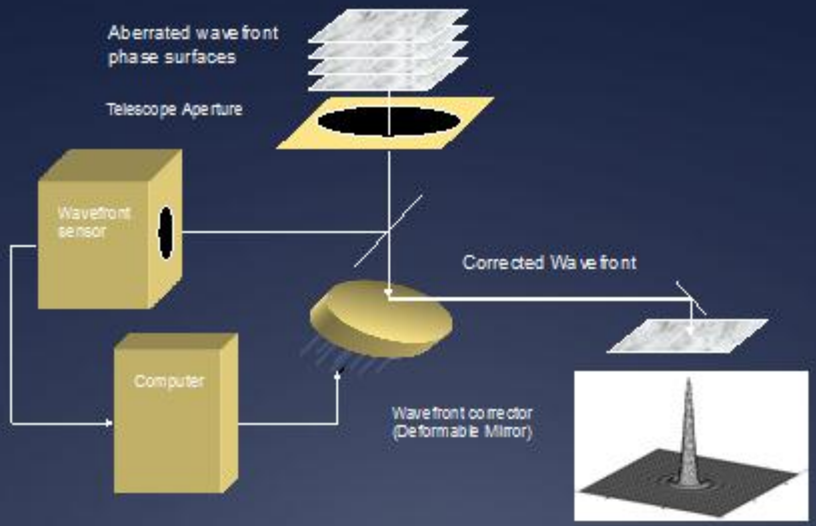
What are control systems?

- Control is the process of making a system variable adhere to a particular value, called the reference value.
- A system designed to follow a changing reference is called tracking control or servo.

What are control systems?

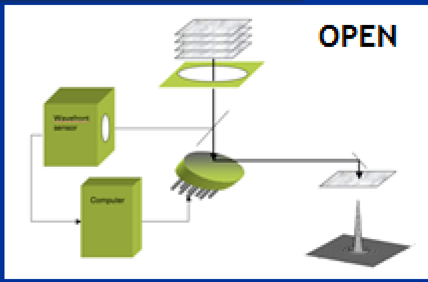
- Control is the process of making a system variable adhere to a particular value, called the reference value.
- A system designed to follow a changing reference is called tracking control or servo.

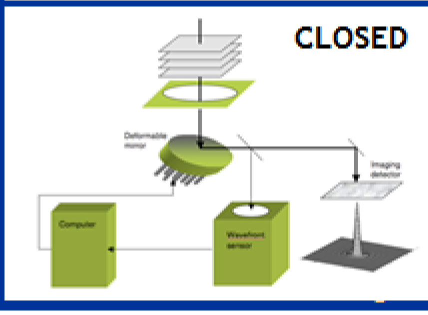
Adaptive Optics Control



Differences between open-loop and closed-loop control systems

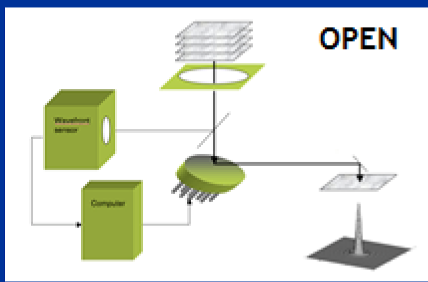
- Open-loop: control system uses no knowledge of the output
- Closed-loop: the control action is dependent on the output in some way
- "Feedback" is what distinguishes open from closed loop
- What other examples can you think of?





More about open-loop systems

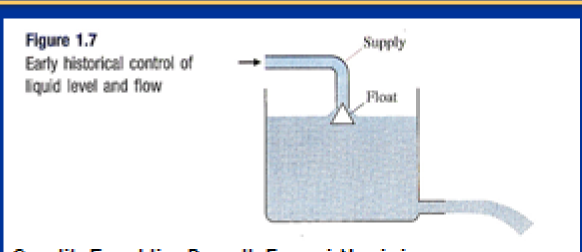
- Need to be carefully calibrated ahead of time:
- Example: for a deformable mirror, need to know exactly what shape the mirror will have if the n actuators are each driven with a voltage V_n
- Question: how might you go about this calibration?



Some Characteristics of Closed- Loop Feedback Control

- Increased accuracy (gets to the desired final position more accurately because small errors will get corrected on subsequent measurement cycles)
- Less sensitivity to nonlinearities (e.g. hysteresis in the deformable mirror) because the system is always making small corrections to get to the right place
- Reduced sensitivity to noise in the input signal
- BUT: can be unstable under some circumstances (e.g. if gain is too high)

Historical control systems: float valve



Credit: Franklin, Powell, Emami-Naeini

- As liquid level falls, so does float, allowing more liquid to flow into tank
- As liquid level rises, flow is reduced and, if needed, cut off entirely
- Sensor and actuator are both “contained” in the combination of the float and supply tube

Block Diagrams: Show Cause and Effect

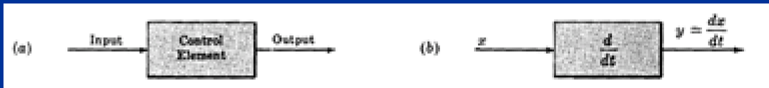


Fig. 2-2

Credit: DiStefano et al. 1990

- Pictorial representation of cause and effect
- Interior of block shows how the input and output are related.
- Example b: output is the time derivative of the input

