$$\gg \int_1^\infty \frac{1}{x}\, dx = \ln x \Big]_1^\infty = \lim_{L\to\infty} (ln x)\Big]_1^L = \lim_{L\to\infty} (\ln L) = \infty$$

The integral is div. $\gg$ *the series is div.*

3-Ratio test :

(Positive term series)

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \rho$$

a) If $\rho < 1$ the series converges
b) If $\rho > 1$ the series diverges
c) If $\rho = 1$ the series may be conv. or div.
another test must be tried.

ملاحظة: يستخدم هذا الاختبار غالبا عند احتواء الحدود على $n!$ او مقادير مرفوعة الى n

Ex.

1- $\sum_{n=1}^{\infty} \frac{1}{n!}$

$$\rho = \frac{a_{n+1}}{a_n} = \frac{\frac{1}{(n+1)!}}{\frac{1}{n!}} = \frac{\frac{1}{(n+1)n!}}{\frac{1}{n!}} = \frac{1}{n+1} = 0 < 1 \ conv.$$

2- $\sum_{n=1}^{\infty} \frac{n}{2^n}$ by ratio test

$$\rho = \lim_{n\to\infty} \frac{\frac{n+1}{2^{n+1}}}{\frac{n}{2^n}} = \lim_{n\to\infty} \frac{n+1}{2^{n+1}} \cdot \frac{2^n}{n} = \lim_{n\to\infty} \frac{n+1}{n} \cdot \frac{2^n}{2.2^n}$$

$$\rho = \frac{1}{2} \lim_{n\to\infty} \frac{n+1}{n} = \frac{1}{2}(1) = \frac{1}{2} \quad < 1 \quad conv.$$

3- $\sum_{n=1}^{\infty} \frac{1}{2n-1}$ *by ratio test*

$$\rho = \lim_{n\to\infty} \frac{\frac{1}{2(n+1)-1}}{\frac{1}{2n-1}} = \lim_{n\to\infty} \frac{1}{2(n+1)-1} \cdot \frac{2n-1}{1}$$

$$= \lim_{n\to\infty} \frac{2n-1}{2n+1} = \lim_{n\to\infty} \frac{2-\frac{1}{n}}{2+\frac{1}{n}} = 1 \qquad test\ is\ fail$$

$Other\ test: by\ integral\ test$

$$\int_1^{\infty} \frac{dx}{2x-1} = \lim_{L\to\infty} \left(\frac{1}{2} ln(2x-1)_1^L\right) = \frac{1}{2} \lim_{n\to\infty} \quad ln(2L-1) = \infty$$

$series\ is\ div.$

4- Root test

(Positive term series)

$$\rho = \lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} (a_n)^{\frac{1}{n}}$$

a) $If \quad \rho \ < 1 \quad the\ series\ converges$

b) $If \quad \rho \ > 1 \ or = \infty \quad the\ series\ diverges$

c) $If \quad \rho \ = 1 \quad the\ series\ may\ be\ conv.\ or\ div.$

another test must be tried.

يستخدم هذا الاختبار عند وجود مقادير مرفوعة الى **n**

Examples:

1- $\sum_{n=1}^{\infty}\left(\frac{4n-5}{2n+1}\right)^{n}$ *by root test*

$$\rho=\lim_{n\to\infty}\left[\left(\frac{4n-5}{2n+1}\right)^{n}\right]^{\frac{1}{n}}=\lim_{n\to\infty}\left[\left(\frac{4n-5}{2n+1}\right)\right]=\lim_{n\to\infty}\left[\frac{4-\frac{5}{n}}{2+\frac{1}{n}}\right]=\mathbf{2}\ >\mathbf{1}\ \ \boldsymbol{div.}$$

2- $\sum_{n=1}^{\infty}\frac{1}{(\ln(n+1))^{n}}$ *by root test*

$$\rho=\lim_{n\to\infty}\left[\left(\frac{1}{\ln(n+1)}\right)^{n}\right]^{\frac{1}{n}}=\lim_{n\to\infty}\frac{1}{\ln(n+1)}=0<1\quad conv.$$

5- Comparison test :

$$\boldsymbol{If\ b_n > a_n\ \ and} \sum b_n\ \boldsymbol{was\ converges}$$

$$\gg \sum a_n\quad \boldsymbol{is\ converges}$$

$$\boldsymbol{If\ b_n < a_n\ \ and} \sum b_n\ \boldsymbol{was\ diverges} \gg \sum a_n\ \ \boldsymbol{is\ diverges}$$