

Circuit - Elements

We may classify circuit elements into two categories:-

*Passive elements: If it contains no source of e.m.f, and if the total energy delivered to it from the rest of the circuit is always non-negative (Resistors, Capacitors, and inductors).

*Active elements: If it contains a source of e.m.f, for example Generator, batteries, and electronic devices which required power supplies.

Voltage and Current Sources

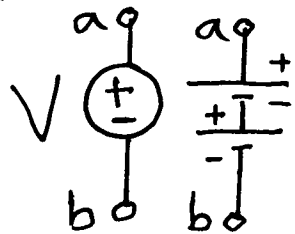
Independent and dependent Voltage and current Sources.

① Independent Sources:

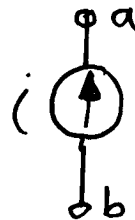
A) The independent Voltage source: Such as a battery or generator that maintain a voltage between its terminals. (a) is at a higher potential than (b).

The voltage (V) may be time varying, and may be constant. The independent voltage source is completely independent of the current through the element.

(B) The independent current source ⁽²⁾: is a two terminal element through which a specified current flows. The current is completely independent of the voltage across the element. (i) is specified current. The direction of current indicated by the arrows.



Voltage Source

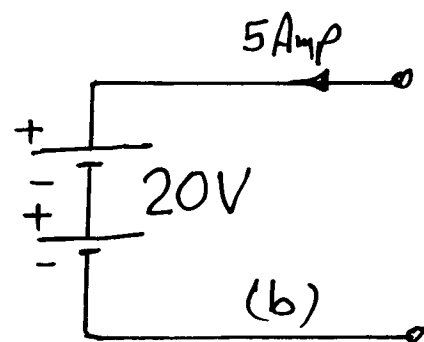
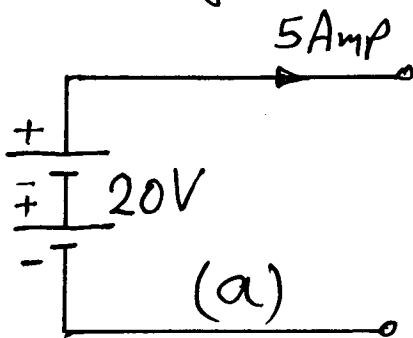


current Source.

* Both independent voltage & current sources are usually meant to deliver power to the external circuit and not to absorb it.

i.e.
$$P = V \cdot I$$

(V) is the voltage across the source, (i) is the current of the source directed out of the +ve terminal, then the source is delivering power. Otherwise it is absorbing power, for example:-



Fig(a), the battery is delivering (100W) to the external circuit, Fig(b) the battery is absorbing (100W) as would be the case when it is being charged.

② Dependent Sources

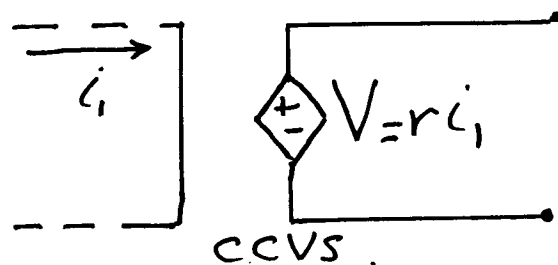
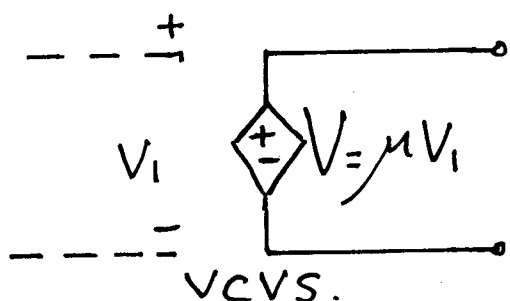
③

Dependent sources whose voltage or current is controlled by another voltage or current somewhere in the circuit.

A) Dependent or Controlled Voltage Source, is one whose terminal voltage depends on or is controlled by a voltage or current.

VCVS: Voltage - controlled voltage source.

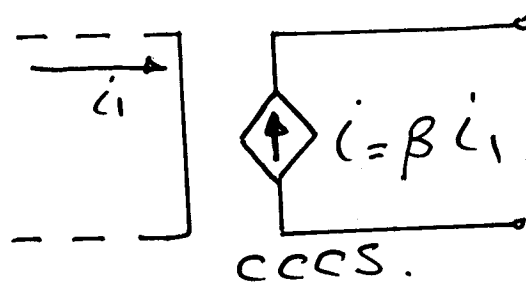
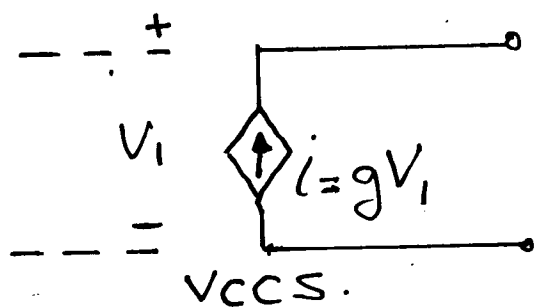
CCVS: current - controlled voltage source. is controlled by a current.



B) Dependent or Controlled Current Source.

VCCS: is controlled by a voltage.

CCCS: is controlled by a current.



The quantities (μ & β) are dimensionless constants commonly referred to as the voltage and current gain.

The constants (r & g) have units of ohms and mhos. Dependent sources are essential components in (Amplifier) circuit.

Resistance (R)

(4)

The unit of resistance is the ohm (Ω), the value of resistance can be determined by the factors:-

- 1- Material
- 2- Cross-Sectional area.
- 3- Length and 4- Temperature.

$$\therefore R \propto \frac{l}{A}$$

Where:

l : The length of conductor in (m).

A : Cross-Sectional area in (m^2).

$$\therefore \boxed{R = \rho \frac{l}{A}}$$

ρ : is the resistivity in ($\Omega \cdot m$).

« Ohm's Law »

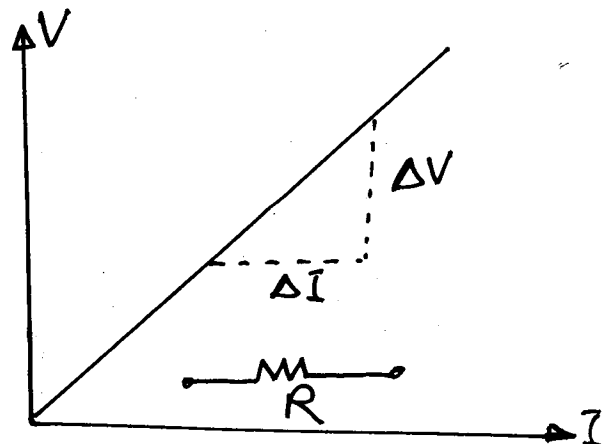
Ohm's Law defined as « The voltage across a resistor is directly proportional to the current flowing through the resistor ».

$$\therefore V \propto I \quad \text{or} \quad \boxed{V = I \cdot R} \quad R \geq 0 \text{ in } \Omega.$$

The function $V = f(I)$, the Voltage-current char- is a straight line passing through the origin.

The slope will be the resistance (R).

$$\text{Slope} = R = \frac{\Delta V}{\Delta I} \text{ ---- ohm.}$$



Electric Power (P)

(5)

Electric power is defined as the time rate at which work is being done or energy is being expended. Since power is the rate of doing work and a rate is any physical quantity divided by time, we may define the average power (P) as:

$$P = \frac{W}{t} = \frac{\text{total work done (J)}}{\text{total time taken (s)}} = \frac{J}{s}$$

$$1 \text{ J/s} = 1 \text{ watt} = 1 \text{ N.m/s}$$

$$\text{But } V = \frac{W}{Q} \quad \text{and} \quad I = \frac{Q}{t}$$

$$\therefore P = \frac{W}{t} = \frac{V \cdot Q}{t} = V \cdot I \quad (\text{Watt}).$$

$$\text{But } V = I \cdot R \quad (\text{ohm's Law}).$$

$$\therefore \boxed{P = V \cdot I = I^2 \cdot R = \frac{V^2}{R}} \quad \text{Watt}.$$

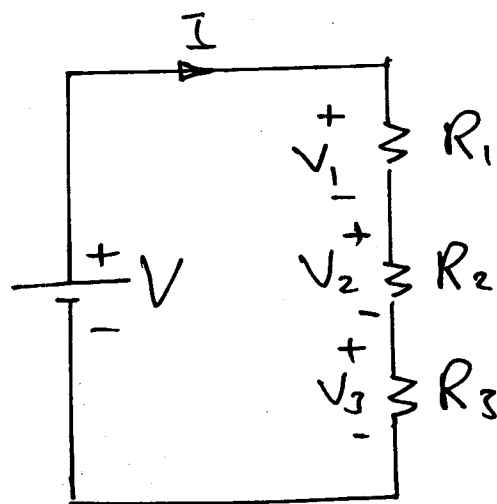
((Resistance Connections))

① Series Connection: If the resistance connected as shown, they are said to be in Series.

$$\begin{aligned} V &= V_1 + V_2 + V_3 \\ &= I \cdot R_1 + I \cdot R_2 + I \cdot R_3 \\ &= I (R_1 + R_2 + R_3). \end{aligned}$$

$$\text{But } V = I \cdot R_{eq}. \quad (\text{ohm's Law}).$$

$$\therefore \boxed{R_{eq} = R_1 + R_2 + R_3}$$



② Parallel connection:

resistances connected as shown, they are said to be in parallel

$$I = I_1 + I_2.$$

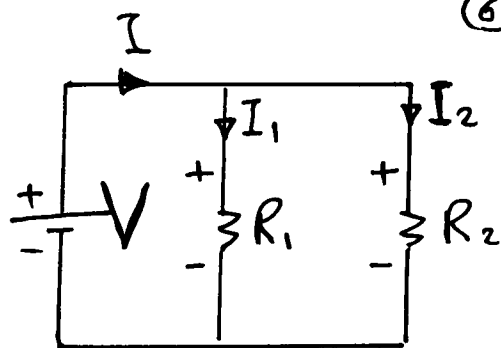
$$\therefore I = \frac{V}{R_1} + \frac{V}{R_2} = V \left(\frac{1}{R_1} + \frac{1}{R_2} \right).$$

But $I = \frac{V}{R_{eq}}$ (ohm's Law).

$$\therefore \boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}}$$

OR

$$\boxed{R_{eq} = \frac{R_1 R_2}{R_1 + R_2}}$$



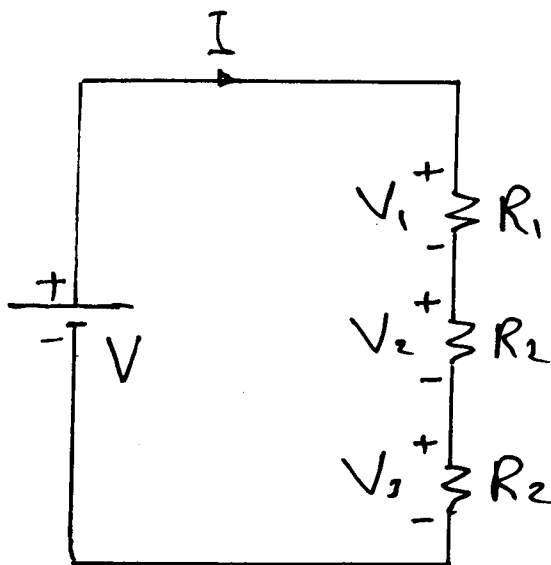
** Voltage Divider Rule

We have $I = \frac{V}{R_{eq}}$

$$= \frac{V}{R_1 + R_2 + R_3}.$$

But $V_1 = I_1 R_1 = I \cdot R_1$

$$\therefore V_1 = V \frac{R_1}{R_1 + R_2 + R_3}.$$



and $V_2 = V \frac{R_2}{R_1 + R_2 + R_3}.$

In General $V_x = V \frac{R_x}{R_1 + R_2 + R_3 + \dots + R_x}.$

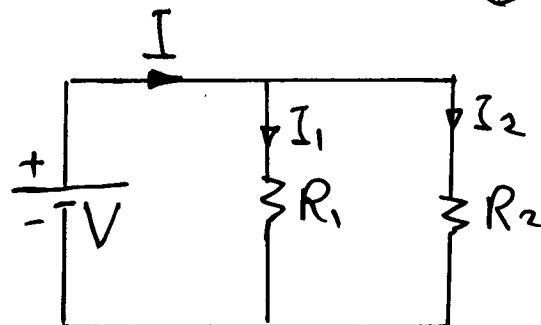
** Current Divider Rule

(7)

We have $I_1 = \frac{V}{R_1}$, $I_2 = \frac{V}{R_2}$.

and $V = I \cdot R_{eq}$

$$= I \frac{R_1 R_2}{R_1 + R_2}$$



$$\therefore I_1 = I \frac{R_2}{R_1 + R_2} \quad \text{or} \quad I_2 = I \frac{R_1}{R_1 + R_2}$$

Home work: Prove that $I_1 = I \frac{R_2 R_3}{R_1 R_2 + R_2 R_3 + R_1 R_3}$
For R_1, R_2, R_3 connected in parallel.

Ex: Find (V) & (I) for the circuit.

$$V_{ab} = 10 \times 5 = 50 \text{ Volt.}$$

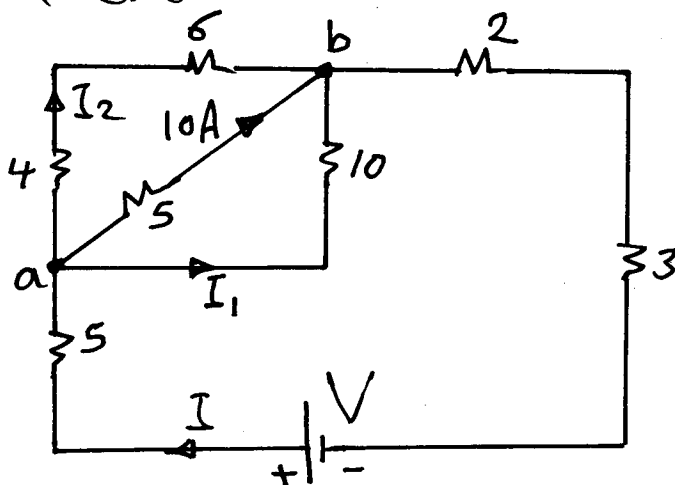
$$\therefore I_1 = 50/10 = 5 \text{ Amp.}$$

$$I_2 = 50/4+6 = 5 \text{ Amp.}$$

$$\therefore I = I_1 + I_2 + I_3 = 10 + 5 + 5 = 20 \text{ Amp.}$$

$$R_{eq} = 5 + (10 \parallel 10 \parallel 5) + 2 + 3 = 12.5 \Omega$$

$$\therefore V = I \cdot R_{eq} = 20 \times 12.5 = 250 \text{ Volt}$$

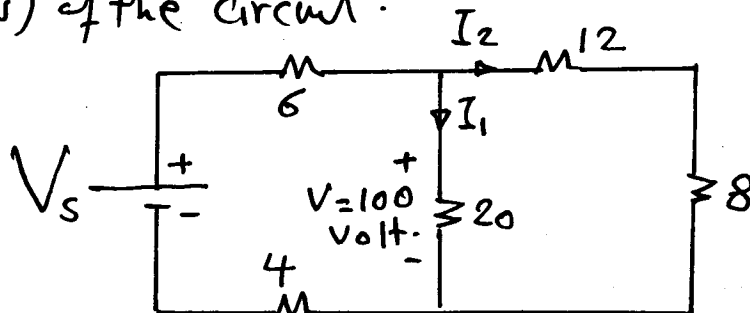


Ex: Find the value of (Vs) of the circuit.

$$I_1 = \frac{100}{20} = 5 \text{ Amp.}$$

$$I_2 = \frac{100}{12+8} = 5 \text{ Amp.}$$

$$I_{Total} = 5 + 5 = 10 \text{ Amp.}$$

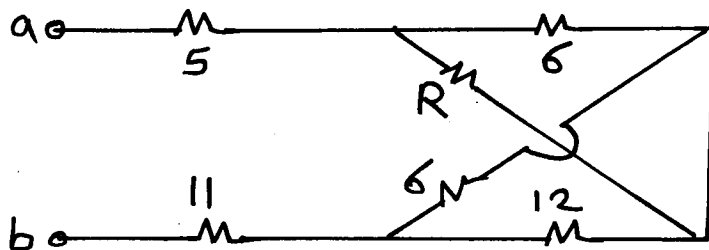


$$R_{eq} = 6 + 4 + (20 \parallel 20) = 20 \Omega.$$

$$\therefore V_s = I \cdot R_{eq} = \underline{200 \text{ Volt.}}$$

Ex: Find the value of (R) , if the equivalent resistance between a & b is $(R_{ab} = 22 \Omega)$.

ans: $R = 3 \Omega.$



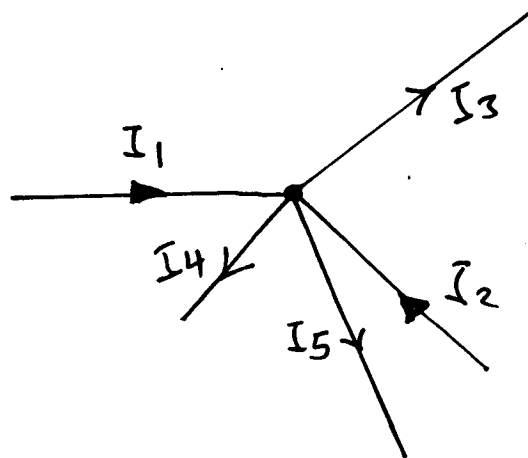
« Kirchhoff's Law »

① Kirchhoff's Current Law (K.C.L).

It states as « The algebraic sum of the incoming currents and Outgoing currents at a node (Point) is Zero ».

$$\sum I_{\text{in coming}} = \sum I_{\text{out going.}}$$

$$I_1 + I_2 = I_3 + I_4 + I_5.$$



② Kirchhoff's Voltage Law (K.V.L).

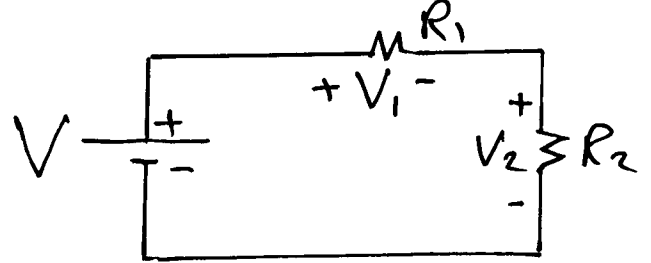
⑨

Kirchhoff's Voltage Law states as ((The algebraic sum of the potential rises and drops around a closed loop (or circuit) is Zero)).

$$\sum V = 0.$$

OR

$$V = V_1 + V_2.$$

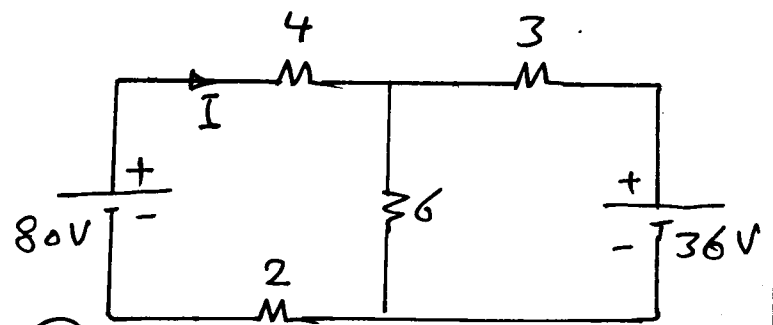


Ex: Find the current (I) for the circuit shown.

For the circuit (abcd).

$$80 = (4 + 2)I + 6I_1$$

$$80 = 6I + 6I_1 \text{ ----- (1)}$$

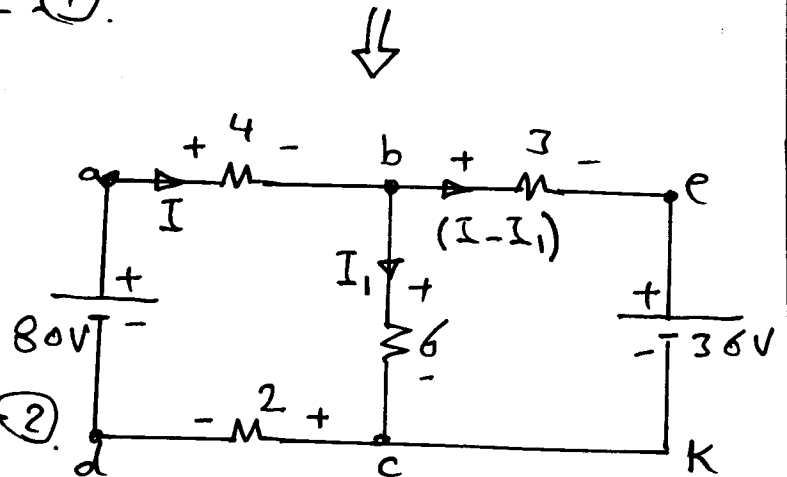


For the circuit (bekc).

$$6I_1 = 36 + 3(I - I_1)$$

$$6I_1 = 36 + 3I - 3I_1$$

$$9I_1 = 36 + 3I \text{ ----- (2)}$$



From eq(2) $I_1 = \frac{12 + I}{3}$

put I_1 in eq(1).

$$80 = 6I + 6\left(\frac{12 + I}{3}\right)$$

$\therefore I = 7 \text{ Amp}$

Ex: Find (I) and power consumed in (8Ω), and power supply by ($320V$)? (10)

by K.V.L the circuit (abcd)

$$320 = (10+6)I + (8+8)(I+10).$$

$$320 = 16I + 16I + 160$$

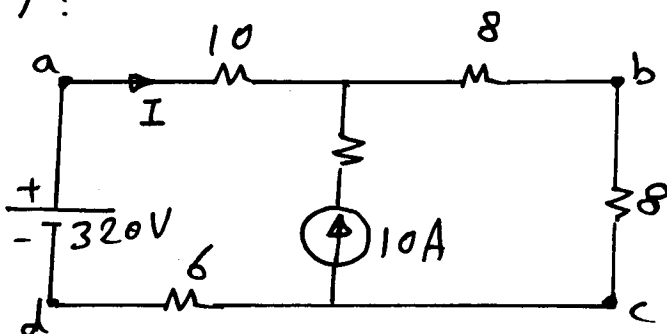
$$160 = 32I.$$

$$\therefore I = 5 \text{ Amp}$$

$$\therefore I \text{ in } 8\Omega = 10 + 5 = 15 \text{ Amp}.$$

$$\therefore P_{in}(8\Omega) = (15)^2 \times 8 = 1800 \text{ Watt}.$$

$$\text{Power Supply by } 320V = V \cdot I = 320 \times 5 = 1600 \text{ Watt}.$$



Ex: Find the value of (I_x)?

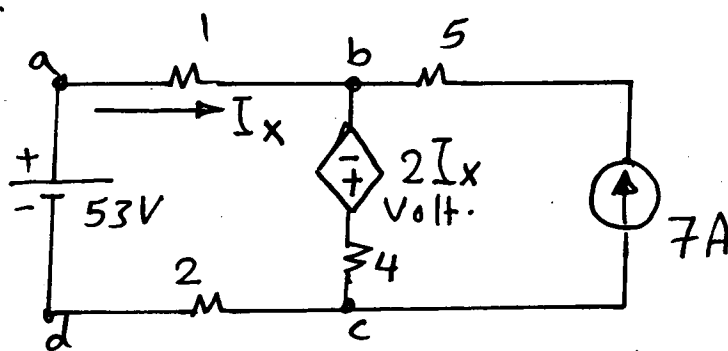
For the circuit of (abcd):

$$53 + 2I_x = 3I_x + 4(I_x + 7).$$

$$53 + 2I_x = 3I_x + 4I_x + 28.$$

$$25 = 5I_x.$$

$$\therefore I_x = 5 \text{ Amp}$$



Home work: Find the power supply by (7 Amp).

ans; $P_{in 7 \text{ amp}} = 511 \text{ Watt}.$

$$P_{1+2} = 5^2 \times 3 = 75 \text{ Watt}.$$

$$P_{4\Omega} = (12)^2 \times 4 = 576 \text{ Watt}.$$

$$P_{5\Omega} = 7^2 \times 5 = 245 \text{ Watt}.$$

$$P_{53V} = V \cdot I = 53 \times 5 = 265 \text{ Watt}.$$

$$P_{in 2I_x} = 10 \times 12 = 120 \text{ Watt}.$$

« Mesh Current (Loop Current) method »

⑪

Ex: Find the current (I) for the circuit shown.

* $I = I_1$

loop(1):

$$80 = (4 + 6 + 2)I_1 - 6I_2$$

$$80 = 12I_1 - 6I_2 \quad \text{--- (1)}$$

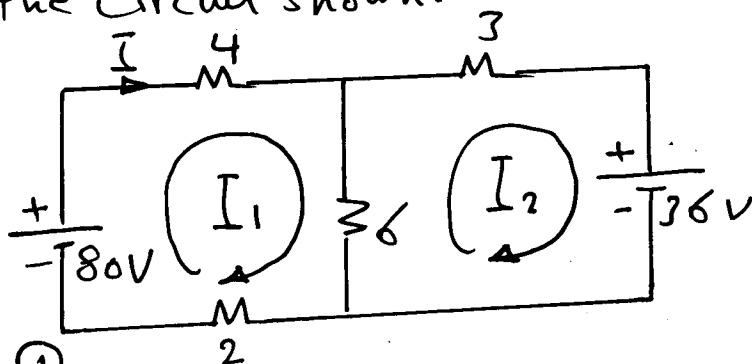
loop(2):

$$-36 = (3 + 6)I_2 - 6I_1$$

$$-36 = 9I_2 - 6I_1 \quad \text{--- (2)}$$

Solving eq(1) & eq(2)

We get $I = I_1 = 7 \text{ Amp}$.



OR

$$I_1 = -I$$

loop(1):

$$-80 = 12I_1 - 6I_2$$

$$-40 = 6I_1 - 3I_2 \quad \text{--- (1)}$$

loop(2):

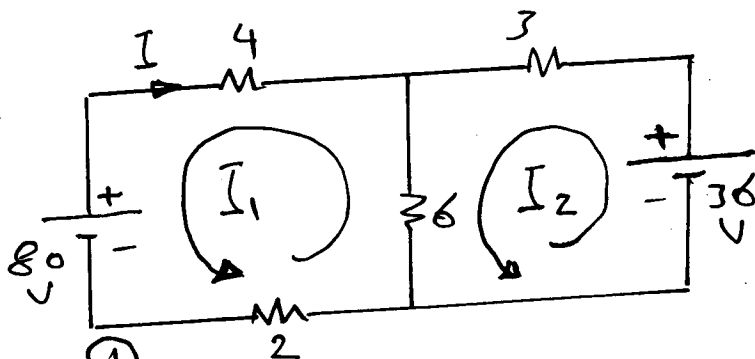
$$36 = 9I_2 - 6I_1$$

$$12 = 3I_2 - 2I_1 \quad \text{--- (2)}$$

Solving eq(1) & eq(2)

We get $I_1 = -I = -7 \text{ Amp}$.

$I = 7 \text{ Amp}$.



*OR $I = I_1$

loop(1):

$$80 = (4 + 6 + 2)I_1 + 6I_2$$

$$80 = 12I_1 + 6I_2 \text{ --- (1)}$$

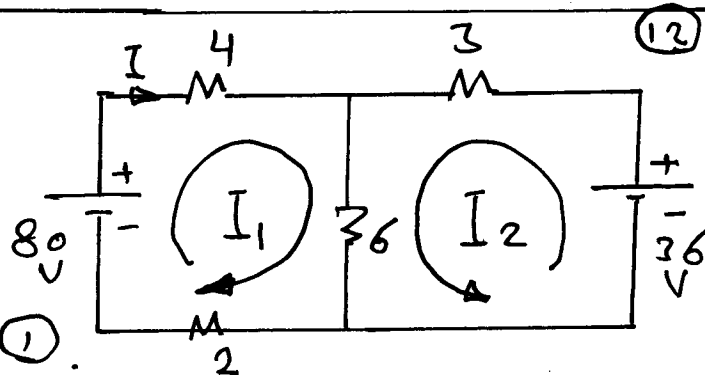
loop(2):

$$36 = (3 + 6)I_2 + 6I_1$$

$$36 = 9I_2 + 6I_1 \text{ --- (2)}$$

Solving eq(1) & eq(2) we get

$I = 7 \text{ Amp}$



*OR $I = I_1 + I_2$

loop(1):

$$80 = (4 + 6 + 2)I_1 + (4 + 2)I_2$$

$$80 = 12I_1 + 6I_2 \text{ --- (1)}$$

loop(2):

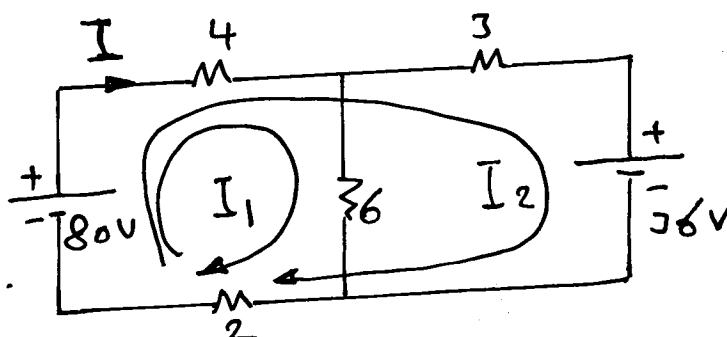
$$80 - 36 = (4 + 3 + 2)I_2 + (4 + 2)I_1$$

$$44 = 9I_2 + 6I_1 \text{ --- (2)}$$

Solving eq(1) & eq(2) we get:

$$I_1 = \frac{76}{12} \text{ Amp}, I_2 = \frac{8}{12} \text{ Amp}$$

$\therefore I = 7 \text{ Amp}$



Ex For the circuit shown find (I)?

$$I = I_2 - I_3$$

loop (1):

$$20 = (10 + 10)I_1 - 10I_2$$

$$20 = 20I_1 - 10I_2 \text{ ----- (1)}$$

loop (2):

$$20 = (20 + 10)I_2 - 10I_1 - 20I_3$$

$$20 = 30I_2 - 10I_1 - 20I_3 \text{ ----- (2)}$$

loop (3):

$$-120 = 40I_3 - 20I_2 \text{ ----- (3)}$$

From eq (1) $I_1 = \frac{2 + I_2}{2}$

put the value of I_1 in eq (2).

$$2 = 3I_2 - \left(\frac{2 + I_2}{2}\right) - 2I_3$$

$$\therefore 6 = 5I_2 - 4I_3 \text{ ----- (4)}$$

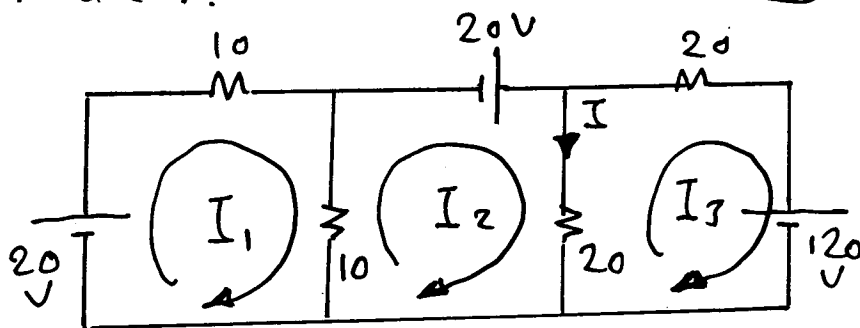
Solving eq (3) & eq (4), we get:

$$I_3 = -4 \text{ Amp.}$$

$$I_2 = -2 \text{ Amp.}$$

$$\therefore I = I_2 - I_3 = \underline{\underline{2 \text{ Amp.}}}$$

Home work: Solve this example by Kirchhoff's Law.



Ex: Find (I_o) for the circuit?

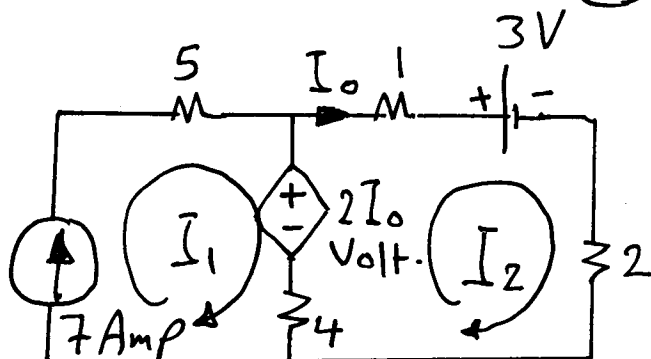
$$I_1 = 7 \text{ Amp.}$$

loop(2): $I_2 = I_o$

$$2I_o - 3 = (1 + 2 + 4)I_o - 4I_1$$

$$25 = 7I_o - 2I_o$$

$$\therefore I_o = 25/5 = 5 \text{ Amp}$$



Ex: Find (I) for the circuit?

$$I_3 = 6 \text{ Amp.}$$

loop(1):

$$100 = 60I_1 - 40I_2 - 20I_3$$

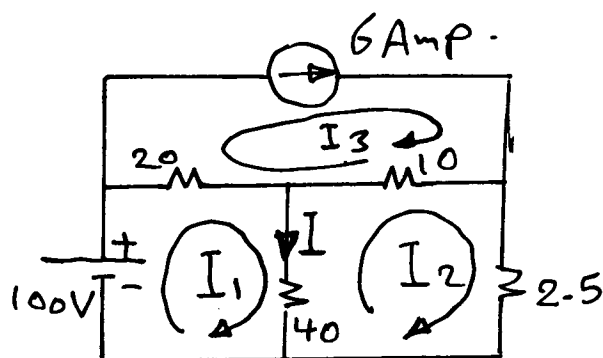
loop(2):

$$0 = (52.5)I_2 - 40I_1 - 10I_3 \quad \text{--- (2)}$$

and $I = I_1 - I_2$.

Solving eq(1) & eq(2) we get

$$I = 1 \text{ Amp}$$

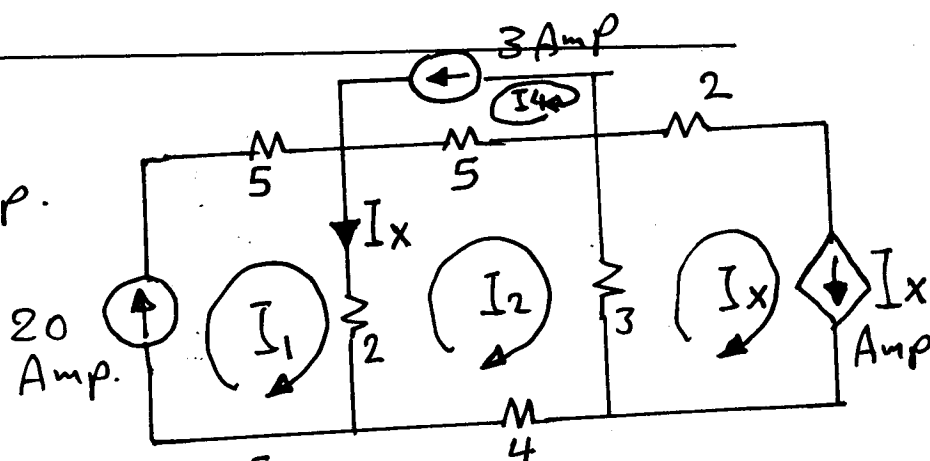


Ex: Find (I_x)?

$$I_1 = 20 \text{ A}, I_4 = -3 \text{ Amp.}$$

$$I_3 = I_x.$$

loop(2):



$$0 = (5 + 3 + 4 + 2)I_2 - 2I_1 - 3I_3 - 5I_4.$$

$$0 = 14I_2 - 2 \times 20 - 3(20 - I_2) - 5(-3).$$

$$0 = 17I_2 - 85$$

$$\therefore I_2 = 5 \text{ Amp.}$$

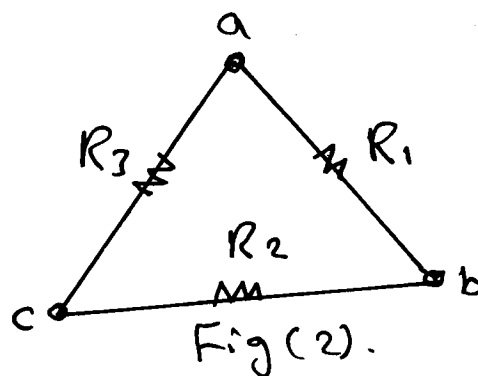
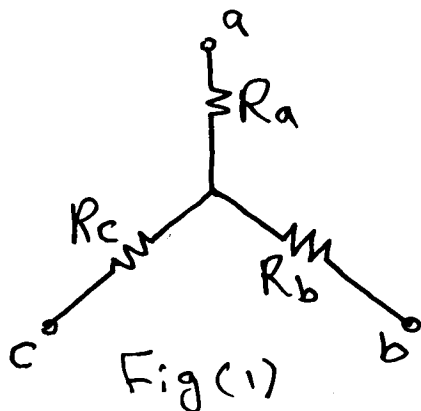
$$\therefore I_x = I_1 - I_2 = 20 - 5 = 15 \text{ Amp.}$$

Star \rightleftharpoons Delta Transformation.

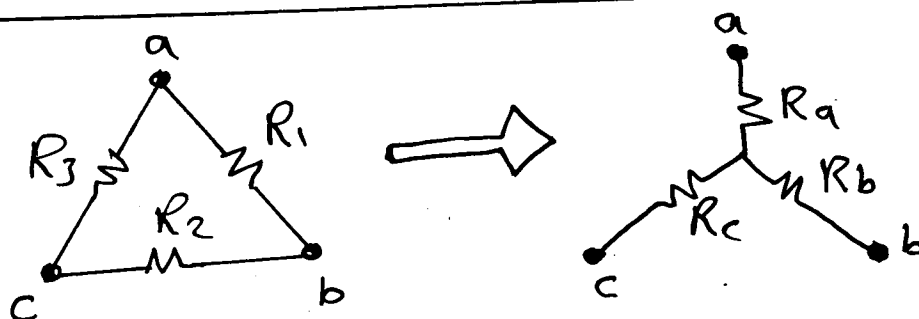
(15)



If the three resistors are connected as shown in Fig (1) they are said to be connected as (Star or Y), and if they are connected as shown in fig (2) they are in (Delta or Δ).



a) Delta \rightarrow Star Transformation $\Delta \rightarrow Y$ Transformation



For Delta Connection:

$$R_{ab} = R_1 \parallel (R_2 + R_3).$$

$$= \frac{R_1 (R_2 + R_3)}{R_1 + R_2 + R_3}$$

$$= \frac{R_1 R_2 + R_1 R_3}{R_1 + R_2 + R_3}$$

$$\therefore R_{bc} = \frac{R_1 R_2 + R_2 R_3}{R_1 + R_2 + R_3}$$

and $R_{ca} = \frac{R_1 R_3 + R_2 R_3}{R_1 + R_2 + R_3}$.