Block Diagrams: Show Cause and Effect

Fig. 2-2

Credit: DiStefano et al. 1990

- Pictorial representation of cause and effect
- Interior of block shows how the input and output are related.
- Example b: output is the time derivative of the input

Page 9

"Summing" Block Diagrams are circles

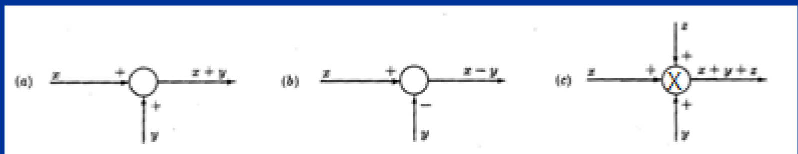


Fig. 2-3

Credit: DiStefano et al. 1990

- Block becomes a circle or "summing point"
- Plus and minus signs indicate addition or subtraction (note that "sum" can include subtraction)
- Arrows show inputs and outputs as before
- Sometimes there is a cross in the circle

A home thermostat from a control theory point of view

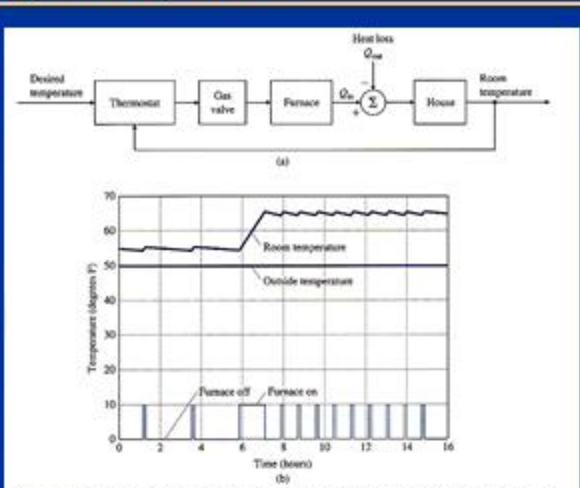
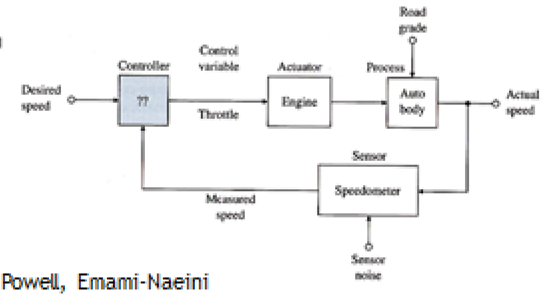


Figure 1.1 (a) Component block diagram of a room temperature control system (b) Plot of room temperature and furnace action

Credit: Franklin, Powell, Emami-Naeini

Block diagram for an automobile cruise control

Figure 1.3
Component block diagram of automobile cruise control



Credit: Franklin, Powell, Emami-Naeini

Example 1

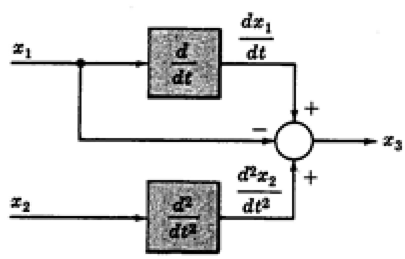
- Draw a block diagram for the equation

$$x_3 = \frac{d^2x_2}{dt^2} + \frac{dx_1}{dt} - x_1$$

Example 1

- Draw a block diagram for the equation

$$x_3 = \frac{d^2x_2}{dt^2} + \frac{dx_1}{dt} - x_1$$



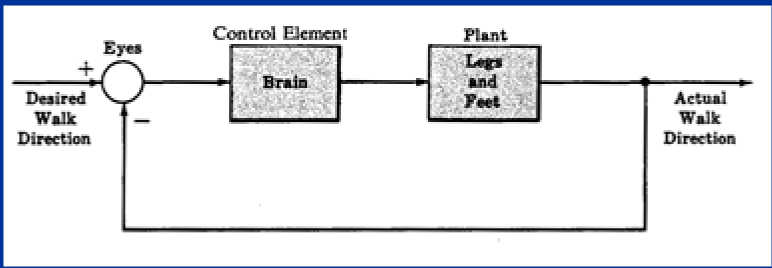
Credit: DiStefano et al. 1990

Example 2

- Draw a block diagram for how your eyes and brain help regulate the direction in which you are walking

Example 2

- Draw a block diagram for how your eyes and brain help regulate the direction in which you are walking



