4-1Conservation of Mass Principle

Consider a control volume of arbitrary shape, as shown in Fig (4-1).

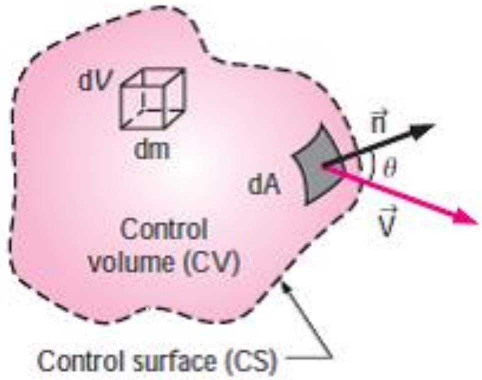


Figure (4-1): the differential control volume and differential control volume

(Total mass entering CV)- (Total mass leaving CV) = Net change in mass within the CV

$$\dot{m}_{in} - \dot{m}_{out} = dm_{cv} \int dt \qquad \frac{kg}{s} \qquad \dots\dots(4-1)$$

Total mass within the CV:

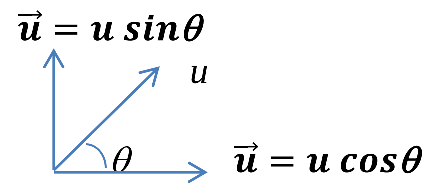
$$m_{cv} = \int_{cv} \rho dV \qquad \dots\dots..(4-2)$$

Rate of change of mass within the CV:

$$\frac{dm_{cv}}{dt} = \frac{d}{dt}\int_{cv} \rho dV \qquad \dots\dots..(4-3)$$

Now consider mass flow into or out of the control volume through a differential area dA on the control surface of a fixed control volume.

Let $\vec{n}$ be the outward unit vector of dA normal to dA and $\vec{u}$ be the flow velocity at dA relative to a fixed coordinate system, as shown in Fig. (4-1). In general, the velocity may cross dA at an angle θ.



normal component of velocity $\quad u = u\ \cos\theta = \vec{u}.\vec{n}$

The mass flow rate through dA is proportional to the fluid density ρ normal velocity u, and the flow area dA, and can be expressed as,

Differential mass flow rate*:*

$$\delta\dot{m} \ = \rho.\mathrm{u}.\mathrm{dA} = \rho.\mathrm{u}\ \cos\theta.\mathrm{dA} = \rho(\vec{\mathrm{u}}.\vec{\mathrm{n}})dA \quad \ldots\ldots(4-4)$$

The net flow rate into or out of the control volume through the entire control surface is obtained by integrating $\boldsymbol{\delta m^{\cdot}}$. Over the entire control surface,

Net mass flow rate:

$$\delta\dot{m} \ = \int_{cs}\rho.\mathrm{u}.\mathrm{dA} = \int_{cs}\rho.\mathrm{u}\ \cos\theta.\mathrm{dA} = \int_{cs}\rho(\vec{u}.\vec{n})dA \quad \ldots..(4-5)$$

The <u>conservation of mass</u> relation for a fixed control volume can then be expressed as

$$\boldsymbol{\frac{d}{dt}\int_{cv}\rho dV = \int_{cs}\rho(\vec{u}.\vec{n})dA = 0 \quad \ldots..(4-6)}$$

The general conservation of mass relation can also be expressed as

$$\sum_{\mathrm{in}}\int_{\mathrm{cs}}\rho.\mathrm{u}.\mathrm{dA} - \sum_{\mathrm{out}}\int_{\mathrm{cs}}\rho.\mathrm{u}.\mathrm{dA} = \frac{\mathrm{d}}{\mathrm{dt}}\int_{\mathrm{cv}}\rho\mathrm{dV} \quad \ldots\ldots(4-7)$$

Or in mass flow rate

$$\sum_{in}\dot{m}-\sum_{out}\dot{m}=\frac{d}{dt}\int_{cv}\rho dV \qquad \dots\dots(4-8)$$

Example (4-1)

A 4ft high, 3ft diameter cylindrical water tank whose top is open to the atmosphere is initially filled with water. Now the discharge plug near the bottom of the tank is pulled out, and water jet whose diameter is 0.5 in streams out. The average velocity of the jet is given by $u=\sqrt{2gh}$, where h is the height of water in the tank measured from the center of the hole (a variable) and g is the gravitational acceleration. Determine how long it will take for the water level in the tank to drop to 2 ft from the bottom.

Solution:

$$\dot{m}_{in}-\dot{m}_{out}=\frac{dm_{cv}}{dt}$$

$\dot{m}_{in}=$ o

$\dot{m}_{out}=\rho uA=\rho\sqrt{2gh}\,A_{jet}$

$m_{cv}=\rho V=\rho\, h\, A_{tank}$

$$0-\rho\sqrt{2gh}\,.\frac{\pi D_{jet}^2}{4}=\frac{\rho\,.\,dh.\frac{\pi D_{tank}^2}{4}}{dt}$$

$$-\sqrt{2gh}\,.\,D_{jet}^2=\frac{dhD_{tank}^2}{dt}$$

$$-\sqrt{2gh}\,.\,D_{jet}^2=\frac{dhD_{tank}^2}{dt}$$

$$dt=-\frac{D_{tank}^2}{D_{jet}^2}\frac{dh}{\sqrt{2gh}}$$

$$\int_0^t \mathrm{dt} = -\frac{\mathrm{D}_{\mathrm{tank}}^2}{\mathrm{D}_{\mathrm{jet}}^2\sqrt{2g}}\int_{h_0}^{h_2}\frac{\mathrm{dh}}{\sqrt{\mathrm{h}}}$$

$$\mathrm{t} = \frac{\sqrt{h_0}-\sqrt{h_2}}{\sqrt{\frac{g}{2}}}(\frac{D_{tank}}{D_{jet}})^2 = \frac{\sqrt{4ft}-\sqrt{2ft}}{\sqrt{\frac{32.2ft^2/s}{2}}}(\frac{3*12\ in}{0.5in})^2 = 757s = 12.6\ min$$

4-2-Bernoulli equation

The **Bernoulli equation** is an approximate relation between pressure, velocity, and elevation, and is valid in regions of steady, incompressible low where net frictional forces are negligible (4-2).