For Star Connection,

$$R_{ab} = R_a + R_b.$$

$$R_{bc} = R_b + R_c.$$

$$R_{ca} = R_c + R_a.$$

Since the points a, b & c for both connection are the same, then.

$$\therefore R_a + R_b = \frac{R_1 R_2 + R_1 R_3}{R_1 + R_2 + R_3} \text{ ----- (1)}$$

$$R_b + R_c = \frac{R_1 R_2 + R_2 R_3}{R_1 + R_2 + R_3} \text{ ----- (2)}$$

$$R_c + R_a = \frac{R_1 R_3 + R_2 R_3}{R_1 + R_2 + R_3} \text{ ----- (3)}$$

** Subtract eq(2) from eq(1) we get :-

$$R_a - R_c = \frac{R_1 R_3 - R_2 R_3}{R_1 + R_2 + R_3} \text{ ----- (4)}$$

** Add eq(3) with eq(4) we get:

$$2R_a = \frac{2R_1 R_3}{R_1 + R_2 + R_3}$$

$$\therefore R_a = \frac{R_1 R_3}{R_1 + R_2 + R_3}$$

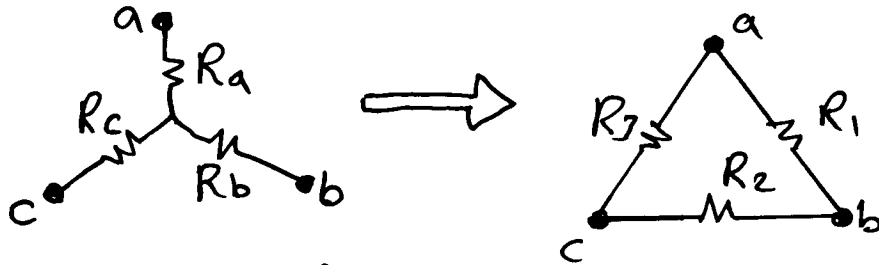
$$R_b = \frac{R_1 R_2}{R_1 + R_2 + R_3}$$

$$R_c = \frac{R_2 R_3}{R_1 + R_2 + R_3}$$

and

b) Star \rightarrow Delta Transformation

(17)



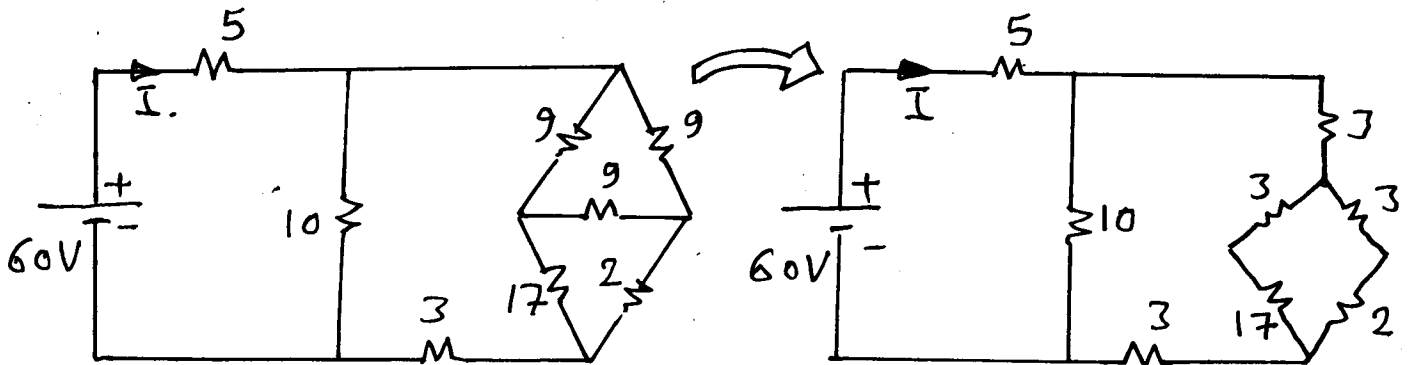
The expressions for the three Delta-connected resistors as function of the three Y-connected resistors are;

$$R_1 = \frac{R_a R_b + R_a R_c + R_b R_c}{R_c}$$

$$R_2 = \frac{R_a R_b + R_a R_c + R_b R_c}{R_a}$$

$$R_3 = \frac{R_a R_b + R_a R_c + R_b R_c}{R_b}$$

Ex: Find the current (I) for the circuit shown?



$$R_{eq} = \left(\left\{ \left[(3+17) \parallel (3+2) \right] + 3 + 3 \right\} \parallel 10 \right) + 5$$

$$= 10 \Omega$$

$$\therefore I = \frac{60}{10} = \underline{\underline{6 \text{ Amp.}}}$$

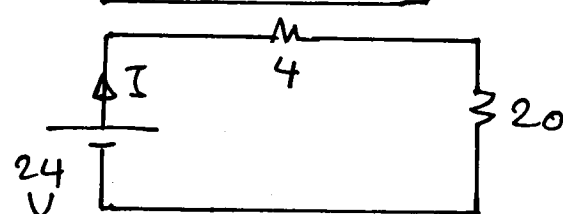
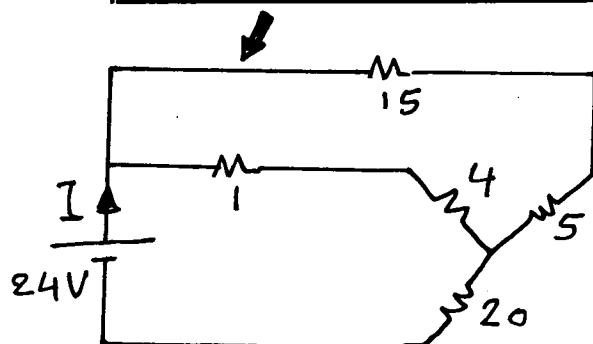
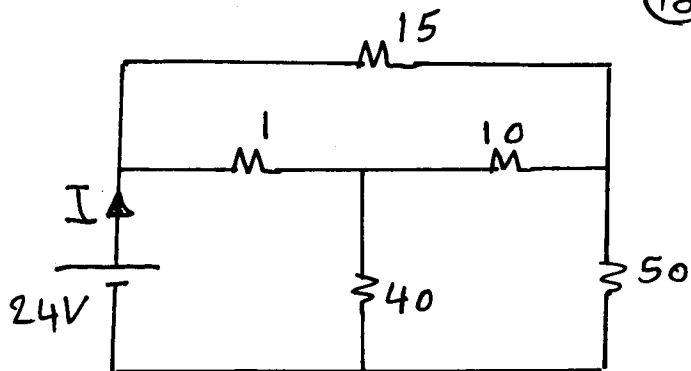
Ex: Use Δ -Y to find (I)?

change the Δ to Y resistance as shown.

$$R_{eq} = [(15+5) \parallel (1+4)] + 20$$

$$= 4 + 20 = 24 \Omega$$

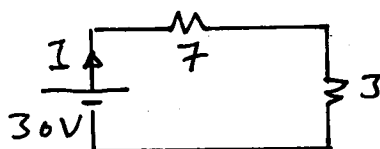
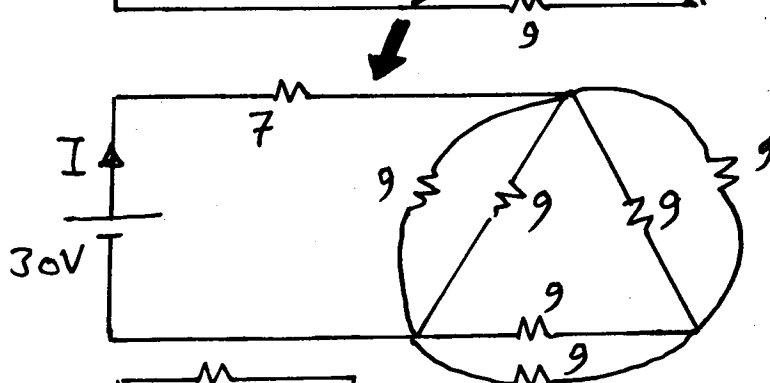
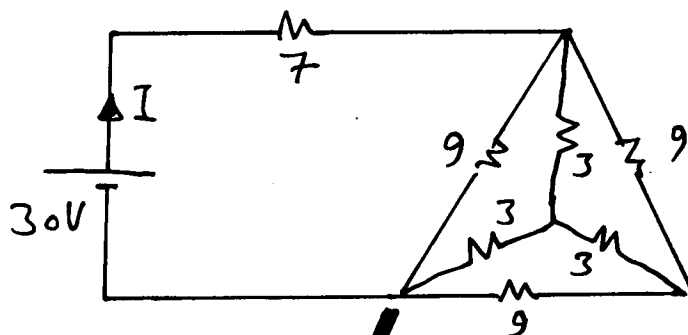
$$\therefore I = \frac{V}{R_{eq}} = \frac{24}{24} = 1 \text{ Amp}$$



Ex: Find (I)?

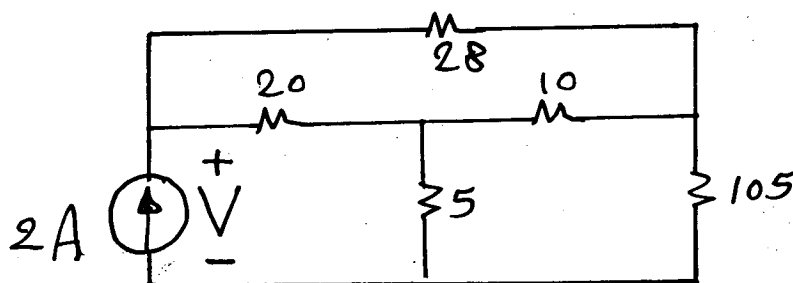
change from Y to Δ .

$$I = \frac{30}{10} = 3 \text{ Amp}$$



Home Work:
Find (V)?

ans: $V = 35 \text{ Volt}$



((Superposition method))

(19)

Ex: Find (I) ?

1) By effect of 100V \rightarrow 80V (s/c).

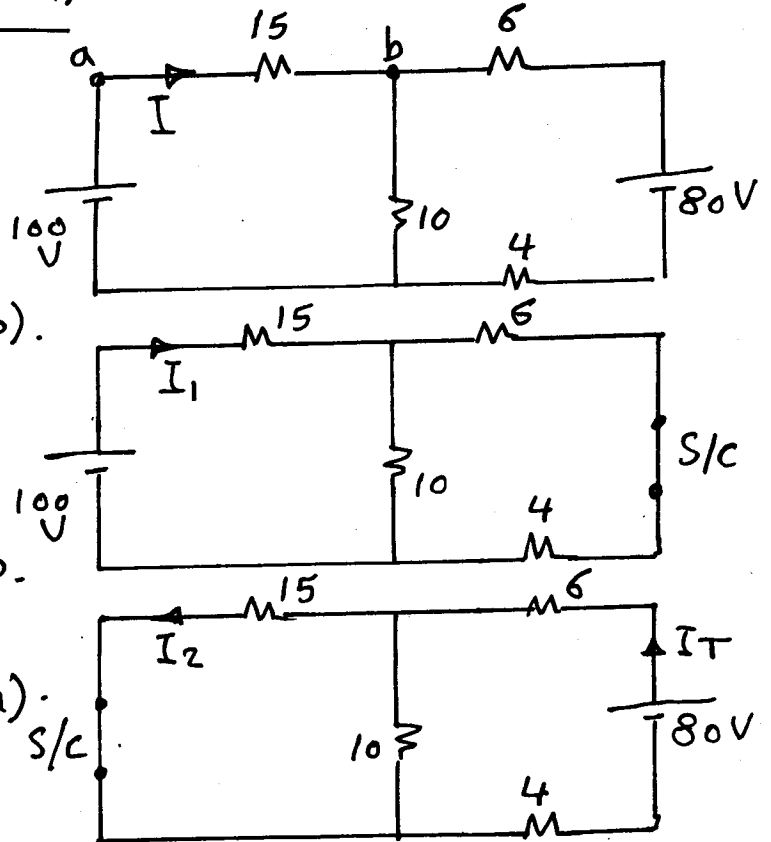
$$I_1 = \frac{100}{15 + (10 \parallel 10)} = 5 \text{ Amp (a} \rightarrow \text{b)}.$$

2) By effect of 80V \rightarrow 100V (s/c).

$$I_T = \frac{80}{6 + 4 + (15 \parallel 10)} = \frac{80}{16} = 5 \text{ Amp.}$$

$$\therefore I_2 = 5 \frac{10}{10 + 15} = 2 \text{ Amp (b} \rightarrow \text{a)}.$$

$$\therefore I = I_1 - I_2 = 5 - 2 = \underline{\underline{3 \text{ Amp}}}$$



Ex: Find (I) ?

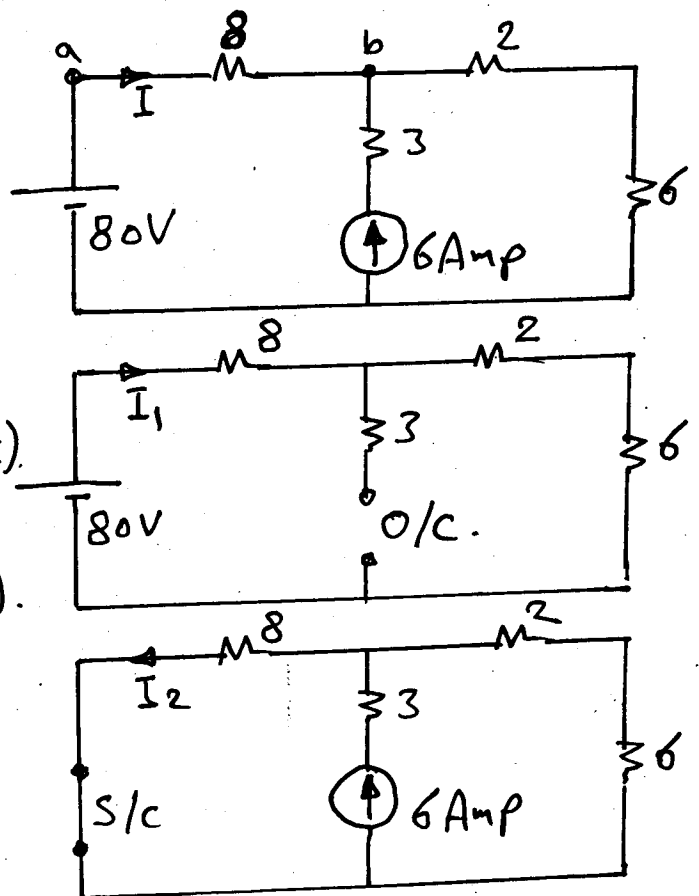
1) By effect of 80V \rightarrow 6A (o/c).

$$I_1 = \frac{80}{16} = 5 \text{ Amp (a} \rightarrow \text{b)}.$$

2) By effect of 6Amp \rightarrow 80V (s/c).

$$I_2 = 6 \frac{8}{8 + 8} = 3 \text{ Amp (b} \rightarrow \text{a)}.$$

$$\therefore I = I_1 - I_2 = 5 - 3 = \underline{\underline{2 \text{ Amp}}}$$



Ex: find (I)?

1) By effect of 9A \rightarrow 54V (s/c).

$$I_{4\Omega} = 9 \frac{2}{2+7} = 2 \text{ Amp.}$$

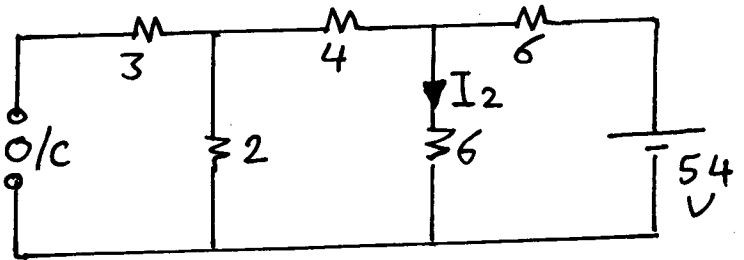
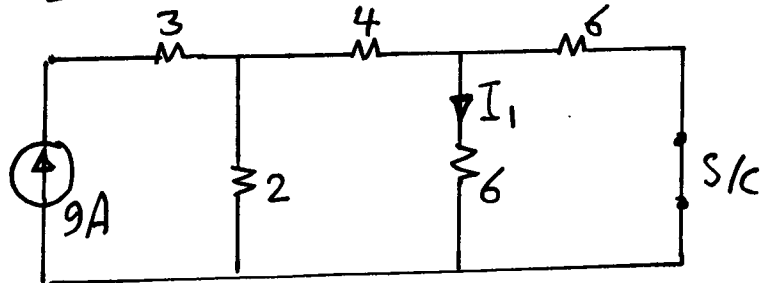
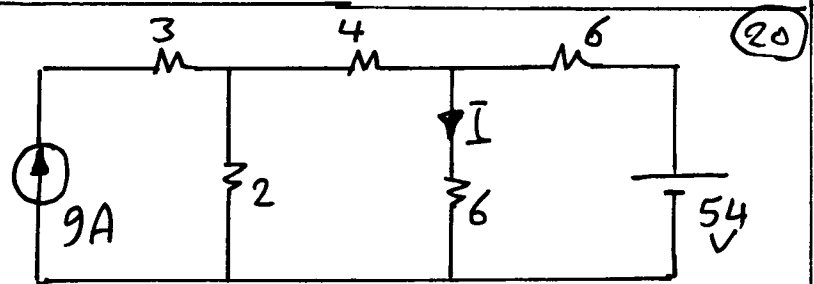
$$\therefore I_1 = 2 \frac{6}{6+6} = 1 \text{ Amp.}$$

2) By effect of 54V \rightarrow 9A (o/c).

$$I_T = \frac{54}{6 + (6//6)} = 6 \text{ Amp.}$$

$$I_2 = 6 \frac{6}{6+6} = 3 \text{ Amp.}$$

$$\therefore I = I_1 + I_2 = \underline{4 \text{ Amp}}$$



Ex: Find (Ix)?

1) By effect of 53V \rightarrow 7A (o/c).

$$I_{X1} = \frac{53 + 2I_{X1}}{1 + 2 + 4}$$

$$7I_{X1} = 53 + 2I_{X1}$$

$$\therefore I_{X1} = \frac{53}{5} \text{ Amp.}$$

2) By effect of 7Amp \rightarrow 53V (s/c).

loop (Ix2):

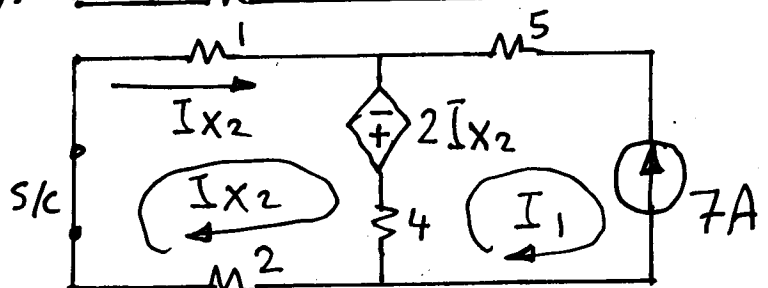
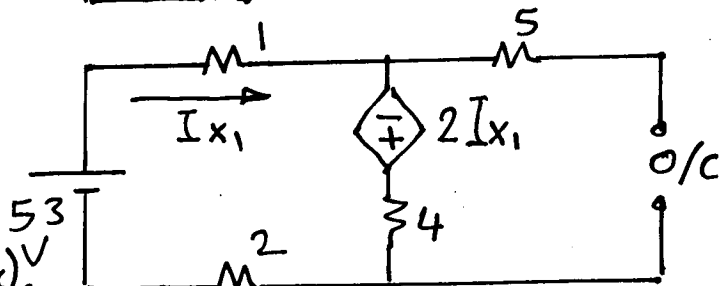
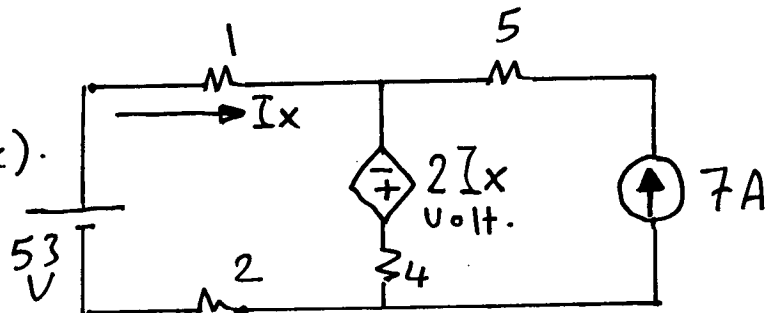
$$2I_X = 7I_{X2} - 4I_1$$

$$2I_{X2} = 7I_{X2} - 4(-7)$$

$$-5I_{X2} = 28$$

$$\therefore I_{X2} = -\frac{28}{5} \text{ Amp.}$$

$$\therefore I_X = I_{X1} + I_{X2} = \frac{53}{5} + \left(-\frac{28}{5}\right) = \frac{25}{5} = \underline{5 \text{ Amp.}}$$



$$I_1 = -7 \text{ Amp.}$$

《Nodal Voltage method》

(21)

Ex: Find (I) for the circuit shown?

$$\left(\frac{1}{3} + \frac{1}{3} + \frac{1}{6}\right)V - \frac{40}{3} - \frac{30}{3} = 0$$

$$(2+2+1)V - 80 - 60 = 0$$

$$5V = 140$$

$$\therefore V = 28 \text{ volt.}$$

$$\text{But } I = \frac{40 - V}{3}$$

$$\therefore I = \frac{40 - 28}{3} = \underline{4 \text{ Amp.}}$$

OR

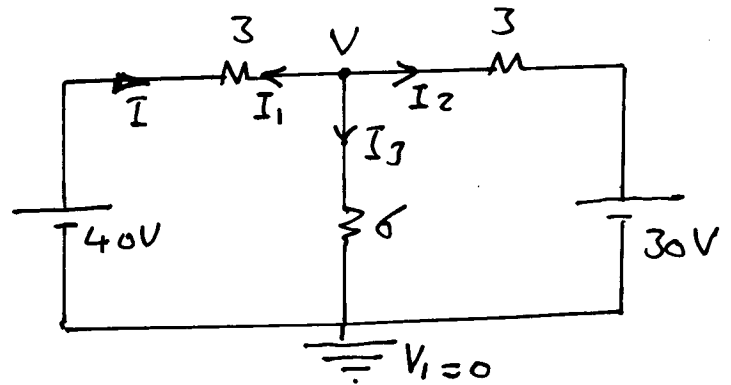
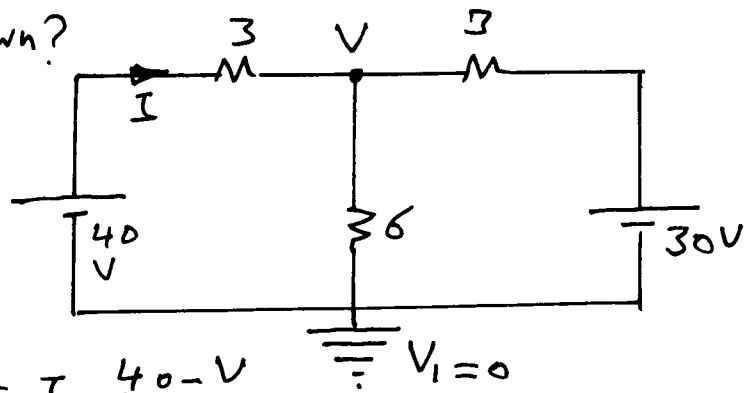
By K.C.L

$$I_1 + I_2 + I_3 = 0$$

$$\frac{V - 40}{3} + \frac{V - 30}{3} + \frac{V}{6} = 0$$

$$\left(\frac{1}{3} + \frac{1}{3} + \frac{1}{6}\right)V - \frac{40}{3} - \frac{30}{3} = 0.$$

$$\therefore V = 28 \text{ volt.}$$



Ex Find (I)?

Node (V):

$$\left(\frac{1}{8} + \frac{1}{8}\right)V - \frac{80}{8} = 6$$

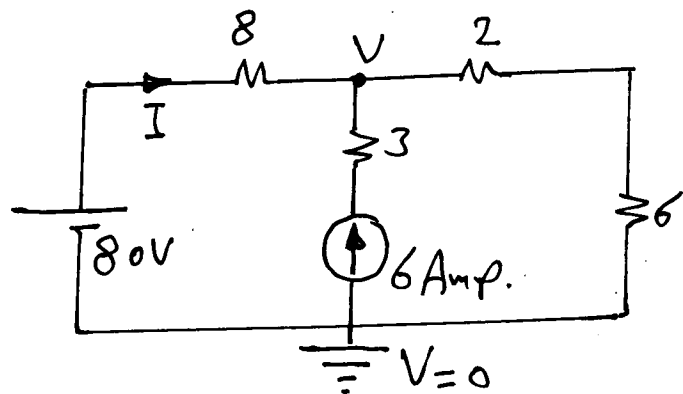
$$(1+1)V - 80 = 48$$

$$2V = 128$$

$$V = 64 \text{ Volt.}$$

$$\text{But } I = \frac{80 - V}{8}$$

$$\therefore I = \frac{80 - 64}{8} = \underline{2 \text{ Amp.}}$$



Ex: Find (I_o)?

(22)

Node(V)

$$\left(\frac{1}{3} + \frac{1}{4}\right)V - \frac{2I_o}{4} - \frac{3}{3} = 7.$$

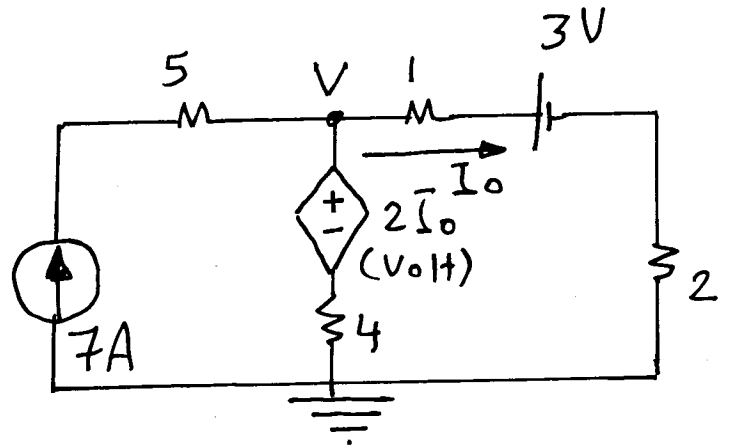
$$(4+3)V - 6I_o - 12 = 84.$$

$$7V - 6I_o = 96$$

$$7V - 2V + 6 = 96$$

$$\therefore V = 18 \text{ Volt.}$$

$$\therefore I_o = \frac{18-3}{3} = \underline{5 \text{ Amp.}}$$



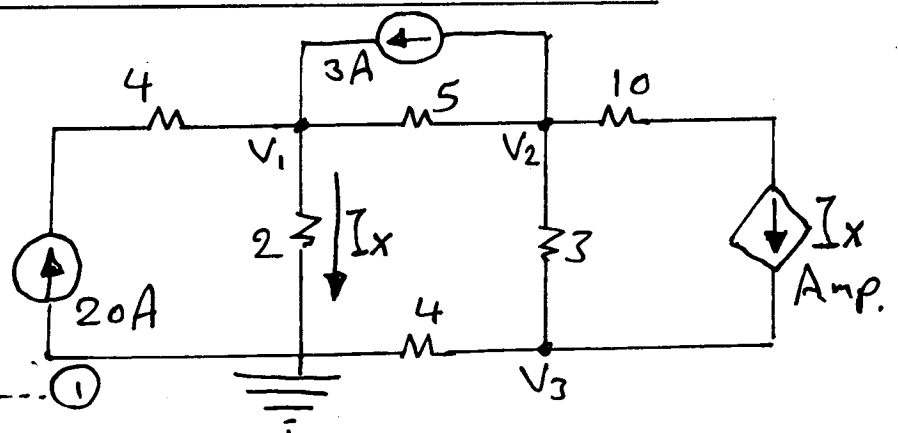
$$\text{But } I_o = \frac{V-3}{3}$$

Ex: Find (I_x)?

Node(V_1):

$$\left(\frac{1}{2} + \frac{1}{5}\right)V_1 - \frac{V_2}{5} = 23$$

$$7V_1 - 2V_2 = 230 \dots\dots (1)$$



Node(V_2):

$$\left(\frac{1}{3} + \frac{1}{5}\right)V_2 - \frac{V_1}{5} - \frac{V_3}{3} = -3 - I_x$$

$$\text{But } I_x = \frac{V_1}{2}$$

$$\therefore 16V_2 + 9V_1 - 10V_3 = -90 \dots\dots (2)$$

Node(V_3):

$$\left(\frac{1}{3} + \frac{1}{4}\right)V_3 - \frac{V_2}{3} = I_x \dots\dots (3)$$

Solving: $V_1 = 30 \text{ Volt.}$

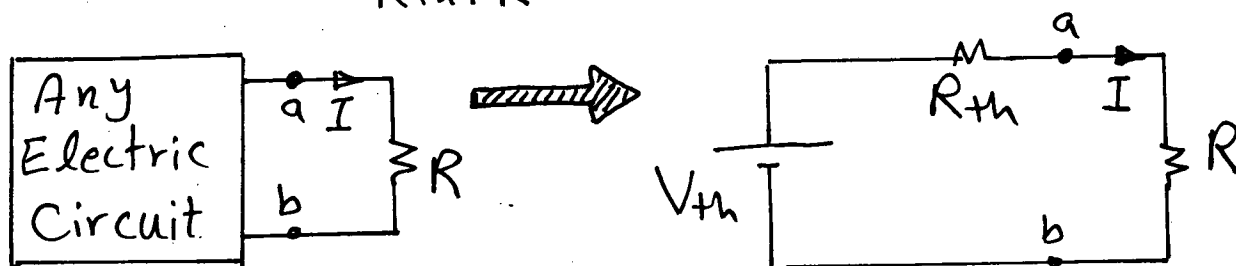
$$\therefore I_x = \frac{30}{2} = \underline{15 \text{ Amp.}}$$

((Thevenin's Theorem))

(23)

This theory states as ((The current (I) passing through a resistance (R) of any electric circuit is

$$I = \frac{V_{th}}{R_{th} + R} .)) .$$

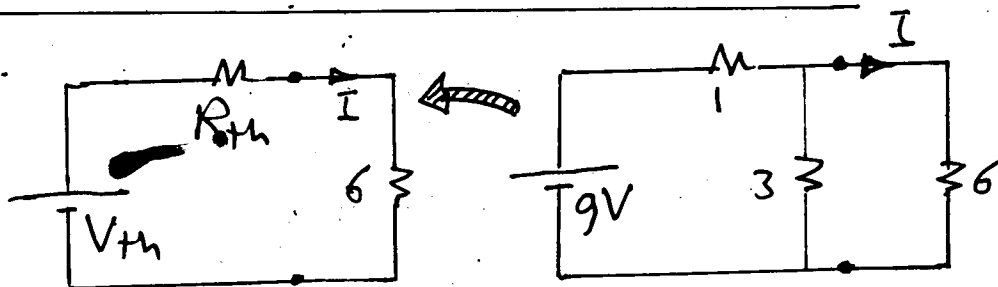


Where: V_{th} : The voltage between a & b with (R) disconnected.

R_{th} : The circuit resistance between a & b with (R) also disconnected.

Ex: Find (I)?

$$I = \frac{V_{th}}{R_{th} + 6}$$

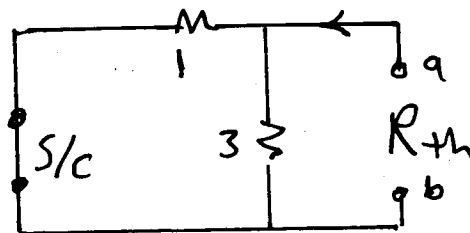


To find R_{th} ?

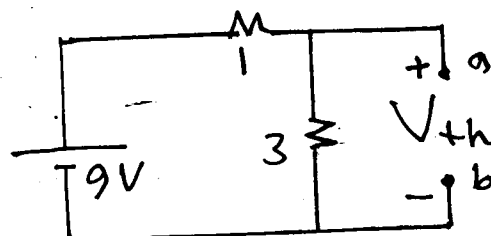
$$* R_{th} = 1 \parallel 3 = \frac{1 \times 3}{1 + 3} = \frac{3}{4} \Omega .$$

$$* V_{th} = V_{3\Omega} = \frac{9}{4} \times 3 = \frac{27}{4} \text{ Volt} .$$

$$\therefore I = \frac{27/4}{\frac{3}{4} + 6} = \frac{27/4}{27/4} = 1 \text{ Amp} .$$

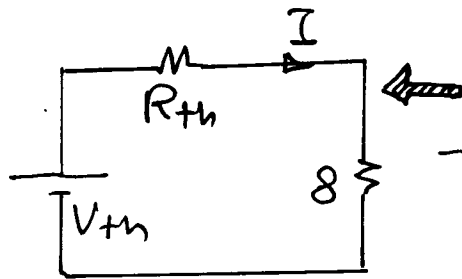


To find V_{th} ?



Ex: Find (I)?

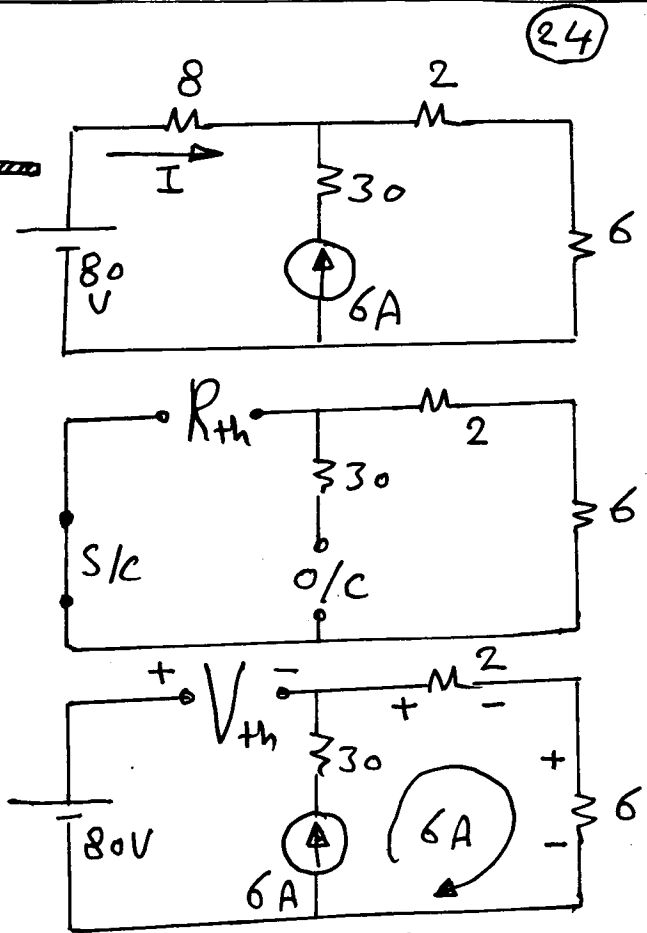
$$I = \frac{V_{th}}{R_{th} + 8}$$



* $R_{th} = 2 + 6 = 8 \Omega$.

* $V_{th} = 80 - V_{2\Omega} - V_{6\Omega}$
 $= 80 - 2 \times 6 - 6 \times 6$
 $= 32 \text{ Volt}$.

$\therefore I = \frac{32}{8+8} = \underline{2 \text{ Amp}}$.



Ex: Find (I)?

$$I = \frac{V_{th}}{R_{th} + 6}$$

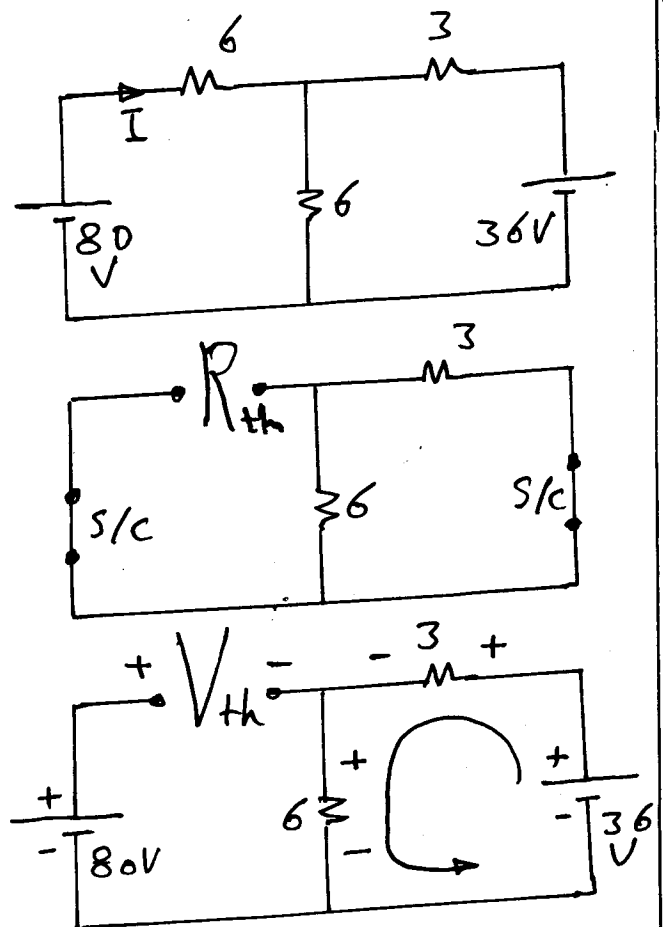
* $R_{th} = 3 \parallel 6 = 2 \Omega$.

* $V_{th} = 80 - V_{6\Omega}$
 $= 80 - \frac{36}{3+6} \times 6 = 56 \text{ Volt}$.

OR $V_{th} + 36 = 80 + V_{3\Omega}$.

$\therefore V_{th} = 80 - 36 + \frac{36}{9} \times 3$
 $= 56 \text{ Volt}$.

$\therefore I = \frac{56}{2+6} = \underline{7 \text{ Amp}}$.