Credit: DiStefano et al. 1990

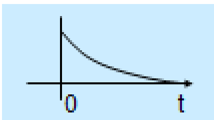
The Laplace Transform Pair

$$H(s) = \int_0^{\infty} h(t) e^{-st}\, dt$$

$$h(t) = \frac{1}{2\pi i} \int_{-i\infty}^{i\infty} H(s) e^{st}\, ds$$

- Example: decaying exponential

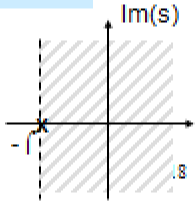
Transform: $h(t) = e^{-\sigma t}$



$$H(s) = \int_0^{\infty} e^{-(s+\sigma)t}\, dt$$

$$= \frac{-1}{s+\sigma} e^{-(s+\sigma)t}\Big|_0^{\infty}$$

$$= \frac{1}{s+\sigma}; \qquad \mathrm{Re}(s) > -\sigma$$



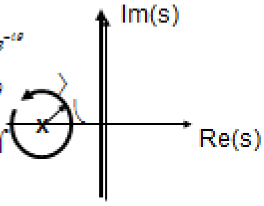
The Laplace Transform Pair

Example (continued), decaying exponential

Inverse Transform:

$$h(t)=\frac{1}{2\pi i}\int_{-i\infty}^{i\infty} e^{st}\frac{1}{s+\sigma}\,ds$$
$$=\frac{1}{2\pi i}\oint_{-\pi}^{\pi}\rho^{-1}e^{-\sigma t}\,i\rho\,d\theta$$
$$=e^{-\sigma t}\frac{1}{2\pi i}\oint_{-\pi}^{\pi} i\,d\theta$$
$$=e^{-\sigma t}$$

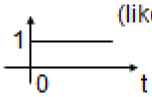
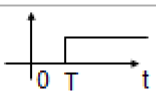
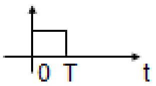
$$s=-\sigma+\rho e^{i\theta}$$
$$\frac{1}{s+\sigma}=\rho^{-1}e^{-i\theta}$$
$$ds=i\rho e^{i\theta}d\theta$$



The above integration makes use of the Cauchy Principal Value Theorem:

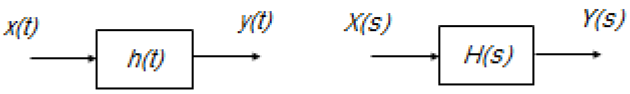
If F(s) is *analytic* then $\oint F(s)\frac{1}{s-a}ds=2\pi i F(a)$

Laplace Transform Pairs

h(t)	H(s)
unit step  (like $\lim_{\sigma\to 0} e^{-\sigma t}$)	$\frac{1}{s}$
$e^{-\sigma t}$	$\frac{1}{s+\sigma}$
$e^{-\sigma t}\cos(\omega t)$	$\frac{1}{2}\left(\frac{1}{s+\sigma-i\omega}+\frac{1}{s+\sigma+i\omega}\right)$
$e^{-\sigma t}\sin(\omega t)$	$\frac{1}{2i}\left(\frac{1}{s+\sigma-i\omega}-\frac{1}{s+\sigma+i\omega}\right)$
delayed step 	$\frac{e^{-sT}}{s}$
unit pulse 	$\frac{1-e^{-sT}}{s}$

Laplace Transform Properties (2)

System Block Diagrams



convolution of input *x(t)* with impulse response *h(t)* ⟷

- Product of input spectrum *X(s)* with frequency response *H(s)*
- *H(s)* in this role is called the *transfer function*

$$L\left[\int_0^t h(t')x(t-t')dt'\right] = H(s)X(s) = Y(s)$$

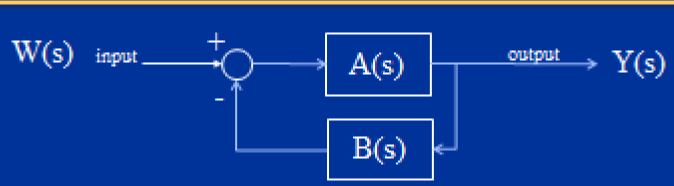
Conservation of Power or Energy

$$\int_0^\infty [h(t)]^2 dt = \frac{1}{2\pi i}\int_{-i\infty}^{i\infty} |H(s)|^2 ds$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} |H(i\omega)|^2 d\omega$$

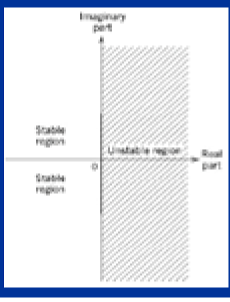
"Parseval's Theorem"

Back Up: Control Loop Arithmetic

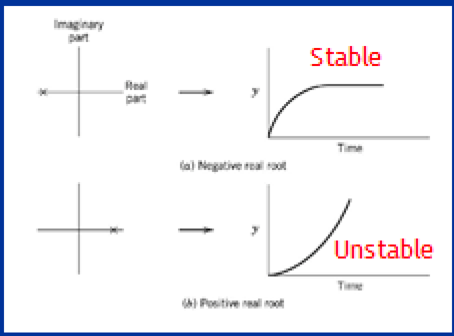


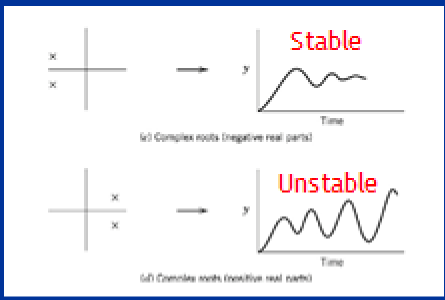
$$Y(s) = A(s)W(s) - A(s)B(s)Y(s) \qquad Y(s) = \frac{A(s)W(s)}{1 + A(s)B(s)}$$