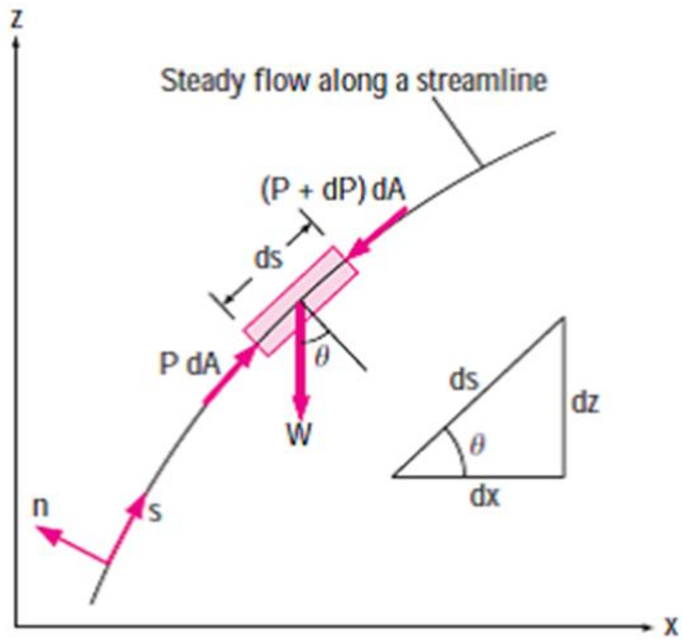


Figure (4-2) the force acting on a fluid particle along a streamline.

Applying Newton's second law in the *s*-direction on a particle moving along a streamline gives

$$\sum F_s = m.a_s \quad \ldots..(4-9)$$

Two component of acceleration

$$a_s = \frac{du}{dt} = \frac{\delta u}{\delta s}\frac{ds}{dt} = u\frac{du}{ds} \qquad \dots\dots(4-10)$$

$$PdA - (P + dp)dA - Wsin\theta = m.u.\frac{du}{ds} \quad \dots\dots\dots\dots(4-11)$$

$$let\ m = \rho V = \rho dAds\ \ and\ \ W = mg = \rho gdAds\ \ and\ \ sin\theta = \frac{dz}{ds} \qquad (4-12)$$

Substituting (4-12) in equation (4-11) yield

$$-dpdA - \rho g.dA.ds.\frac{dz}{ds} = \rho g.dA.ds.u.\frac{du}{ds} \qquad \dots\dots\dots\ (4-13)$$

Canceling dA from each term and simplifying,

$$-dp - \rho gdz = \rho gudu \qquad \dots\dots(4-14)$$

Noting $udu = \frac{1}{2}d(u^2)$ that and dividing each term by ρ gives

$$\frac{dp}{\rho} + \frac{du^2}{2} + gdz = 0\ \ \dots\dots\dots\ (4-15) \qquad Euler's\ equation\ of\ motion$$

By integration

$$\int \frac{p}{\rho} + \frac{u^2}{2} + gz = constant \qquad \dots\dots\dots(4-16)$$

For incompressible

$$\frac{p}{\rho} + \frac{u^2}{2} + gz = constant \qquad \dots\dots(4-17)$$

This is the famous **Bernoulli equation,** which is commonly used in fluid mechanics for steady, incompressible flow.

$$\frac{p}{\rho} = flow\ energy$$

$$\frac{u^2}{2} = kinetic\ energy$$

$$gz = potential\ energy$$

The Bernoulli equation can also be written between any two points on the same streamline as

$$\frac{\mathrm{p}_1}{\rho} + \frac{u_1^2}{2} + z_1.g = \frac{p_2}{\rho} + \frac{u_2^2}{2} + z_2.g \qquad \dots\dots(4-18)$$

Static, Dynamic, and Stagnation Pressures

The kinetic and potential energies of the fluid can be converted to flow energy (and vice versa) during flow, causing the pressure to change. This phenomenon can be made more visible by multiplying the Bernoulli equation by the density ρ

$$\mathrm{P} + \rho\frac{u^2}{2} + \rho gz = constant$$

Each term in this equation has pressure units, and thus each term represents some kind of pressure:

- *P* is the **static pressure;** it is represents the actual thermodynamic pressure of the fluid.
- *$\rho u^2 2$ is the **dynamic pressure;** it represents the pressure rise when the fluid in motion is brought to a stop isentropically*
- *ρgz is the **hydrostatic pressure,** which is not pressure in a real sense since its value depends on the reference level selected; it accounts for the elevation effects, i.e., of fluid weight on pressure*

Limitations on the Use of the Bernoulli Equation

- **Steady flow:** it should not be used during the transient start-up and shut-down periods,

- **Frictionless flow:** (valve and sharp entrance are disturbs the streamlined structure of flow)
- **No shaft work:** pump, turbine, fan, or any other machine or impeller since such devices destroy the streamlines
- **No heat transfer**
- **Flow along a streamline:** no irrotational region of the flow

Hydraulic Grade Line (HGL) and Energy Grade Line (EGL)

This is done by dividing each term of the Bernoulli equation by *g to give*

$$\frac{p}{\rho g}+\frac{u^2}{2g}+z=H=constant \qquad (4-21)$$

Where

- $P/\rho g$ is the **pressure head;** it represents the height of a fluid column that produces the static pressure P.
- $u^2/2g$ is the **velocity head**; it represents the elevation needed for a fluid to reach the velocity u during frictionless free fall.
- z is the **elevation head**; it represents the potential energy of the fluid
- Also, H is the **total head** for the flow.

$$\begin{aligned}\textbf{Hydraulic Grade Line (HGL)} &= \frac{p}{\rho g}+z \\ \textbf{Energy Grade Line (EGL)} &= \frac{p}{\rho g}+\frac{u^2}{2g}+z\end{aligned} \qquad (4\text{-}22)$$