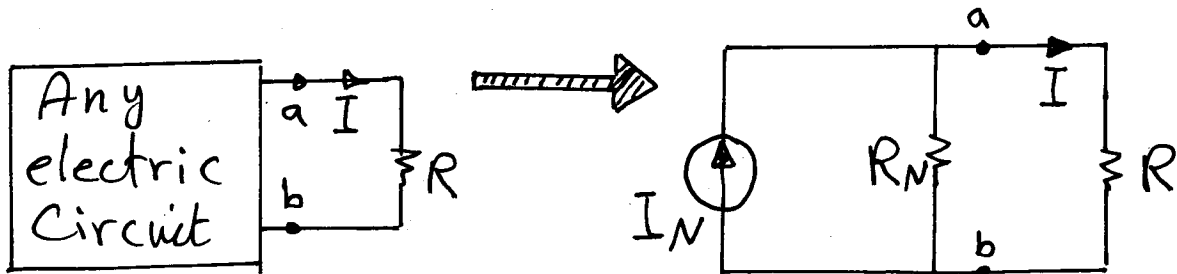


(Norton Theory)

(25)

This theory states as ((The current (I) passing through a resistance (R) of any electric circuit is

$$I = I_N \frac{R_N}{R_N + R} .)) .$$



Where:

I_N : (The current passing through a & b after replacing the resistor (R) by a short-circuit)

R_N : The circuit resistance between a & b with (R) disconnected.

$$R_N = R_{th}$$

Ex:

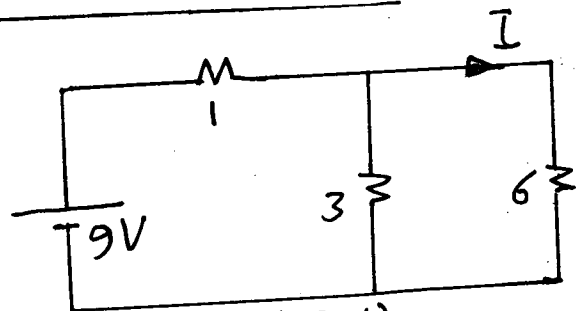
$$I = I_N \frac{R_N}{R_N + 6}$$

$$* R_N = R_{th} = 1 \parallel 3 = 3/4 \Omega .$$

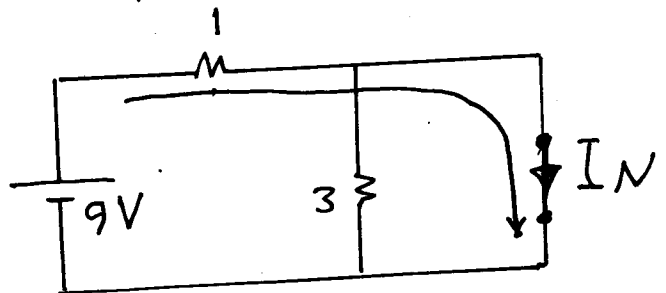
$$* I_N \rightarrow$$

$$I_N = \frac{9}{1} = 9 \text{ Amp} .$$

$$\therefore I = 9 \frac{3/4}{3/4 + 6} = \frac{27/4}{27/4} = 1 \text{ Amp} .$$



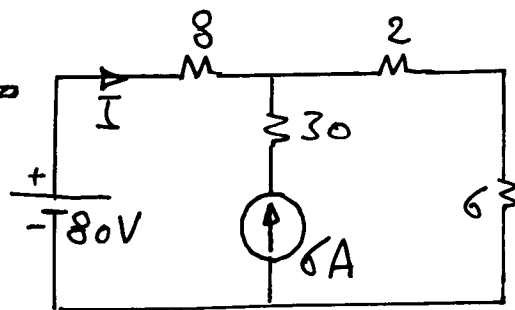
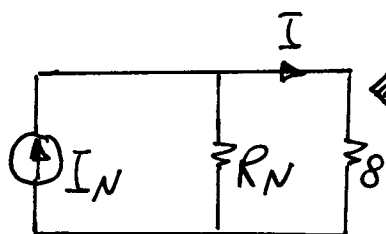
To find (I_N).



Ex: Find (I)?

(26)

$$I = I_N \frac{R_N}{R_N + 8}$$



* $R_N = R_{th} = 2 + 6 = 8 \Omega$.

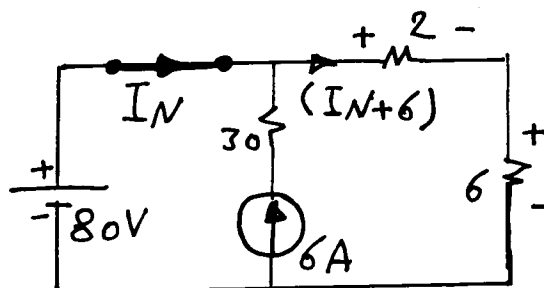
* $I_N \rightarrow$

$$80 = 2(I_N + 6) + 6(I_N + 6)$$

$$32 = 8 I_N$$

$$\therefore I_N = 4 \text{ Amp}$$

$$\therefore I = 4 \frac{8}{8+8} = 2 \text{ Amp}$$



Ex: Find (I)?

$$I = I_N \frac{R_N}{R_N + 6}$$

* $R_N = R_{th} = 3 // 6 = 2 \Omega$.

* $I_N \rightarrow$

for circuit abcd

$$80 = 3(I_N - I_1) + 36$$

$$44 = 3 I_N - 3 I_1 \text{ ----- (1)}$$

for circuit afed

$$80 = 6 I_1 \text{ ----- (2)}$$

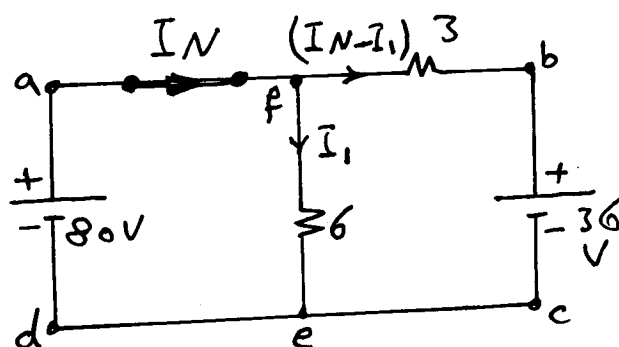
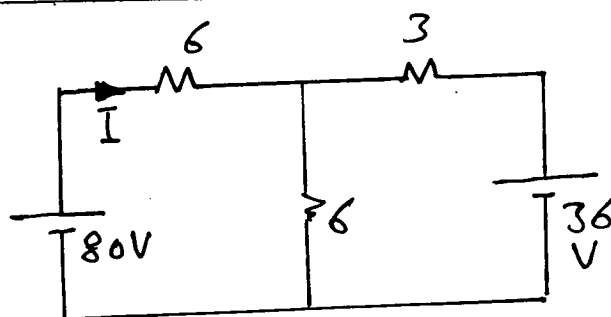
$$\therefore I_1 = 80/6$$

from eq (1):

$$44 = 3 I_N - 3 \left(\frac{80}{6} \right)$$

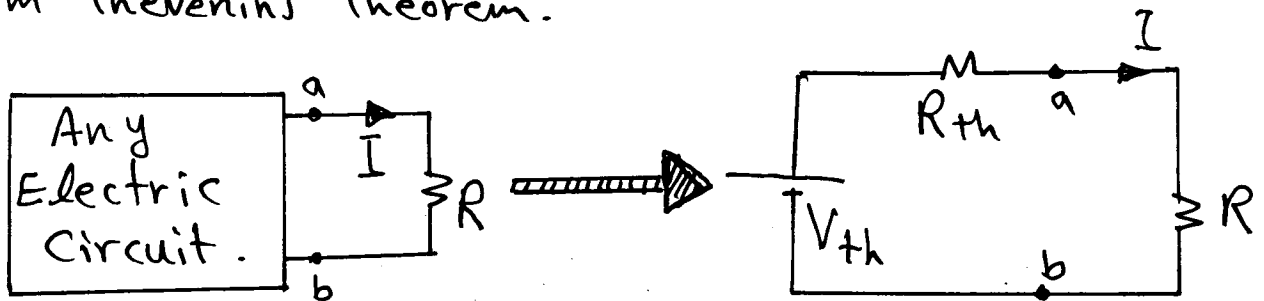
$$\therefore I_N = 28 \text{ Amp}$$

$$\therefore I = 28 \frac{2}{2+6} = 7 \text{ Amp}$$



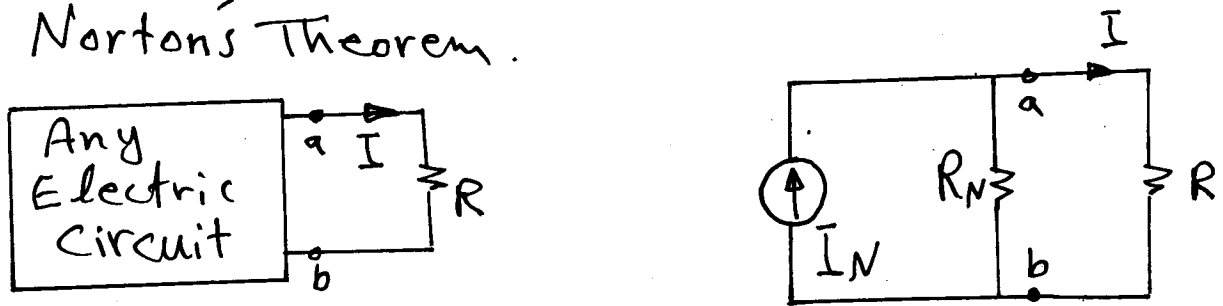
Voltage Sources \rightleftharpoons Current Sources Transformation (27) V.S \rightleftharpoons C.S Transformations

* From Thevenin's Theorem.



$$I = \frac{V_{th}}{R_{th} + R} \quad \text{--- (1)}$$

* From Norton's Theorem.



$$I = I_N \frac{R_N}{R_N + R} \quad \text{--- (2)}$$

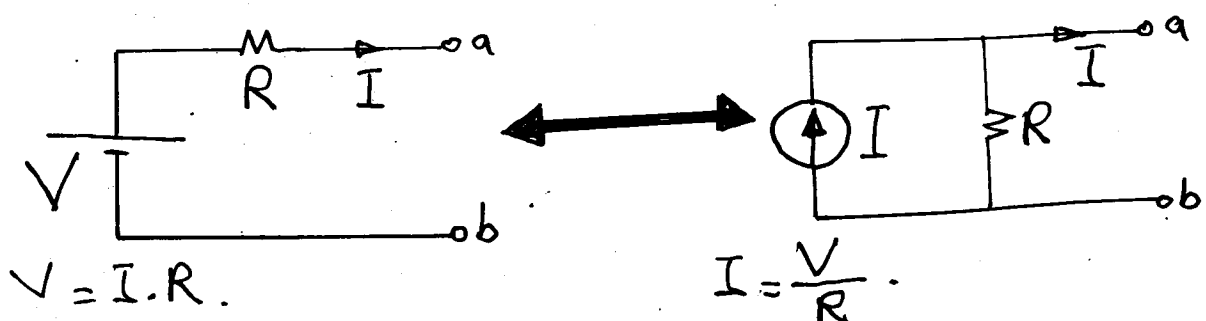
** Since $R_{th} = R_N$

\therefore From eq(1) & eq(2).

$$V_{th} = I_N R_N$$

$$\therefore \boxed{R_{th} = R_N = \frac{V_{th}}{I_N}}$$

* In General:



Ex: Find (I_o) By using Thevenin's Theory?

(28)

$$R_{th} = R_N = \frac{V_{th}}{I_N}$$

$$\therefore V_{th} = 100 - 4 \times 2.5 = 90V.$$

$$I_N \rightarrow$$

$$I_N = I_1$$

$$I_2 = -2.5A.$$

loop(1):

$$100 - 4I_N = 4I_N - 4(-2.5).$$

$$90 = 8I_N.$$

$$\therefore I_N = 90/8.$$

$$\therefore R_{th} = R_N = 8\Omega.$$

$$\therefore I = \frac{V_{th}}{R_{th} + 10} = \frac{90}{8 + 10} = \underline{\underline{5Amp.}}$$

$$\text{OR } I = I_N \frac{R_N}{R_N + 10} = \frac{90}{8} \times \frac{8}{10 + 8} = \underline{\underline{5Amp.}}$$

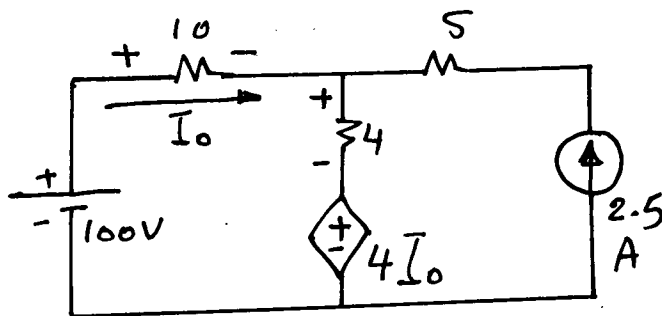
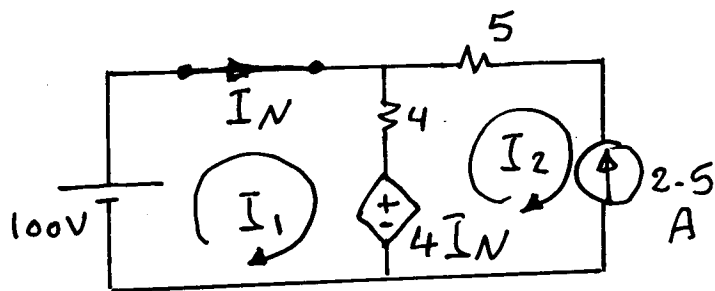
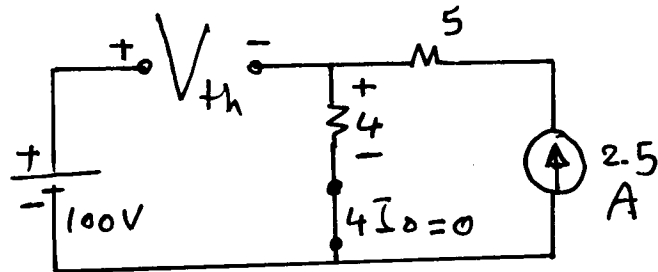
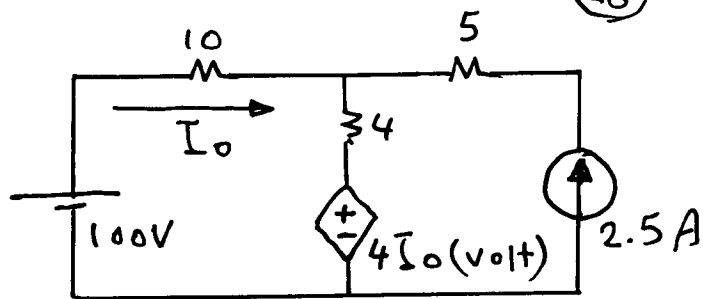
Solve the Same Ex by Kirchhoff's Law.

abcd

$$100 = 10I_o + 4(I_o + 2.5) + 4I_o$$

$$90 = 18I_o.$$

$$\therefore I_o = \underline{\underline{5Amp.}}$$



Ex: Find the current (I)?

* By Node method

$$\left(\frac{1}{20} + \frac{1}{20} + \frac{1}{5}\right)V - \frac{30}{5} - \frac{120}{20} = 0$$

$$6V - 120 - 120 = 0$$

$$6V = 240$$

$$\therefore V = 40 \text{ Volt}$$

$$I = \frac{V}{20} = \frac{40}{20} = 2 \text{ Amp}$$

* By Thevening Theory

$$R_{th} = 5 // 20 = 4 \Omega$$

* $V_{th} \rightarrow$

$$I = \frac{120 - 30}{25} = \frac{90}{25} \text{ Amp}$$

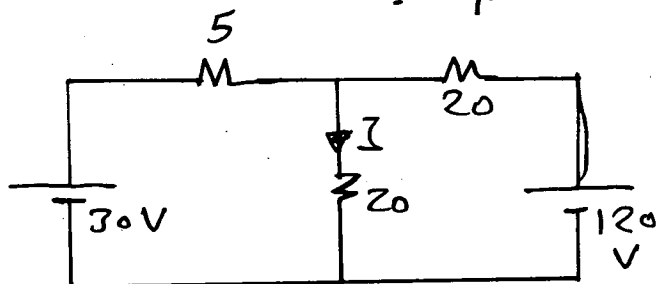
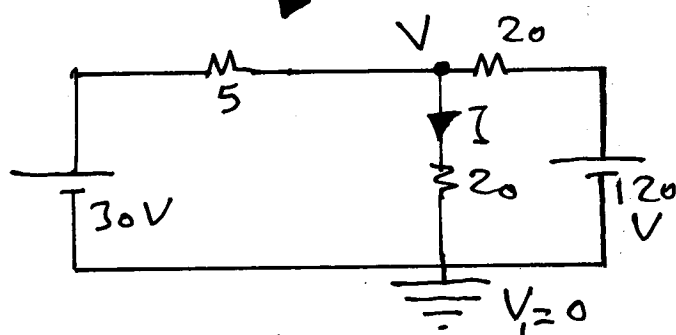
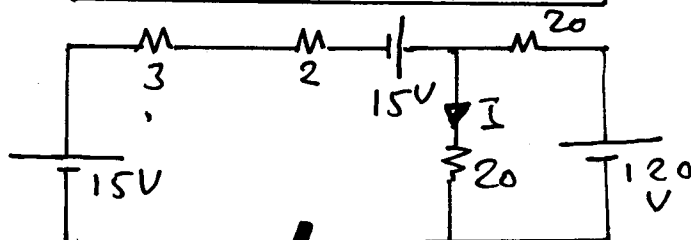
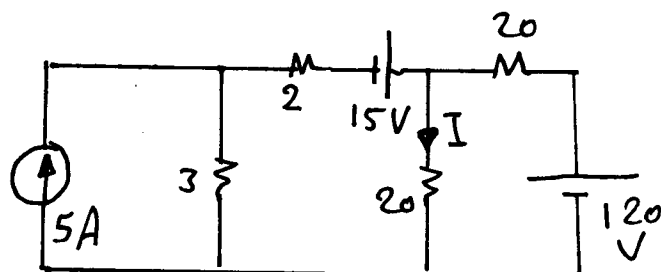
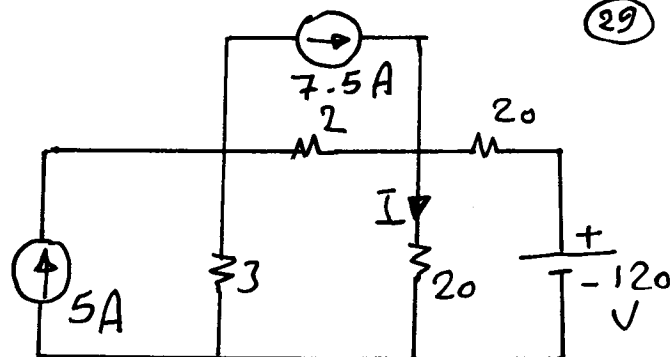
$$\therefore V_{th} = 30 + V_{5\Omega}$$

$$= 30 + \frac{90}{25} \times 5 = 48 \text{ Volt}$$

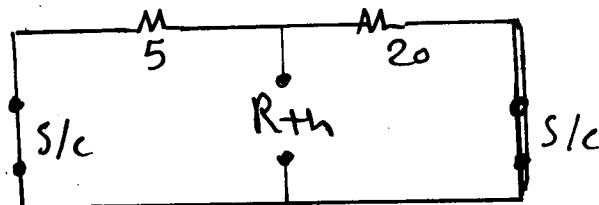
OR $V_{th} = 120 - V_{20\Omega}$

$$= 120 - \frac{90}{25} \times 20 = 48 \text{ Volt}$$

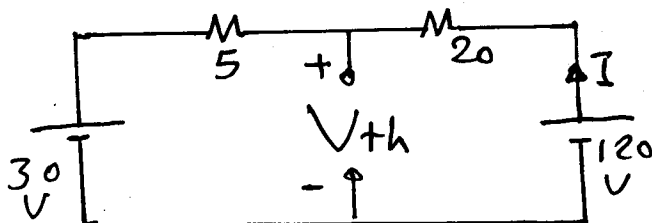
$$\therefore I = \frac{V_{th}}{R_{th} + 20} = \frac{48}{4 + 20} = 2 \text{ Amp}$$



To find R_{th}



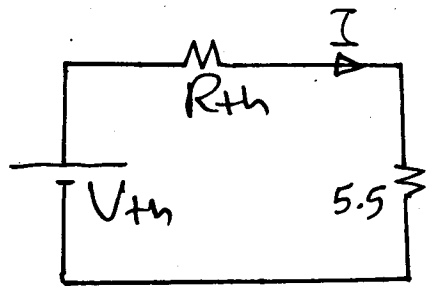
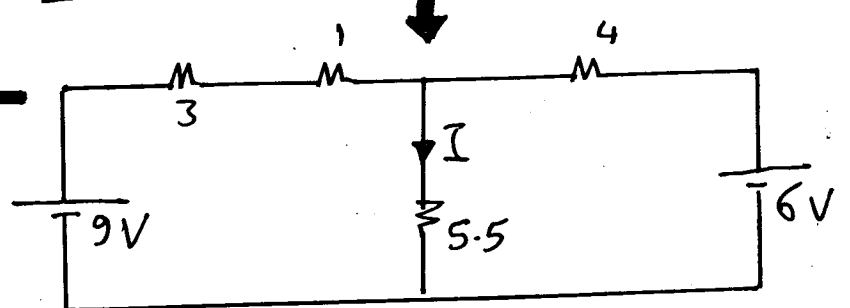
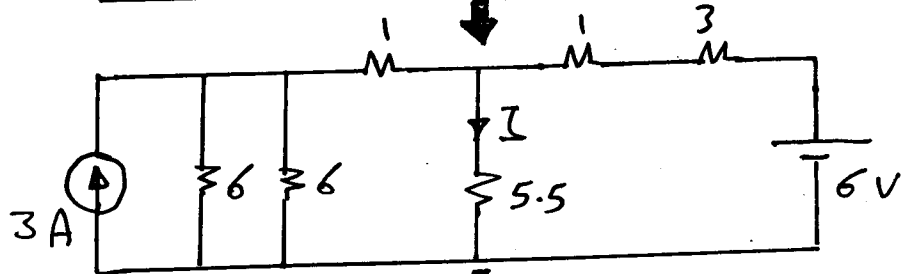
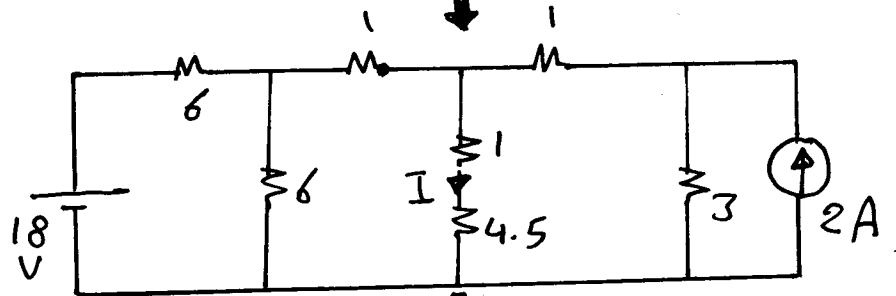
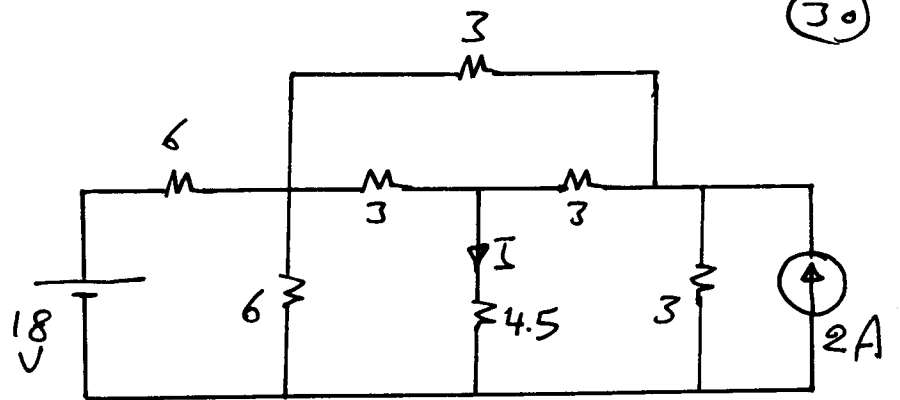
To find V_{th}



Ex: Find (I)?

(30)

By Thevenin's



$$I = \frac{V_{th}}{R_{th} + 5.5}$$

* $R_{th} = 4 \parallel 4 = 2 \Omega$

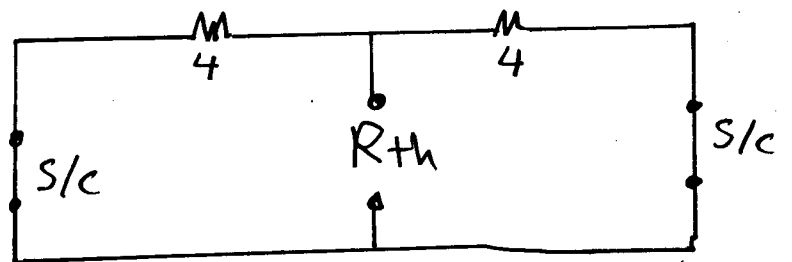
* $V_{th} \rightarrow$

$$V_{th} = 9 - V_{4\Omega}$$

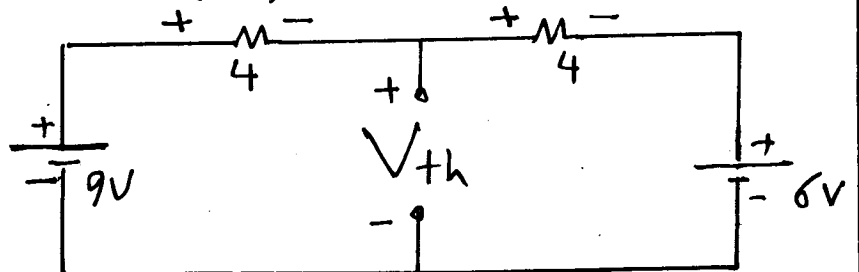
$$= 9 - \frac{3}{8} \times 4 = 7.5 \text{ Volt}$$

$$\therefore I = \frac{7.5}{5.5 + 2} = 1 \text{ Amp}$$

To find R_{th}



To find V_{th}



Maximum Power Transfer

(31)

The power delivered to the load (R_L) $= I^2 \cdot R_L$.

$$\text{But } I = \frac{V_{in}}{R_{in} + R_L}$$

$$\therefore P_L = \left[\frac{V_{in}}{(R_{in} + R_L)} \right]^2 \times R_L$$

$$\text{Since } P = \frac{V_{in}^2}{(R_{in} + R_L)^2} \times R_L$$

$$\frac{dP_{max}}{dR_L} = \frac{(R_{in} + R_L)^2 - R_L(R_{in} + R_L) \times 2}{(R_{in} + R_L)^4}$$

$$\text{But } \frac{dP_{max}}{dR_L} = 0$$

$$\therefore (R_{in} + R_L)^2 - 2R_L(R_{in} + R_L) = 0$$

$$(R_{in} + R_L)[(R_{in} + R_L) - 2R_L] = 0$$

$$\therefore (R_{in} + R_L)(R_{in} - R_L) = 0$$

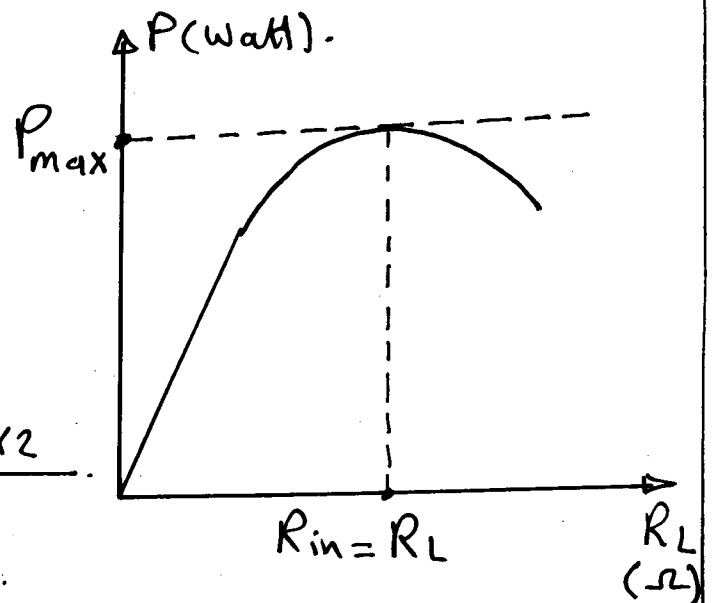
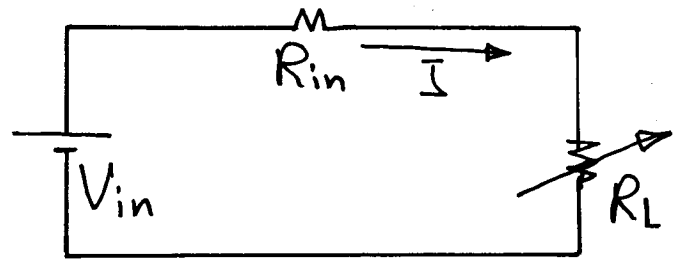
* either $R_{in} = -R_L$ (neglected).

* OR $R_{in} = R_L$

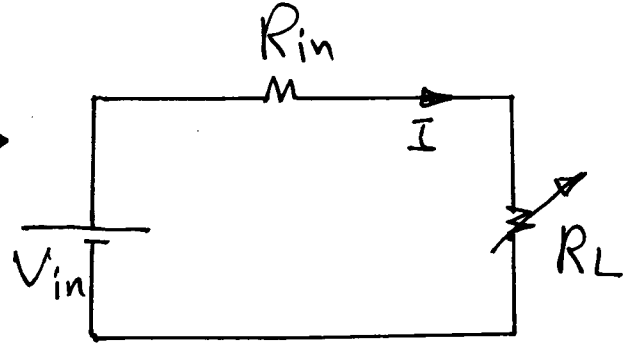
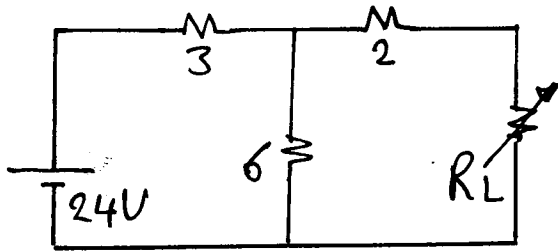
\therefore Maximum power transfer take place only when

$$R_L = R_{in}$$

$$\text{and } R_L = R_{in} = R_{th}$$



Ex: Find the value of (R_L) for maximum power to (R_L) , and then find this max. power. (32)



at maximum power.

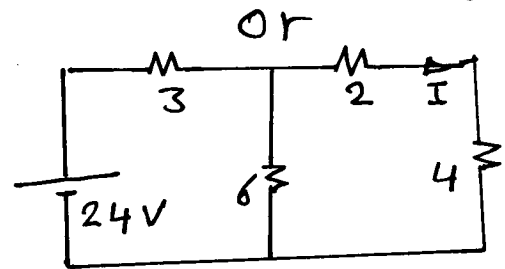
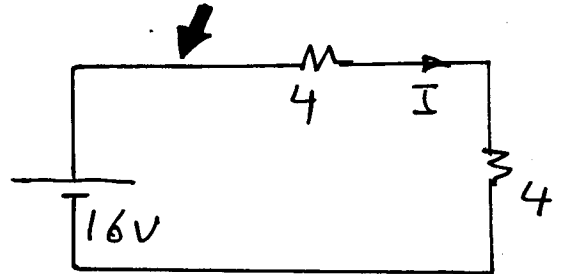
$$R_L = R_{in} = R_{th} \\ = (3//6) + 2 = \underline{4\Omega}$$

$$\therefore V_{in} = V_{th} = V_{6\Omega}$$

$$= \frac{24}{3+6} \times 6 = 16 \text{ Volt.}$$

$$\text{and } I = \frac{V_{th}}{R_{th} + 4} = \frac{16}{4+4} = 2 \text{ Amp.}$$

$$\text{and } P = I^2 \cdot R = (2)^2 \times 4 = \underline{16 \text{ Watt}}$$

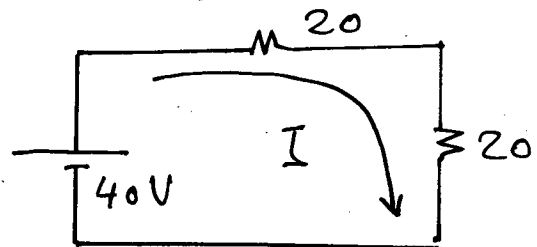
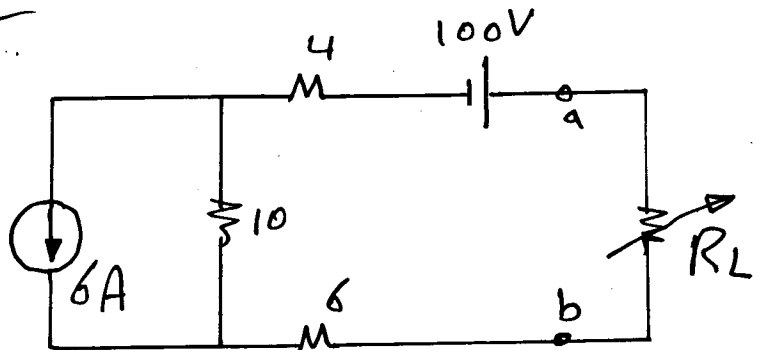


Ex: Find (R_L) for max. power

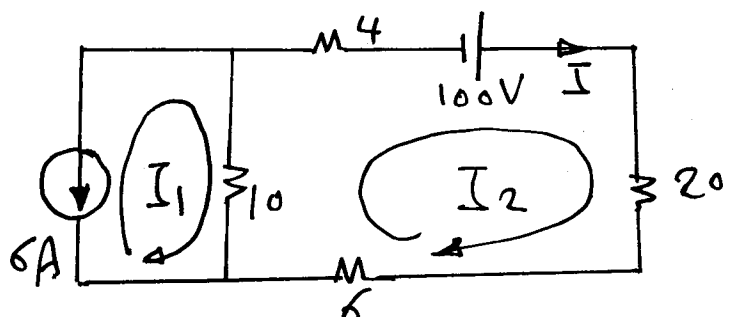
$$R_L = R_{th} = 6 + 10 + 4 = \underline{20\Omega}$$

$$V_{in} = V_{th} = V_{ab} = 100 - 60 \\ = 40 \text{ Volt.}$$

$$\therefore I = \frac{40}{20+20} = 1 \text{ Amp.}$$



By loop method.



OR

$$\text{loop (1)} \rightarrow I_1 = -6 \text{ A.}$$

$$\text{loop (2)}$$

$$100 = (20 + 4 + 6 + 10)I_2 - 10I_1$$

$$100 = 40I_2 - 10(-6)$$

$$40 = 40I_2$$

$$\therefore I_2 = I = \underline{1 \text{ Amp}}$$

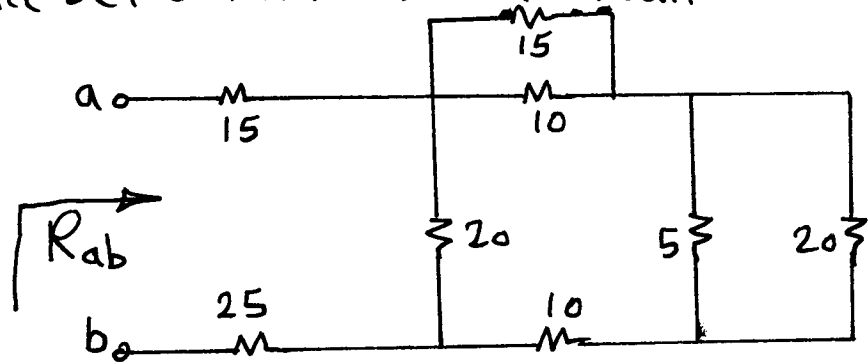
Q1 Find the resistance between a & b for the circuit shown?

Solution:

$$5 \parallel 20 = \frac{5 \times 20}{5 + 20} = 4 \Omega.$$

$$10 \parallel 15 = \frac{10 \times 15}{10 + 15} = 6 \Omega.$$

$$\therefore R_{ab} = 15 + (20 \parallel (4 + 6 + 10) + 25 = 50 \Omega.$$



Q2 Find the resistance between a & b for the circuit shown?

Solution:

$$X = [(45 + 15) \parallel 40] + 6$$

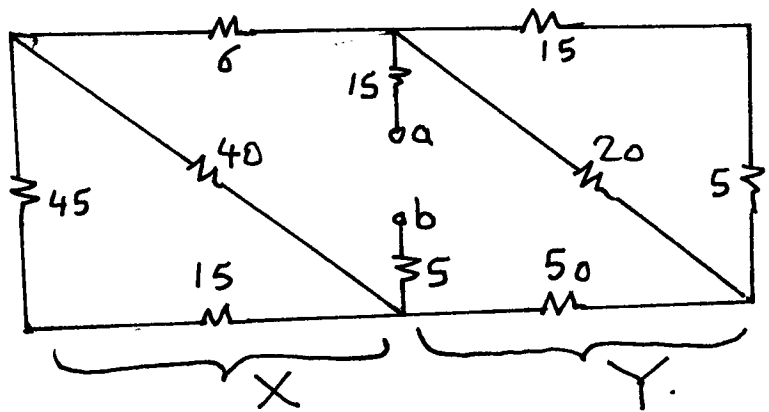
$$= (60 \parallel 40) + 6 = 30 \Omega.$$

$$Y = [(15 + 5) \parallel 20] + 50$$

$$= (20 \parallel 20) + 50 = 60 \Omega.$$

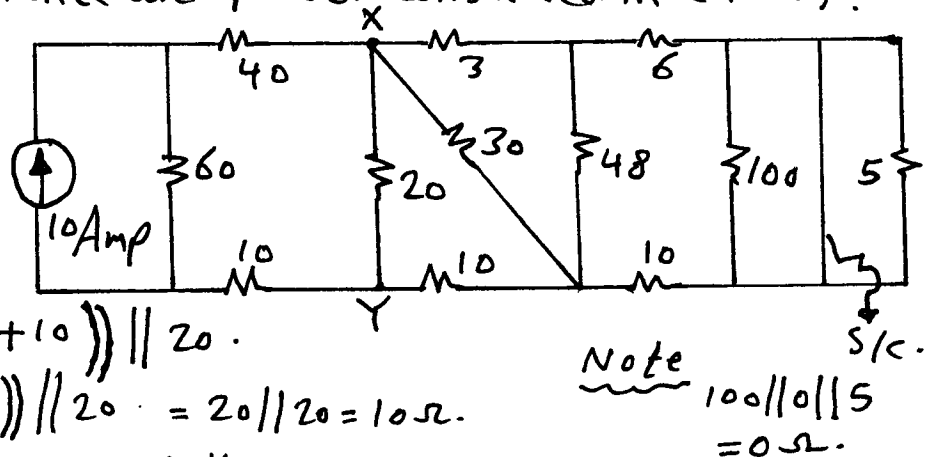
$$X \parallel Y = 30 \parallel 60 = \frac{30 \times 60}{30 + 60} = 20 \Omega.$$

$$\therefore R_{ab} = 5 + 15 + 20 = 40 \Omega$$



Q3 Find the total resistance and power consumed in (40 ohm)?

Solution:



$$R_{XY} = \left[\left(\left((6 + 10) \parallel 48 \right) + 3 \right) \parallel 30 \right] + 10 \parallel 20.$$

$$= \left(\left[\left((16 \parallel 48) + 3 \right) \parallel 30 \right] + 10 \right) \parallel 20 = 20 \parallel 20 = 10 \Omega.$$

$$\therefore R_{eq} = 60 \parallel (40 + 10 + R_{XY}) = 60 \parallel 60 = 30 \Omega.$$

$$\therefore I_{in 40 \Omega} = 10 \frac{60}{60 + 60} = 5 \text{ Amp (Current Divider rule) (C.D.R.)}$$

$$\therefore P_{in 40 \Omega} = I^2 \cdot R = (5)^2 \times 40 = 1000 \text{ watt} = 1 \text{ Kwatt}.$$

Q4] Find the Value of (V_s) and power consumed in (60Ω)?