Unstable if any roots of
$1+A(s)B(s) = 0$
are in right-half of the s-plane:
exponential growth $\exp(st)$



Stable and unstable behavior



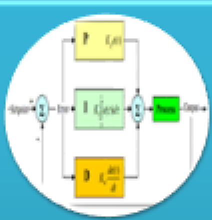


Feedback Control Systems (FCS)

Lecture 4 & 5
Introduction to the Subject

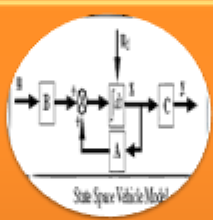
Dr. Ahmed A. Oglah
email: 60008@uotechnology.edu.iq

Course Outline



Classical Control

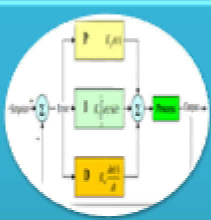
- System Modelling
  - Transfer Function
  - Block Diagrams
  - Signal Flow Graphs
- System Analysis
  - Time Domain Analysis (Test Signals)
  - Frequency Domain Analysis
    - Bode Plots
    - Nyquist Plots
    - Nichol's Chart



Modern Control

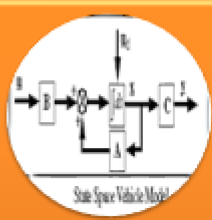
- State Space Modelling
- Eigenvalue Analysis
- Observability and Controllability
- Solution of State Equations (state Transition Matrix)
- State Space to Transfer Function
- Transfer Function to State Space
  - Direct Decomposition of Transfer Function
  - Cascade Decomposition of Transfer Function
  - Parallel Decomposition of Transfer Function

Course Outline



Classical Control

- System Modelling
  - Transfer Function
  - Block Diagrams
  - Signal Flow Graphs
- System Analysis
  - Time Domain Analysis (Test Signals)
  - Frequency Domain Analysis
    - Bode Plots
    - Nyquist Plots
    - Nichol's Chart



Modern Control

- State Space Modelling
- Eigenvalue Analysis
- Observability and Controllability
- Solution of State Equations (state Transition Matrix)
- State Space to Transfer Function
- Transfer Function to State Space
  - Direct Decomposition of Transfer Function
  - Cascade Decomposition of Transfer Function
  - Parallel Decomposition of Transfer Function

Text Books

1. Modern Control Engineering, (5th Edition)

   By: Katsuhiko Ogata.

   (Prof Emeritus)

   Mechanical Engineering

   University of Minnesota

2. Control Systems Engineering, (6th Edition)

   By: Norman S. Nise. (Professor Emeritus)

   Electrical and Computer

   Engineering Department

   at California State Polytechnic University

Reference Books

1. Modern Control Systems, (12th Edition)
   By: Richard C. Dorf and Robert H. Bishop.

2. Automatic Control Systems, (9th Edition)
   By: Golnaraghi and B. C. Kuo.

Prerequisites

- For Classical Control Theory
  - Differential Equations
  - Laplace Transform
  - Basic Physics
  - Ordinary and Semi-logarithimic graph papers
- For Modern Control theory above &
  - Linear Algebra
  - Matrices

Practical Sessions

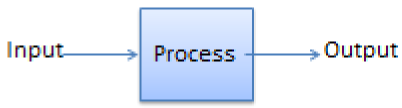
- Practicals are divided into two sessions
  - Software Based (Cyber Interactive Lab)
    - Matlab
    - Simulink
    - Control System Toolbox
  - Hardware Based (Instrument & Control Lab)
    - Modular Servo System (MS150 Available in)

Definitions

System – An interconnection of elements and devices for a desired purpose.

Control System – An interconnection of components forming a system configuration that will provide a desired response.

Process – The device, plant, or system under control. The input and output relationship represents the cause-and-effect relationship of the process.



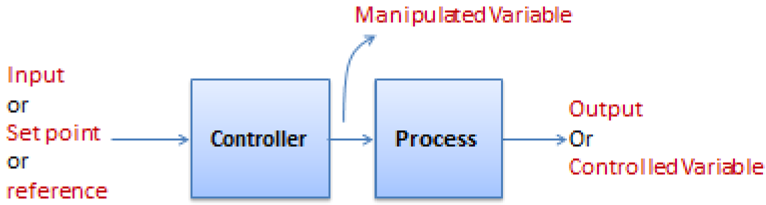
Definitions

Controlled Variable– It is the quantity or condition that is measured and Controlled. Normally *controlled variable* is the output of the control system.

Manipulated Variable– It is the quantity of the condition that is varied by the controller so as to affect the value of *controlled variable*.

Control – Control means measuring the value of *controlled variable* of the system and applying the *manipulated variable* to the system to correct or limit the deviation of the measured value from a desired value.

Definitions



Disturbances– A disturbance is a signal that tends to adversely affect the value of the system. It is an unwanted input of the system.

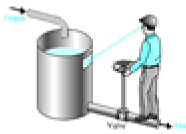
- If a disturbance is generated within the system, it is called *internal disturbance*. While an *external disturbance* is generated outside the system.