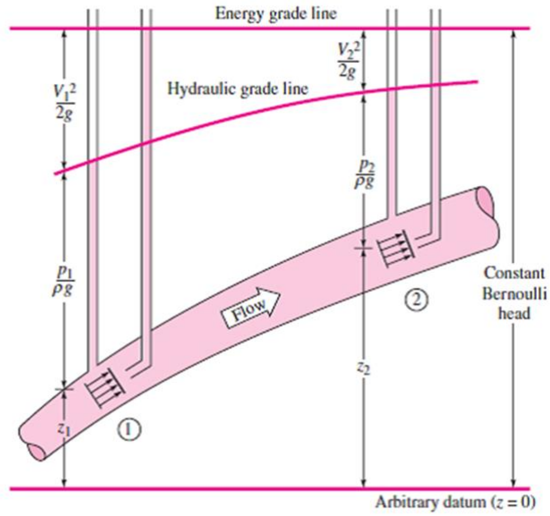


Figure (4-4): HGL and EGL for frictionless flow in a duct

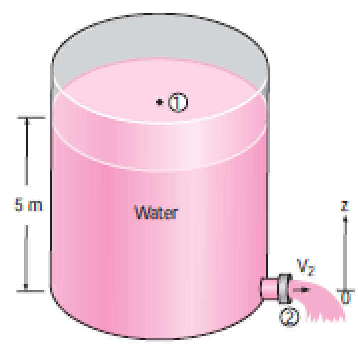
Example (4-2)

A large tank open to the atmosphere is filled with water to a height of 5 m from the outlet tap. A tap near the bottom of the tank is now opened, and water flows out from the smooth and rounded outlet. Determine the water velocity at the outlet.

Solution:

$P_1 = P_{atm}$ (open to the atmosphere), $u_1 = 0$ (the tank is large relative to the outlet) and z_2=0 Also, $P_2 = P_{atm}$ (water discharges into the atmosphere).

$$\cancel{\frac{p_1}{\rho}} + \cancel{\frac{u_1^2}{2}} + z_1 . g = \cancel{\frac{p_2}{\rho}} + \frac{u_2^2}{2} + \cancel{z_2 . g}$$



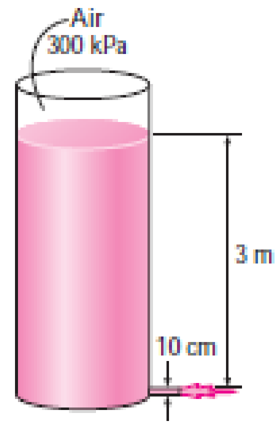
Then the Bernoulli equation simplifies to

$$z_1 . g = \frac{u_2^2}{2} \qquad \rightarrow u = \sqrt{2gz} = \sqrt{2 * 9.81 \frac{m^2}{s} * 5m} = \frac{9.9m}{s}$$

$$\textit{the relation } \boldsymbol{u} = \sqrt{\boldsymbol{2gz}} \quad \textit{is called the toricelli equation}$$

Example (4-3)

A pressurized tank of water has a 10cmdiameter orifice at the bottom, where water discharges to the atmosphere. The water level is 3 m above the outlet. The tank air pressure above the water level is 300 kPa (absolute) while the atmospheric pressure is 100 kPa. Neglecting frictional effects, determine the initial discharge rate of water from the tank.



Solution:

$$\frac{p_1}{\rho} + \cancel{\frac{u_1^2}{2}} + z_1 . g = \frac{p_2}{\rho} + \frac{u_2^2}{2} + \cancel{z_2 . g}$$

$$u = \sqrt{2\left(\frac{p_2 - p_1}{\rho} + z.g\right)}$$

$$u = \sqrt{2\left(\frac{(300-100)kpa}{1000\frac{kg}{m^3}}\left(\frac{1000\frac{N}{m^2}}{kpa}\right)\left(\frac{kg.\frac{m}{s^2}}{N}\right) + 9.81\frac{m}{s^2} * 3m\right)} = 21.4\, m/s$$

$$Q = uA = u\frac{\pi D^2}{4} = 21.4 * \frac{\pi (0.1)^2}{4} = 0.168\frac{m^3}{s}$$

4-3-Conservation of Energy

One of the most fundamental laws in nature is the **first law of thermodynamics,** also known as the **conservation of energy principle.**

The conservation of energy principle for any system can be expressed simply as

$$E_{in} - E_{out} = \Delta E$$

Then the conservation of energy for a fixed quantity of mass can be expressed in rate form as (Fig. 4-5)

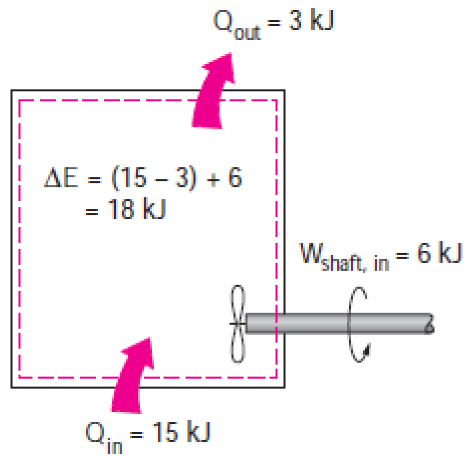


Figure (4-5): A closed system

$$\dot{Q}_{net.in} + \dot{W}_{net.in} = \frac{dE_{sys}}{dt} \quad or \quad \dot{Q}_{net.in} + \dot{W}_{net.in} = \frac{d}{dt}\int_{sys} \rho\, e\, du \qquad \ldots..(4-23)$$

$$where \qquad \dot{Q}_{net.in} = \dot{Q}_{in} - \dot{Q}_{out} \qquad and \qquad \dot{W}_{net.in} = \dot{W}_{in} - \dot{W}_{out}$$

Total energy consists of <u>internal, kinetic and potential energies</u>, and it is expressed on a unit-mass basis as

$$e = U + p_e + k_e = U + \frac{u^2}{2} + zg \qquad \ldots(4-24)$$

<u>Energy Transfer by Heat, *Q*</u>

- For adiabatic process Q=0
- For isothermal process there is change in temperature