Solution:

$$30 \parallel 60 = 20\Omega.$$

$$\therefore R_{eq} = 10 + \left[(30 + 20) \parallel (50 + 25) \right] \times 25 \text{ Amp}$$

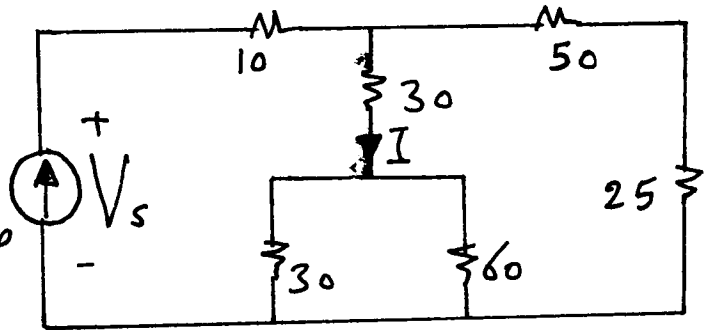
$$= 10 + (50 \parallel 75) = 10 + 30 = 40\Omega.$$

$$\therefore V_s = I \cdot R_{eq} = 25 \times 40 = \underline{1000 \text{ Volt}}.$$

$$I = 25 \frac{75}{75 + 50} = 15 \text{ Amp (C.D.R.)}$$

$$\therefore I_{60\Omega} = 15 \frac{30}{30 + 60} = 5 \text{ Amp. (C.D.R.)}$$

$$\therefore P_{in 60\Omega} = I^2 \cdot R = (5)^2 \times 60 = \underline{1500 \text{ Watt}} = \underline{1.5 \text{ kW}}.$$



Q5] Find the resistance between a & b for the circuit?

Solution:

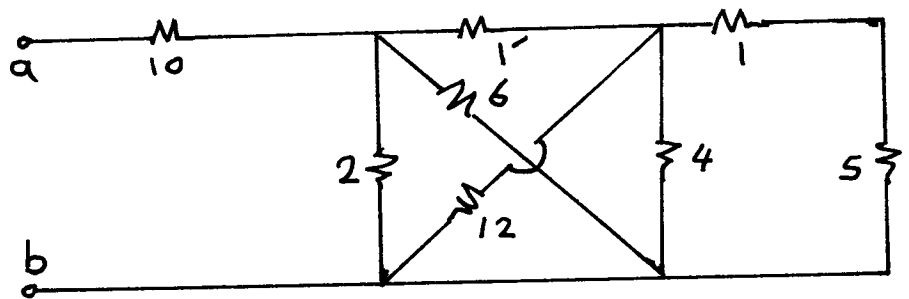
$$1 + 5 = 6\Omega.$$

$$12 \parallel 4 \parallel (1 + 5)$$

$$12 \parallel 4 \parallel 6 = 2\Omega.$$

$$2 + 1' = 3\Omega$$

$$\therefore R_{ab} = 10 + (2 \parallel 3 \parallel 6) = \underline{11\Omega}.$$



Q6] Find the resistance between a & b for the circuit?

Solution:

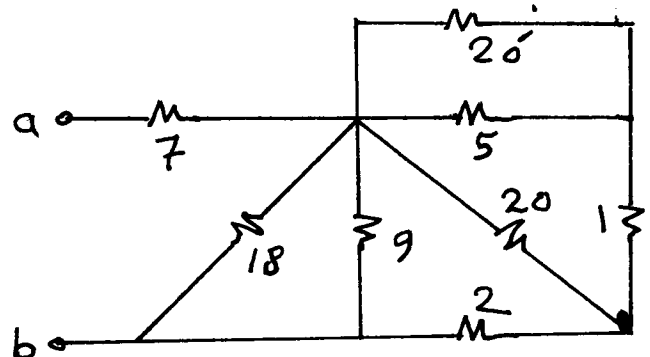
$$20' \parallel 5 = \frac{20' \times 5}{20' + 5} = 4\Omega.$$

$$4 + 1 = 5\Omega.$$

$$5 \parallel 20 = 4\Omega.$$

$$4 + 2 = 6\Omega.$$

$$\therefore R_{ab} = 7 + (18 \parallel 9 \parallel 6) = \underline{10\Omega}$$



Q7/ Find the current (I) and Power Consumed in (5-Ω)? when

- 1- The switch (S) open.
- 2- " " (S) closed.

Solution:

1- $10 \parallel 40 = \frac{10 \times 40}{10 + 40} = 8 \Omega$.

$\therefore I_x = \frac{150}{5 + 2 + 8} = 10 \text{ Amp.}$

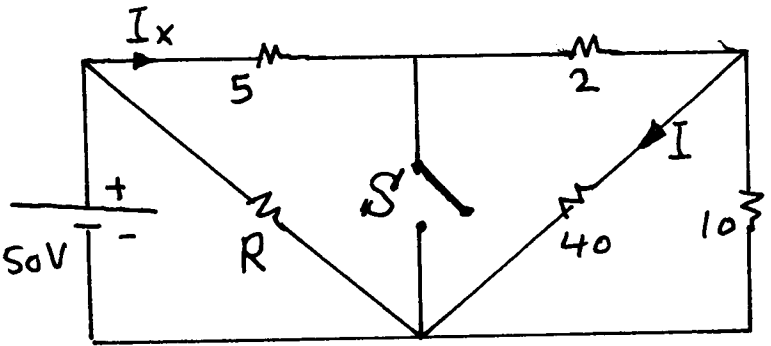
$\therefore P_{in 5\Omega} = 10^2 \times 5 = \underline{500 \text{ Watt.}}$

$\therefore I = 10 \times \frac{10}{10 + 40} = \underline{2 \text{ Amp (C.D.R.)}}$

2- $I_x = \frac{150}{5} = 30 \text{ Amp.}$

$\therefore P_{in 5\Omega} = 30^2 \times 5 = \underline{4500 \text{ Watt.}}$

$\therefore I = 0 \rightarrow$ because (S) closed (S/c).



Q8/ Find the value of (V), i_1 & i_2 ?

Solution:

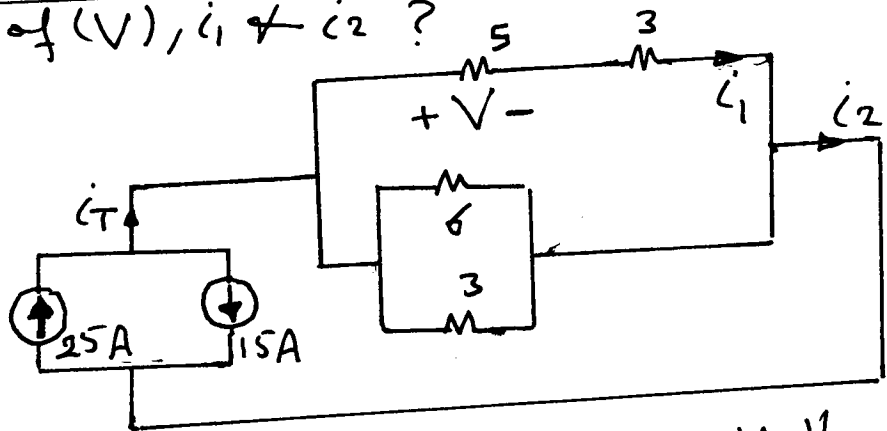
$i_T = i_2$ (same current)
 $= 25 - 15 = 10 \text{ Amp.}$

$5 + 3 = 8 \Omega$.

$6 \parallel 3 = 2 \Omega$.

$\therefore i_1 = 10 \times \frac{2}{2 + 8} = 2 \text{ Amp.}$

$\therefore V = I \cdot R = 2 \times 5 = \underline{10 \text{ Volt.}}$



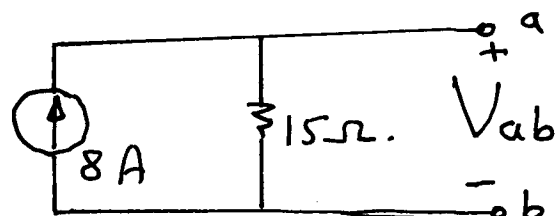
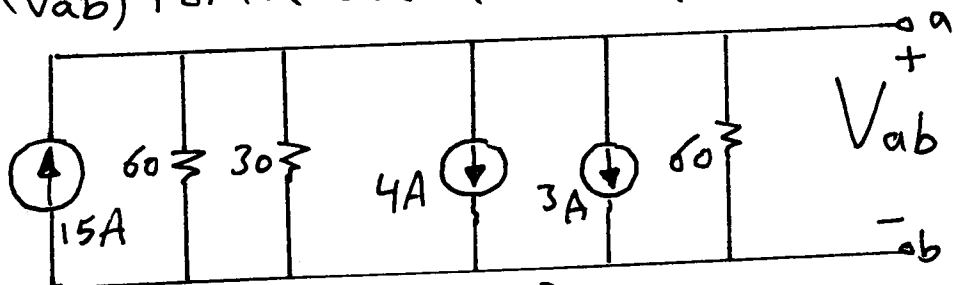
Q9/ Find the Voltage (V_{ab}) For the circuit shown?

Solution:

$I_T = 15 - 4 - 3 = 8 \text{ A.}$

$R_{eq} = 60 \parallel 30 \parallel 60$
 $= 15 \Omega$.

$\therefore V_{ab} = I_T \times R_{eq}$
 $= 8 \times 15 = \underline{120 \text{ Volt}}$



Q10] Given that the total power supply is (250W), find R, i_1, i_2 & i_3 ?

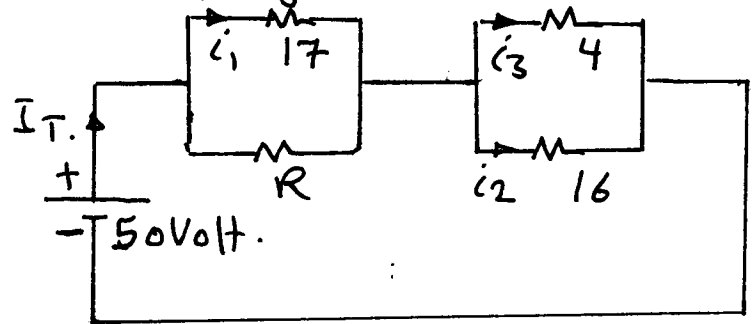
Solution:

Since $P_T = 250 \text{ Watt}$.

$$\therefore P_T = V_T \cdot I_T$$

$$250 = 50 \times I_T$$

$$\therefore I_T = \frac{250}{50} = 5 \text{ Amp.}$$



$$\therefore I_T = i_2 + i_3 \longrightarrow i_2 = 5 \frac{4}{4+16} = 1 \text{ Amp. (C.D.R.)}$$

$$\therefore i_3 = 5 - 1 = 4 \text{ Amp OR } i_3 = 5 \frac{16}{4+16} = 4 \text{ Amp.}$$

$$V_{4\Omega} = V_{16\Omega} = 4 \times 4 = 16 \text{ Volt OR } 1 \times 16 = 16 \text{ Volt.}$$

$$V_{17\Omega} = V_R = 50 - 16 = 34 \text{ Volt.}$$

$$\therefore i_1 = \frac{34}{17} = 2 \text{ Amp. } \& R = \frac{34}{3} = 11.333 \Omega.$$

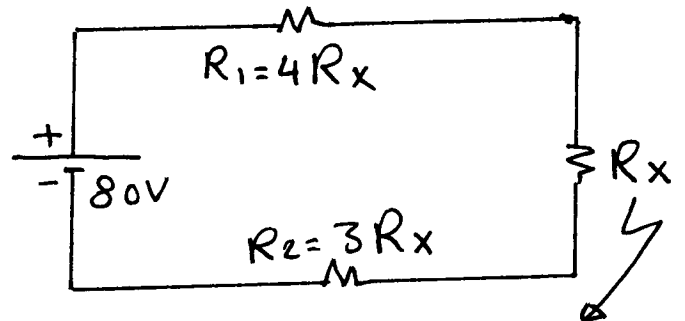
$$\longrightarrow 3 = 5 - 2 = 3 \text{ Amp.}$$

Q11] Find the value of (R_x) and the power supply by (80V)?

Solution:

$$R_T = 4R_x + R_x + 3R_x = 8R_x.$$

$$\therefore I_T = \frac{V_T}{R_T} = \frac{80}{8R_x} = \frac{10}{R_x} \text{ Amp.}$$



$$50 = (I_T)^2 \times R_x$$

$$50 = \left(\frac{10}{R_x}\right)^2 \cdot R_x.$$

$$\therefore R_x = \frac{100}{50} = 2 \Omega.$$

$$\therefore I_T = \frac{10}{2} = 5 \text{ Amp.}$$

$$\& \text{ Power supply } (P_T) = V \cdot I = 80 \times 5 = 400 \text{ Watt.}$$

Q12] Find the value of R_1 & R_2 ?

Solution:

$$I_T = I_{5\Omega} = \frac{20}{5} = 4 \text{ Amp.}$$

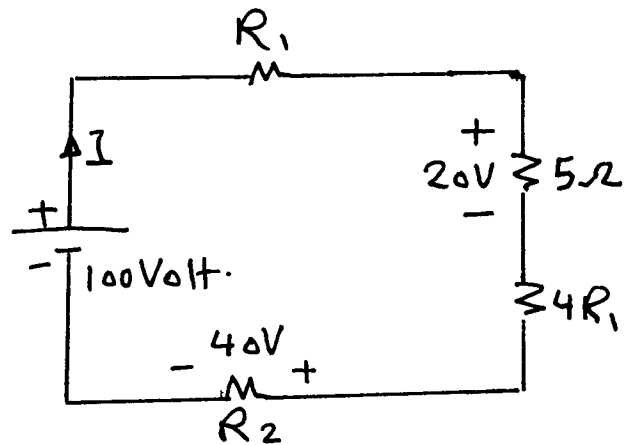
$$\therefore 40 = 4 \times R_2.$$

$$\therefore R_2 = 10 \Omega$$

$$R_{eq} = \frac{V_T}{I_T} = \frac{100}{4} = 25 \Omega.$$

$$25 = R_1 + 5 + 4R_1 + R_2 = 5R_1 + 15.$$

$$\therefore 10 = 5R_1 \longrightarrow \therefore R_1 = 2 \Omega.$$



Q13] For the circuit find i_1, i_2, i_3 & V ?

Solution:

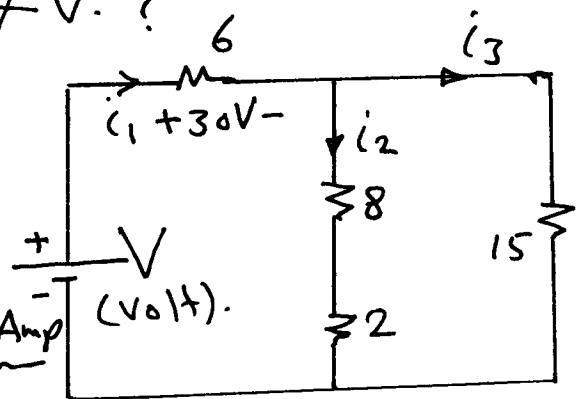
$$i_1 = \frac{30}{6} = 5 \text{ Amp.}$$

$$i_2 = 5 \frac{15}{15 + (8+2)} = 3 \text{ Amp.}$$

$$i_3 = 5 - 3 = 2 \text{ Amp OR } i_3 = 5 \frac{10}{25} = 2 \text{ Amp}$$

$$R_{eq} = 15 \parallel (8+2) + 6 = 12 \Omega.$$

$$\therefore V = I \cdot R_{eq} = 5 \times 12 = 60 \text{ Volt.}$$



Q14] For the circuit, find (I & I_s)?

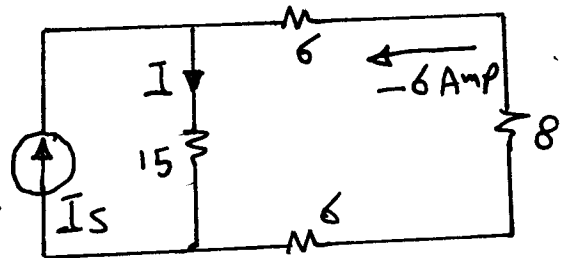
Solution: $6+8+6 = 20 \Omega.$

$$\therefore V_{20\Omega} = 20 \times 6 = 120 \text{ Volt.}$$

$$V_{20\Omega} = V_{15\Omega} = 120 \text{ Volt (parallel).}$$

$$\therefore I = \frac{120}{15} = 8 \text{ Amp}$$

$$\text{& } I_s = 8 + 6 = 14 \text{ Amp (Kirchhoff current Law) (K.C.L.)}$$



Q15] For the circuit, find (V) & Power supply by (10 Amp)?

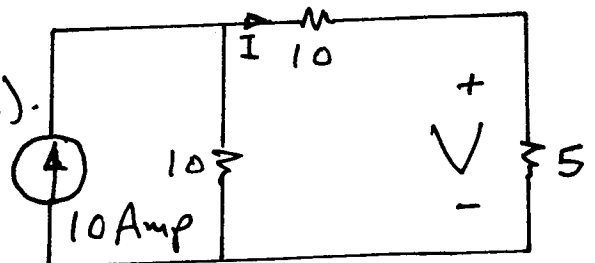
Solution:

$$I = 10 \frac{10}{10 + (10+5)} = 4 \text{ Amp (C.D.R.)}$$

$$\therefore V_{5\Omega} = 5 \times 4 = 20 \text{ Volt.}$$

$$R_{eq} = 10 \parallel (10+5) = 10 \parallel 15 = 6 \Omega.$$

$$\therefore P_T = I^2 \cdot R = (10)^2 \times 6 = 600 \text{ Watt.}$$



Q16 Find the Unknown quantities for the circuit?

Solution:

$$V_{12} = V_{9\Omega} = V_R \text{ (parallel)}$$

$$= 3 \times 12 = 36 \text{ Volt.}$$

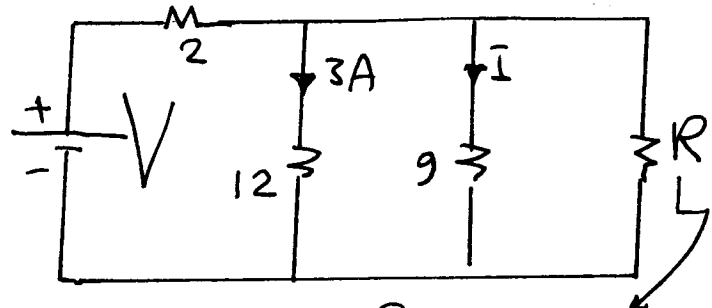
$$\therefore I = \frac{36}{9} = 4 \text{ Amp.}$$

$$P = 72 = \frac{V^2}{R} = \frac{(36)^2}{R} \rightarrow \therefore R = \frac{(36)^2}{72} = 18 \Omega$$

$$\therefore I_{18\Omega} = \frac{36}{18} = 2 \text{ Amp} \rightarrow \therefore I_T = 3 + 4 + 2 = 9 \text{ Amp.}$$

$$R_{eq} = 2 + (12 \parallel 9 \parallel 18) = 2 + 4 = 6 \Omega.$$

$$\therefore V = I_T \cdot R_{eq} = 9 \times 6 = 54 \text{ Volt.}$$



$$P_{in R} = 72 \text{ watt.}$$

Q17 Find the resistance between (a & b)?

$\Delta \Rightarrow Y$
xyz

$$R_x = \frac{3 \times 3}{3 + 3 + 3} = 1 \Omega.$$

$$1 + 1 + 6 = 8 \Omega.$$

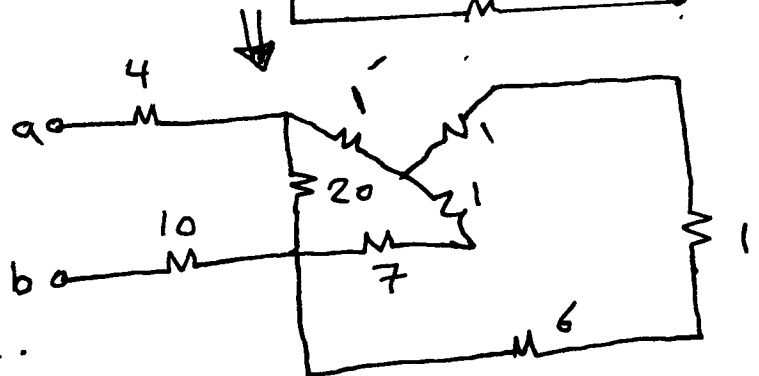
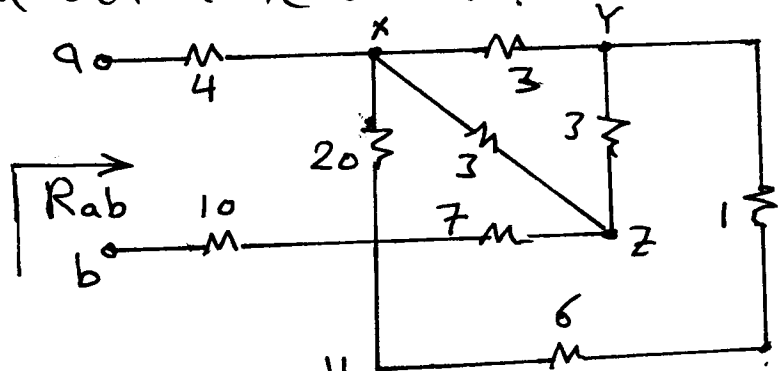
$$1 + 7 = 8 \Omega.$$

$$8 \parallel 8 = 4 \Omega.$$

$$4 + 1 = 5 \Omega.$$

$$5 \parallel 20 = \frac{5 \times 20}{5 + 20} = 4 \Omega.$$

$$\therefore R_{eq} = 4 + 4 + 10 = 18 \Omega.$$



Q18 Use $\Delta \Rightarrow Y$ transformation, find the voltage (V)?

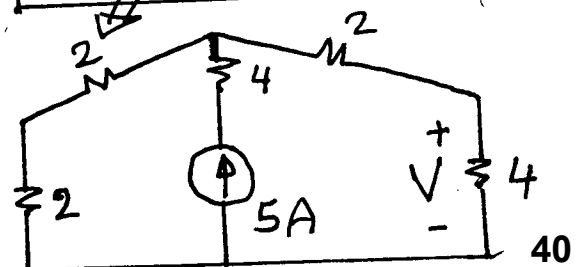
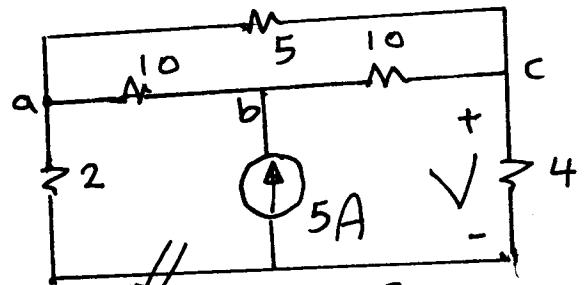
$$\left. \begin{aligned} R_a &= \frac{5 \times 10}{5 + 10 + 10} = 2 \Omega \\ R_b &= \frac{10 \times 10}{5 + 10 + 10} = 4 \Omega \end{aligned} \right\} R_c = \frac{5 \times 10}{5 + 10 + 10} = 2 \Omega.$$

$$2 + 4 = 6 \Omega$$

$$2 + 2 = 4 \Omega.$$

$$\therefore I_{4\Omega} = 5 \frac{4}{4 + 6} = 2 \text{ Amp.}$$

$$\therefore V = 2 \times 4 = 8 \text{ Volt.}$$



Q19] If $R_{eq} = 50\Omega$, find the value of (R) ?

Solution:

$$12 \parallel 12 \parallel 12 = 4\Omega$$

$$10 + R + 4 = 14 + R$$

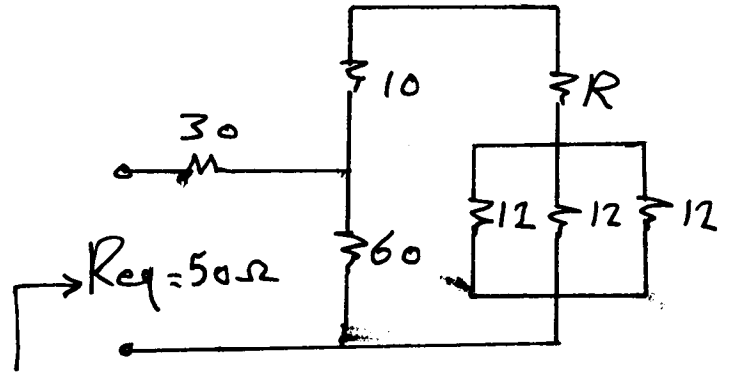
$$R_{eq} = 50 = 30 + (60 \parallel (14 + R))$$

$$20 = 60 \parallel (14 + R)$$

$$20 = \frac{60(14 + R)}{60 + (14 + R)}$$

$$1480 + 20R = 840 + 60R$$

$$\rightarrow \therefore 40R = 640 \rightarrow \therefore R = 16\Omega$$



Q20] Find R_{ab} ?

Solution:

$$12 \parallel 60 = \frac{12 \times 60}{12 + 60} = 10\Omega$$

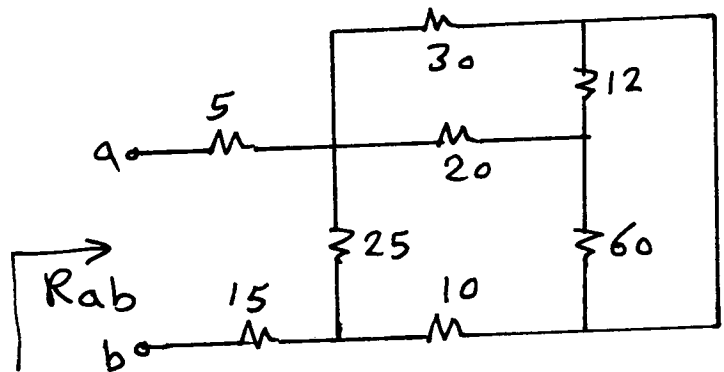
$$10 + 20 = 30\Omega$$

$$30 \parallel 30 = 15\Omega$$

$$15 + 10 = 25$$

$$\therefore R_{ab} = 5 + 15 + 25 \parallel 25 = 32.5\Omega$$

$$\text{OR } R_{ab} = 5 + 15 + \left[25 \parallel \left(\{ (12 \parallel 60) + 20 \} \parallel 30 \right) + 10 \right] = 32.5\Omega$$



Q21] Find the value of (R) ?

Solution:

$$V_{10\Omega} = 10 \times 5 = 50\text{ Volt}$$

$$\therefore V_{(4+5)} = 140 - 50 = 90\text{ V}$$

$$\therefore I_{(4+5)} = \frac{90}{9} = 10\text{ Amp}$$

$(a \rightarrow b)$

(K.C.L) at point a.

$$\therefore I_{2\Omega} = 10 - 5 = 5\text{ Amp}$$

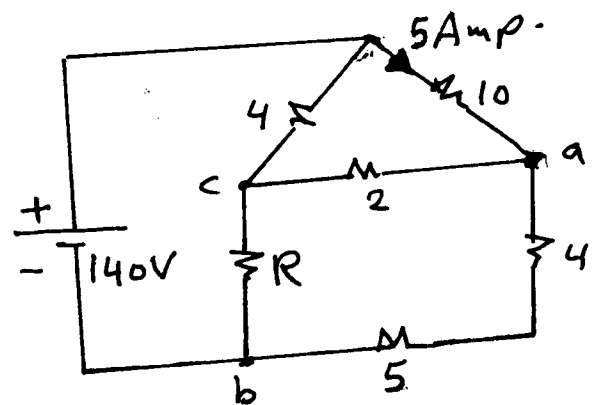
$$\therefore V_{2\Omega} = 10\text{ Volt}$$

$$\therefore V_{4\Omega} = 50 - 10 = 40\text{ Volt} \rightarrow \therefore I_{4\Omega} = \frac{40}{4} = 10\text{ Amp}$$

$$\therefore \text{K.C.L at b} \rightarrow I_R = 10 - 5 = 5\text{ Amp}$$

$$\text{K.V.L at (cab)} \rightarrow V_R = V_{2\Omega} + V_{(4-5)} = 10 + 90 = 100\text{ V}$$

$$\therefore R = \frac{V_R}{I_R} = \frac{100}{5} = 20\Omega$$



Q22 Find the power consumed in (10Ω) ?

Solution:

$$6 + 4 = 10\Omega.$$

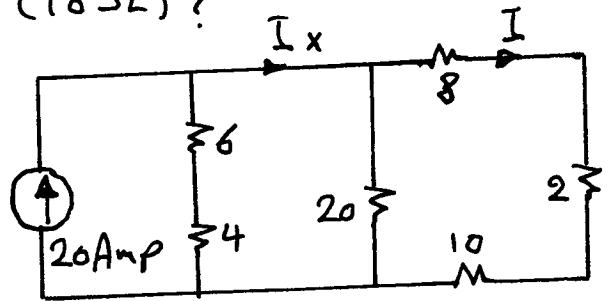
$$8 + 2 + 10 = 20\Omega.$$

$$I_x = 20 \frac{10}{10 + (20 \parallel 20)} = 20 \frac{10}{10 + 10}$$

$$= 10 \text{ Amp. (C.D.R.)}$$

$$I = 10 \frac{20}{20 + 20} = 5 \text{ Amp.}$$

$$\therefore P_{in 10\Omega} = (5)^2 \times 10 = 250 \text{ Watt.}$$



Q23 For the circuit find the value of (I_S) ?

Solution:

$$8 + 12 + 4 = 24\Omega.$$

$$\therefore V_{24\Omega} = 2 \times 24 = 48 \text{ Volt.}$$

$$V_{12\Omega} = 12 \times 4 = 48 \text{ Volt.}$$

$$6 + 10 = 16\Omega.$$

$$\therefore V_{16\Omega} = 80 \text{ Volt.}$$

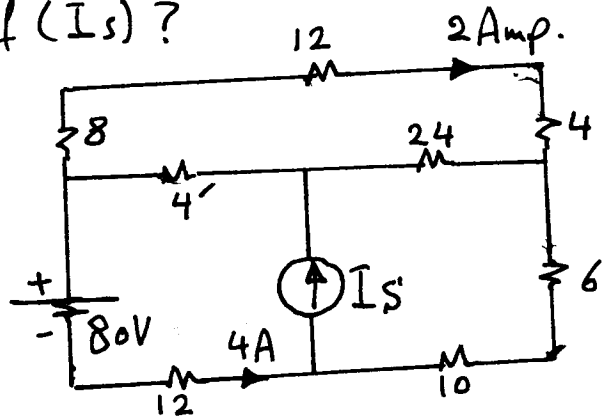
$$\therefore I_{16\Omega} = \frac{80}{16} = 5 \text{ Amp.}$$

$$\therefore I_{24\Omega} = 5 - 2 = 3 \text{ Amp.}$$

$$\therefore V_{24\Omega} = 3 \times 24 = 72 \text{ Volt.}$$

$$\therefore V_{4\Omega} = 72 - 48 = 24 \text{ Volt} \rightarrow \therefore I_{in 4\Omega} = \frac{24}{4} = 6 \text{ Amp.}$$

$$\therefore I_S = 3 + 6 = 9 \text{ Amp (Kirchhoff current Law) (K.C.L.)}$$



Q24 Find the value of (V) ?

Solution:

$$I_{15\Omega} = \frac{60}{15} = 4 \text{ Amp.}$$

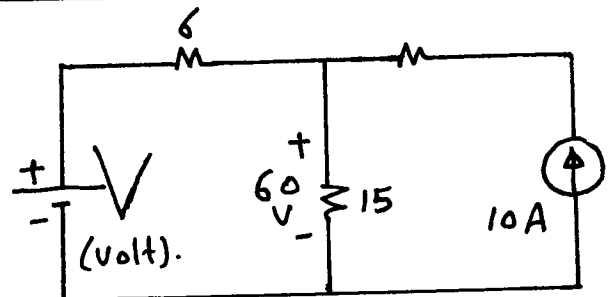
$$\therefore I_{6\Omega} = 10 - 4 = 6 \text{ Amp.}$$

$$\therefore V_{6\Omega} = 6 \times 6 = 36 \text{ Volt.}$$

By Kirchhoff Voltage Law (K.V.L).

$$V + 36 = 60$$

$$\therefore \underline{V = 24 \text{ Volt.}}$$



Q25 Find the current (I)?

Solution:

①

By Kirchhoff's Law:

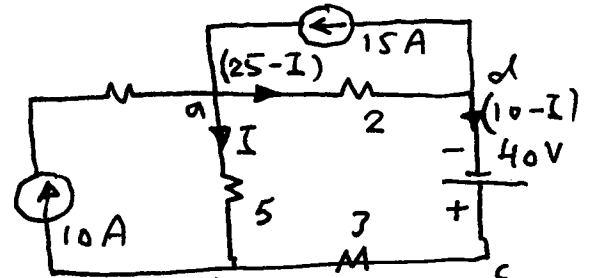
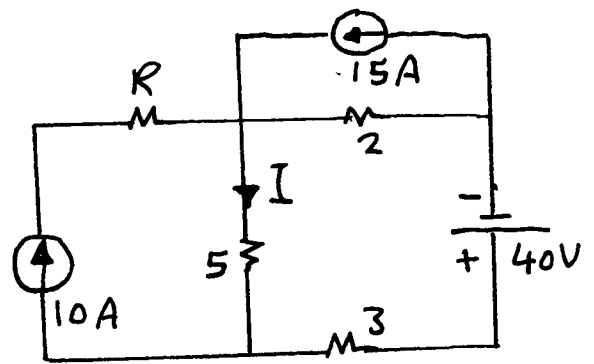
for the circuit of a, b, c & d:

$$40 + 5I = 2(25 - I) + 3(10 - I)$$

$$40 + 5I = 50 - 2I + 30 - 3I$$

$$10I = 40$$

$$\therefore I = 4 \text{ Amp.}$$



K-L.

② By loop method:

$$I = 10 - I_1$$

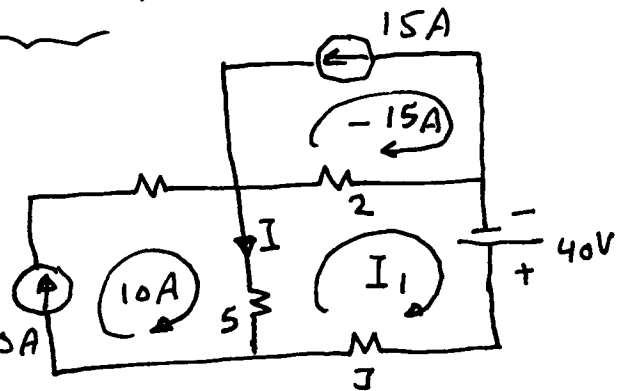
loop (1):

$$40 = (2 + 3 + 5)I_1 - 5 \times 10 - 2(-15)$$

$$40 = 10I_1 - 50 + 30$$

$$60 = 10I_1 \longrightarrow I_1 = 6 \text{ Amp.}$$

$$\therefore I = 10 - 6 = 4 \text{ Amp.}$$



③ By superposition:

* I by 40Volt \rightarrow 10A & 15A (o/c).

$$= \frac{40}{2+5+3} = \frac{40}{10} = 4 \text{ Amp} \uparrow$$

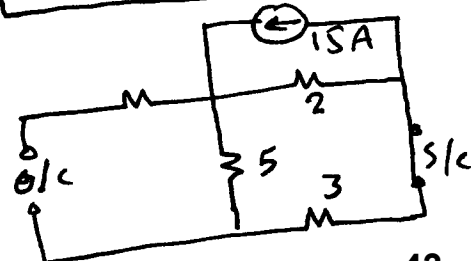
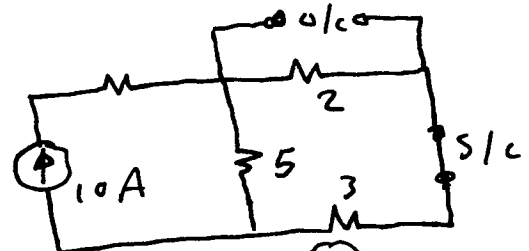
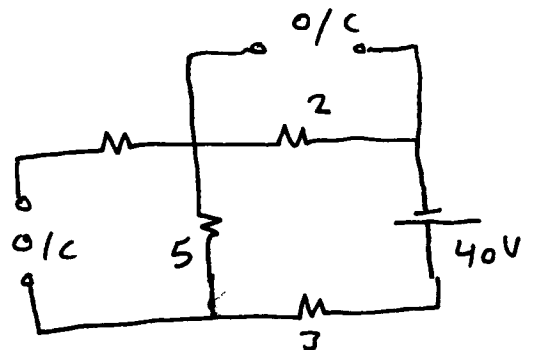
* I by 10Amp \rightarrow 15A (o/c) & 40V (s/c).

$$= 10 \frac{(2+3)}{5+(2+3)} = 5 \text{ Amp} \downarrow \text{ (C.D.R.)}$$

* I by 15Amp \rightarrow 10Amp (o/c) & 40V (s/c).

$$= 15 \frac{2}{2+(3+5)} = 3 \text{ Amp} \downarrow \text{ (C.D.R.)}$$

$$\therefore I = 5 + 3 - 4 = 4 \text{ Amp.}$$



Q26 Find the current (I)?