Types of Control System

- Natural Control System
  - Universe
  - Human Body
- Manmade Control System
  - Vehicles
  - Aeroplanes

Types of Control System

- Manual Control Systems
  - Room Temperature regulation Via Electric Fan
  - Water Level Control



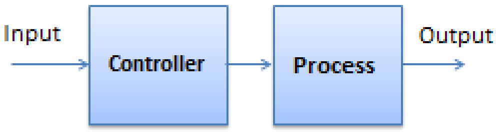
- Automatic Control System
  - Room Temperature regulation Via A.C
  - Human Body Temperature Control

Types of Control System

Open-Loop Control Systems

Open-Loop Control Systems utilize a controller or control actuator to obtain the desired response.

- Output has no effect on the control action.
- In other words output is neither measured nor fed back.



Open-loop control system (without feedback).

Examples:- Washing Machine, Toaster, Electric Fan

Types of Control System

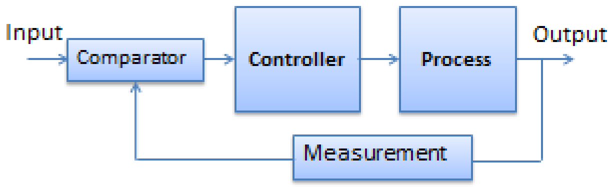
Open-Loop Control Systems

- Since in open loop control systems reference input is not compared with measured output, for each reference input there is fixed operating condition.
- Therefore, the accuracy of the system depends on calibration.
- The performance of open loop system is severely affected by the presence of disturbances, or variation in operating/ environmental conditions.

Types of Control System

Closed-Loop Control Systems

Closed-Loop Control Systems utilizes feedback to compare the actual output to the desired output response.

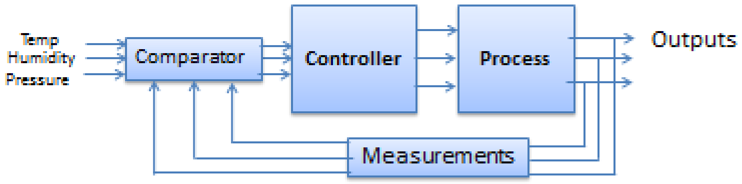


Closed-loop feedback control system (with feedback).

Examples:- Refrigerator, Iron

Types of Control System

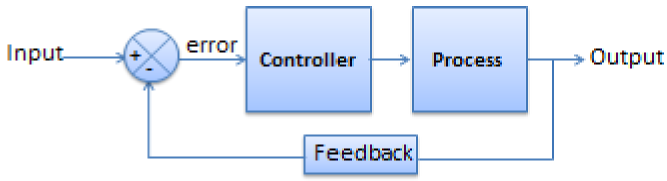
Multivariable Control System



Types of Control System

Feedback Control System

- A system that maintains a prescribed relationship between the output and some reference input by comparing them and using the difference (i.e. error) as a means of control is called a feedback control system.

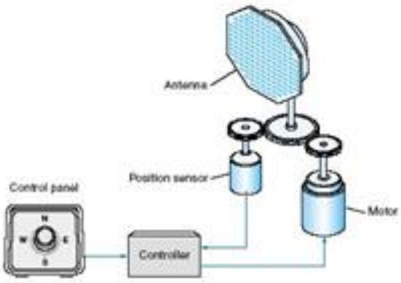


- Feedback can be positive or negative.

Types of Control System

Servo System

- A Servo System (or servomechanism) is a feedback control system in which the output is some mechanical position, velocity or acceleration.



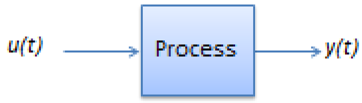
Antenna Positioning System

Modular Servo System (MS150)

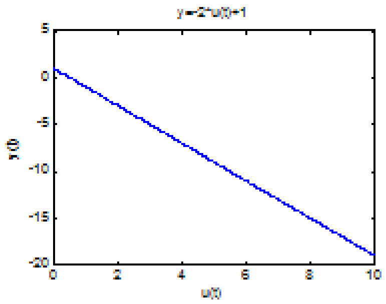
Types of Control System

Linear Vs Nonlinear Control System

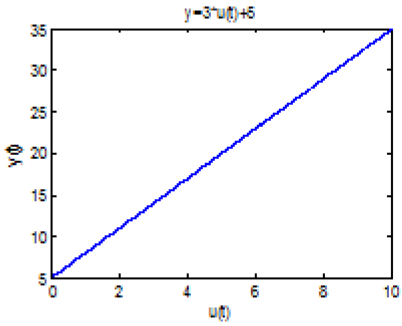
- A Control System in which output varies linearly with the input is called a linear control system.



$$y(t) = -2u(t) + 1$$



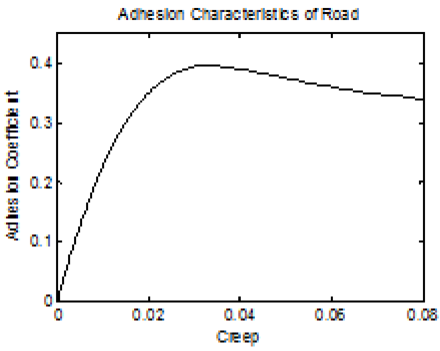
$$y(t) = 3u(t) + 5$$



Types of Control System

Linear Vs Nonlinear Control System

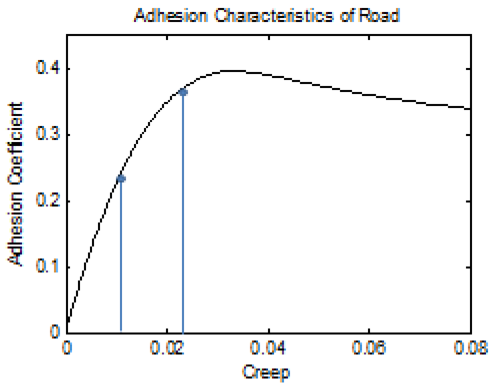
- When the input and output has nonlinear relationship the system is said to be nonlinear.