<u>Energy Transfer by Work, *W*</u>

A system may involve numerous forms of work, and the total work can be expressed as

$$W_{total} = W_{shaft} + W_{pressure} + W_{viscous} + W_{other}$$

<u>Shaft Work</u>

Many flow systems involve a machine such as a pump, a turbine, a fan, or a compressor whose shaft protrudes through the control surface, and the work transfer associated with all such devices is simply referred to as *shaft work* W_{shaft}

$$W_{shaft} = \omega T_{shaft} = 2\pi\dot{n}T_{shaft} \quad \ldots..(4-25)$$

Where:
ω = angular speed of the shaft in rad/s

n = number of revolutions of the shaft per unit time rev/ min or rpm

T_{shaft} = torque of shaft N.m

Work Done by Pressure Forces

Consider a gas being compressed in the piston-cylinder device shown in Fig. (4-6)

$$\delta W = PAds$$

$$\delta W_{pressure} = PAu_{piston}$$

Where u= ds/dt is the piston velocity
So

$$\delta \dot{W}_{pressure} = -PdAu = -PdA(\vec{u}.\vec{n}) \quad \ldots..(4-26)$$

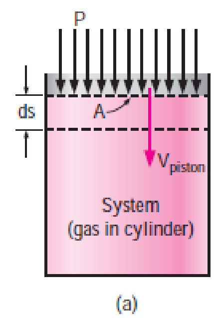


Figure (4-6) piston-cylinder

Work done by pressure forces is <u>positive</u> when it is done *on* the system and <u>negative</u> when it is done *by* the system,

$$\delta \dot{W}_{pressure} = -PdAu = -\int_A PdA(\vec{u}.\vec{n}) = -\int_A \frac{P}{\rho}\rho dA(\vec{u}.\vec{n}) \quad \ldots..(4-27)$$

Then the rate form of the conservation of energy relation for a closed system becomes

$$\dot{Q}_{net.in} + \dot{W}_{net.in} + \dot{W}_{pressure.in} = \frac{dE_{sys}}{dt} \qquad \ldots\ldots(4-28)$$

(Net rate of energy into CV)+(time rate of change of CV)=net flow rate of energy CS

$$\frac{dE_{sys}}{dt} = \frac{d}{dt}\int_{cv} e\rho dV = \int_{cs} \mathrm{e}\rho(\vec{u}.\vec{n})dA \qquad \ldots\ldots(4-29)$$

$$\dot{Q}_{net.in} + \dot{W}_{net.in} + \dot{W}_{pressure.in} = \frac{d}{dt}\int_{\mathrm{cv}} \mathrm{e}\rho\mathrm{dV} + \int_{cs} \mathrm{e}\rho(\vec{u}.\vec{n})dA \quad ..(4-30)$$

Substituting the surface integral for the rate of pressure work from Eq.(4-27) into Eq.(4-30) and combining it with the surface integral on the right give

$$\dot{Q}_{net.in} + \dot{W}_{net.in} = \frac{d}{dt}\int_{\mathrm{cv}} \mathrm{e}\rho\mathrm{dV} + \int_{cs} (\frac{P}{\rho} + \mathrm{e})\rho(\vec{u}.\vec{n})dA \quad ..(4-31)$$

$$noting\ that \quad m^{\cdot} = \int_{cs} \rho(\vec{u}.\vec{n})dA \ \ is\ the\ mass\ flow\ rate\ inlet\ and\ outlet$$

Then equation (4-31) become

$$\dot{Q}_{net.in} + \dot{W}_{net.in} = \frac{d}{dt}\int_{\mathrm{cv}} \mathrm{e}\rho\mathrm{dV} + \sum_{out} \dot{m}\left(\frac{P}{\rho} + \mathrm{e}\right) - \sum_{in} \dot{m}\,(\frac{P}{\rho} + e) \quad ..(4-32)$$

$$\text{where} \quad \mathrm{e} = \mathrm{U} + \frac{\mathrm{u}^2}{2} + \mathrm{zg} \quad \text{then eq}(4-32)\text{becoms}$$

$$\dot{Q}_{net.in} + \dot{W}_{net.in} = \frac{d}{dt}\int_{\mathrm{cv}} \mathrm{e}\rho\mathrm{dV} + \sum_{out} \dot{m}\left(\frac{P}{\rho} + U + \frac{u^2}{2} + zg\right) - \sum_{in} \dot{m}\,(\frac{P}{\rho} + U + \frac{u^2}{2} + zg) \quad ..(4-33)$$

or

$$\dot{Q}_{net.in} + \dot{W}_{net.in} = \frac{d}{dt}\int_{\mathrm{cv}} \mathrm{e}\rho\mathrm{dV} + \sum_{out} \dot{m}\left(h + \frac{u^2}{2} + zg\right) - \sum_{in} \dot{m}\,(h + \frac{u^2}{2} + zg) \quad .(4-34)$$

where $h = \frac{P}{\rho} + U$

The last two equations are fairly general expressions of conservation of energy,

ENERGY ANALYSIS OF STEADY FLOWS

For steady flows, the time rate of change of the energy content of the control volume is zero, and Eq. (4-34) simplifies to

$$\dot{Q}_{net.in} + \dot{W}_{net.in} = \sum_{out} \dot{m}\left(h + \frac{u^2}{2} + zg\right) - \sum_{in} \dot{m}\,(h + \frac{u^2}{2} + zg) \;.(4-34)$$

Many practical problems involve just one inlet and one outlet. The mass flow rate for such single-stream devices remains constant, and Eq. (4-34) reduces to

$$\dot{Q}_{net.in} + \dot{W}_{net.in} = \dot{m}\left(h_2 - h_1 + \frac{u_2^2 - u_1^2}{2} + g(z_2 - z_1)\right) \ldots\ldots\ldots(4-35)$$