Solution:

① By K.L:

take (abcd)

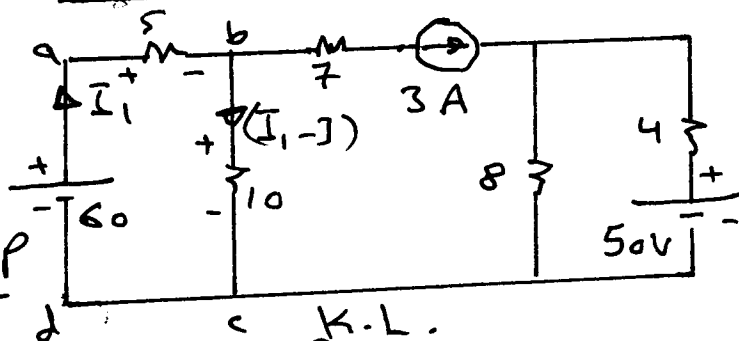
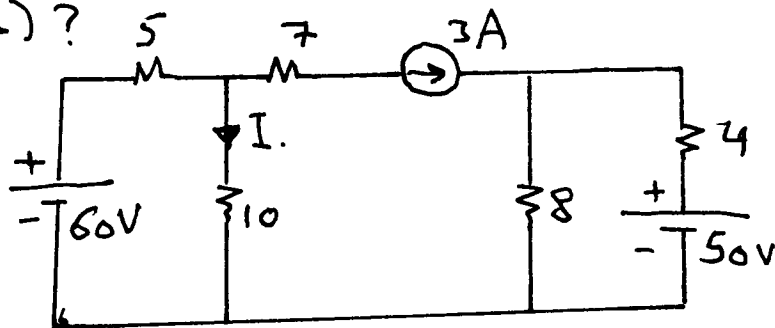
$$60 = 5I_1 + 10(I_1 - 3)$$

$$60 = 15I_1 - 30$$

$$\therefore 90 = 15I_1$$

$$I_1 = 6 \text{ Amp}$$

$$\therefore I = (I_1 - 3) = 6 - 3 = 3 \text{ Amp}$$



② By loop method:

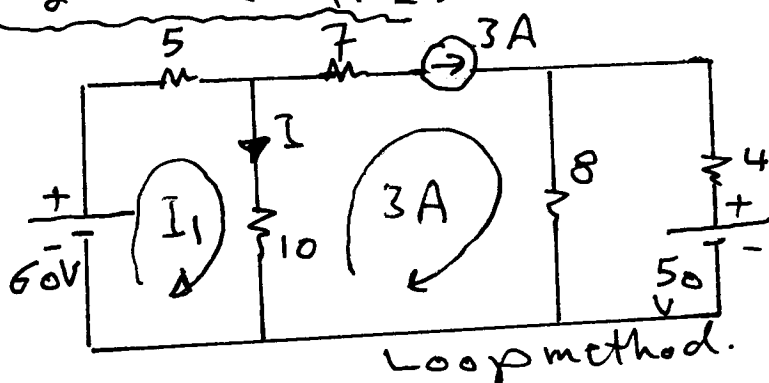
$$I = I_1 - 3$$

loop (1):

$$60 = (5 + 10)I_1 - 10 \times 3$$

$$90 = 15I_1$$

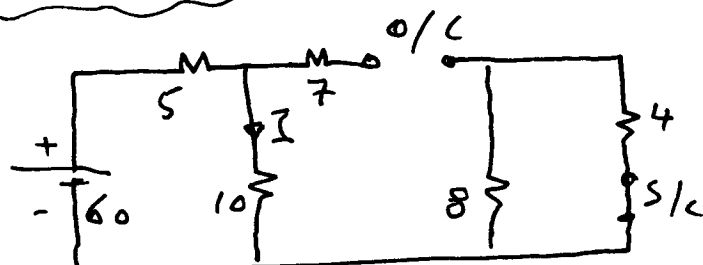
$$\therefore I_1 = 6 \text{ Amp} \rightarrow \therefore I = 6 - 3 = 3 \text{ Amp}$$



③ By superposition.

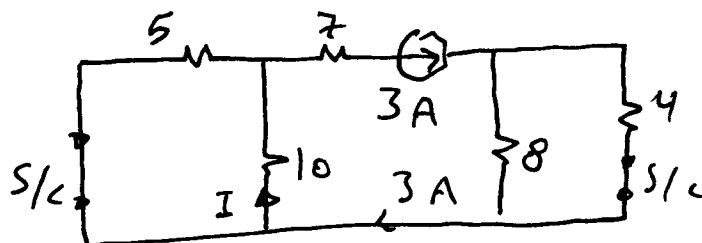
* By 60V \rightarrow 3A (o/c), 50V (s/c).

$$I = \frac{60}{5 + 10} = 4 \text{ Amp} \downarrow$$



* By 3A \rightarrow 60V & 50V (s/c)

$$I = 3 \frac{5}{5 + 10} = 1 \text{ Amp} \uparrow$$



* By 50V \rightarrow 60V (s/c), 3A (o/c).

$$I = 0 \text{ (open circuit)}$$

$$\therefore I = 4 - 1 = 3 \text{ Amp}$$

④ By Nodal Voltage

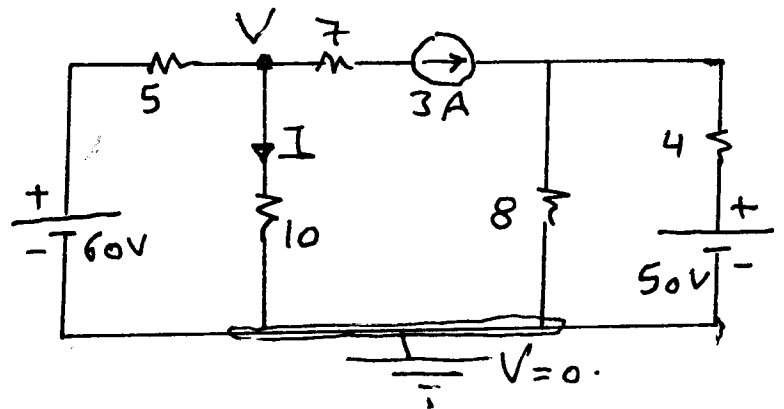
$$I = \frac{V}{10}$$

Node (V):

$$\left(\frac{1}{5} + \frac{1}{10}\right)V - \frac{60}{5} = -3$$

$$(2+1)V - 120 = -30$$

$$3V = 90 \rightarrow V = 30 \text{ Volt} \rightarrow \therefore I = \frac{30}{10} = 3 \text{ Amp}$$

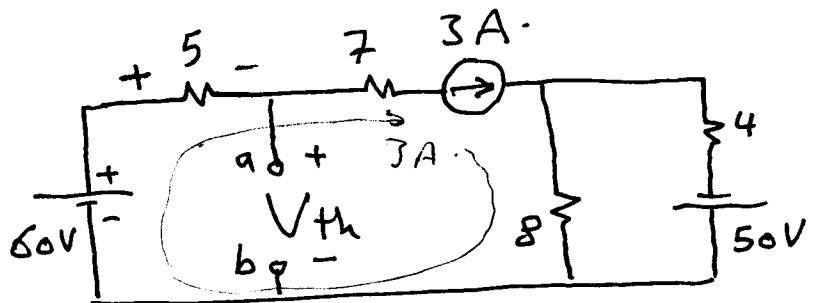


⑤ By Thevenin's Theorem

$$I = \frac{V_{th}}{R_{th} + 10}$$

* $V_{th} \rightarrow$

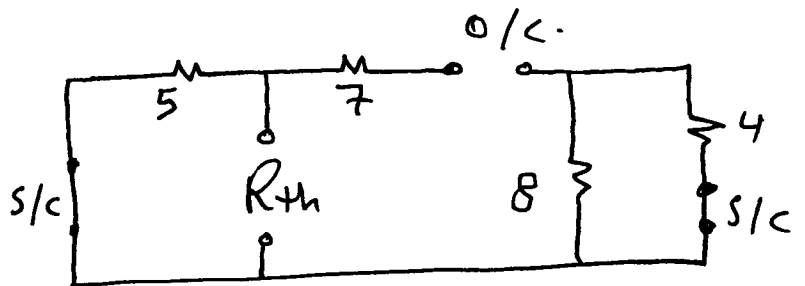
$$V_{th} = 60 - 5 \times 3 \quad \left. \begin{array}{l} \\ \end{array} \right\} \text{K.V.L.} \\ = 45 \text{ Volt}$$



* $R_{th} \rightarrow$

$$\therefore R_{th} = 5 \Omega \text{ (only)}$$

$$\therefore I = \frac{45}{5+10} = 3 \text{ Amp}$$



Q27 Find the power delivered by (6 Amp)?

Solution:

$$P = V \times I = V \times 6 = 6V \text{ (Watt)}$$

\therefore find (V) across the 6Amp.

① By Nodal Voltage

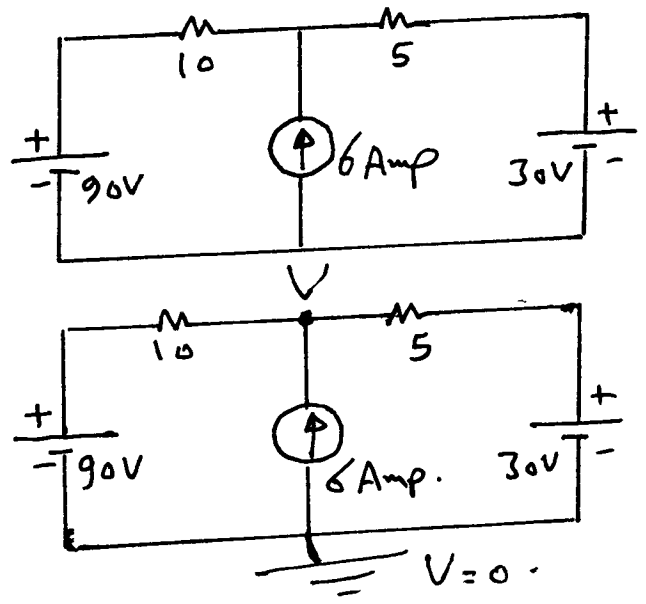
$$\left(\frac{1}{5} + \frac{1}{10}\right)V - \frac{90}{10} - \frac{30}{5} = 6$$

$$(2+1)V - 90 - 60 = 60$$

$$3V = 210$$

$$\therefore V = 70 \text{ Volt}$$

$$\therefore P = 70 \times 6 = 420 \text{ Watt}$$



② By K-L

By the circuit (a, b, c & d).

$$90 = 10I + 5(I+6) + 30$$

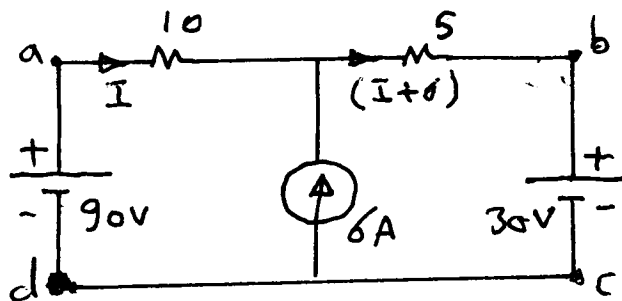
$$30 = 15I$$

$$\therefore I = 2 \text{ Amp}$$

$$\therefore \text{The voltage across (6Amp)} = 90 - 10 \times I = 70 \text{ Volt}$$

$$\text{OR } V_{6\text{Amp}} = 30 + 5(I+6) = 30 + 40 = 70 \text{ Volt}$$

$$\therefore P = 6 \times 70 = 420 \text{ Watt}$$



Q28 Find the value of I & V for the circuit.

① By K-L

take abcd:

$$40 + 20 = 5I + 3(5I)$$

$$60 = 20I$$

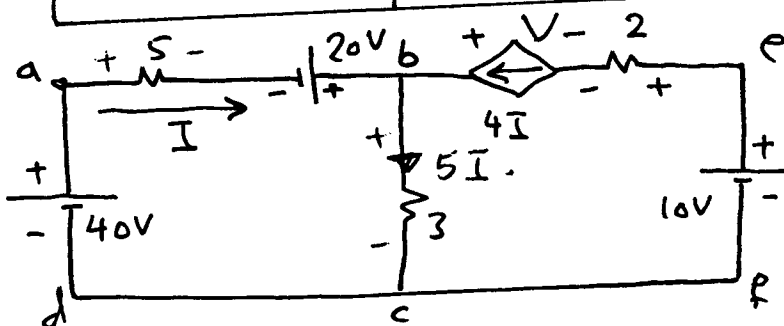
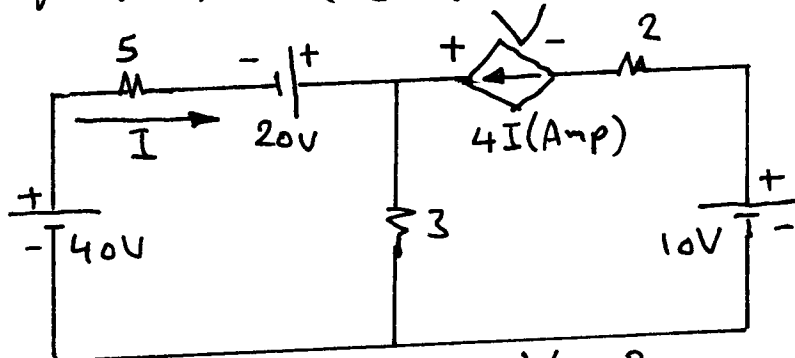
$$\therefore I = 3 \text{ Amp}$$

to find (V) take aefcd

$$10 + 15 + V = 40 + 20 + 2 \times 12$$

$$25 + V = 84$$

$$\therefore V = 59 \text{ Volt}$$



② By Nodal method

to find (I).

$$I = \frac{40 + 20 - V}{5} = \frac{60 - V}{5}$$

Node (V)

$$\left(\frac{1}{5} + \frac{1}{3}\right)V - \frac{60}{5} = 4I$$

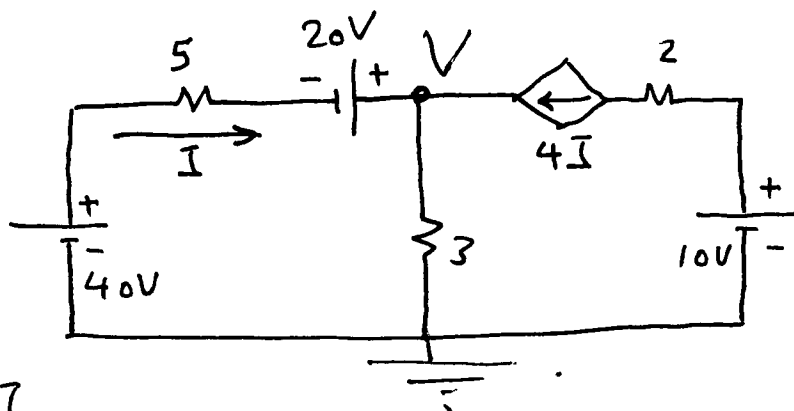
$$(3+5)V - 180 = 60I$$

$$8V - 180 = 60 \left(\frac{60 - V}{5}\right)$$

$$8V - 180 = 12(60 - V)$$

$$\therefore 20V = 900 \rightarrow \therefore V = 45 \text{ Volt}$$

$$\therefore I = \frac{60 - 45}{5} = 3 \text{ Amp}$$



Q29] Find (V_x) for the circuit shown

Solution:

① By Nodal Voltage

$$V_x = 12 - V.$$

Node (V):

$$\left(\frac{1}{3} + \frac{1}{6} + \frac{1}{8}\right)V - \frac{12}{3} - \frac{2V_x}{8} = 0$$

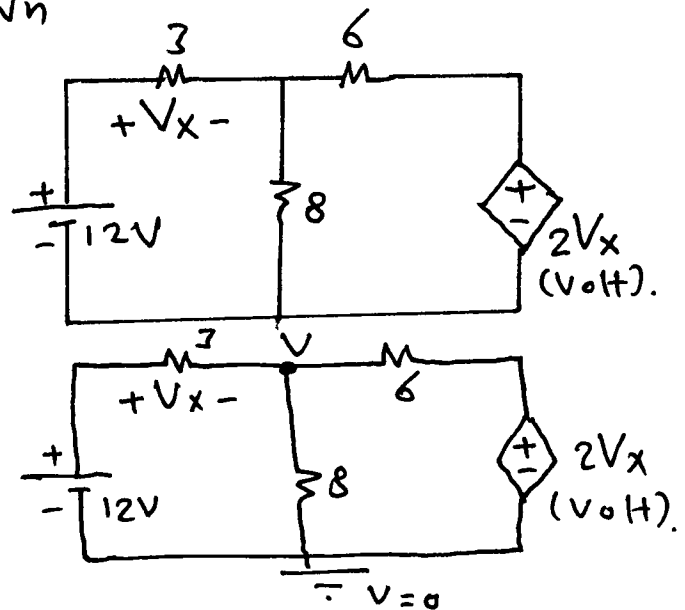
$$(8 + 4 + 3)V - 96 - 8V_x = 0.$$

$$15V - 8V_x = 96.$$

$$\therefore 15V - 8(12 - V) = 96.$$

$$23V = 192 \longrightarrow \therefore V = \frac{192}{23} = 8.347$$

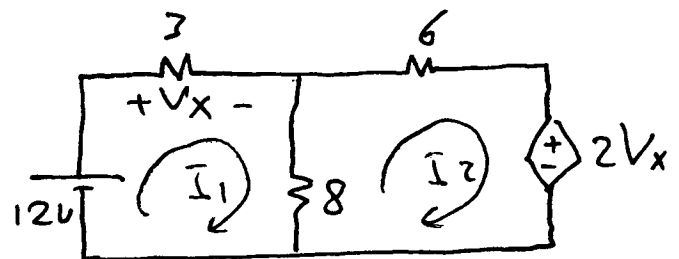
$$\text{and } V_x = 12 - 8.347 = \underline{3.653 \text{ Volt.}}$$



② By loop method.

Loop (1):

$$12 = (3 + 8)I_1 - 8I_2 \dots \dots \textcircled{1}.$$



Loop (2):

$$-2V_x = (6 + 8)I_2 - 8I_1 \dots \dots \textcircled{2}.$$

$$\text{But } V_x = 3I_1$$

$$\therefore -2(3I_1) = 14I_2 - 8I_1$$

$$\text{Solving eq (1) \& (2)} \longrightarrow I_1 = 1.217 \text{ Amp.}$$

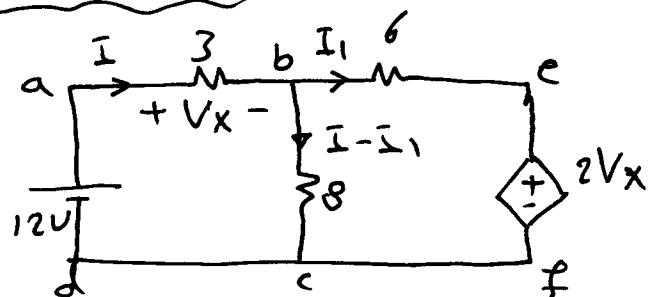
③ By K.L.

take abcd

$$12 = 3I + 8(I - I_1) \dots \dots \textcircled{1}.$$

take aefcd

$$12 = 3I + 6I_1 + 2V_x \dots \dots \textcircled{2}$$



$$\text{But } V_x = 3I.$$

$$\text{Solving eq (1) \& eq (2)} \longrightarrow I = 1.217 \text{ Amp.}$$

Q30 Use superposition to find (I)?

Solution:

* take (16V) \rightarrow 4A (o/c), 12V (s/c).

$$\therefore I = \frac{16}{6+2+8} = \frac{16}{16} \text{ Amp} \uparrow$$

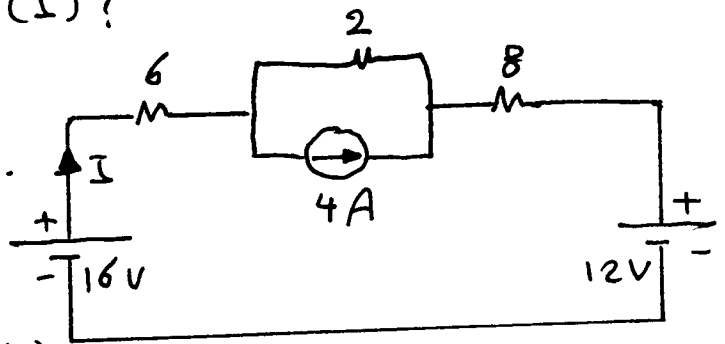
* take (12V) \rightarrow 16V (s/c), 4A (o/c).

$$\therefore I = \frac{12}{6+2+8} = \frac{12}{16} \text{ A} \downarrow$$

* take (4A) \rightarrow 12V & 16V (s/c).

$$\therefore I = 4 \frac{2}{2+6+8} = \frac{8}{16} \text{ A} \uparrow \text{ (C.D.R.)}$$

$$\therefore I = \frac{16}{16} + \frac{8}{16} - \frac{12}{16} = \frac{12}{16} = \underline{\underline{0.75 \text{ Amp}}}$$



Q31 Find V_{th} & R_{th} of the circuit between a & b?

Solution:

$$V_{th} = V_{ab} = V_{5\Omega}$$

Use superposition to find (V_{th}).

* take (12V) \rightarrow 2A (o/c).

$$V_{th1} = \frac{12}{4+8+5+8} \times 5 = \frac{12}{5} \text{ Volt}$$

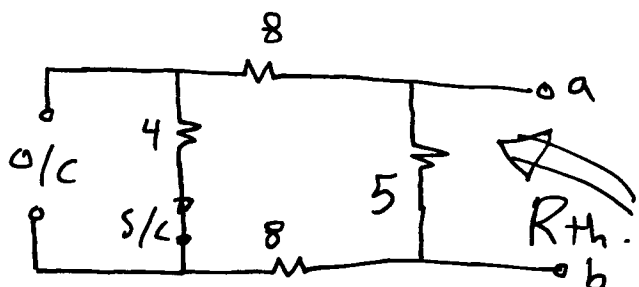
* take (2A) \rightarrow 12V (s/c).

$$V_{th2} = 2 \frac{4}{4+8+5+8} \times 5 = \frac{8}{5} \text{ Volt} \rightarrow \text{Use current divider rule.}$$

$$\therefore V_{th} = V_{th1} + V_{th2} = \frac{12}{5} + \frac{8}{5} = \frac{20}{5} = \underline{\underline{4 \text{ Volt}}}$$

$R_{th} \rightarrow$

$$\begin{aligned} R_{th} &= 5 // (8+4+8) \\ &= 5 // 20 \\ &= \frac{5 \times 20}{5+20} = \underline{\underline{4 \Omega}} \end{aligned}$$



Q32] Find (V_o) for the circuit?

Solution:

① By K.L. take (abcd)

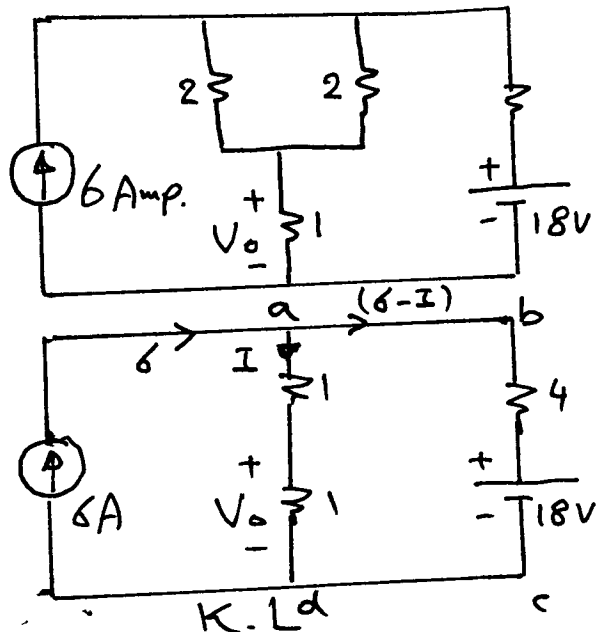
$$(1+1)I = 4(6-I) + 18$$

$$2I = 24 - 4I + 18$$

$$\therefore 6I = 42$$

$$\therefore I = 7 \text{ Amp}$$

$$\text{and } V_o = 7 \times 1 = 7 \text{ Volt}$$



② By loop method.

loop (I):

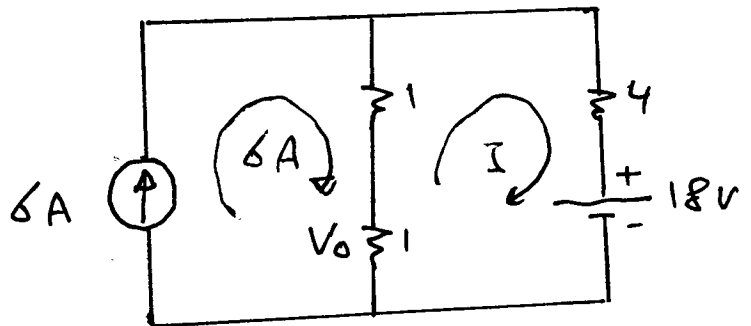
$$-18 = (4+1+1)I - 6 \times 2$$

$$-6 = 6I$$

$$\therefore I = -1 \text{ Amp}$$

But the current in $1\Omega = 6 - I = 6 - (-1) = 7 \text{ Amp}$

$$\therefore V_o = 7 \text{ Volt}$$



③ By Nodal method

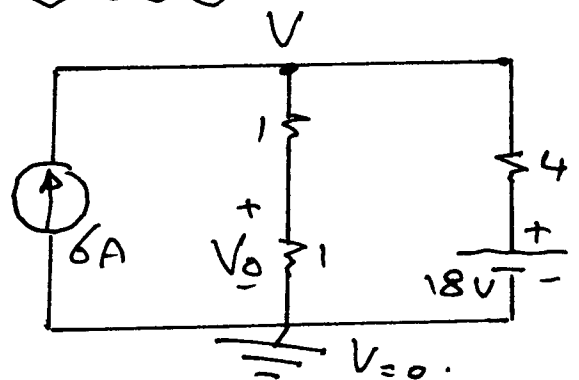
Node (V):

$$\left(\frac{1}{2} + \frac{1}{4}\right)V - \frac{18}{4} = 6$$

$$(2+1)V - 18 = 24$$

$$3V = 42$$

$$\therefore V = 14 \text{ Volt} \rightarrow \therefore V_o = \frac{14}{1+1} \times 1 = 7 \text{ Volt (voltage Divider rule)}$$



④ By superposition

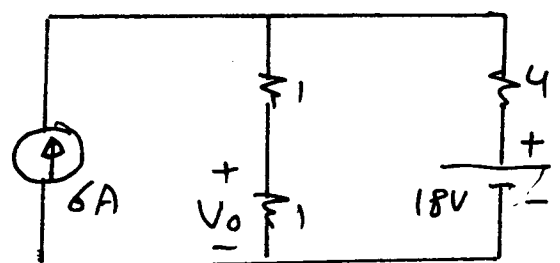
* By 18V \rightarrow 6Amp (o/c).

$$V_o = \frac{18}{4+1+1} \times 1 = 3 \text{ Volt (V.D.R.)}$$

* By 6A \rightarrow 18V (s/c).

$$V_o = \left(6 \times \frac{4}{4+1+1}\right) \times 1 = \frac{24}{6} = 4 \text{ Volt}$$

$$\therefore V_o = 4 + 3 = 7 \text{ Volt}$$



Q33 Find (V_o) for the circuit shown?

Solution

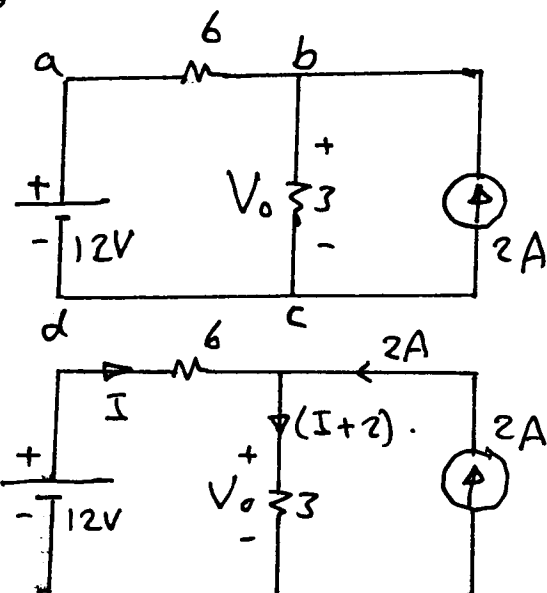
① By Kirchhoff Law take abcd

$$12 = 6I + 3(I + 2)$$

$$6 = 9I$$

$$\therefore I = \frac{6}{9} \text{ Amp}$$

$$\therefore V_o = 3(I + 2) = 3\left(\frac{6}{9} + 2\right) = 8 \text{ Volt}$$



② By loop method:

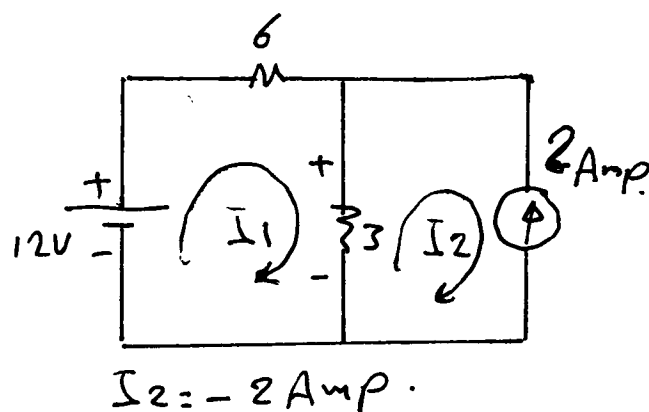
loop (1):

$$12 = (6 + 3)I_1 - 3(-2)$$

$$6 = 9I_1$$

$$\therefore I_1 = \frac{6}{9} \text{ Amp}$$

$$\text{But } V_o = 3(I_1 - I_2) = 3\left(\frac{6}{9} - (-2)\right) = 8 \text{ Volt}$$



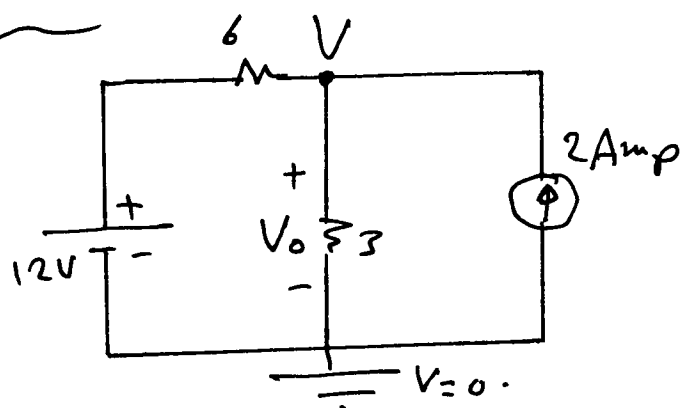
③ By Nodal method.

$$\left(\frac{1}{3} + \frac{1}{6}\right)V - \frac{12}{6} = 2$$

$$(2 + 1)V - 12 = 12$$

$$\therefore 3V = 24$$

$$\text{and } V = V_o = 8 \text{ Volt}$$



④ By Super position:

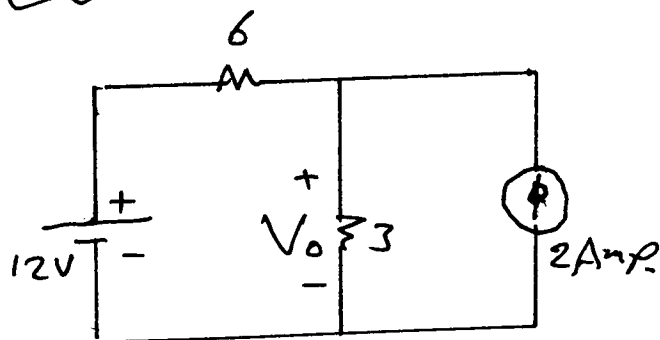
* By 12V \rightarrow 2Amp (o/c).

$$V_o = \frac{12}{6 + 3} \times 3 = 4 \text{ Volt (V.D.R.)}$$

* By 2Amp \rightarrow 12V (s/c).

$$V_o = \left(2 \times \frac{6}{6 + 3}\right) \times 3 = 4 \text{ Volt (C.D.R.)}$$

$$\therefore V_o = 4 + 4 = 8 \text{ Volt}$$



Q34 Find V_{th} between a & b for the circuit?

Solution:

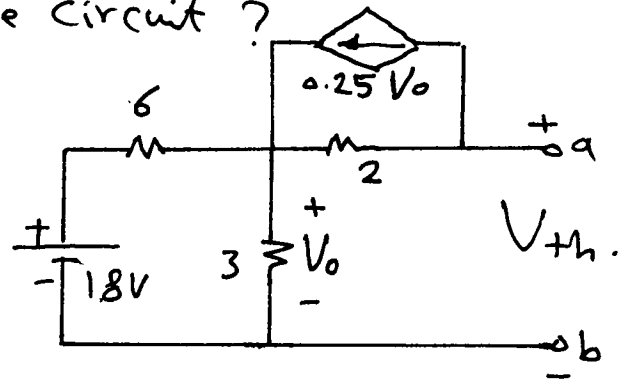
$$V_o = V_{th} + V_{2\Omega}$$

$$\therefore V_{th} = V_o - V_{2\Omega}$$

$$V_o = \left(\frac{18}{6+3} \right) \times 3 = \underline{6 \text{ Volt}} \cdot (\text{V.D.R}).$$

$$V_{2\Omega} = 2(0.25V_o) = 2 \times (0.25 \times 6) = 2 \times 3 = 3 \text{ Volt}.$$

$$\therefore V_{th} = 6 - 3 = \underline{3 \text{ Volt}}.$$



Q35 Find V_{th} between a & b for the circuit?

Solution:

* By Kirchhoff's Law:

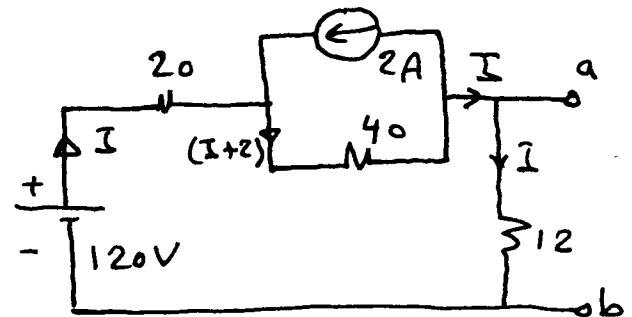
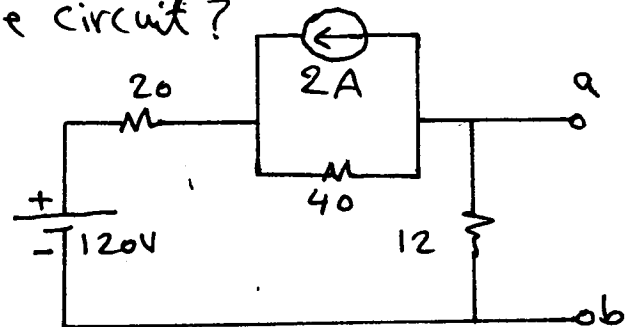
$$120 = 20I + 40(I+2) + 12I$$

$$= 20I + 80I + 80 + 12I$$

$$40 = 72I \longrightarrow I = \frac{40}{72} \text{ Amp.}$$

$$\text{But } V_{th} = V_{ab} = V_{12} = \frac{40}{72} \times 12 = \frac{40}{6}$$

$$V_{th} = \underline{6.666 \text{ Volt}}$$



* By loop method:

Loop (I):

$$120 = (20 + 40 + 12)I - 40(-2)$$

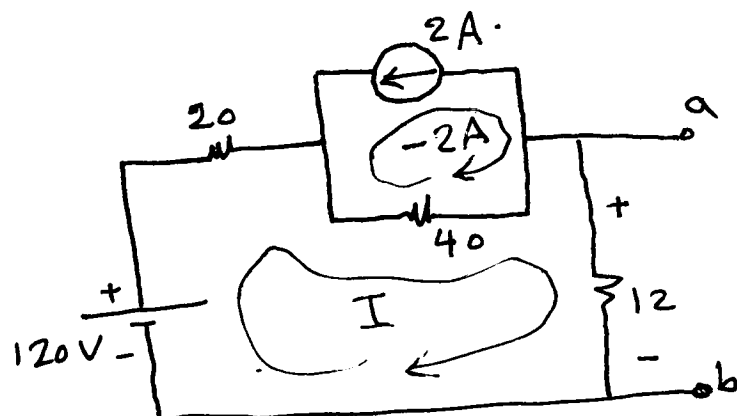
$$40 = 72I$$

$$\therefore I = \frac{40}{72} \text{ Amp.}$$

$$V_{th} = V_{ab} = V_{12\Omega}$$

$$= \left(\frac{40}{72} \right) \times 12 = \frac{40}{6} \text{ Volt.}$$

$$= \underline{6.666 \text{ Volt}}$$



Q36 Find (I_ϕ) for the circuit shown?

Solution:

* By loop method.

$$I_\phi = 5 - I.$$

loop(I):

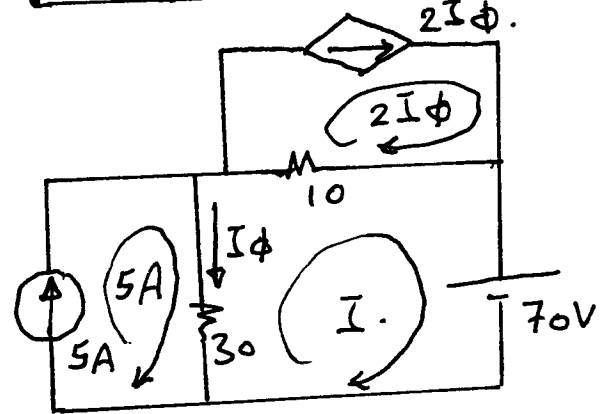
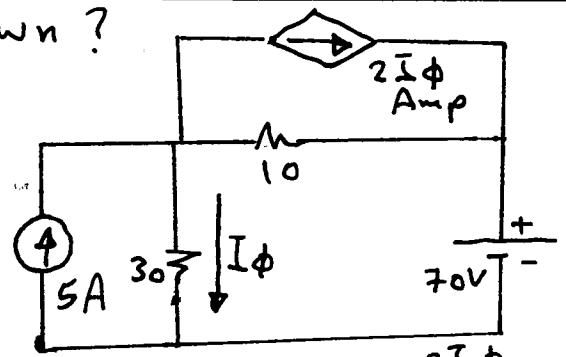
$$-70 = (10 + 30)I - 30 \times 5 - 10(2I_\phi).$$

$$80 = 40I - 20I_\phi.$$

$$\therefore 80 = 40(5 - I_\phi) - 20I_\phi.$$

$$\therefore 60I_\phi = 120$$

$$\therefore I_\phi = 2 \text{ Amp.}$$



* By Node:

$$V_1 = 70 \text{ Volt.}$$

Node(V).

$$\left(\frac{1}{10} + \frac{1}{30}\right)V - \frac{70}{10} = 5 - 2I_\phi.$$

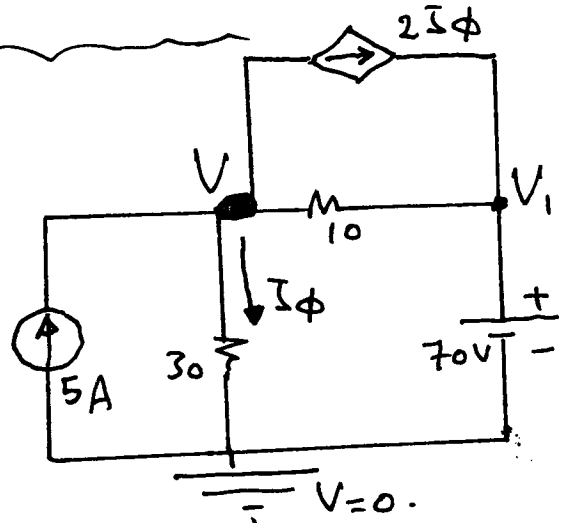
$$(3+1)V - 210 = 150 - 60I_\phi.$$

$$\text{But } I_\phi = \frac{V}{30} \rightarrow \therefore V = 30I_\phi.$$

$$\therefore 4(30I_\phi) - 210 = 150 - 60I_\phi.$$

$$180I_\phi = 360.$$

$$\therefore I_\phi = 2 \text{ Amp.}$$



Q37 Find V_{th} & R_{th} between a & b for the circuit?