Types of Control System

Linear Vs Nonlinear Control System

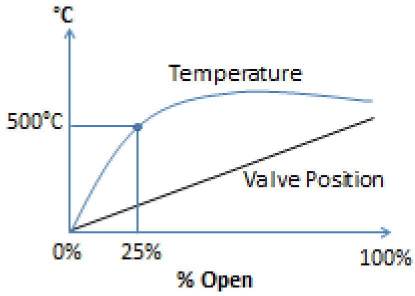
- Linear control System Does not exist in practice.
- Linear control systems are idealized models fabricated by the analyst purely for the simplicity of analysis and design.
- When the magnitude of signals in a control system are limited to range in which system components exhibit linear characteristics the system is essentially linear.



Types of Control System

Linear Vs Nonlinear Control System

- Temperature control of petroleum product in a distillation column.



Types of Control System

Time invariant vs Time variant

- When the characteristics of the system do not depend upon time itself then the system is said to time invariant control system.

$$y(t) = -2u(t) + 1$$

- Time varying control system is a system in which one or more parameters vary with time.

$$y(t) = 2u(t) - 3t$$

Types of Control System

Lumped parameter vs Distributed Parameter

- Control system that can be described by ordinary differential equations are lumped-parameter control systems.

$$M\frac{d^2x}{dt^2} = C\frac{dx}{dt} + kx$$

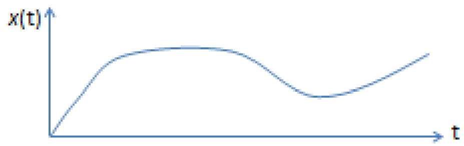
- Whereas the distributed parameter control systems are described by partial differential equations.

$$f_1\frac{\partial x}{dy} + f_2\frac{\partial x}{dz} = g\frac{\partial^2 x}{dz^2}$$

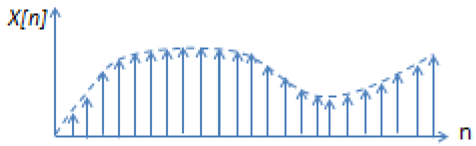
Types of Control System

Continuous Data Vs Discrete Data System

- In continuous data control system all system variables are function of a continuous time t.



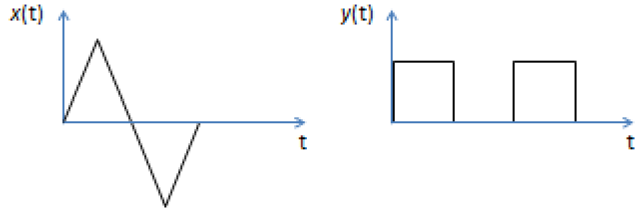
- A discrete time control system involves one or more variables that are known only at discrete time intervals.



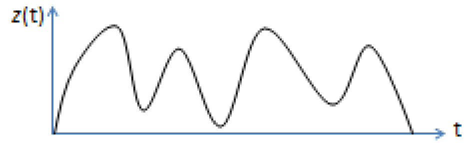
Types of Control System

Deterministic vs Stochastic Control System

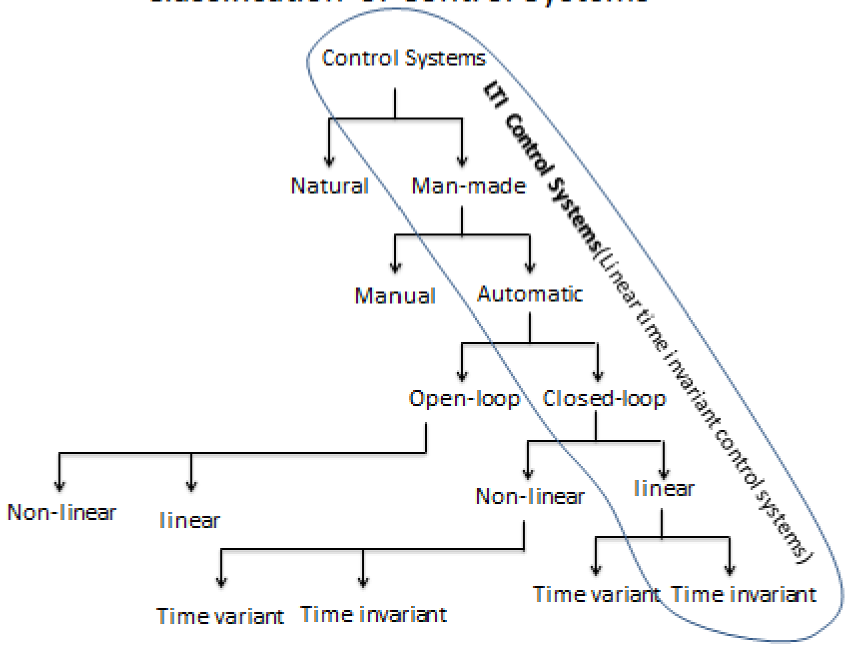
- A control System is deterministic if the response to input is predictable and repeatable.