Divided by mass flowrate m·

$$q_{net.in} + w_{net.in} = \left(h_2 - h_1 + \frac{u_2^2 - u_1^2}{2} + g(z_2 - z_1)\right) \ldots\ldots\ldots(4-36)$$

where $q_{net.in} = \frac{\dot{Q}_{net.in}}{\dot{m}}$ *is the net heat transfer to the fluid per unit mass*

$w_{net.in} = \frac{\dot{W}_{net.in}(\frac{kj}{s})}{\dot{m}(\frac{kg}{s})}$ *is the net shaft work input to the fluid per unit mass*

Using the definition of enthalpy $h = \frac{P}{\rho} + U$ and rearranging, the steady-flow energy equation can also be expressed as

$$w_{net.in} + \frac{p_1}{\rho} + \frac{u_1^2}{2} + z_1.g = \frac{p_2}{\rho} + \frac{u_2^2}{2} + z_2.g + (U_2 - U_1 - q_{net.in}) \quad \dots.(4-37)$$

thus $(U_2 - U_1 - q_{net.in})$ represent the mechanical energu loss , so

$$w_{net.in} + \frac{p_1}{\rho} + \frac{u_1^2}{2} + z_1.g = \frac{p_2}{\rho} + \frac{u_2^2}{2} + z_2.g + e_{mech.loss} \quad \dots.(4-38)$$

For single-phase fluids (a gas or a liquid), we have

$$U_2 - U_1 = cv\,(T_2 - T_1)$$

Where cv is the constant-volume specific heat.

Noting that $w_{shaft,\ net\ in} = w_{shaft,\ in} - w_{shaft,\ out} = w_{pump} - w_{turbine}$, the mechanical energy balance can be written more explicitly as

$$\frac{p_1}{\rho} + \frac{u_1^2}{2} + z_1.g + w_{pump} = \frac{p_2}{\rho} + \frac{u_2^2}{2} + z_2.g + w_{turbine} + e_{mech.loss} \quad \frac{kj}{kg}\, or\ \frac{m^2}{s^2}.(4-39)$$

Multiplying Eq. (4-39) by the mass flow rate $\dot{m}$ gives

$$\dot{m}(\frac{p_1}{\rho} + \frac{u_1^2}{2} + z_1.g) + \dot{W}_{pump} = \dot{m}(\frac{p_2}{\rho} + \frac{u_2^2}{2} + z_2.g) + \dot{W}_{turbine} + \dot{E}_{mech.loss} \quad \frac{kj}{s} \quad \dots.(4-40)$$

The *total* mechanical power loss which consists of pump, turbine losses and frictional losses in the piping network. That is,

$$\boldsymbol{\dot{E}_{mech.loss} = \dot{E}_{mech.loss.pump} + \dot{E}_{mech.loss.turbine} + \dot{E}_{mech.loss.pipe}}$$

Divided equation (4-39) by g

$$\frac{p_1}{\rho g}+\frac{u_1^2}{2g}+z_1+h_{pump}=\frac{p_2}{\rho g}+\frac{u_2^2}{2g}+z_2+h_{turbine}+h_{mech.loss}\quad (m) \quad ....(4-41)$$

Where

$$h_{pump}=\frac{w_{pump}(\frac{kj}{kg})}{g(\frac{m}{s^2})}=\frac{\dot{W}_{pump}}{\dot{m}g}=\frac{\eta_{pump}\dot{W}_{pump}}{\dot{m}g}$$

Is the *useful head delivered to the fluid by the pump.* Because of irreversible losses in the pump, h_{pump}, is *less* than $\frac{\dot{W}_{pump}}{\dot{m}g}$ by the factor η_{pump}. similarly

$$h_{turbine}=\frac{w_{turbine}}{g}=\frac{\dot{W}_{turbine}}{\dot{m}g}=\frac{\dot{W}_{turbine}}{\eta_{turbine}\,\dot{m}g}$$

Is the *extracted head removed from the fluid by the turbine.* Because of irreversible losses in the turbine, $h_{turbine}$, *e* is greater than $\frac{\dot{W}_{turbine}}{\dot{m}g}$ by the factor $\eta_{turbine}$. Finally

$$h_{mech.loss}=\frac{e_{mech.loss.pipe}}{g}=\frac{\dot{E}_{mech.loss.pipe}}{m\cdot g}$$

Is the irreversible *head loss* between 1 and 2 due to all components of the piping system other than the pump or turbine? Note that the head loss $\boldsymbol{h_L}$ represents the frictional losses associated with fluid flow in piping, and it does not include the losses that occur within the pump or turbine due to the inefficiencies of these devices

Equation (4-41) is illustrated schematically in Fig. (4-7).

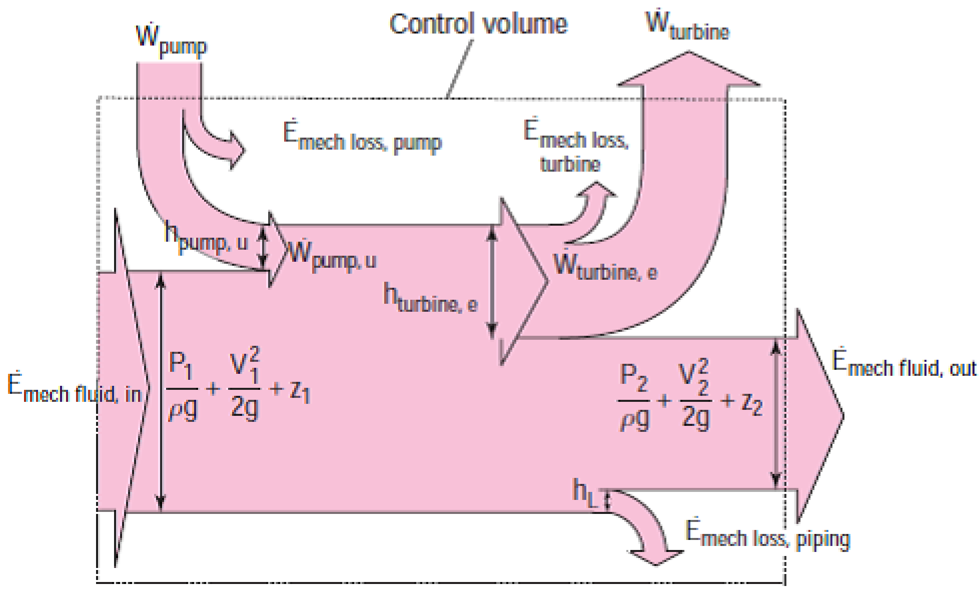
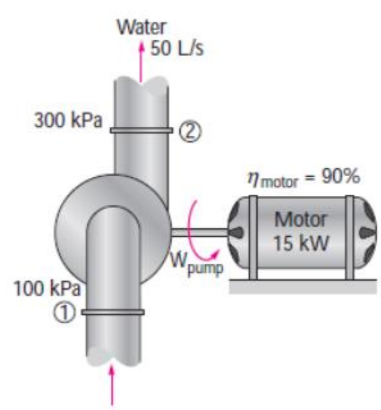