Solution:

$R_{th} \rightarrow$

$$5 \parallel 20 = \frac{5 \times 20}{5 + 20} = 4 \Omega$$

$$\therefore R_{th} = (4 + 8) \parallel 12 = 6 \Omega$$

$V_{th} \rightarrow$

$$V_{th} = V_{20\Omega} + V_{8\Omega}$$

$$I = \frac{72}{R_{eq}}$$

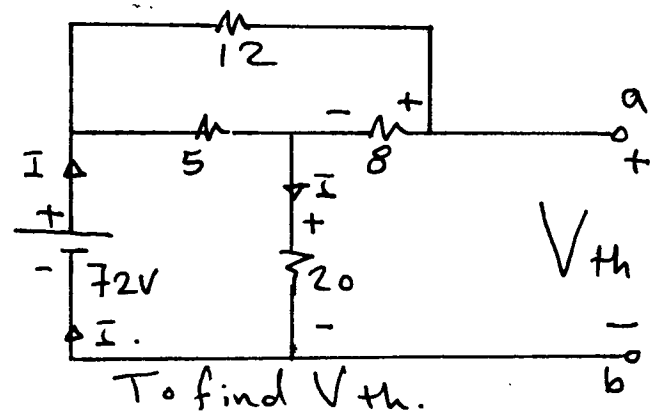
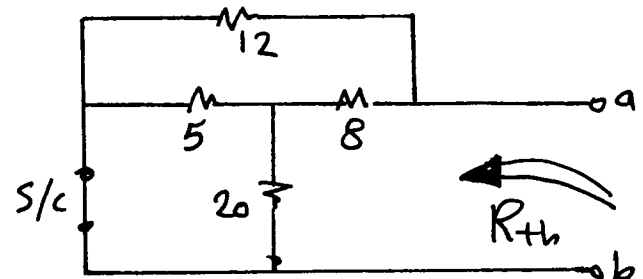
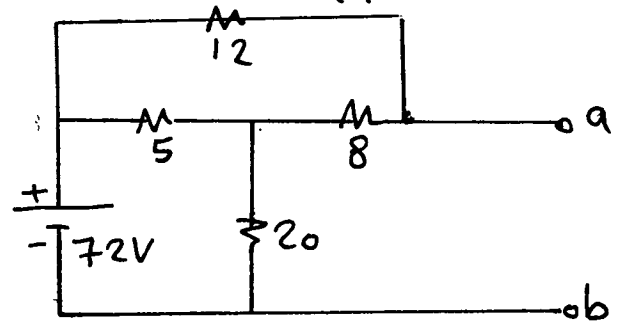
$$= \frac{72}{((12+8) \parallel 5) + 20}$$

$$= \frac{72}{4 + 20} = \frac{72}{24} = 3 \text{ Amp}$$

$$\therefore V_{20\Omega} = 3 \times 20 = \underline{60 \text{ Volt}}$$

$$V_{8\Omega} = 8 \left(3 \frac{5}{5+12+8} \right) = 8 \frac{15}{25} = \frac{24}{5} = \underline{4.8 \text{ Volt}} \quad (\text{C.D.R.})$$

$$\therefore V_{th} = 60 + 4.8 = \underline{64.8 \text{ Volt}}$$



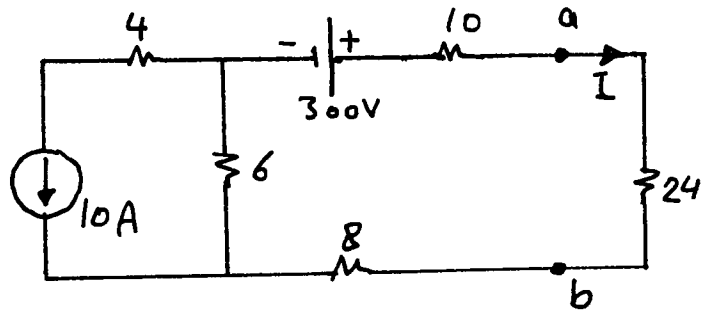
Q38 Find the current (I) and power consumed in (10Ω) ?

Solution:

* By Superposition

** By $300V \rightarrow 10A$ (o/c).

$$I_1 = \frac{300}{6+10+8+24} = \frac{300}{48} \text{ Amp.} \downarrow$$



** By $10A \rightarrow 300V$ (S/c).

$$I_2 = 10 \frac{6}{6+10+8+24} = \frac{60}{48} \uparrow$$

$$\therefore I = I_1 - I_2 = \frac{300}{48} - \frac{60}{48} = \frac{240}{48} = 5 \text{ Amp.}$$

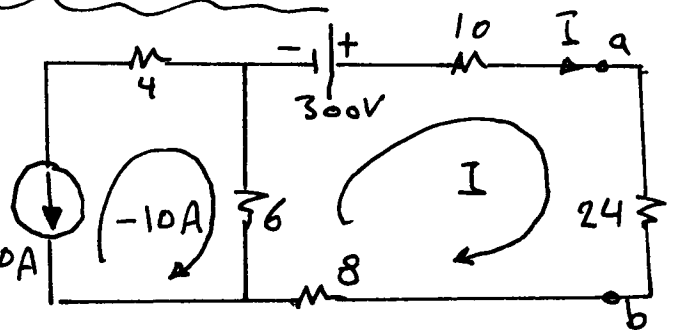
* By loop.

loop (I) :-

$$300 = (6+8+10+24)I - 6(-10). \quad 10A$$

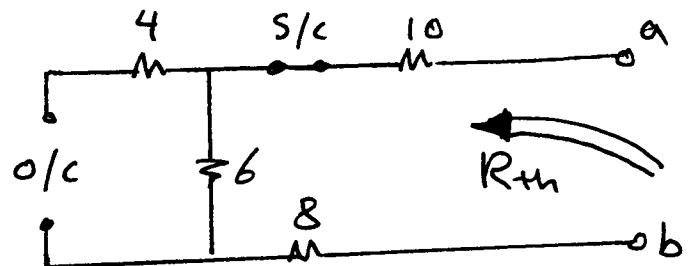
$$300 = 48I + 60.$$

$$\therefore I = \frac{300-60}{48} = \frac{240}{48} = 5 \text{ Amp.}$$



* By Thevenin's Theorem

$$I = \frac{V_{th}}{R_{th} + 24}$$

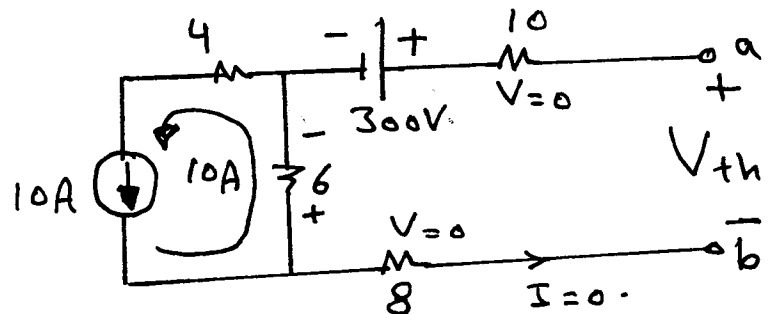


$$R_{th} \rightarrow R_{th} = 6+8+10 = 24\Omega.$$

$V_{th} \rightarrow$

$$V_{th} = 300 - V_{6\Omega}.$$

$$= 300 - 60 = 240 \text{ Volt.}$$



$$\therefore I = \frac{240}{24+24} = \frac{240}{48} = 5 \text{ Amp.}$$

Q39 Find (I) for the circuit shown?

Solution

① By Nodal method:

$$I = \frac{V_1 - V_2}{2}$$

Node (V_1):

$$\left(\frac{1}{2} + \frac{1}{10} + \frac{1}{15}\right)V_1 - \frac{V_2}{2} - \frac{100}{10} = 0$$

$$(15 + 3 + 2)V_1 - 15V_2 - 300 = 0$$

$$20V_1 - 15V_2 = 300$$

$$4V_1 - 3V_2 = 60 \text{ --- (1)}$$

Node (V_2):

$$\left(\frac{1}{2} + \frac{1}{4} + \frac{1}{4}\right)V_2 - \frac{V_1}{2} - \frac{20}{4} = 0$$

$$(2 + 1 + 1)V_2 - 2V_1 = 20$$

$$4V_2 - 2V_1 = 20 \text{ --- (2)}$$

Solving eq (1) & (2)

$$V_1 = 30 \text{ Volt} \quad \& \quad V_2 = 20 \text{ Volt}$$

$$\therefore I = \frac{30 - 20}{2} = 5 \text{ Amp}$$

② By Thevenin's Theorem:

$$I = \frac{V_{th}}{R_{th} + 2}$$

$$R_{th} \rightarrow R_{th} = (10 \parallel 15) + (4 \parallel 4) \\ = 6 + 2 = 8 \Omega$$

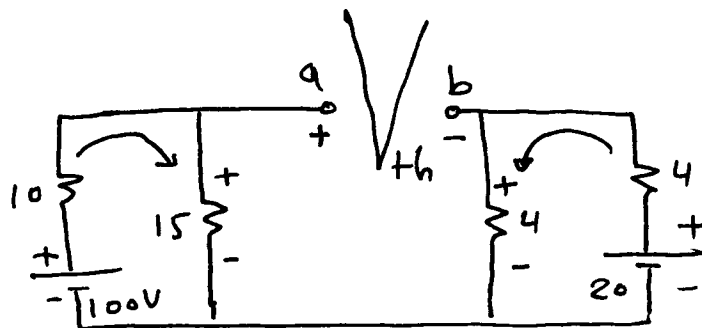
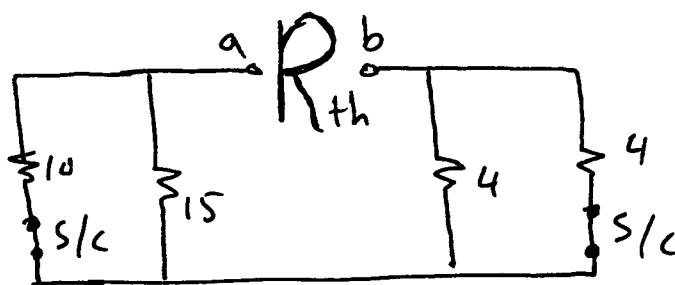
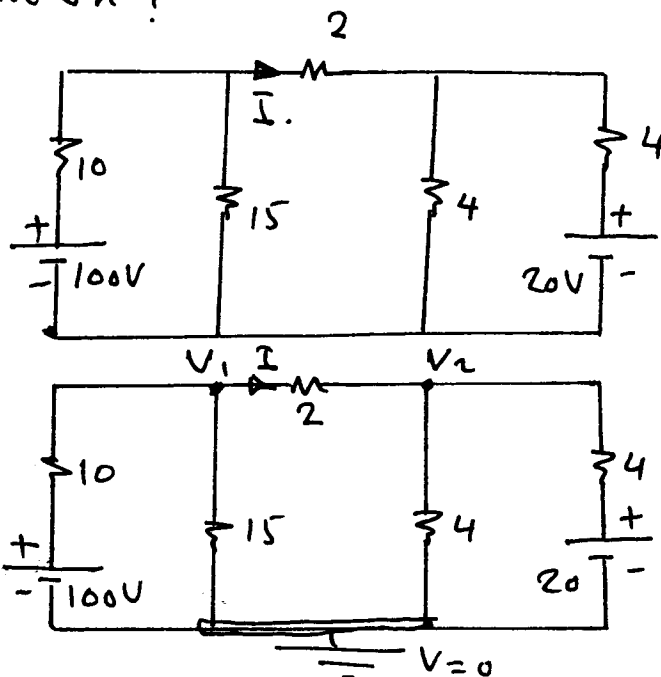
$V_{th} \rightarrow$

$$V_{th} = V_{15\Omega} - V_{4\Omega}$$

$$= \left(\frac{100}{10+15}\right) \times 15 - \left(\frac{20}{4+4}\right) \times 4$$

$$= 60 - 10 = 50 \text{ Volt}$$

$$\therefore I = \frac{50}{2+8} = 5 \text{ Amp}$$



Q.40 Find the current (I) for the circuit ?

Solution:

① By loop method:-

$$I = I_1 - I_2.$$

loop(1):

$$30 = 11I_1 - 2I_2 \text{ ----- ①}$$

loop(2):

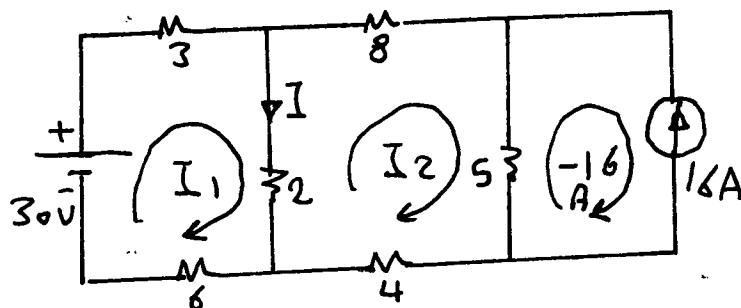
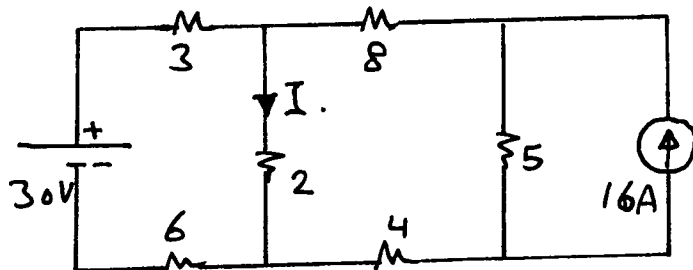
$$0 = 19I_2 - 2I_1 - 5(-16).$$

$$-80 = 19I_2 - 2I_1 \text{ ----- ②}$$

Solving eq(1) & (2)

$$I_1 = 2 \text{ Amp} \quad \& \quad I_2 = -4 \text{ Amp}.$$

$$\therefore I = 2 - (-4) = 6 \text{ Amp}.$$

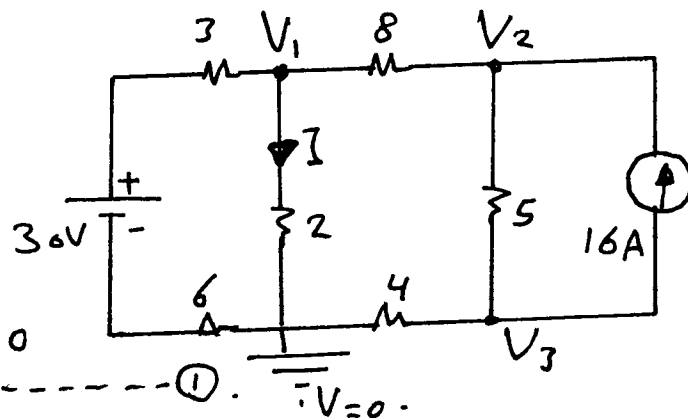


② By Nodal voltage method:

$$I = \frac{V_1}{2} \text{ Amp}.$$

Node(V1):

$$\left(\frac{1}{9} + \frac{1}{2} + \frac{1}{8}\right)V_1 - \frac{V_2}{8} - \frac{30}{9} = 0 \text{ ----- ①}$$



Node(V2): $\left(\frac{1}{8} + \frac{1}{5}\right)V_2 - \frac{V_1}{8} - \frac{V_3}{5} = 16 \text{ ----- ②}$

Node(V3): $\left(\frac{1}{4} + \frac{1}{5}\right)V_3 - \frac{V_2}{5} = -16 \text{ ----- ③}$

Solving eq(1), (2) & (3).

$$V_1 = 12 \text{ Volt}.$$

$$\therefore I = \frac{12}{2} = 6 \text{ Amp}.$$

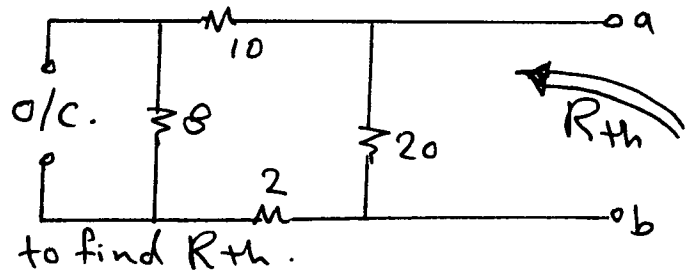
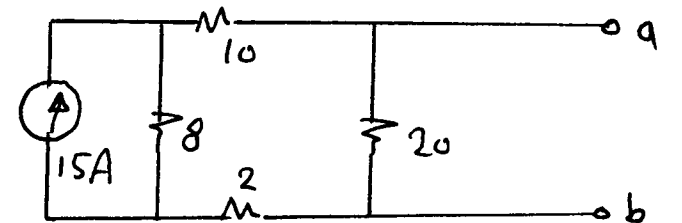
Q.41 Find V_{th} & R_{th} between a & b for the Circuit?

Solution:

$R_{th} \rightarrow$

$$R_{th} = (2 + 8 + 10) \parallel 20$$

$$= 10 \Omega.$$



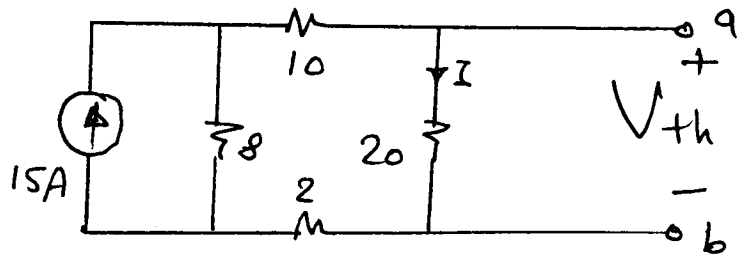
$V_{th} \rightarrow$

$$V_{th} = V_{ab} = V_{20\Omega}.$$

$$= 20 I.$$

$$= 20 \left(15 \frac{8}{8 + 10 + 20 + 2} \right)$$

$$= 60 \text{ Volt}$$



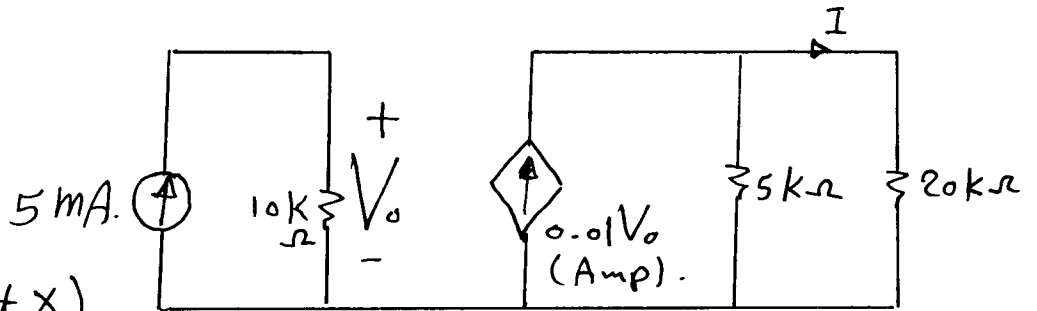
Q.42 Find the current, Voltage & Power Consumed in ($20k\Omega$)?

Solution:

$$V_0 = 5 \times 10^{-3} \times 10 \times 10^3$$

$$= 50 \text{ Volt.}$$

(from the circuit X)



\rightarrow Circuit (X).

\rightarrow Circuit (Y).

from Circuit (Y).

$$I = (0.01 V_0) \frac{5}{5 + 20} = 0.5 \frac{5}{25}.$$

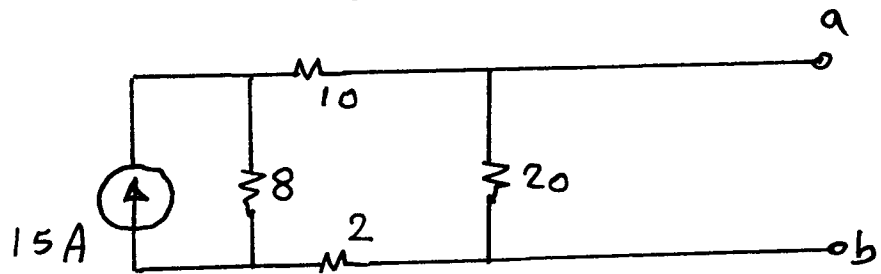
$$= 0.1 \text{ Amp.}$$

$$V_{20k\Omega} = 0.1 \times 20 \times 10^3 = 2000 \text{ Volt} = 2 \text{ kV.}$$

$$P_{20k\Omega} = I^2 \cdot R = (0.1)^2 \times 20 \times 1000 = 200 \text{ Watt} = 0.2 \text{ kW.}$$

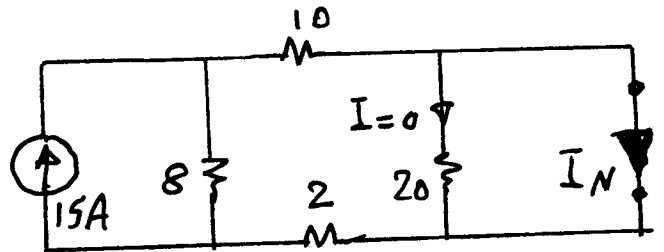
Q43 Find (I_N) between a & b?

Solution:



$$I_N = 15 \frac{8}{8+10+2} = \frac{120}{20} = 6 \text{ Amp.}$$

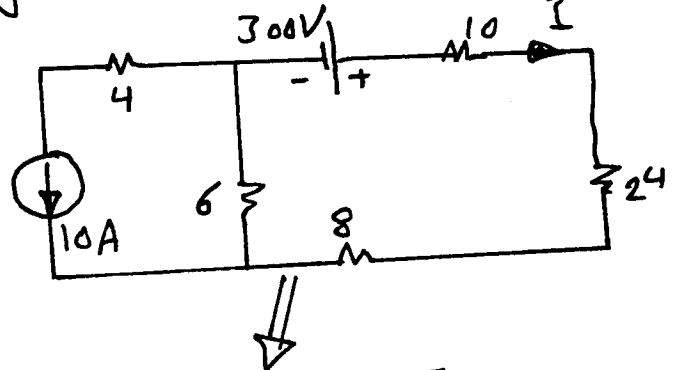
(current Divide rule).



Q44 Find the current (I) by using (Norton) Theorem?

Solution:

$$I = I_N \frac{R_N}{R_N + 24}$$



$$R_N = R_{th} = 6 + 8 + 10 = 24 \Omega.$$

See Q(38).

$$\therefore I = I_N \frac{24}{24+24} = \frac{I_N}{2} \text{ Amp.}$$

* To find (I_N) →

By loop:

$$I_1 = -10 \text{ Amp.}$$

$$I_2 = I_N.$$

Loop(2):

$$300 = (6+8+10) I_2 - 6 \times I_1$$

$$300 = 24 I_2 + 60$$

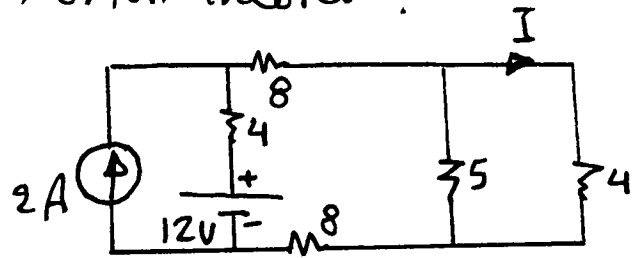
$$\rightarrow I_2 = I_N = \frac{240}{24} = 10 \text{ Amp}$$

$$\therefore I = \frac{10}{2} = \underline{5 \text{ Amp.}}$$

Q45 Find the current (I) by Norton Theorem?

Solution:

$$I = I_N \frac{R_N}{R_N + 4}$$



$$R_N = R_{th} = 5 \parallel (8 + 4 + 8) \\ = 5 \parallel 20 = 4 \Omega \quad (\text{See Q 31}).$$

$$\therefore I = I_N \frac{4}{4 + 4} = \frac{I_N}{2} \text{ Amp.}$$

$I_N \rightarrow$

By using Superposition: 2A.

* By (2Amp) $\rightarrow 12V \rightarrow (s/c)$.

$$\therefore I_{N1} = 2 \frac{4}{4 + 8 + 8} = \frac{8}{20} \text{ Amp.}$$

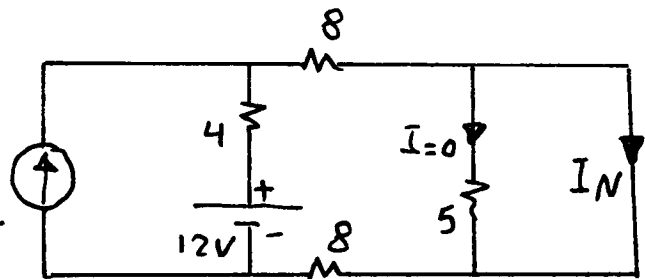
* By (12 Volt) $\rightarrow 2\text{Amp} \rightarrow (o/c)$.

$$I_{N2} = \frac{12}{4 + 8 + 8} = \frac{12}{20} \text{ Amp.}$$

$$\therefore I_N = I_{N1} + I_{N2} = \frac{8}{20} + \frac{12}{20} = 1 \text{ Amp.}$$

check $R_{th} = R_N = \frac{V_{th}}{I_N} = 4 = \frac{V_{th}}{1}$

$$\therefore V_{th} = 4 \text{ Volt. (See Q 31).}$$



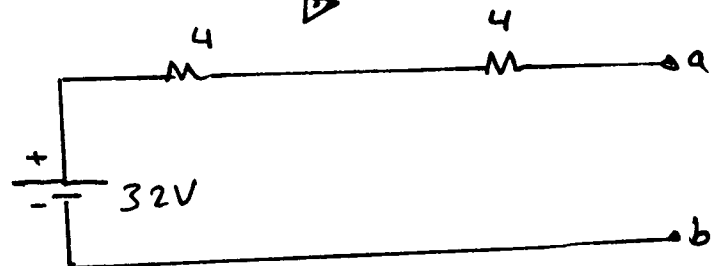
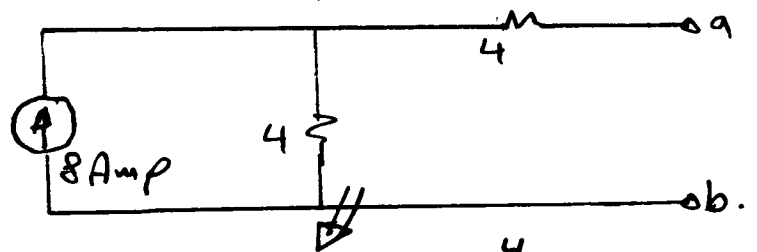
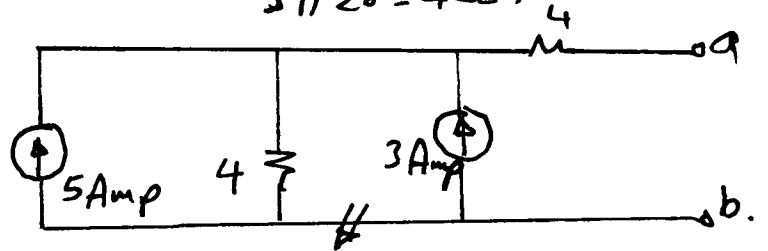
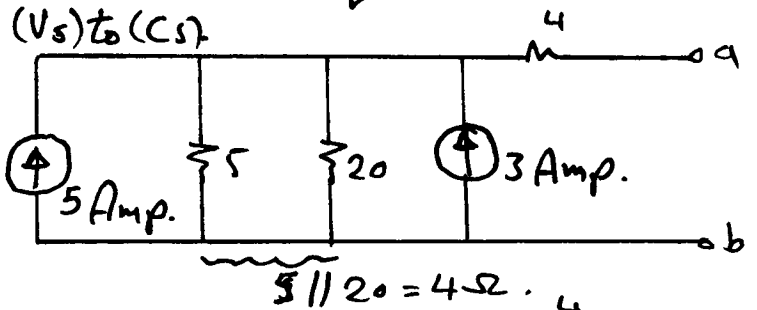
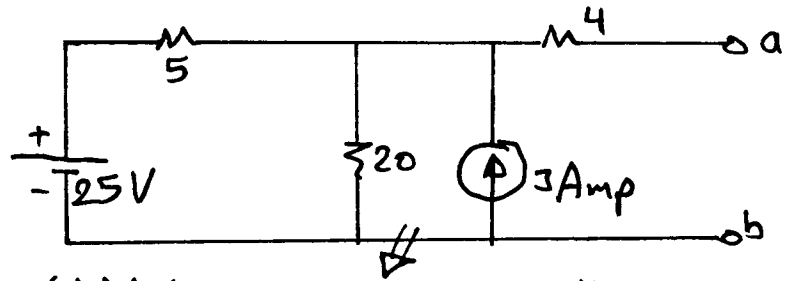
Q46 Find V_{th} , I_N & $R_N = R_{th}$ between a & b?

Solution:

$$R_{th} \rightarrow 4 + 4 = 8\Omega.$$

$$V_{th} = 32 \text{ Volt.}$$

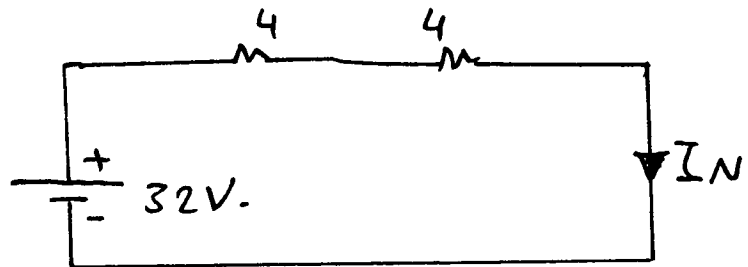
$$I_N = \frac{V_{th}}{R_{th}} = \frac{32}{8} = 4 \text{ Amp.}$$



OR

$I_N \rightarrow$

$$I_N = \frac{32}{4+4} = 4 \text{ Amp.}$$



Q47 Find (I) by using Norton Theorem?

Solution:

$$R_N = R_{th} = \frac{V_{th}}{I_N}$$

$$I = I_N \frac{R_N}{R_N + 30}$$

* $V_{th} \rightarrow$

$$V_{th} = 10 + 10 I_x \quad (\text{Kirch. Voltage Law})$$

But;

$$I_x = \frac{10}{4} = 2.5 \text{ Amp.}$$

$$\therefore V_{th} = 10 + 10(2.5) = \underline{35 \text{ Volt.}}$$

* $I_N \rightarrow$

loop(I_N):

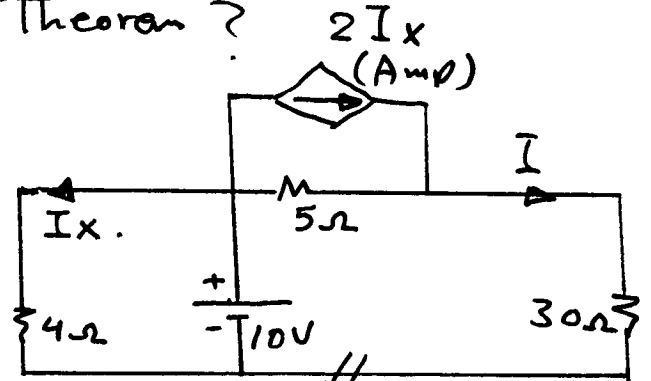
$$10 + 10 I_x = 5 I_N$$

$$\text{But } I_x = \frac{10}{4} = 2.5 \text{ Amp.}$$

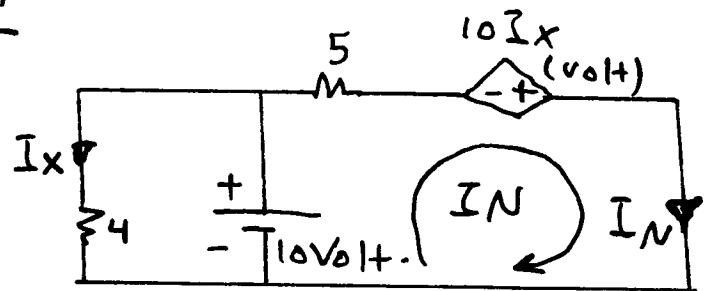
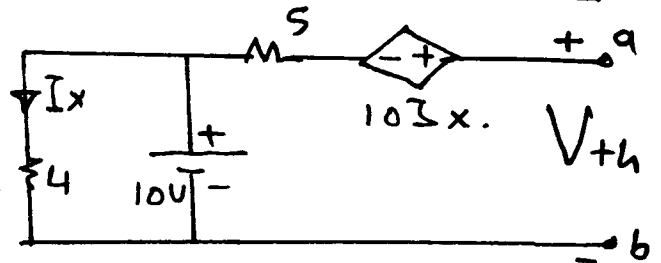
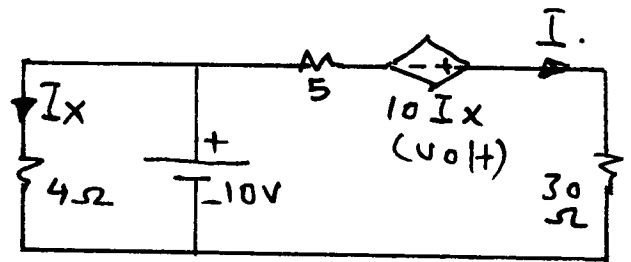
$$\therefore 35 = 5 I_N$$

$$\therefore I_N = \underline{7 \text{ Amp.}}$$

$$I = 7 \frac{5}{5+30} = \underline{1 \text{ Amp.}}$$



(C.D) to $(V_s \rightarrow V_s)$.



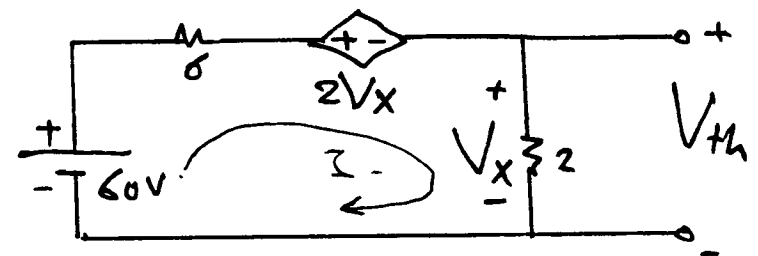
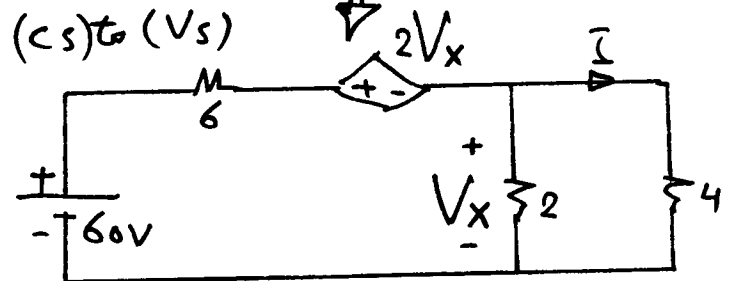
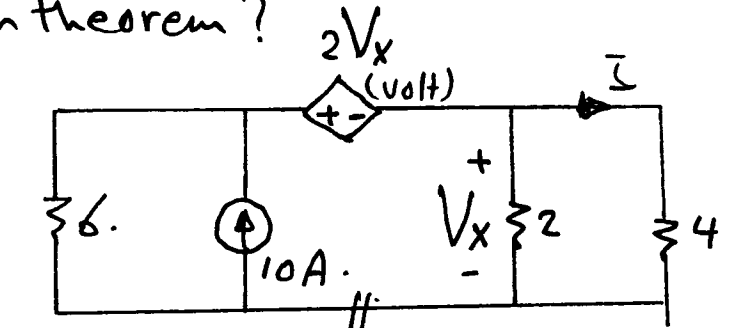
$$\therefore R_N = R_{th} = \frac{35}{7} = \underline{5 \Omega}$$

Q48 Find the (I) by Norton theorem?

Solution

$$I = \bar{I}_N \frac{R_N}{R_N + 4}$$

$$R_N = R_{th} = \frac{V_{th}}{I_N}$$



* $V_{th} \rightarrow$

$$V_{th} = V_{2\Omega} = V_x$$

But $V_x = 2I$ (Ohm's Law).

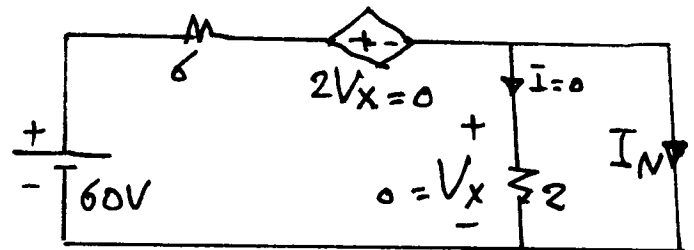
$$\therefore V_x = 2 \left(\frac{60 - 2V_x}{6 + 2} \right)$$

$$V_x = \frac{120 - 4V_x}{8} \rightarrow 8V_x = 120 - 4V_x$$

$$\therefore 12V_x = 120 \rightarrow V_x = V_{th} = 10 \text{ Volt}$$

* $I_N \rightarrow$

$$I_N = \frac{60}{6} = 10 \text{ Amp. (Ohm's Law)}$$



$$\therefore R_N = \frac{V_{th}}{I_N} = \frac{10}{10} = 1\Omega$$

$$\therefore I = 10 \frac{1}{1 + 4} = 2 \text{ Amp}$$

Q49 Find the Load resistance (R_L) for maximum power to the Load, then find this maximum power?

Solution:

$$R_L = R_{th} \\ = 15 \Omega.$$

* $R_{th} \rightarrow$

$$\therefore I = \frac{V_{th}}{R_{th} + R_L} = \frac{V_{th}}{15 + 15}.$$

* $V_{th} \rightarrow$

By K.V.L.