- If not, the control system is a stochastic control system

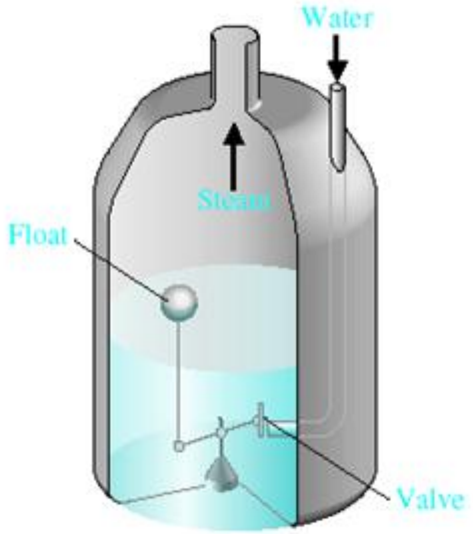


Classification of Control Systems

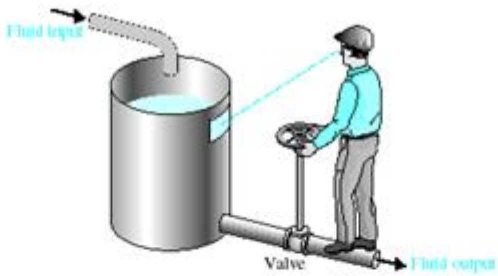


Examples of Control Systems

Water-level float regulator

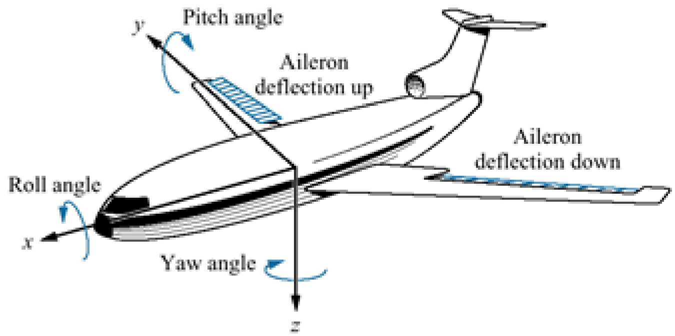


Examples of Control Systems



A manual control system for regulating the level of fluid in a tank by adjusting the output valve. The operator views the level of fluid through a port in the side of the tank.

Examples of Modern Control Systems



PID Control

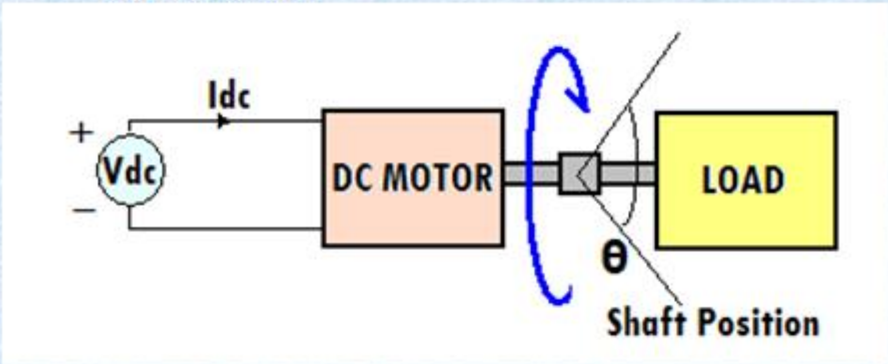
- A closed loop (feedback) control system, generally with Single Input-Single Output (SISO)
- A portion of the signal being fed back is:
  - Proportional to the signal (**P**)
  - Proportional to integral of the signal (**I**)
  - Proportional to the derivative of the signal (**D**)

When PID Control is Used

- PID control works well on SISO systems of 2^{nd} Order, where a desired Set Point can be supplied to the system control input
- PID control handles step changes to the Set Point especially well:
  - Fast Rise Times
  - Little or No Overshoot
  - Fast settling Times
  - Zero Steady State Error
- PID controllers are often fine tuned on-site, using established guidelines

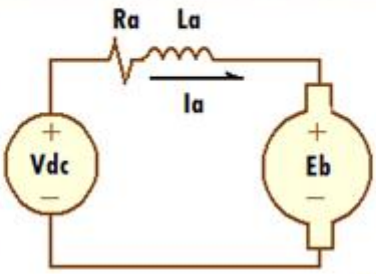
Control Theory

- Consider a DC Motor turning a Load:
  - Shaft Position, Theta, is proportional to the input voltage



Looking at the Motor:

- Electrically (for Permanent Magnet DC):



