$\boldsymbol{sin^2 \leq 1}$
$\boldsymbol{cos^2 \leq 1}$
$\boldsymbol{|sin| \leq 1}$
$\boldsymbol{|cos| \leq 1}$
$\boldsymbol{ln\, n > 1}$
$\boldsymbol{ln\, n < n}$
$\boldsymbol{n^{\frac{1}{2}} < n}$
$\boldsymbol{n^{\frac{2}{5}} < n^{\frac{3}{2}}}$

حفظ

Examples:

1- $\sum_{n=1}^{\infty} \frac{sin^2}{n^4}$ $\quad Comparison\ test$

$sin^2 < 1 \quad] \div n^4$

$\frac{sin^2}{n^4} < \frac{1}{n^4} \quad b_n = \frac{1}{n^4} \ \ldots. \ P-series$

$P = 4 > 1 \ \ conv.$

$\therefore \frac{sin^2}{n^4} \quad conv.$

2- $\sum_{n=1}^{\infty} \frac{|\cos n|}{n^{\frac{3}{2}}}$

$|\cos n| \leq 1 \quad] \div n^{\frac{3}{2}}$

$\frac{|\cos n|}{n^{\frac{3}{2}}} \leq \frac{1}{n^{\frac{3}{2}}} \quad b_n = \frac{1}{n^{\frac{3}{2}}} \ \ldots. \ P-series$

$P = \frac{3}{2} < 1 \ \ div \ .$

6- Limit comparison test :

$Let \sum a_n \ and \sum b_n \ be \ series \ \ with \ positive \ term \ and$

$suppose \qquad \rho = \lim_{n\to\infty} \frac{a_n}{b_n}$

$If \ \rho \ is \ finit \ and \ \rho \neq 0 \ \ then \ the \ sereis \ both \ conv. or \ div.$

Examples:

1- $\sum_{n=1}^{\infty} \frac{1}{2n^2-n}$ $\qquad$ *Limit comparison*

$let \ b_n = \frac{1}{n^2} \qquad\qquad P-series \ \rightarrow \ P>1 \ \ conv.$

$$\rho = \lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\frac{1}{2n^2-n}}{\frac{1}{n^2}} = \lim_{n\to\infty} \frac{n^2}{2n^2-n} \qquad] \div n^2$$

$$= \lim_{n\to\infty} \frac{1}{2-\frac{1}{n}} = \frac{1}{2} \qquad \rightarrow \rho \neq 0 \ \ \therefore \textbf{the series is conv.}$$

2- $\sum_{n=1}^{\infty} \frac{1}{n-\frac{1}{4}}$ $\qquad$ *Limit comparison*

$Let \ b_n = \sum_{n=1}^{\infty} \frac{1}{n} \qquad\qquad P-series \ \rightarrow P=1 \quad div.$

$$\rho = \lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\frac{1}{n-\frac{1}{4}}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n}{n-\frac{1}{4}} \qquad] \div n$$

$$\lim_{n\to\infty} \frac{1}{1-\frac{\frac{1}{4}}{n}} = 1 \qquad div.$$

Problems

1- $\sum_{n=1}^{\infty} \frac{1}{(n-1)\sqrt{(n-1)}}$ $ans.: \rho = 1 \quad conv.$

2- $\sum_{n=1}^{\infty} \frac{1}{3^{(n-1)}-1}$ $ans.: \rho = \frac{1}{3} \quad conv.$

3- $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2+1}$ $ans.: conv.$

4- $\sum_{n=2}^{\infty} \frac{1}{n\ln(n)}$ $ans.: div.$

5- $\sum_{n=1}^{\infty} \frac{n^2-2n+1}{n^4-5n+2}$ $ans.: \rho = 1 \quad conv.$

6- $\sum_{n=1}^{\infty} sin\frac{1}{n}$ $ans.: div.$