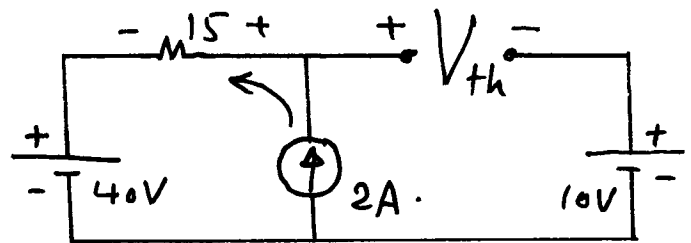
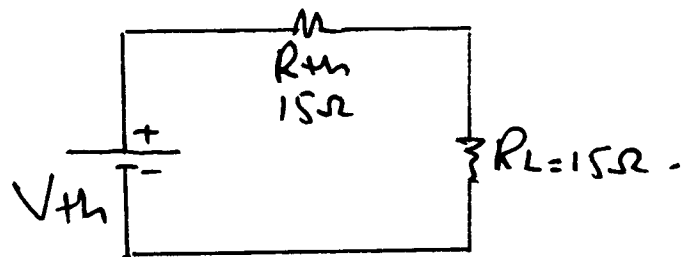
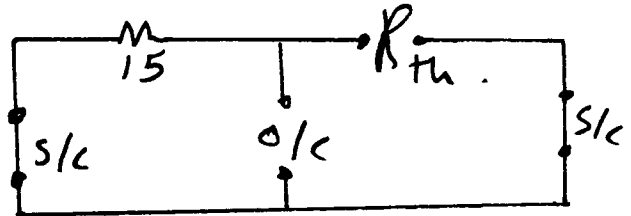
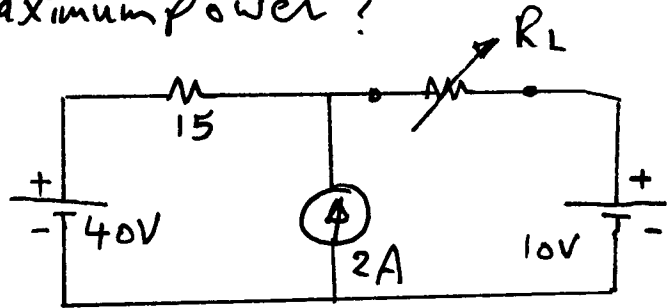
$$40 + 15 \times 2 = V_{th} + 10$$

$$70 = V_{th} + 10$$

$$\therefore V_{th} = \underline{60 \text{ Volt.}}$$

$$\therefore I = \frac{V_{th}}{R_{th} + R_L} = \frac{60}{15 + 15} = \underline{2 \text{ Amp.}}$$

$$P = I^2 \cdot R_L = (2)^2 \times 15 = \underline{60 \text{ Watt.}}$$



Q50 Find (R_L) for max. power & find this P_{max} ?

Solution:

$$R_L = R_{th}$$

$$\therefore R_L = 5 + 5 + 3 + 2 = 15 \Omega.$$

$$\therefore I = \frac{V_{th}}{R_{th} + R_L} = \frac{V_{th}}{15 + 15}.$$

* $V_{th} \rightarrow$

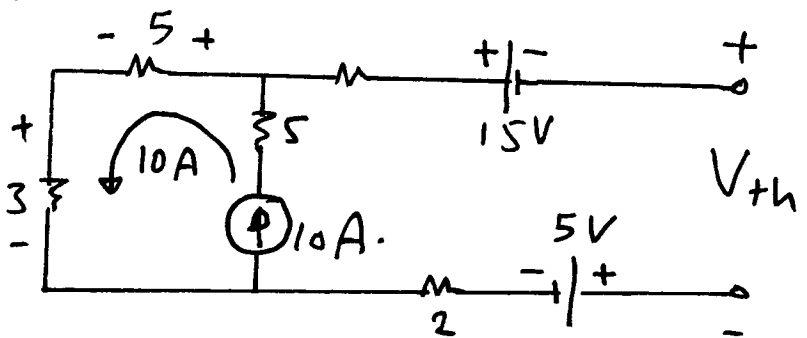
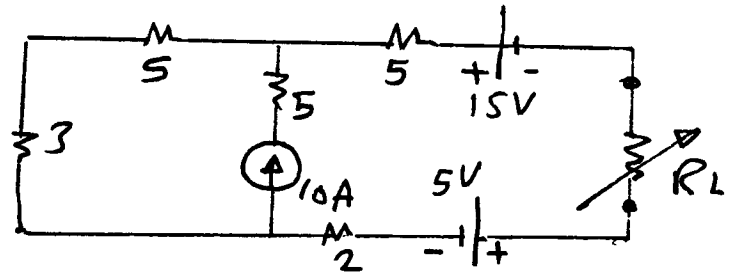
$$V_{5\Omega} + V_{3\Omega} = 15 + 5 + V_{th}.$$

$$10 \times 5 + 10 \times 3 = 20 + V_{th}.$$

$$80 = 20 + V_{th} \rightarrow \therefore V_{th} = \underline{60 \text{ Volt}}.$$

$$\therefore I = \frac{60}{15 + 15} = \underline{2 \text{ Amp}}.$$

$$\therefore P = I^2 \cdot R_L = (2)^2 \times 15 = \underline{60 \text{ Watt}}.$$



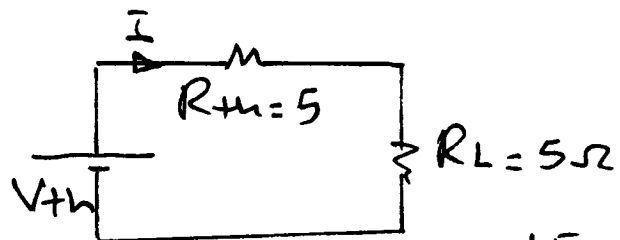
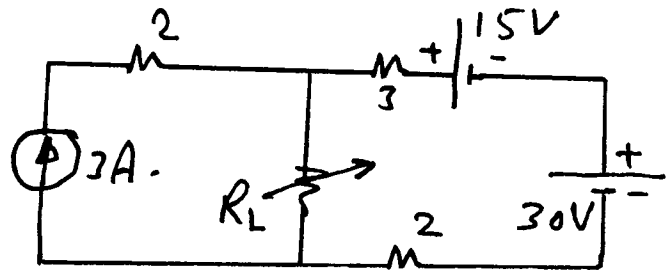
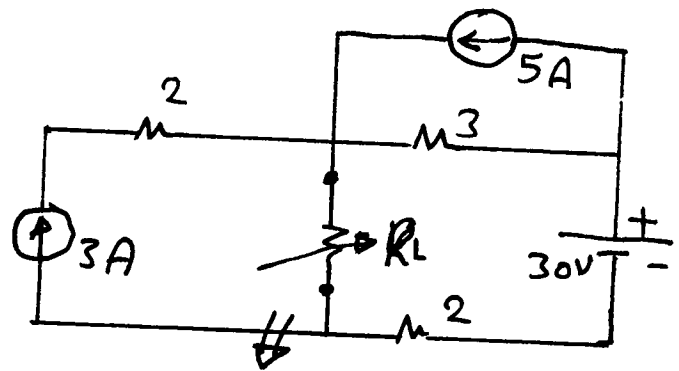
Q51) find (R_L) for max. power & find this P_{max} ?

Solution

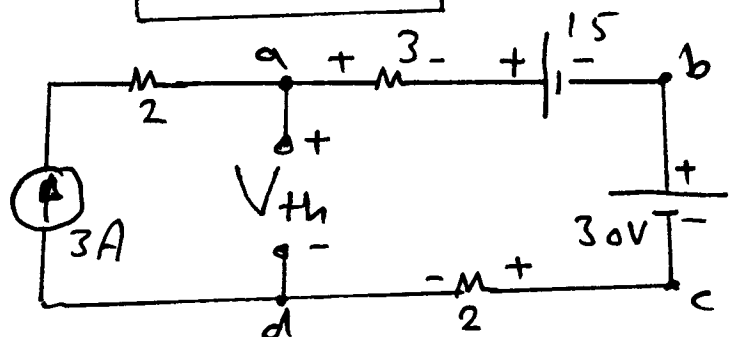
$$R_L = R_{th}$$

$$\therefore R_L = 3 + 2 = 5 \Omega$$

$$I = \frac{V_{th}}{R_{th} + R_L} = \frac{V_{th}}{5 + 5}$$



* $V_{th} \rightarrow$
take a b c d



$$\begin{aligned} V_{th} &= V_{3\Omega} + 15 + 30 + V_{2\Omega} \\ &= 3 \times 3 + 15 + 30 + 3 \times 2 \\ &= \underline{60 \text{ Volt}} \end{aligned}$$

$$\therefore I = \frac{60}{5 + 5} = \underline{6 \text{ Amp}}$$

$$P = I^2 \cdot R_L = (6)^2 \times 5 = \underline{180 \text{ watt}}$$

Q52 Find (R_L) for max. power & find this P_{max} ?

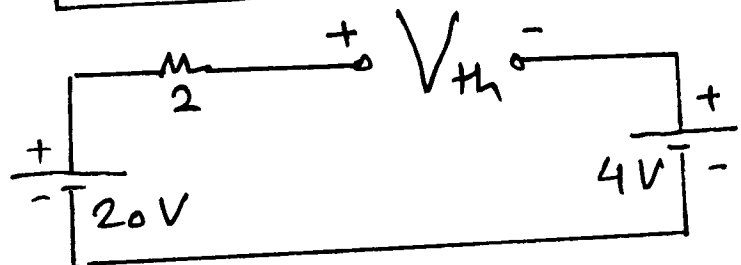
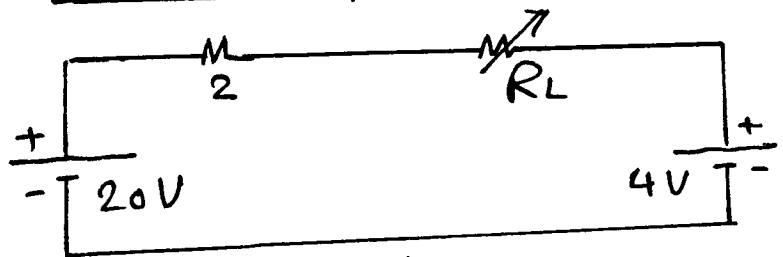
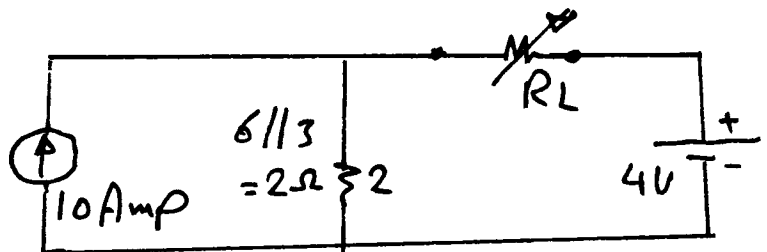
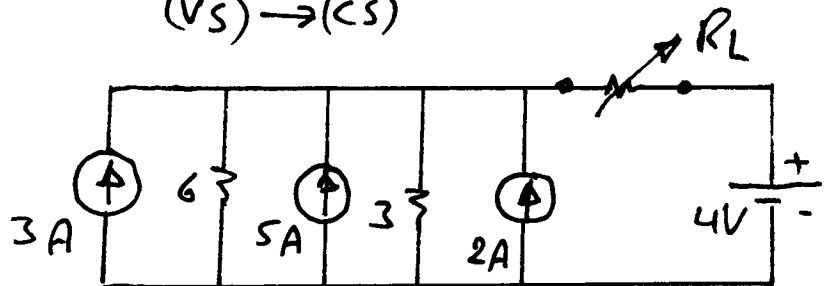
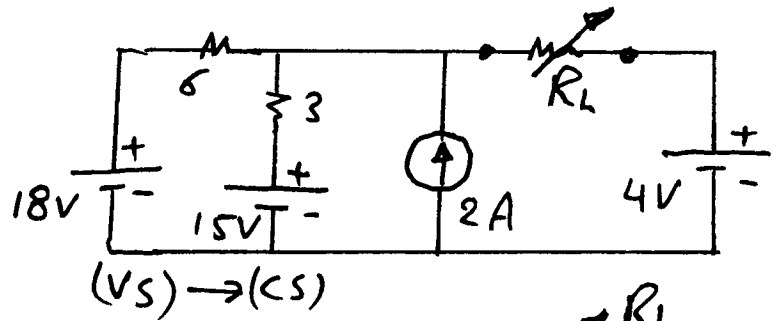
Solution:

$$R_L = R_{th}$$

$$\therefore R_L = 2\Omega$$

$$I = \frac{V_{th}}{R_{th} + R_L}$$

$$I = \frac{V_{th}}{2+2}$$



* $V_{th} \rightarrow$

$$20 = V_{th} + 4$$

$$\therefore V_{th} = \underline{16 \text{ volt}}$$

$$I = \frac{16}{2+2} = \underline{4 \text{ Amp}}$$

$$\therefore P = I^2 \cdot R_L = (4)^2 \times 2 = \underline{32 \text{ watt}}$$

Q53 | Find (R_L) for max. power, & find this P_{max} ?

Solution:

$$R_L = R_{th}$$

* $\therefore R_L \rightarrow$

$$= 15 // (5 + 10)$$

$$= 15 // 15 = \underline{7.5 \Omega}$$

$$\therefore I = \frac{V_{th}}{R_{th} + R_L}$$

$$= \frac{V_{th}}{7.5 + 7.5}$$

* $V_{th} \rightarrow$

$$60 = V_{th} + V_{15\Omega}$$

$$I_{in 15\Omega} = 6 \frac{10}{10 + 5 + 15} = \frac{60}{30} = 2 \text{ Amp} \rightarrow (\text{C.D.R}).$$

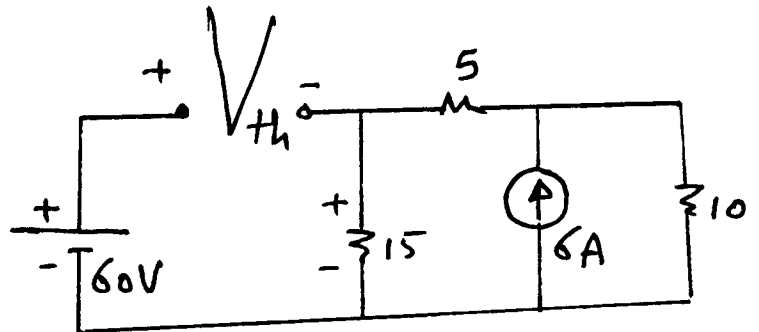
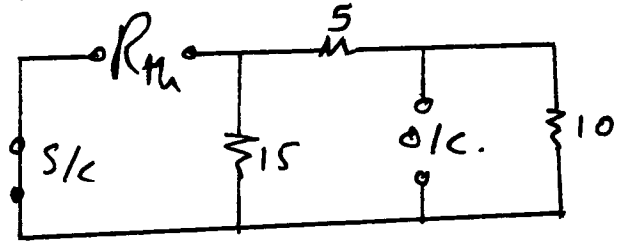
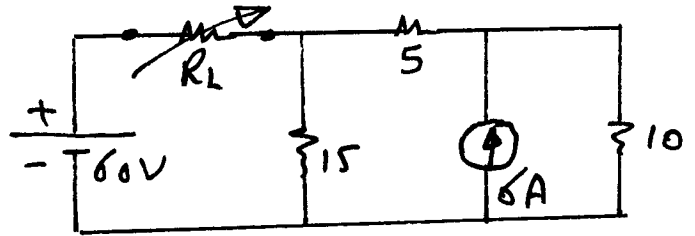
$$\therefore 60 = V_{th} + 15 \times 2$$

$$\therefore V_{th} = \underline{30 \text{ volt.}}$$

$$\therefore I = \frac{30}{7.5 + 7.5} = \underline{2 \text{ Amp.}}$$

$$P = I^2 \cdot R_L$$

$$= (2)^2 \times 7.5 = \underline{30 \text{ Watt.}}$$



Q54 Find (R_L) for max. power & find this P_{max} ?

Solution:

$$R_L = R_{th}$$

* $R_L \rightarrow$

$$\therefore R_L = R_{th} = 5 + (10 \parallel 10) = 10 \Omega$$

$$\therefore I = \frac{V_{th}}{R_{th} + R_L} = \frac{V_{th}}{10 + 10}$$

* $V_{th} \rightarrow$

take abcd

$$V_{th} = V_{5\Omega} + V_{10\Omega}$$

$$V_{5\Omega} = 5 \times 5 = \underline{25 \text{ Volt}}$$

$$I_2 = 5 \text{ Amp}$$

\therefore loop (1):

$$100 = (10 + 10) I_1 - 10 I_2$$

$$100 = 20 I_1 - 10 \times 5$$

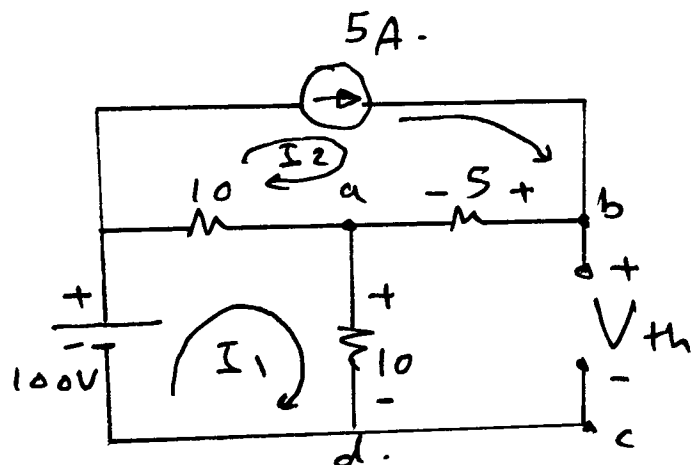
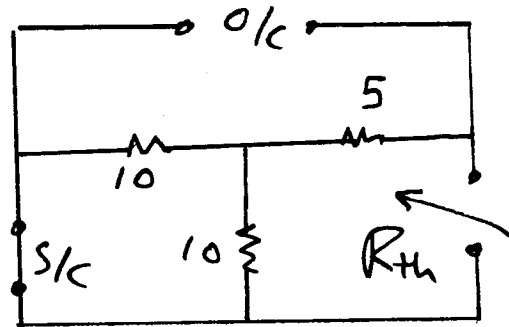
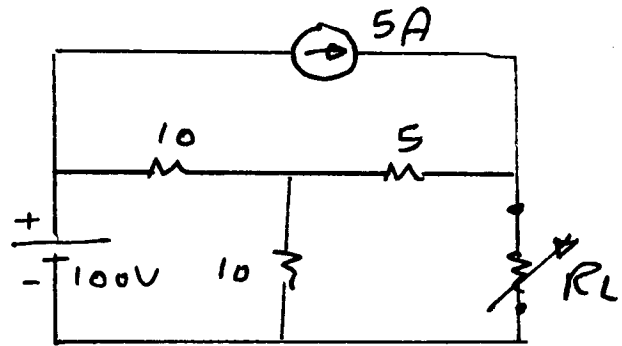
$$150 = 20 I_1 \longrightarrow \therefore I_1 = \frac{150}{20} = 7.5$$

$$V_{10\Omega} = 7.5 \times 10 = \underline{75 \text{ Volt}}$$

$$\therefore V_{th} = 25 + 75 = \underline{100 \text{ Volt}}$$

$$\therefore I = \frac{100}{10 + 10} = \underline{5 \text{ Amp}}$$

$$P = I^2 R_L = (5)^2 \times 10 = \underline{250 \text{ Watts}}$$



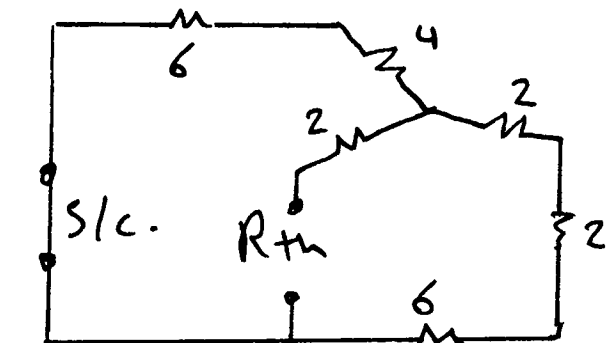
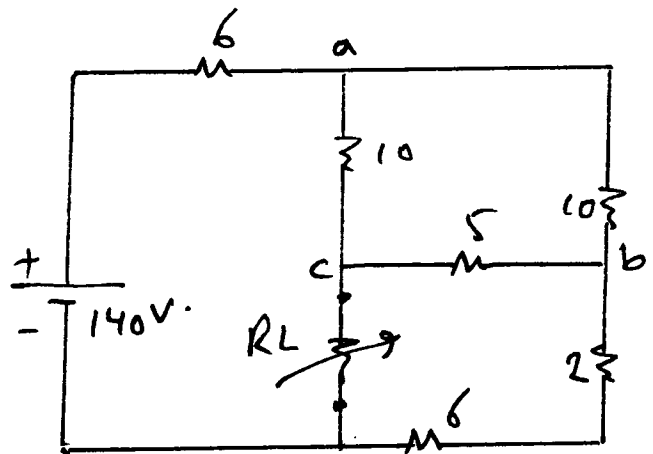
Q55 Find (R_L) for max. power?

Solution:

$$R_L = R_{th}$$

$R_L \rightarrow$

$$\begin{aligned} \therefore R_L &= 2 + \left[(6+4) \parallel (2+2+6) \right] \\ &= 2 + (10 \parallel 10) \\ &= \underline{7 \Omega} \end{aligned}$$



$\Delta \Rightarrow Y$

$$R_a = \frac{10 \times 10}{10+10+5} = 4\Omega$$

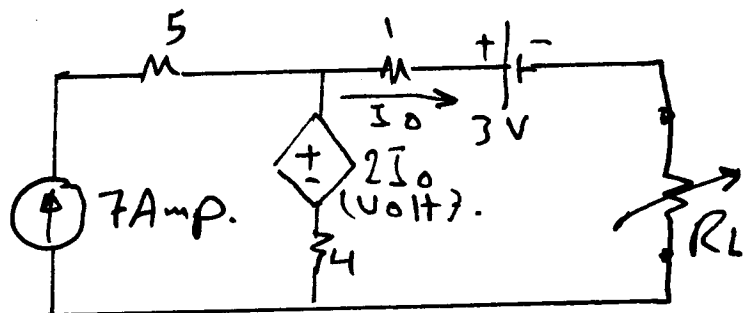
$$R_b = \frac{5 \times 10}{25} = 2\Omega$$

$$R_c = \frac{5 \times 10}{25} = 2\Omega$$

Q56 Find (R_L) for max. power then find this P_{max} ?

Solution:

$$R_L = R_{th} = \frac{V_{th}}{I_N}$$



* $V_{th} \rightarrow$

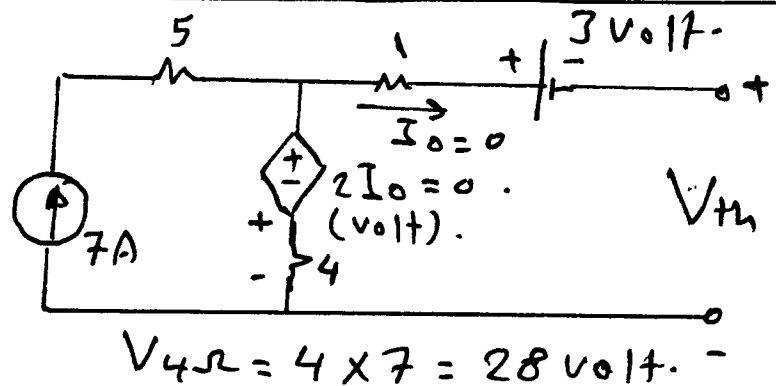
$I_o = \text{Zero (open circuit)}$.

$$\therefore 2I_o \rightarrow 0.$$

$$V_{1\Omega} = 0.$$

$$\therefore V_{th} + 3 = 4 \times 7.$$

$$\therefore V_{th} = \underline{25 \text{ Volt}}.$$



* $I_N \rightarrow$

$$I_N = I_o.$$

$$\therefore 2I_o = 2I_N$$

$$\text{loop (1)} = 7 \text{ Amp.}$$

loop (I_N):

$$2I_N - 3 = 5I_N - 4 \times 7.$$

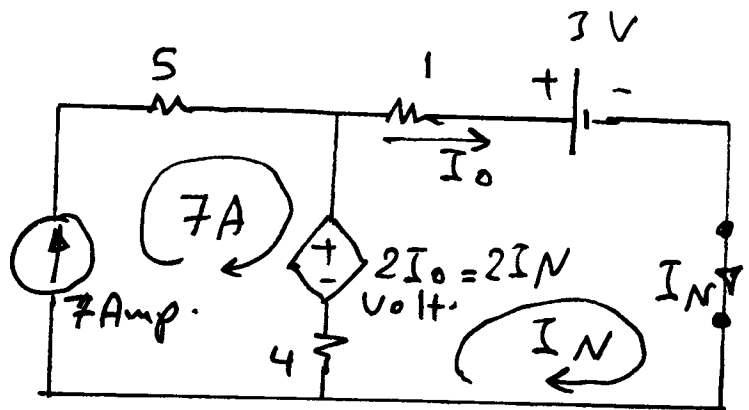
$$25 = 3I_N.$$

$$\therefore I_N = 25/3 \text{ Amp.}$$

$$\therefore R_L = R_{th} = \frac{25}{25/3} = \underline{3 \Omega}.$$

$$\therefore I = \frac{V_{th}}{R_{th} + R_L} = \frac{25}{3+3} \text{ Amp.}$$

$$\therefore P = I^2 \cdot R_L = \left(\frac{25}{6}\right)^2 \times 3 = \underline{52 \text{ Watt}}.$$



Q57 | Find (R_L) for P_{max} . & Find this P_{max} ?

Solution

$$R_L = R_{th} = \frac{V_{th}}{I_N}$$

* $V_{th} \rightarrow$

$$V_o = V_{6\Omega} = 6 \times 1.5V_o$$

$$\therefore V_o = 9V_o$$

$\therefore V_o$ must be zero

$$\therefore V_{th} = 20 \text{ Volt}$$

* $I_N \rightarrow$

By K.C.L

$$I + 1.5V_o = I_N$$

$$\therefore I_N = \frac{20}{6+4} + 1.5V_o$$

$$= 2 + 1.5V_o$$

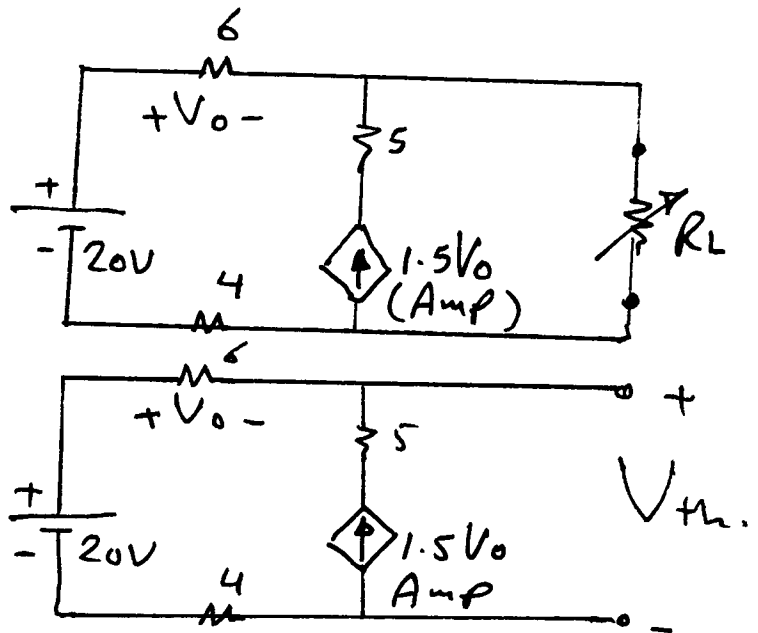
$$\text{But } V_o = 6I = 6 \times 2 = 12 \text{ Volt}$$

$$\therefore I_N = 2 + 1.5 \times 12 = 20 \text{ Amp}$$

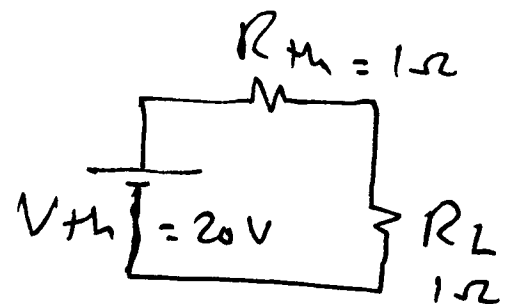
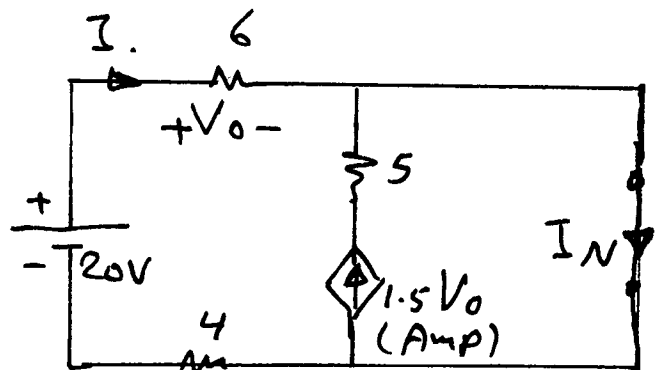
$$\therefore R_L = R_{th} = \frac{20}{20} = 1\Omega$$

$$\therefore I = \frac{20}{1+1} = 10 \text{ Amp}$$

$$P = I^2 \cdot R_L = (10)^2 \times 1 = 100 \text{ Watt}$$



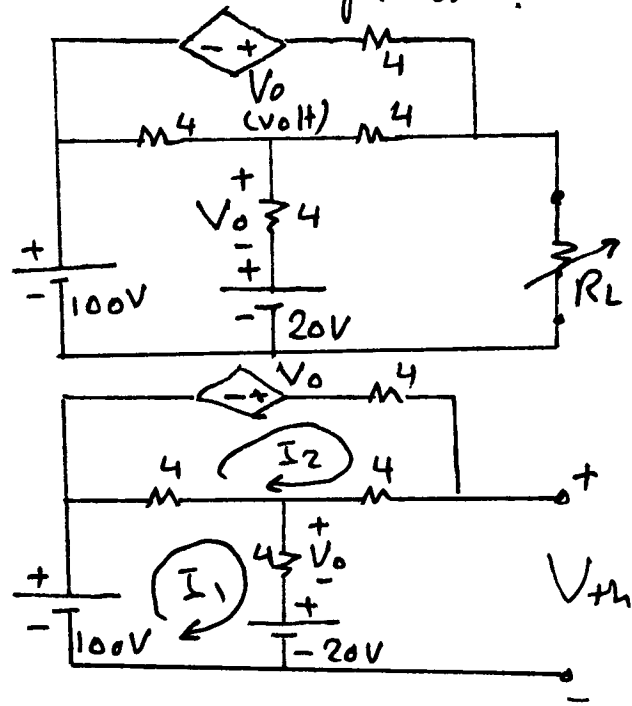
$$\therefore 1.5V_o \rightarrow 0$$



Q58 Find (R_L) for max. power then find the P_{max} ?

Solution:

$$R_L = R_{th} = \frac{V_{th}}{I_N}$$



* $V_{th} \rightarrow$

$$V_{th} = 20 + V_{4\Omega} + V_{4\Omega}$$

loop(1):

$$100 - 20 = (4 + 4)I_1 - 4I_2$$

$$80 = 8I_1 - 4I_2$$

$$20 = 2I_1 - I_2 \quad \text{--- (1)}$$

loop(2):

$$V_o = (4 + 4 + 4)I_2 - 4I_1$$

$$\text{But } V_o = 4I_1$$

$$\therefore 4I_1 = 12I_2 - 4I_1$$

$$8I_1 = 12I_2 \quad \text{--- (2)} \rightarrow I_1 = \frac{3I_2}{2}$$

$$\therefore 20 = 2\left(\frac{3I_2}{2}\right) - I_2$$

$$I_2 = 10 \text{ Amp} \rightarrow \therefore I_1 = \frac{3 \times 10}{2} = 15 \text{ Amp}$$

$$\therefore V_{th} = 20 + 10 \times 4 + 10 \times 15 = 120 \text{ Volt}$$

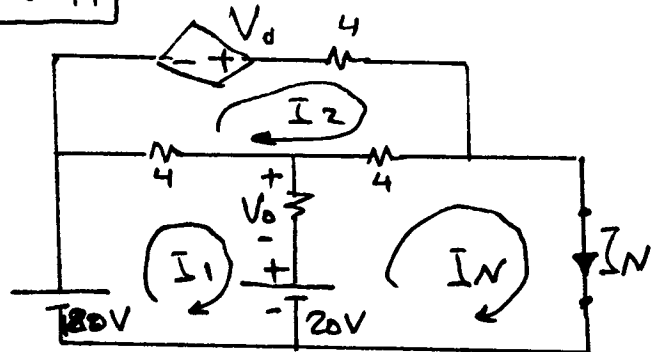
* $I_N \rightarrow$

loop(1):

$$100 - 20 = 8I_1 - 4I_2 - 4I_N$$

$$80 = 8I_1 - 4I_2 - 4I_N$$

$$20 = 2I_1 - I_2 - I_N \quad \text{--- (1)}$$



loop(N):

$$20 = 8I_N - 4I_1 - 4I_2$$

$$5 = 2I_N - I_1 - I_2 \quad \text{--- (2)}$$

Loop (2):

$$V_o = 12I_2 - 4I_1 - 4I_N.$$

$$\text{But } V_o = 4(I_1 - I_N).$$

$$\therefore 4(I_1 - I_N) = 12I_2 - 4I_1 - 4I_N.$$

$$4I_1 - 4I_N = 12I_2 - 4I_1 - 4I_N.$$

$$\therefore \boxed{8I_1 = 12I_2} \longrightarrow \textcircled{3}.$$

$$I_1 = \frac{3I_2}{2}.$$

Solving: $I_1 = 45 \text{ Amp}.$

$$I_2 = 30 \text{ Amp}.$$

put I_1 & I_2 in eq (2).

$$5 = 2I_N - 45 - 30$$

$$80 = 2I_N \longrightarrow \boxed{I_N = 40 \text{ Amp}}$$

$$\therefore R_{th} = R_L = \frac{120}{40} = \underline{\underline{3 \Omega}}$$

$$\therefore I = \frac{120}{3+3} = \frac{120}{6} = \underline{\underline{20 \text{ Amp}}}.$$

$$\therefore P = I^2 \cdot R_L = (20)^2 \times 3 = \underline{\underline{1200 \text{ watt}}}.$$

Q1] For the circuit shown, Find (I_o)?

a: By using Kirchhoff's Law:

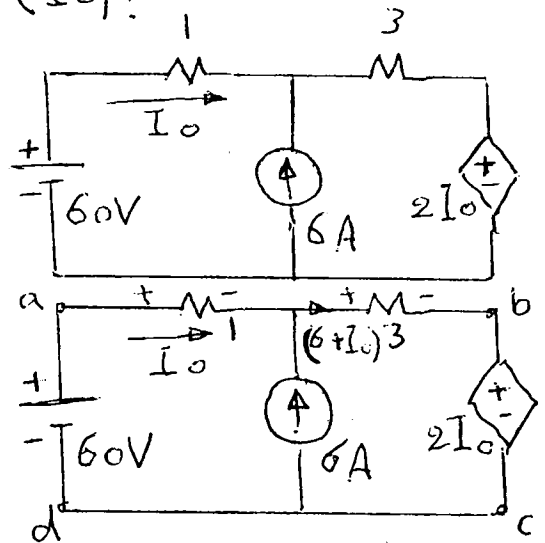
abcd

$$60 = I_o + 3(6 + I_o) + 2I_o$$

$$60 = I_o + 18 + 3I_o + 2I_o$$

$$42 = 6I_o$$

$$\therefore \underline{I_o = 7 \text{ Amp}}$$



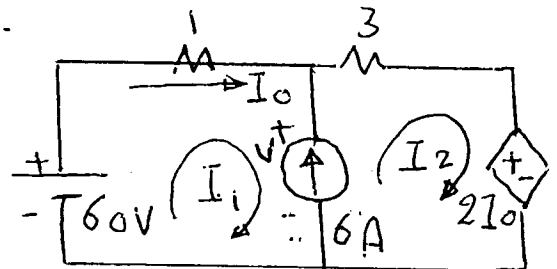
b: By mesh (loop current) method.

loop (1): $I_o = I_1$

$$60 - V = I_o \quad \text{--- (1)}$$

loop (2):

$$V - 2I_o = 3I_2 \quad \text{--- (2)}$$



From eq (1): $V = 60 - I_o$

$$\therefore (60 - I_o) - 2I_o = 3I_2$$

$$\text{But: } 6 = I_2 - I_1 = I_2 - I_o$$

$$\therefore (60 - I_o) - 2I_o = 3(6 + I_o)$$

$$\therefore 42 = 6I_o$$

$$\therefore \underline{I_o = 7 \text{ Amp}}$$

OR: take K.L for abcd

$$\therefore 60 = I_1 + 3I_2 + 2I_o$$

$$\text{But } I_1 = I_o \text{ \& } I_2 = 6 + I_1 = 6 + I_o$$

$$\therefore 60 = I_o + 3(I_o + 6) + 2I_o$$

$$42 = 6I_o$$

$$I_o = 7 \text{ Amp}$$

c: By Superposition method.

1- By effect of 60V \rightarrow 6A (o/c).

$$\therefore I_{01} = \frac{60 - 2I_{01}}{4}$$

$$4I_{01} = 60 - 2I_{01}$$

$$\therefore I_{01} = 10 \text{ Amp.}$$

2- By effect of 6Amp \rightarrow 60V (s/c).

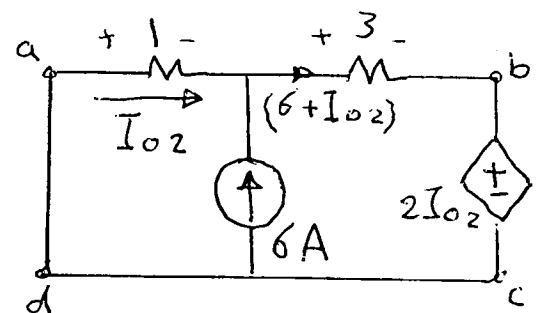
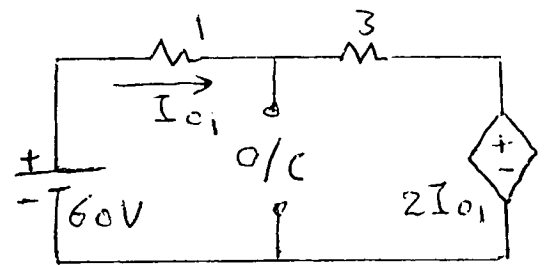
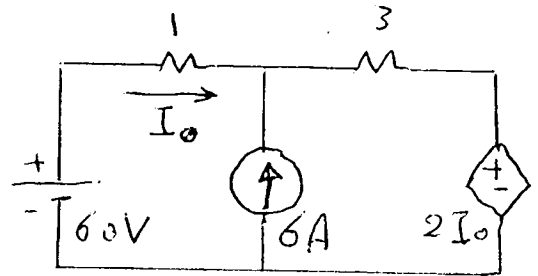
take abcd

$$I_{02} + 3(6 + I_{02}) + 2I_{02} = 0.$$

$$18 = -6I_{02}$$

$$\therefore I_{02} = -3 \text{ Amp.}$$

But $I_0 = I_{01} + I_{02} = 7 \text{ Amp}$



d: By Nodal equation method.

