}{10S+1}}{1 + 2 * 1.5 * \frac{3}{10S+1}}$$

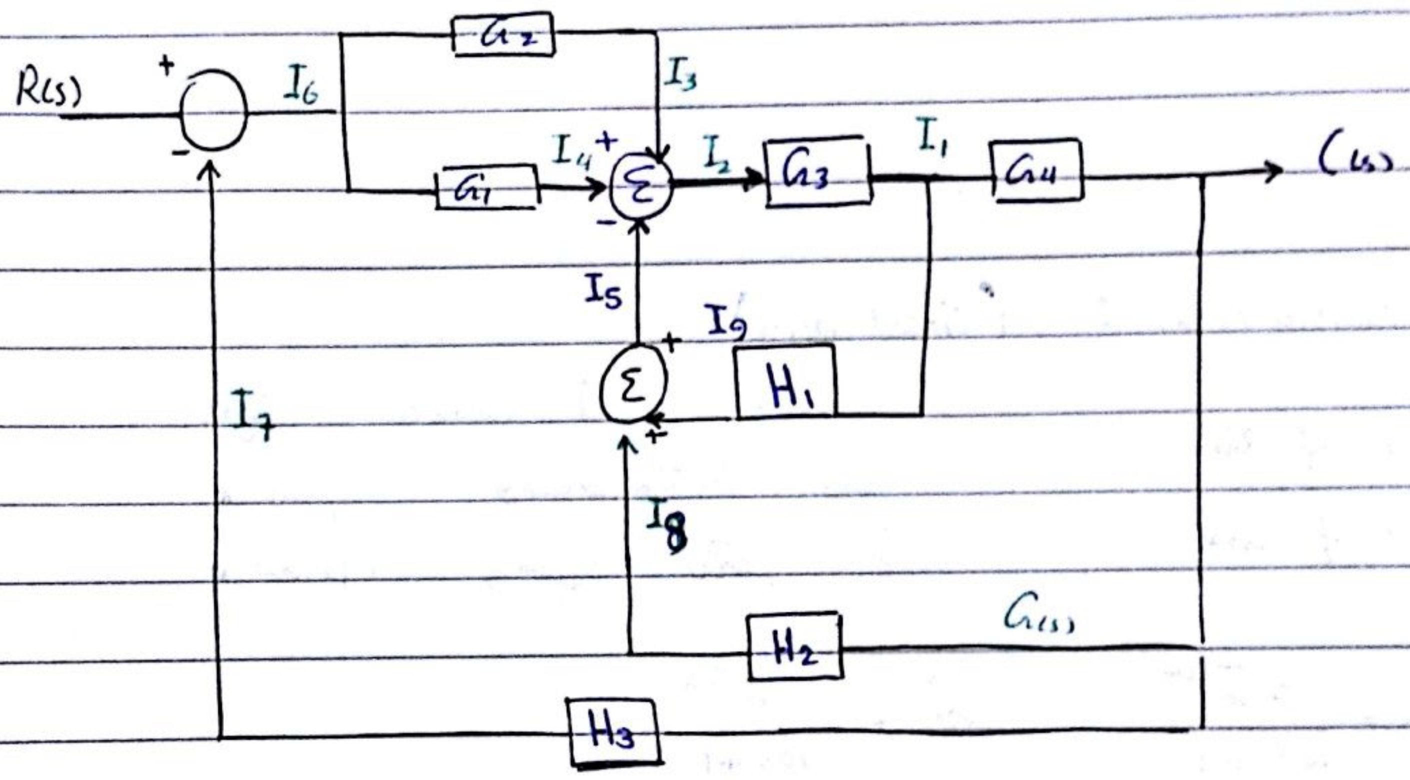
$$= \frac{1}{S} * \frac{\frac{1}{10S+1}}{\frac{10S+1+9}{10S+1}}$$



$$T(t) = \frac{1}{10}\left(1 - e^{-t}\right)$$

2- Special closed loop System

Ex: Find transfer Function $C(s)/R(s)$ for the System shown below.