Figure (4-7): Mechanical energy flow chart for a fluid flow system

Example (4-4)

The pump of a water distribution system is powered by a 15kW electric motor whose efficiency is 90 percent (Fig. below). The water flow rate through the pump is 50 L/s. The diameters of the inlet and outlet pipes are the same, and the elevation difference across the pump is negligible. If the pressures at the inlet and outlet of the pump are measured to be 100 kPa and 300 kPa (absolute), respectively, determine (*a*) the mechanical efficiency of the pump and (*b*) the temperature rise of water as it flow through the pump due to the mechanical inefficiency.

SOLUTION:



$$\dot{m} = \rho Q = \frac{1\ kg}{lit} . \frac{50\ lit}{s} = \frac{50\ kg}{s}$$

$$\dot{W}_{pump.shaft} = \eta_{pump} \dot{W}_{electric} = (0.9)(15)$$
$$= 13.5 \text{ kw}$$

$$\dot{m}(\frac{\mathrm{p}_1}{\rho}+\frac{u_1^2}{2}+z_1.g)+\dot{W}_{pump}$$

$$=\dot{m}(\frac{p_2}{\rho}+\frac{u_2^2}{2}+z_2.g)+\dot{W}_{turbine}+\dot{E}_{mech.loss} \quad \frac{kj}{s} \quad \ldots.(4-40)$$

$u_1=u_2=0$ and $z_1=z_2$ so

$$\dot{E}_{mech.loss}=m\cdot\left(\frac{p_2-p_1}{\rho}\right) = \frac{50\ kg}{s}\left(\frac{300-100\ kpa}{1000\frac{kg}{m^3}}\right)\left(\frac{kj}{kpa.m^3}\right)=10kw$$

Then the mechanical efficiency of the pump becomes

$$\eta_{\mathrm{pump}}=\frac{\dot{W}_{pump}}{\dot{\mathrm{W}}_{\mathrm{pump.shaft}}}=\frac{\dot{E}_{mech.fluid}}{\dot{\mathrm{W}}_{\mathrm{pump.shaft}}}=\frac{10}{13.5}=0.741=74.1\%$$

Of the 13.5kW mechanical power supplied by the pump, only 10 kW is imparted to the fluid as mechanical energy. The remaining 3.5 kW is converted to thermal energy due to frictional effects, and this “lost” mechanical energy manifests itself as a heating effect in the fluid,

$$\dot{E}_{mech.loss}=\dot{W}_{pump.shaft}-\triangle\dot{E}_{mech.fluid}$$

$$\dot{E}_{mech.loss}=13.5-10=3.5\ KW$$

The temperature rise of water due to this mechanical inefficiency is determined from the thermal energy balance,

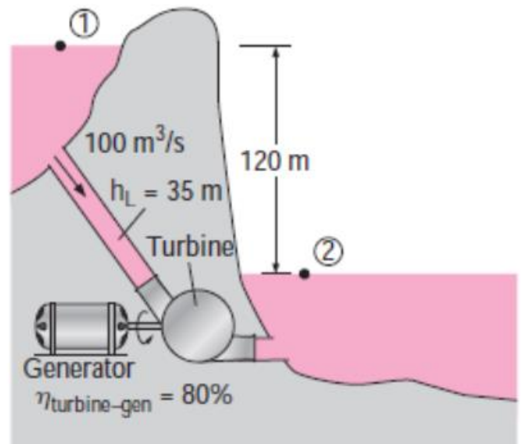
$$\dot{E}_{mech.loss}=U_2-U_1=m\cdot cp\Delta T$$

Solving for $\triangle T$,

$$\dot{E}_{mech.loss}=U_2-U_1=\dot{m}cp\Delta T=\frac{\dot{E}_{mech.loss}}{\dot{m}cp}=\frac{3.5KW}{\left(50\frac{Kg}{s}\right)\left(4.18\frac{kj}{kg.c}\right)}=0.017\ C$$

Example (4-5)

In a hydroelectric power plant, 100 m3/s of water flow from an elevation of 120 m to a turbine, where electric power is generated (Fig. below). The total irreversible head loss in the piping system from point 1 to point 2 (excluding the turbine unit) is determined to be 35 m. If the overall efficiency of the turbine–generator is 80 percent, estimate the electric power output



Solution:

$$\dot{m} = \rho Q = \frac{1000\ kg}{m^3} . \frac{100\ m^3}{s} = \frac{100000\ kg}{s}$$

$P_1 = P_2 =$atm

$u_1 = u_2 = 0$

Equation (4-41) reduce to

$$h_{turbine} = z - h_{mech.loss} = 120 - 35 = 85m$$

$$\dot{W}_{turbine} = \dot{m} g h_{turbine} = \frac{100000\ kg}{s} * 9.81 \frac{m}{s^2} * 85m \left(\frac{\frac{kj}{kg}}{1000 \frac{m^2}{s^2}} \right) = 83400kw$$

The electric power generated by the actual unit

$$\dot{W}_{electric} = \eta_{turbine} \dot{W}_{turbine} = 83.4\text{MW} * 0.8 = 667\text{MW}$$

4-4-Conservation of Momentum Principle

Newton's second law for a system of mass *m* subjected to a net force *F* is expressed as

$$\sum \vec{F} = m\,\vec{a} = m \frac{d\vec{u}}{dt} = \frac{d}{dt}(m\,\vec{u}) \quad \ldots\ldots (4-42)$$

Where $m\,\vec{u}$ is the **linear momentum** of the system, Newton's second law can be expressed more generally as

$$\sum \vec{F} = \frac{d}{dt}\int_{sys} \rho u \, dV \qquad \dots\dots(4-43)$$

where $\delta m = \rho dV$ is the mass of a differential volume element dV and $\rho u \, dV$ is its momentum.