Node(V):

$$\left(\frac{1}{3} + \frac{1}{1}\right)V - \frac{60}{1} - \frac{2I_0}{3} = 0.$$

$$(1 + 3)V - 180 - 2I_0 = 18$$

$$4V - 2I_0 = 198.$$

But $I_0 = \frac{60 - V}{1}$

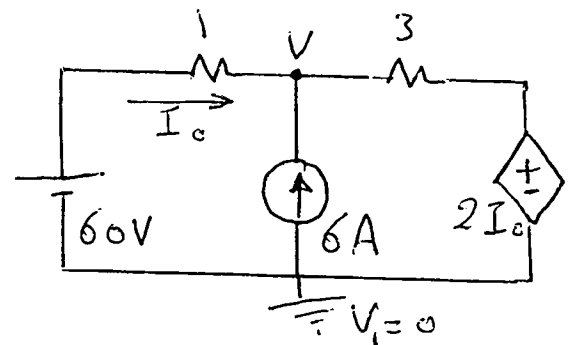
$$\therefore 4V - 2(60 - V) = 198.$$

$$6V = 318$$

$$\therefore V = 53 \text{ Volt.}$$

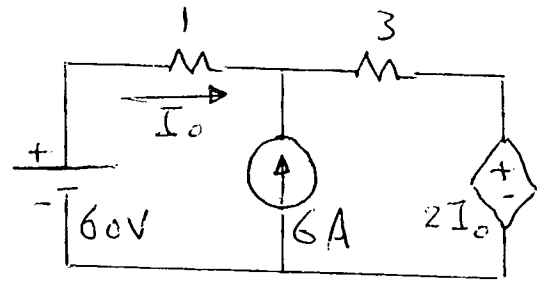
$$\therefore I_0 = \frac{60 - V}{1}$$

$$\underline{I_0 = 7 \text{ Amp.}}$$



e: By Thevening or Norton Theory

$$R_{th} = R_N = \frac{V_{th}}{I_N} = \frac{V_{o/c}}{I_{s/c}}$$



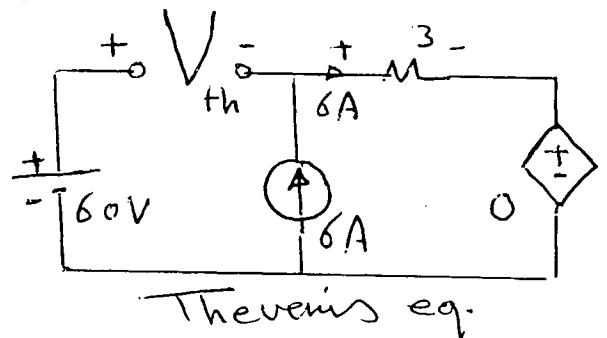
* V_{th} :

Since $I_o = 0$

\therefore the dependent source $(2I_o) = 0$.

$$\therefore V_{th} = 60 - 3 \times 6$$

$$V_{th} = 42 \text{ Volt.}$$



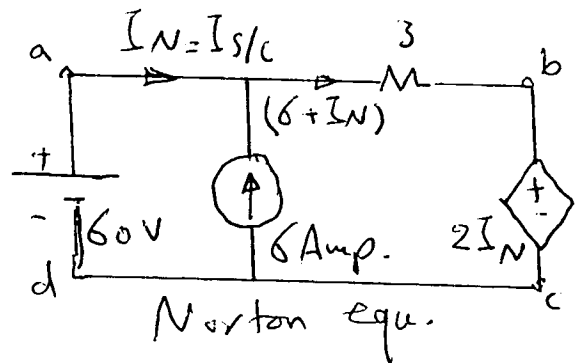
* I_N

abcd

$$60 = 3(6 + I_N) + 2I_N.$$

$$42 = 5I_N.$$

$$\therefore I_N = \frac{42}{5} \text{ Amp.}$$



$$\therefore R_{th} = R_N = \frac{42}{42/5} = 5 \Omega.$$

\therefore By Thevenin equivalent

$$I = \frac{V_{th}}{R_{th} + 1} = \frac{42}{5 + 1} = \underline{\underline{7 \text{ Amp}}}$$

By Norton's equivalent

$$I = I_N \frac{R_N}{R_N + 1} = \frac{42}{5} \times \frac{5}{5 + 1} = \underline{\underline{7 \text{ Amp}}}$$

Q.2 Find (I_ϕ) for the circuit shown

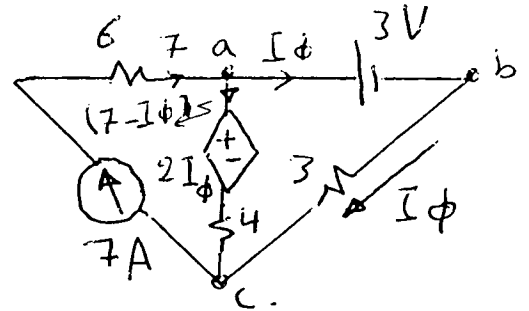
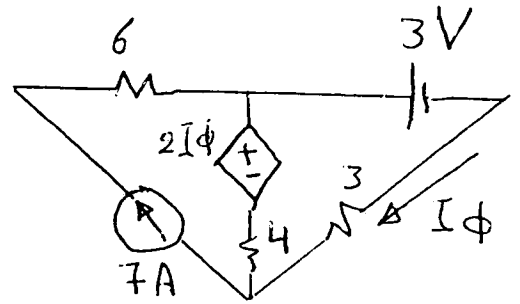
a: By K-L:

abc

$$4(7 - I_\phi) + 2I_\phi = 3 + 3I_\phi$$

$$25 = 5I_\phi$$

$$\therefore \underline{I_\phi = 5 \text{ Amp.}}$$



b: By loop current method.

$$\text{loop (1)} : I_1 = 7 \text{ Amp.}$$

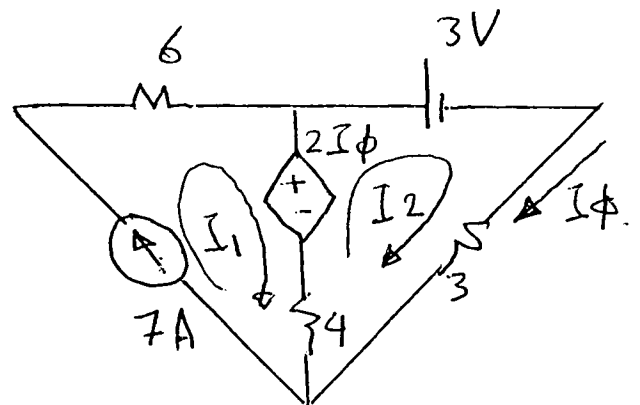
$$\text{loop (2)} : I_2 = I_\phi$$

$$2I_\phi - 3 = (3 + 4)I_2 - 4I_1$$

$$2I_\phi - 3 = 7I_\phi - 4 \times 7$$

$$25 = 5I_\phi$$

$$\therefore \underline{I_\phi = 5 \text{ Amp}}$$



C: By Superposition method

1. By effect of 7Amp \rightarrow 3V(s/c).

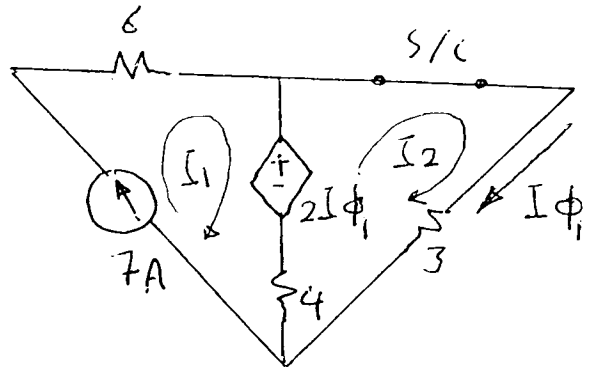
$$\text{loop (1)} : I_1 = 7 \text{ Amp.}$$

$$\text{loop (2)} : I_{\phi_1} = I_2.$$

$$2 I_{\phi_1} = 7 I_{\phi_1} - 4 I_1$$

$$\therefore 5 I_{\phi_1} = 28.$$

$$I_{\phi_1} = 28/5 \text{ Amp.}$$



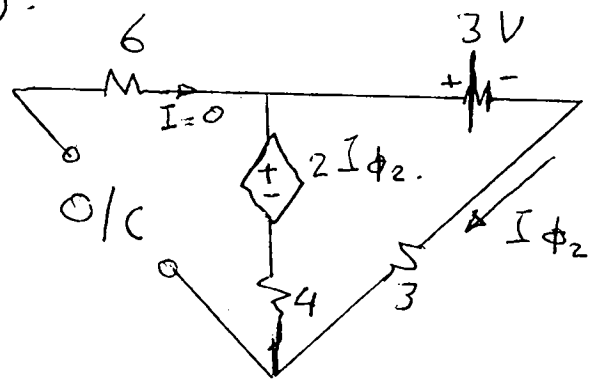
2. By effect of 3V \rightarrow 7A(o/c).

$$\therefore I_{\phi_2} = \frac{2 I_{\phi_2} - 3}{7}$$

$$7 I_{\phi_2} = 2 I_{\phi_2} - 3$$

$$\therefore 5 I_{\phi_2} = -3.$$

$$\therefore I_{\phi_2} = -\frac{3}{5}.$$



$$\text{But } I_{\phi} = I_{\phi_1} + I_{\phi_2}.$$

$$= \frac{28}{5} + \left(-\frac{3}{5}\right).$$

$$= \frac{25}{5} = 5 \text{ Amp}$$

d. By Nodal Voltage method.

Node (v):

$$\left(\frac{1}{4} + \frac{1}{3}\right)V - \frac{2I\phi}{4} - \frac{3}{3} = 7.$$

$$(3+4)V - 6I\phi - 12 = 84$$

$$7V - 6I\phi = 96.$$

$$\text{But } I\phi = \frac{V-3}{3}.$$

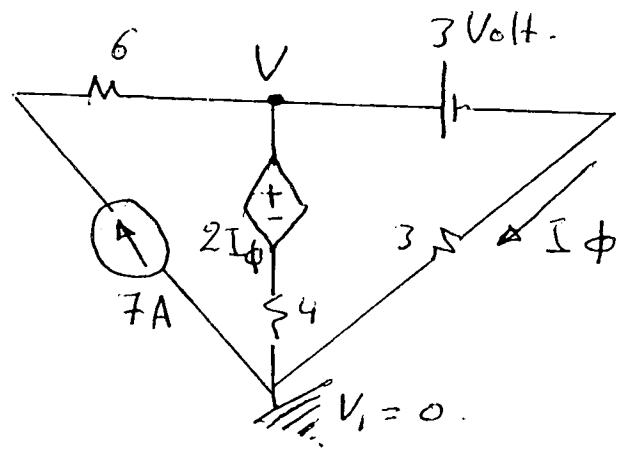
$$\therefore 7V - 6\left(\frac{V-3}{3}\right) = 96.$$

$$7V - 2V + 6 = 96$$

$$5V = 90.$$

$$\therefore V = 18 \text{ Volt.}$$

$$\therefore I\phi = \frac{18-3}{3} = \underline{\underline{5 \text{ Amp}}}$$



e: Isy Thevening or Norton Theory

$$R_{th} = R_N = \frac{V_{th}}{I_N}$$

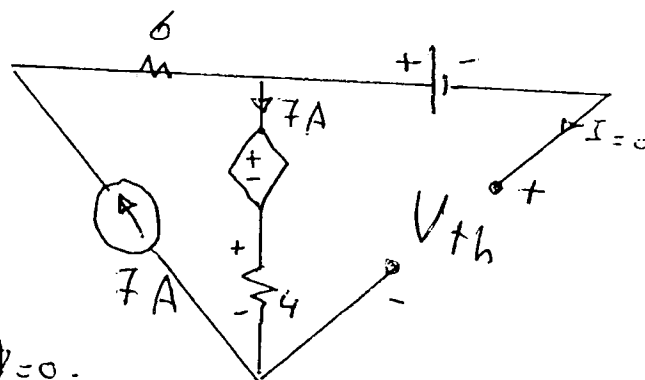
* V_{th} :

Since $I_\phi = 0$.

\therefore the dependent source $= 2I_\phi = 0$.

$$\therefore V_{th} + 3 = 4 \times 7.$$

$$V_{th} = 25 \text{ Volt.}$$



* I_N :

$$I_1 = 7 \text{ Amp.}$$

$$I_2 = I_N.$$

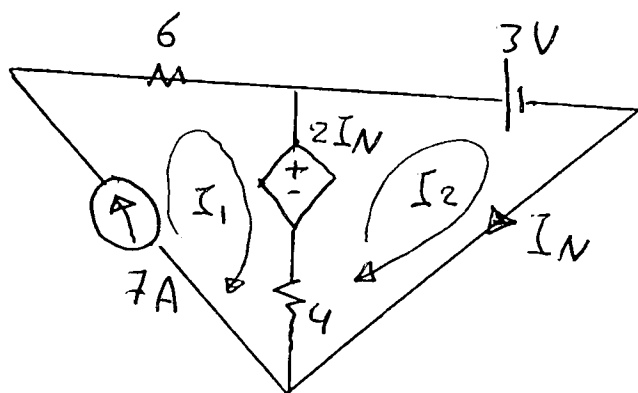
loop (2)

$$2I_N - 3 = 4I_2 - 4I_1$$

$$2I_N - 3 = 4I_N - 4 \times 7.$$

$$25 = 2I_N$$

$$\therefore I_N = \frac{25}{2} \text{ Amp.}$$



$$\therefore R_{th} = R_N = \frac{25}{25/2} = 2 \Omega.$$

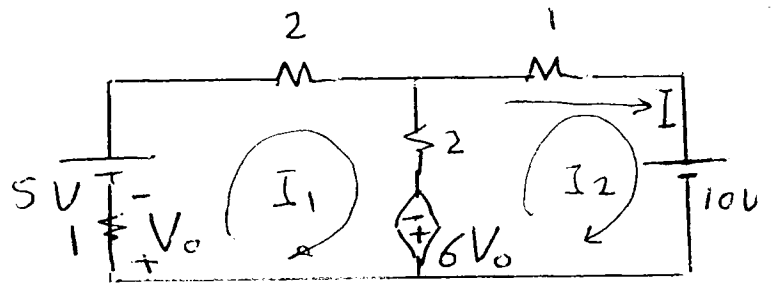
$$\therefore I = \frac{V_{th}}{R_{th} + 3} = \frac{25}{2 + 3} = \underline{\underline{5 \text{ Amp.}}}$$

$$\text{OR } I = I_N \frac{R_N}{R_N + 3}$$

$$= \underline{\underline{5 \text{ Amp}}}$$

Q3] Find (I) For the circuit shown.

a) By mesh (loop method).



loop (1):

$$5 + 6V_o = (2 + 2 + 1)I_1 - 2I_2 \quad \text{But } V_o = I_1 \times 1$$

$$\therefore 5 + 6I_1 = 5I_1 - 2I_2$$

$$\therefore 5 + I_1 = -2I_2 \quad \text{--- (1) here } I_2 = I$$

loop (2):

$$-6V_o - 10 = 3I_2 - 2I_1$$

$$-6I_1 - 10 = 3I_2 - 2I_1$$

$$-4I_1 - 3I_2 = 10 \quad \text{--- (2)}$$

From eq (1) $I_1 = -2I_2 - 5$

$$\therefore -4(-2I_2 - 5) - 3I_2 = 10$$

$$8I_2 + 20 - 3I_2 = 10$$

$$5I_2 = -10$$

$$\therefore I_2 = I = -2 \text{ Amp}$$

B) By K.L.

abcd

$$5 + 6V_o = (2 + 1)I_1 + 2(I_1 - I)$$

also $V_o = I_1$

$$\therefore 5 + 6I_1 = 3I_1 + 2I_1 - 2I$$

$$5 = -I_1 - 2I \quad \text{--- (1)}$$

befc

$$2(I_1 - I) = I + 10 + 6V_o$$

$$2(I_1 - I) = I + 10 + 6I_1$$

$$\therefore 10 = -3I - 4I_1 \quad \text{--- (2)}$$

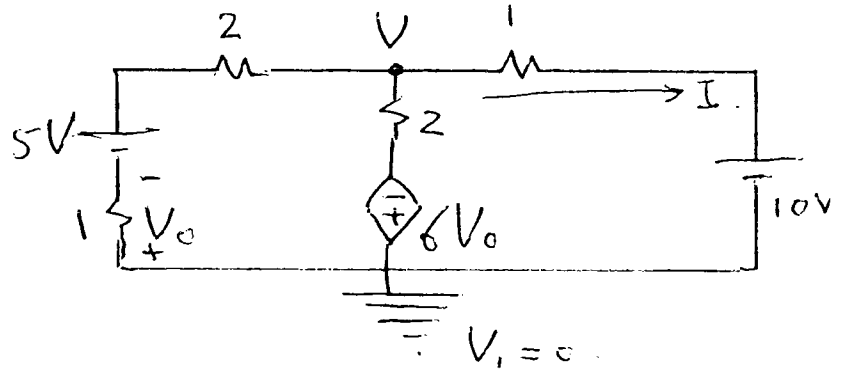
$$\therefore 10 = -3I - 4(-5 - 2I)$$

$$10 = 5I + 20$$

$$\therefore 5I = -10$$

$$\therefore I = -2 \text{ Amp}$$

c) By Nodal equation method:



Node (V)

$$\left(\frac{1}{1} + \frac{1}{2} + \frac{1}{2}\right)V - \frac{5}{3} - \frac{10}{1} + \frac{6V_o}{2} = 0.$$

$$11V - 10 - 60 + 18V_o = 0.$$

$$11V + 18V_o = 70$$

$$\text{But } V_o = \frac{5-V}{3} \times 1.$$

$$\therefore 11V + 6(5-V) = 70.$$

$$5V = 40.$$

$$\therefore V = 8 \text{ Volt.}$$

$$\therefore \underline{I = \frac{8-10}{1} = -2 \text{ Amp.}}$$

Q4] Find (I) for the Circuit shown.

a) By K.L.

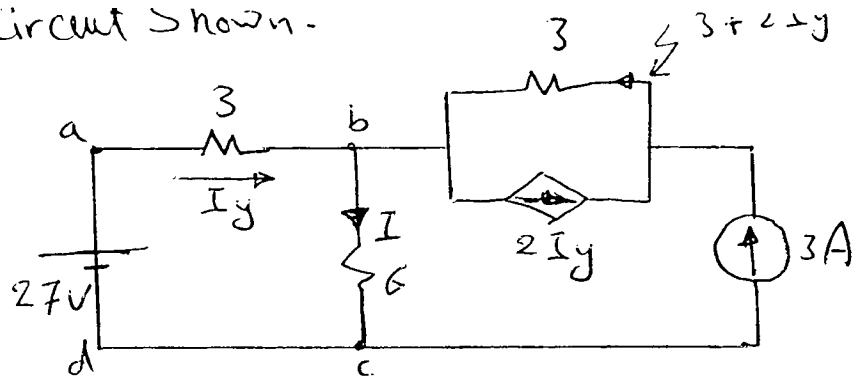
abcd

$$27 = 3 I_y + 6 I.$$

$$\therefore 27 = 3(I - 3) + 6I.$$

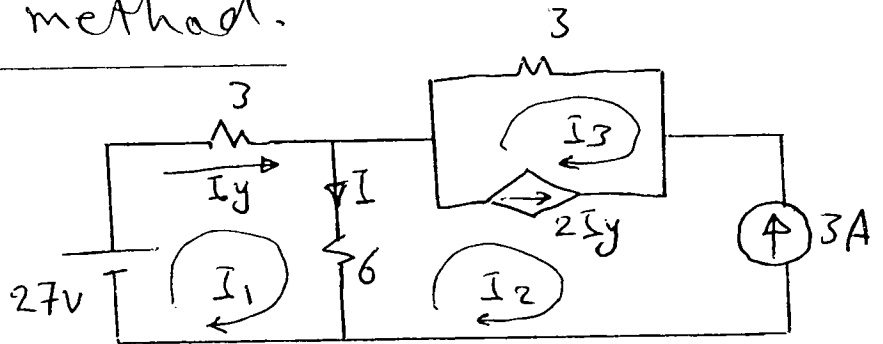
$$36 = 9I.$$

$$\therefore \underline{I = 4 \text{ Amp}}$$



at point C, we have $I = 3 + I_y$.
 $\therefore I_y = I - 3$.

b) By loop current method.



loop (1):

$$27 = (3 + 6) I_1 - 6 I_2.$$

$$27 = 9 I_1 - 6 I_2.$$

from loop (2) $I_2 = -3 \text{ Amp}.$

$$\therefore 27 = 9 I_1 - 6(-3).$$

$$9 = 9 I_1$$

$$\therefore I_1 = 1 \text{ Amp} = I_y.$$

$$\text{But } I = I_1 - I_2.$$

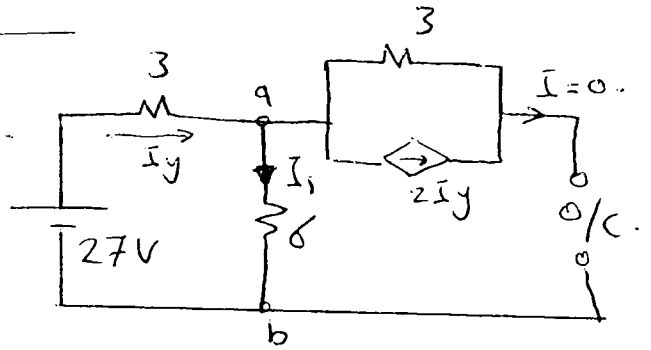
$$= 1 - (-3)$$

$$\therefore \underline{I = 4 \text{ Amp}}$$

c) By Superposition method.

1) By effect of $27V \rightarrow 3A (o/c)$.

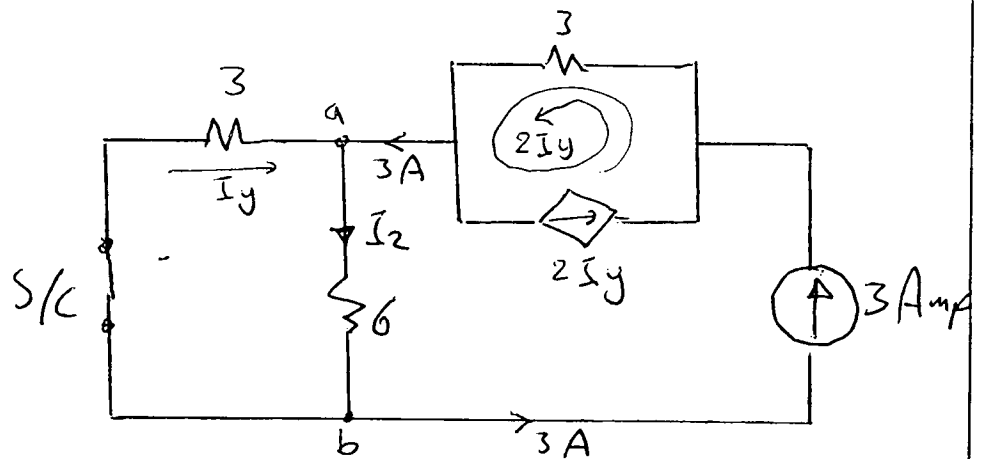
$$\therefore I_1 = \frac{27}{3+6} = \underline{\underline{3Amp}} (a \rightarrow b).$$



2) By effect of $3Amp \rightarrow 27 (S/c)$.

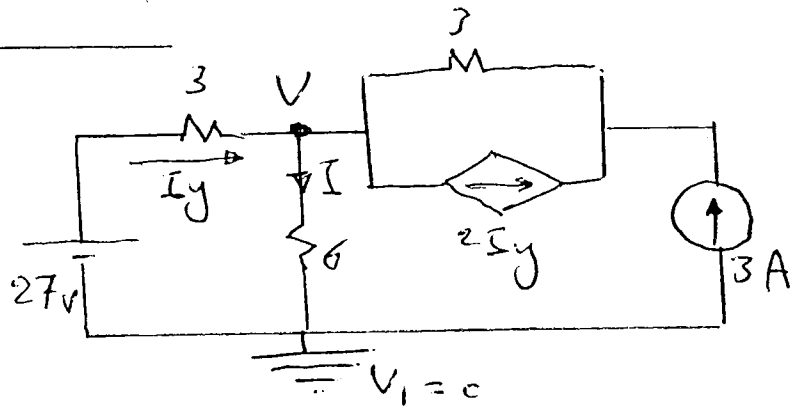
$$I_2 = 3 \frac{3}{3+6} = \underline{\underline{1Amp}} (a \rightarrow b).$$

$$\therefore \underline{\underline{I = I_1 + I_2 = 4Amp}}$$



d) By Nodal voltage method

Node (V):



$$\left(\frac{1}{3} + \frac{1}{6}\right)V - \frac{27}{3} = 3$$

$$(2+1)V - 54 = 18$$

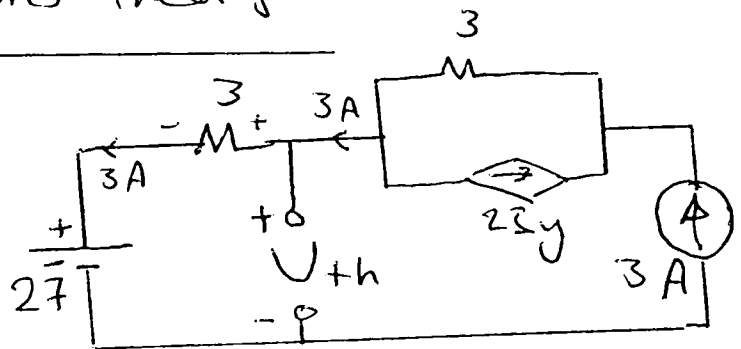
$$3V = 72$$

$$\therefore V = 24 \text{ Volt}$$

$$\therefore I = \frac{V}{6} = 4 \text{ Amp}$$

e) By Thevenin's or Norton's Theory

$$V_{th} = R_N = \frac{V_{th}}{I_N}$$



$$* V_{th} = 27 + 3 \times 3$$

$$\therefore V_{th} = 36 \text{ Volt}$$

* I_N :

1) $I_{N1} \rightarrow$ By effect of 27 Volt ($3A \rightarrow o/c$).

$$I_{N1} = \frac{27}{3} = 9 \text{ Amp (a} \rightarrow \text{b)}$$

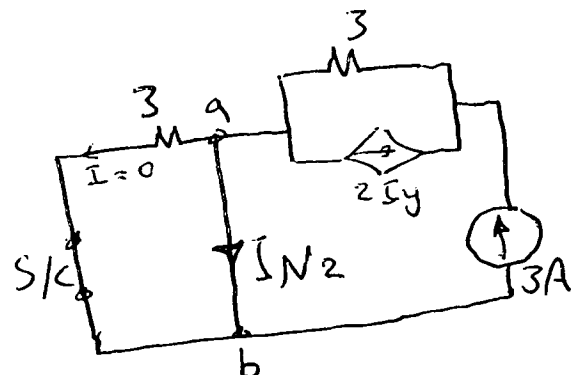
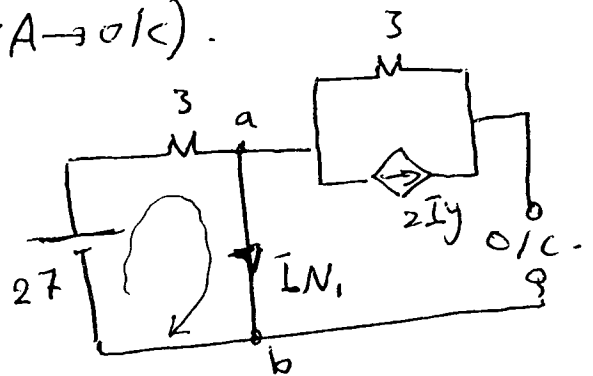
2) $I_{N2} \rightarrow$ By 3 Amp $\rightarrow 27 \rightarrow S/c$

$$I_{N2} = 3 \text{ Amp (a} \rightarrow \text{b)}$$

$$\therefore I_N = I_{N1} + I_{N2} = 12 \text{ Amp}$$

$$\therefore R_{th} = \frac{36}{12} = 3 \Omega$$

$$\therefore I = \frac{V_{th}}{R_{th} + 6} = \frac{36}{3+9} = 4 \text{ Amp}$$



Q.5 Find the current (I_x).

a) By K.L.

abcd

$$50 = 5 I_1 + 20 I_x$$

$$\therefore 10 = I_1 + 4 I_x \quad \text{--- (1)}$$

befc

$$20 I_x = 15 I_x + 4 (I_1 - I_x)$$

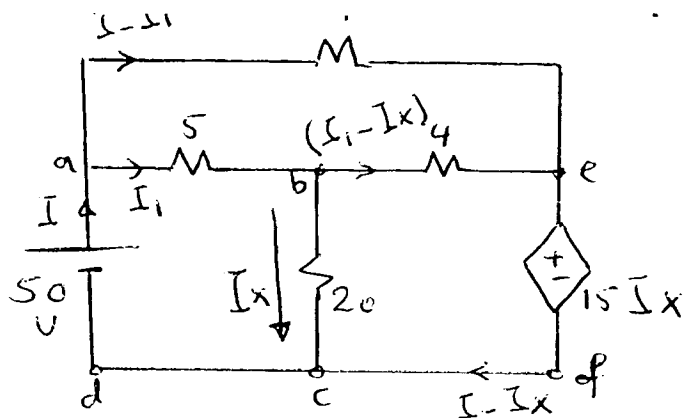
$$\therefore 9 I_x = 4 I_1 \quad \text{--- (2)} \quad \therefore I_1 = \frac{9}{4} I_x$$

put I_1 in eq (1).

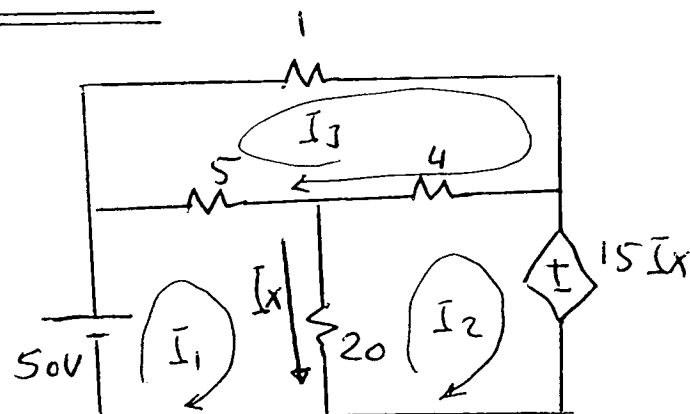
$$\therefore 10 = \frac{9}{4} I_x + 4 I_x$$

$$40 = 25 I_x$$

$$\therefore I_x = \frac{40}{25} = 1.6 \text{ Amp.}$$



b) By loop current method



loop (1):

$$50 = 25 I_1 - 20 I_2 - 5 I_3$$

$$10 = 5 I_1 - 4 I_2 - I_3 \quad \text{--- (1)}$$

loop (2):

$$-15 I_x = 24 I_2 - 20 I_1 - 4 I_3 \quad \text{--- (2)}$$

loop (3):

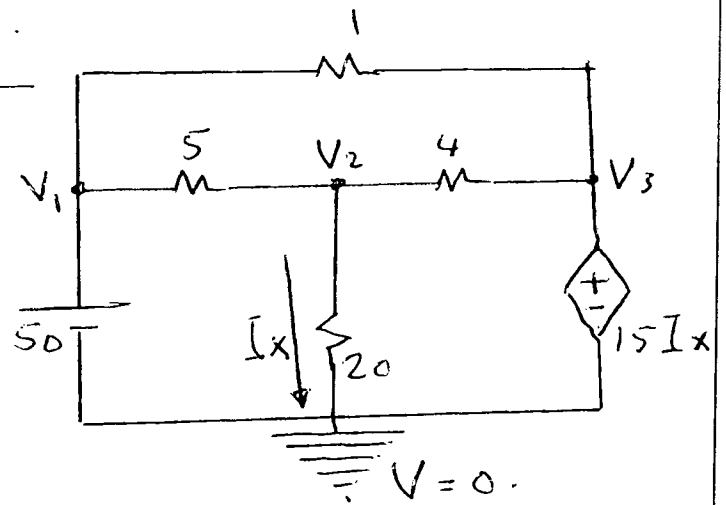
$$0 = 10 I_3 - 5 I_1 - 4 I_2 \quad \text{--- (3)}$$

$$\text{But } I_x = I_1 - I_2$$

Solving eq (1), (2) & (3).

$$\text{we get } I_x = 1.6 \text{ Amp.}$$

c) By Nodal Voltage method.



$$V_1 = 50 \text{ Volt.}$$

$$V_3 = 15 I_x$$

$$\text{But } I_x = \frac{V_2 - 0}{20} = \frac{V_2}{20}$$

Node (V_2):

$$\left(\frac{1}{4} + \frac{1}{5} + \frac{1}{20}\right)V_2 - \frac{50}{5} - \frac{15 I_x}{4} = 0.$$

$$(5 + 4 + 1)V_2 - 200 - 75 I_x = 0.$$

$$10 V_2 - 75 I_x = 200.$$

$$\therefore 10 V_2 - 75 \left(\frac{V_2}{20}\right) = 200.$$

$$40 V_2 - 15 V_2 = 800.$$

$$25 V_2 = 800.$$

$$\therefore V_2 = \frac{800}{25} = 32 \text{ Volt.}$$

$$\therefore \underline{\underline{I_x = \frac{32}{20} = 1.6 \text{ Amp.}}}$$

Q.6] Find (I_x) for the circuit

a) By Nodal voltage method.

Node (V_1):

$$\left(\frac{1}{2} + \frac{1}{5} + \frac{1}{20}\right)V_1 - \frac{20}{2} - \frac{V_2}{5} = 0$$

$$(10 + 4 + 1)V_1 - 200 - 4V_2 = 0$$

$$200 = 15V_1 - 4V_2 \quad \text{--- (1)}$$

Node (V_2):

$$\left(\frac{1}{2} + \frac{1}{5} + \frac{1}{10}\right)V_2 - \frac{V_1}{5} - \frac{8I_x}{2} = 0$$

$$\therefore 4V_2 - V_1 - 20I_x = 0 \quad \text{--- (2)}$$

$$\therefore 4V_2 - V_1 - 20\left(\frac{V_1 - V_2}{5}\right) = 0$$

$$8V_2 - 5V_1 = 0$$

$$V_1 = \frac{8}{5}V_2$$

put V_1 in eq (1)

$$\therefore 200 = 15\left(\frac{8}{5}V_2\right) - 4V_2$$

$$\therefore V_2 = 10 \text{ Volt} \quad \text{--- } V_1 = 16 \text{ Volt}$$

$$\therefore I_x = \frac{16 - 10}{5} = \frac{6}{5} = \underline{\underline{1.2 \text{ Amp}}}$$

b) By loop current method.

loop (1):

$$20 = 22I_1 - 20I_2 \quad \text{--- (1)}$$

loop (2):

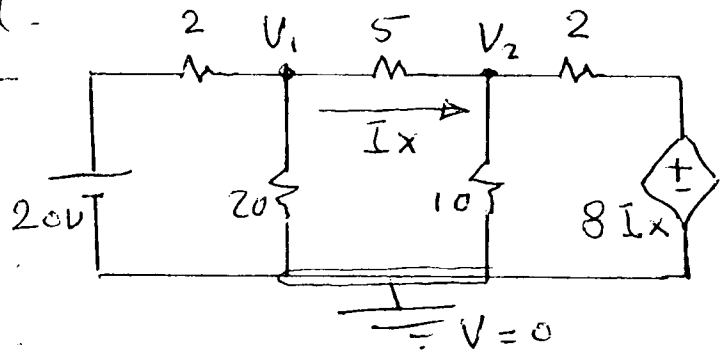
$$I_2 = I_x$$

$$0 = 35I_2 - 20I_1 - 10I_3 \quad \text{--- (2)}$$

loop (3):

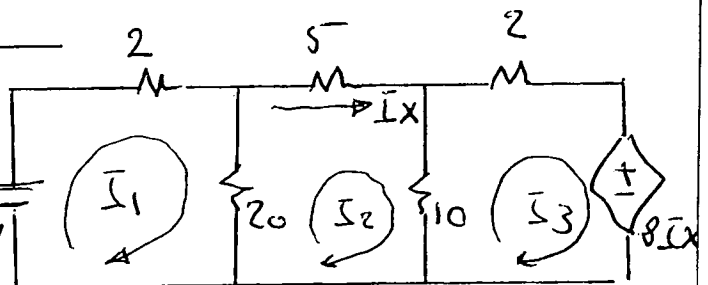
$$-8I_x = 12I_3 - 10I_2 \quad \text{--- (3)}$$

$$\text{Solving: } \underline{\underline{I_2 = I_x = 1.2 \text{ Amp}}}$$



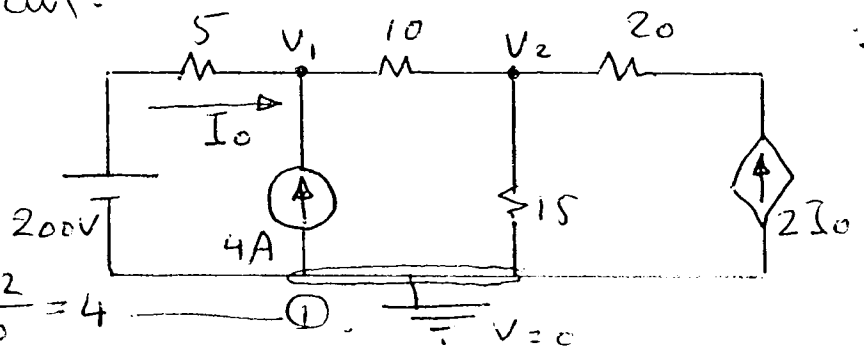
But

$$I_x = \frac{V_1 - V_2}{5}$$



Q7) Find (I_o) for the circuit.

a) By Nodal method.



Node (V_1):

$$\left(\frac{1}{5} + \frac{1}{10}\right)V_1 - \frac{200}{5} - \frac{V_2}{10} = 4 \quad \text{--- (1)}$$

$$3V_1 - V_2 = 440 \quad \text{--- (1)}$$

Node (V_2):

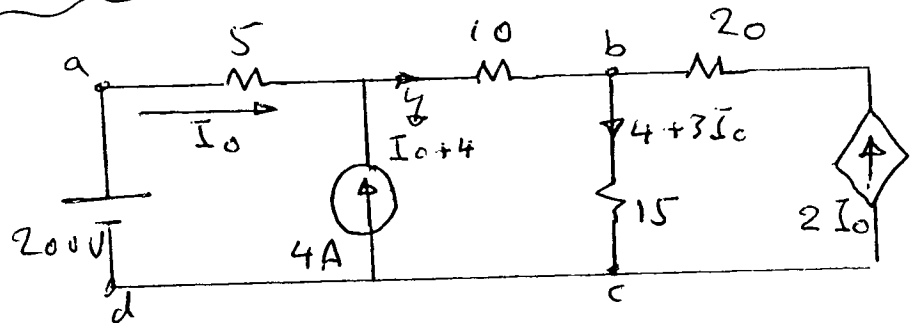
$$\left(\frac{1}{10} + \frac{1}{15}\right)V_2 - \frac{V_1}{10} = 2I_o. \quad \text{But } I_o = \frac{200 - V_1}{5}$$

$$\therefore 5V_2 + 9V_1 = 2400 \quad \text{--- (2)}$$

Solving: $V_1 = 191.6667 \text{ Volt}$.

$$\therefore I_o = \frac{200 - 191.6667}{5} = 1.6667 \text{ Amp.}$$

b) By K-L



abcd

$$200 = 5I_o + 10(I_o + 4) + 15(4 + 3I_o)$$

$$100 = 60I_o$$

$$\therefore I_o = \frac{100}{60} = 1.6667$$