$$C_{(s)} = G_4 \, I_1$$

$$C(s) = G_4 G_3 \, I_2 = G_4 G_3 (I_3 + I_4 \mp I_5)$$

$$C(s) = G_4 G_3 G_2 I_6 + G_4 G_3 G_1 I_6 - G_4 G_3 (I_9 + I_8)$$

$$C(s) = G_4 G_3 G_2 (R_{(s)} - I_7) + G_4 G_3 G_2 (R_{(s)} - I_7) - G_4 G_3 H_1 I_1 - G_4 G_3 H_2 C_{(s)}$$

$$C(s) = G_4 G_3 G_2 R_{(s)} - G_4 G_3 G_2 \underbrace{H_3 C_{(s)}}_{I_7} + G_4 G_3 G_1 R_{(s)} - G_4 G_3 G_1 H_3 C_{(s)} - \cancel{G_4} G_3 H_1 \cdot \frac{C_{(s)}}{\cancel{G_4}} - G_4 G_3 H_2 C_{(s)}$$

$$(1 + G_4 G_3 G_2 H_3 + G_4 G_3 G_1 H_3 + G_3 H_1 + G_4 G_3 H_2) \, C_{(s)}$$

$$= (G_4 G_3 G_2 + G_4 G_3 G_1) R_{(s)}$$

$$\frac{C_{(s)}}{R_{(s)}} = \frac{G_4 G_3 G_2 + G_4 G_3 G_1}{1 + G_4 G_3 G_2 H_3 + G_4 G_3 G_1 H_3 + G_3 H_1 + G_4 G_3 H_2}$$

$$C(s) = I_1 G_4$$

$$I_1 = \frac{C(s)}{G_4(s)}$$

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Pade Approximation : (closed loops).

لمعرفة Closed أو Open

- Open : وجود فقط G_p
- Closed : وجود G_v , G_c

$$e^{-\tau_D s} = \frac{1 - \frac{1}{2}\tau_D s}{1 + \frac{1}{2}\tau_D s}$$

Example:

$$G_p = \frac{3e^{-\tau_D s}}{10s+1} \quad , \quad G_L = \frac{e^{-\tau_D s}}{10s+1}$$

$G_m(s) = 1$, $G_v(s) = 1.5$, $G_c(s) = 2$, $\tau_D = 2$

Determine the Response For Unit step ; load ?

Controllers

A Controllers is a device that recieves data From a measurment instrument Compares that data to programmed Set point, and if necessary Signals a Control element to take Convective actions.
↑ Valve

The actions of Controllers Can be divided into groups based Upon the Functions of their Control mechanisum. Each type of Controller has advantage and disadvantage and will meet the needs of different applications. Grouped by Control mechanism function

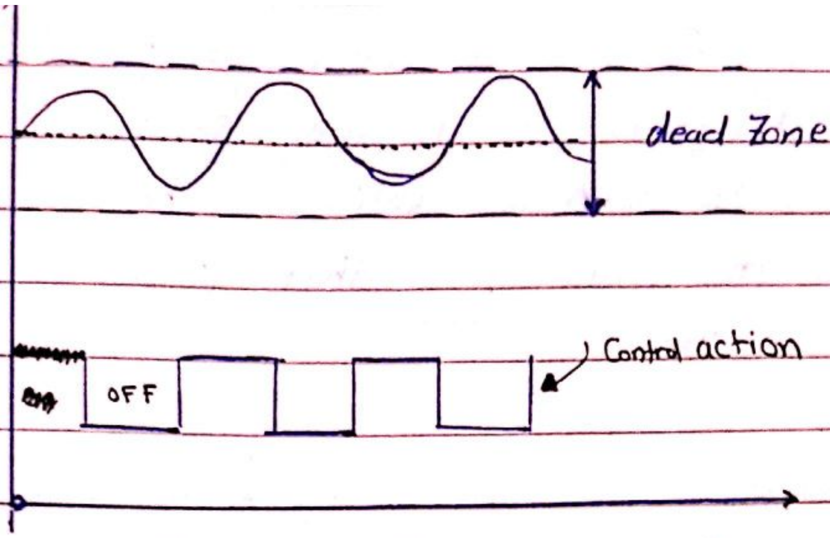
Controller Are:

1- Discrete Controllers.
2- Multi step Controllers.
3 Continuous Controllers.