(Sum of force acting CV)=(Rate of change of momentum of CV)+(net flow rate at CS)

$$\frac{d(m\,u^{\rightarrow})_{sys}}{dt} = \frac{d}{dt}\int_{cv} \rho\vec{u} dV = \int_{cs} \rho\vec{u}(\vec{u}.\vec{n}) dA \qquad \dots..(4-44)$$

$$General \qquad \sum \vec{F} = \frac{d}{dt}\int_{cv} \rho\vec{u} dV = \int_{cs} \rho\vec{u}(\vec{u}.\vec{n}) dA \qquad \dots..(4-45)$$

During *steady flow*, the amount of momentum within the control volume remains constant (the second term of Eq. 4-45) is zero. It gives

$$Steady\ flow \qquad \sum \vec{F} = \int_{cs} \rho\vec{u}(\vec{u}.\vec{n}) dA \qquad \dots..(4-46)$$

Mass flow rate across an inlet or outlet

$$\dot{m} = \int_{Ac} \rho(\vec{u}.\vec{n}) dA_c = \rho u dA_c \qquad \dots..(4-47)$$

Momentum flow rate across a uniform inlet or outlet:

$$= \int_{Ac} \rho\vec{u}(\vec{u}.\vec{n}) dA_c = \rho u A_c\,\vec{u} = \dot{m}\,\vec{u} \ \dots..(4-48)$$

so

$$\sum \vec{F} = \sum_{out} \dot{m}\,\vec{u} - \sum_{in} \dot{m}\,\vec{u} \qquad \dots..(4-49)$$

Many practical problems involve just one inlet and one outlet, and Eq. (4-49) reduces to

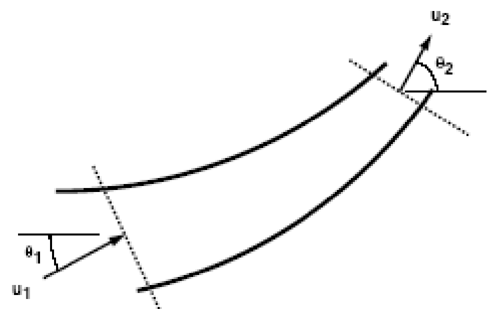
$$\sum \vec{F} = m^{\cdot}(u_2^{\rightarrow} - u_1^{\rightarrow}) \qquad \dots..(4-50)$$

Momentum Equation for Two and three dimensional flow along a streamlin

Consider the two dimensional system shown, since both momentum and force are vector quantities, they can be resolving into components in the x and y directions

$$F_x = \dot{m}(u_2\, cos\theta - u_1 cos\theta)$$

$$F_y = \dot{m}(u_2\, sin\theta - u_1 sin\theta)$$

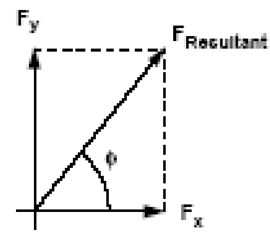


These components can be combined to give the resultant force

$$F = \sqrt{F_y + F_x}$$

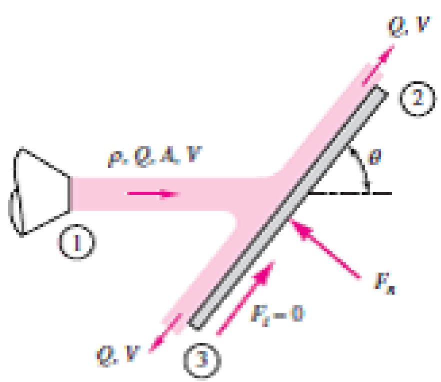
And the angle of this force

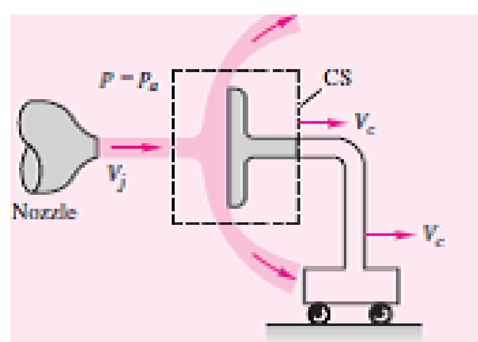
$$\theta = \frac{F_y}{F_x}$$



Force exerted by a jet striking flat plate

Consider a jet striking a flat plate that may be *perpendicular* or *inclined* to the direction of the jet.





The general term of the jet velocity component normal to the plate can be written as:

$$u_n = \left(\vec{u_j} - \vec{u_c}\right) = (u_j - u_c)cos\theta$$

The mass flow entering the control volume

$$\dot{m} = \rho A(u_j - u_c)$$

If the plate is stationary:

$$\dot{m} = \rho A(u_j)$$

Thus the rate of change of momentum normal to the plate

$$\dot{m}\, u_n = \rho A(u_j - u_c)\,(u_j - u_c)\cos\theta$$

Force exerted normal to the plate = The rate of change of momentum normal to the plate**:**

$$F = \rho A(u_j - u_c)\,(u_j - u_c)\cos\theta$$

- if the plate is stationary and inclined

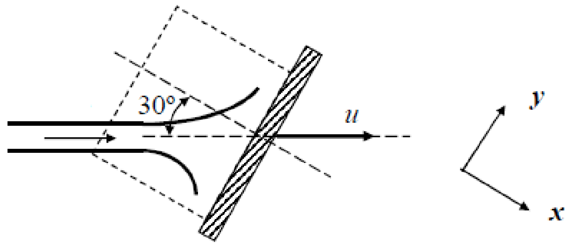
$$F = \rho A(u_j^2)\cos\theta$$

- if the plate is both stationary and perpendicular

$$F = \rho A(u_j^2)$$

<u>Example (4-6)</u>

A jet of water from a fixed nozzle has a diameter d of 25mm and strikes a flat plate at angle θ of 30^o to the normal to the plate. The velocity of the jet is 5m/s, and the surface of the plate can be assumed to be frictionless. Calculate the force exerted normal to the plate (a) if the plate is stationary, (b) if the plate is moving with velocity u of 2m/s in the same direction as the jet.