Discrete Controllers: (تحكم بس On + OFF فقط)

Are Controllers that have Only two positions : On and OFF. A Common example of discrete Controller is a home hot Water heater.

(لا يصل بالفعل الى set point)
This type of Control doesn't actually hold the Variable at Set point, but keeps the Variable within approximity of Set point in what is known as a dead Zone.



(حيثية الـ Control)

Discrete Control

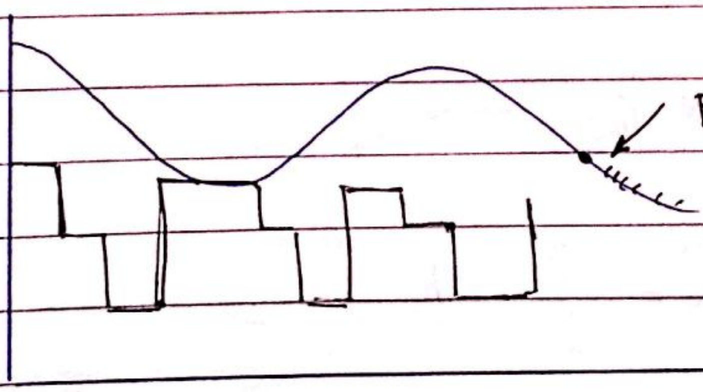
MultiStep Controllers :

(عدة درجات)

Are Controllers that have at least One other possible position in addition to On and oFF Multistep Controllers Operate Similary to disCrete Controllers. but as a Setpoint is approached, the multistep Controllers takes intermediate steps.

خطوة وسطية

(أحق)



Multistep Controller

(كلما كانت خطوات (أفعال) ولكن كلما كانت التفصيلات أكثر أقل تموج)

Continuous Controllers:

A Controllers automatically Compare the Value of p.v (Process Variable) to the Set point to determine if an error exists. If there is an error, the Controller adjusts (سيغير) the Output according to the parameters (المتغيرات) that have been Set in the Controller.

Types of Continuous Controllers:

يقارن p.v مع s.p
يوجد error ⇐ تدخل
لا يوجد error ⇐ يستمر

1. proportional Controller (P)
2. Integral Controller (I)
3. Derivation Controller (D)
4. proportional Integral (PI) ⇒ من افضل الـ Controllers المستخدمة صناعياً
5. proportional Derivate (PD).
6. proportional-Integral-Derivate (PID) ⇐ معقد الاستخدام

(A) Proportional Controller:

$$E \longrightarrow \boxed{G_c} \longrightarrow P$$

Error   process

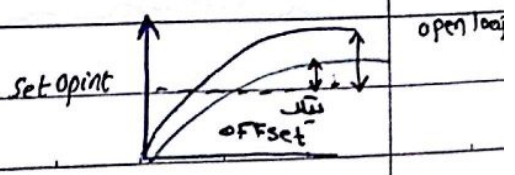
$P \propto E$

$$P = K_c E \Rightarrow G_c = \frac{P}{E} = K_c$$

عند وجود رقم = K_c
فهو يعني proportional

K_c: Gain Or Sensitivity of Controller.

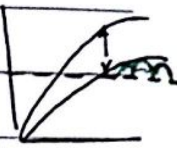
1. Proportional Controller reduce the effect without eleminating it



② When k_c InCreasing the effect will reduce Unitil we reach k_{max}, which at it the System starts to regulate (التذبذب) for that we Use k_{opt}.

* قل الـ error
يقل الحيّز (offset)

$$k_{c\,optimum} = \frac{k_{c,max}}{2}$$ ⇐ * أقل حيّز + تكون قبل التذبذب



③ System time Constant (τ) is less than process time Constant (τ_p) so that the system response will be Faster than the process response.

يقلل زمن الاستجابة
فيكون شغله عكسي

④ System time Constant (τ) deCreasing as k_c InCreasing

الـ error يقل ⇐ K_c يزداد ⇐ τ يقل
يصل الى الاستقرارية بأقرب وقت

B) Integrall Controller :