Solution:

a) Force exerted normal to the plate = The rate of change of momentum normal to the plate**:**

$$F = \rho A(u_j - u_c)\ (u_j - u_c)cos\theta$$

- if the plate is stationary and inclined

$$F = \rho A(u_j{}^2)cos\theta = 1000\frac{kg}{m^3}\left(\frac{\pi(0.025)^2}{4}\ m^2\right)(5^2)cos30 = 10.36N$$

- if the plate is moving with velocity 2m/s

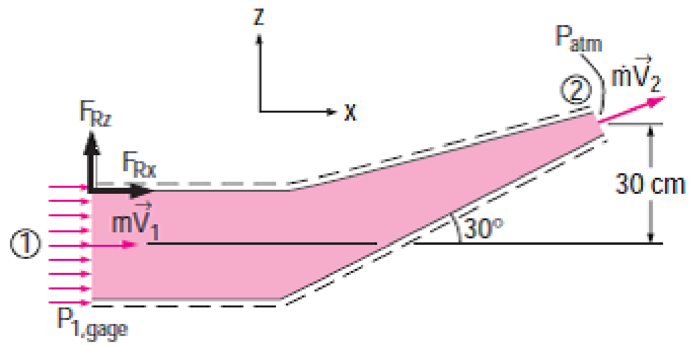
$$F = \rho A\ (u_j - u_c)^2 cos\theta = 1000\frac{kg}{m^3}\left(\frac{\pi(0.025)^2}{4}\ m^2\right)(5-2)^2 cos30 = 3.83N$$

Example (4.7)

A reducing elbow is used to deflect water flow at a rate of 14 kg/s in a horizontal pipe upward 30° while accelerating it (Fig. below). The elbow discharges water into the atmosphere. The cross-sectional area of the elbow is 113 cm^2 at the inlet and 7 cm^2 at the outlet. The elevation difference between the centers of the outlet and the inlet is 30 cm. The weight of the elbow and the water in it is considered to be negligible. Determine (*a*) the gage pressure at the center of the inlet of the elbow and (*b*) the anchoring force needed to hold the elbow in place.

$$u_1 = \frac{\dot{m}}{\rho A_1} =$$

$$\frac{14\ kg/s}{1000\frac{kg}{m^3}(0.0113m^2\ )} = 1.24\ m/s$$



$$u_2 = \frac{\dot{m}}{\rho A_2} =$$

$$\frac{14\ kg/s}{1000\frac{kg}{m^3}(0.0007m^2\ )} = 20\ m/s$$

level ($z_1 = 0$) and noting that $P_2 = P_{atm}$, the Bernoulli equation for a streamline going through the center of the elbow is expressed as

$$\frac{p_1}{\rho g} + \frac{u_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{u_2^2}{2g} + z_2$$

$$p_1 - p_2 = \rho g(\frac{u_2^2 - u_1^2}{2g} + z_2 - z_2)$$

$$p_{1\,gage} = 1000\frac{kg}{m^3}9.81\frac{m}{s^2}(\frac{(20)^2 - (1.24)^2}{2 * 9.81\frac{m}{s^2}} + 0.3m - 0)(\frac{kN}{1000kg.\frac{m}{s^2}})$$

$$p_{1\,gage} = 202.2\frac{kN}{m^2}$$

(*b*) The momentum equation for steady one-dimensional flow is

$$\sum \vec{F} = \dot{m}(\vec{u_2} - \vec{u_1})$$

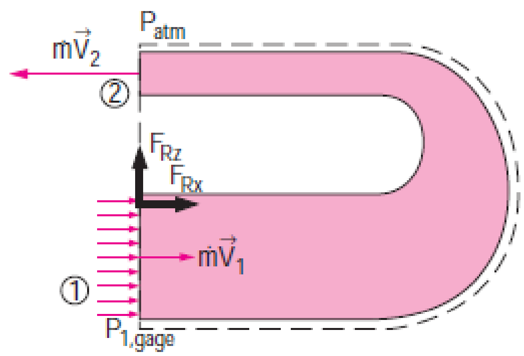
$$F_{Rx} + p_1 A_1 - p_2 cos\theta A_2 = \dot{m}(u_2\,cos\theta - u_1)$$

$$F_{Rz} - p_2 sin\theta A_2 = \dot{m}(u_2\,sin\theta)$$

$$F_{Rx} = \left(\frac{14kg}{s}\left(20cos30 - 1.24\frac{m}{s}\right) - (202.2kpa * 0.0113m)\right)\left(\frac{kN}{kg.\frac{m}{s^2}}\right)$$
$$= -2053N$$

$$F_{Rz} = (\frac{14kg}{s}\left(\frac{20sin30m}{s}\right) - 0)\ (\frac{kN}{kg.\frac{m}{s^2}}) \ = \ 144N$$

If we repeated example above for the figure below



Noting that the outlet velocity is negative since it is in the negative *x*-direction, we have

$$F_{Rx} + p_1A_1 - p_2A_2 = \dot{m}(-u_2 \ - u_1)$$
$$F_{Rx} = -\dot{m}(u_2 \ + u_1) - p_1A_1$$
$$F_{Rx} = -(\frac{14kg}{s}\left(20 + 1.24\frac{m}{s}\right) - (202.2kpa * 0.0113m))(\frac{kN}{kg.\frac{m}{s^2}})$$

$$\boldsymbol{F_{Rx} = -306 - 2285 = -2591N}$$