KVL:

$$-Vdc + IaRa + V_L + Eb = 0$$

$$V_L = L\,\dot{Ia}$$

Substituting:

$$\boxed{L\,\dot{Ia} + RaIa = Vdc - Ka\,\dot{\theta}}$$

Motor Equations:

Back-emf:

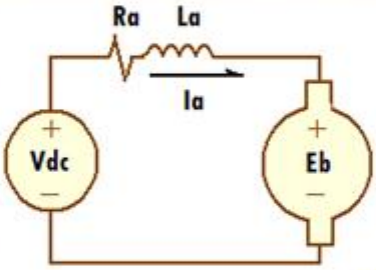
$$Eb = Ka\,\omega = Ka\,\dot{\theta}$$

Elec. Torque:

$$\boxed{T_e = Kt\,Ia}$$

Looking at the Motor:

- Electrically (for Permanent Magnet DC):



KVL:

$-Vdc + IaRa + V_L + Eb = 0$

$V_L = L\,\dot{Ia}$

Substituting:

$\boxed{L\,\dot{Ia} + RaIa = Vdc - Ka\,\dot{\theta}}$

Motor Equations:

Back-emf:

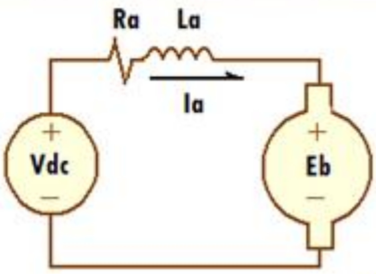
$Eb = Ka\,\omega = Ka\,\dot{\theta}$

Elec. Torque:

$\boxed{T_e = Kt\,Ia}$

Looking at the Motor:

- Electrically (for Permanent Magnet DC):



KVL:

$-Vdc + IaRa + V_L + Eb = 0$

$V_L = L\,\dot{Ia}$

Substituting:

$\boxed{L\,\dot{Ia} + RaIa = Vdc - Ka\,\dot{\theta}}$

Motor Equations:

Back-emf:

$Eb = Ka\,\omega = Ka\,\dot{\theta}$

Elec. Torque:

$\boxed{T_e = Kt\,Ia}$

Combining Elect/Mech

Torque is Conserved: Tm = Te

$$\text{Mech. Torque, } T_m = J\ddot{\theta} + B\dot{\theta} \quad = \quad \text{Elec. Torque: } T_e = K_t I_a$$

(1) $J\ddot{\theta} + B\dot{\theta} = K_t I_a$ (2) $L\dot{I}_a + R_a I_a = V_{dc} - K_a\dot{\theta}$

In LAPLACE Domain:

$$s(Js + B)\,\theta(s) = K\,I_a(s) \qquad (Ls + R)\,I_a(s) = V_{dc} - K\,s\,\theta(s)$$

Combining and forming Open Loop Transfer Function:

$$\frac{\theta}{V_{dc}} = \frac{K}{(Js + B)(Ls + R_a) + K^2}$$

Shaft Position as a Function of Input Voltage

1 and 2 above are a basis for the state-space description

Combining Elect/Mech

Torque is Conserved: Tm = Te

$$\text{Mech. Torque, } T_m = J\ddot{\theta} + B\dot{\theta} \quad = \quad \text{Elec. Torque: } T_e = K_t I_a$$

(1) $J\ddot{\theta} + B\dot{\theta} = K_t I_a$ (2) $L\dot{I}_a + R_a I_a = V_{dc} - K_a\dot{\theta}$

In LAPLACE Domain:

$$s(Js + B)\,\theta(s) = K\,I_a(s) \qquad (Ls + R)\,I_a(s) = V_{dc} - K\,s\,\theta(s)$$

Combining and forming Open Loop Transfer Function:

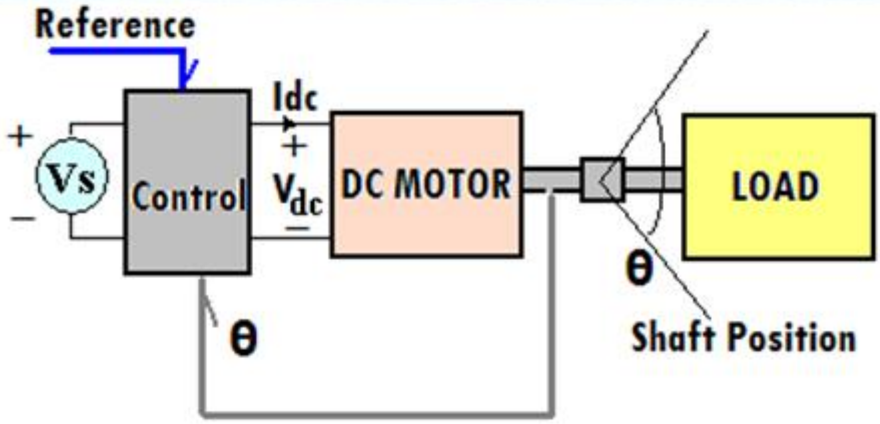
$$\frac{\theta}{V_{dc}} = \frac{K}{(Js + B)(Ls + R_a) + K^2}$$

Shaft Position as a Function of Input Voltage

1 and 2 above are a basis for the state-space description

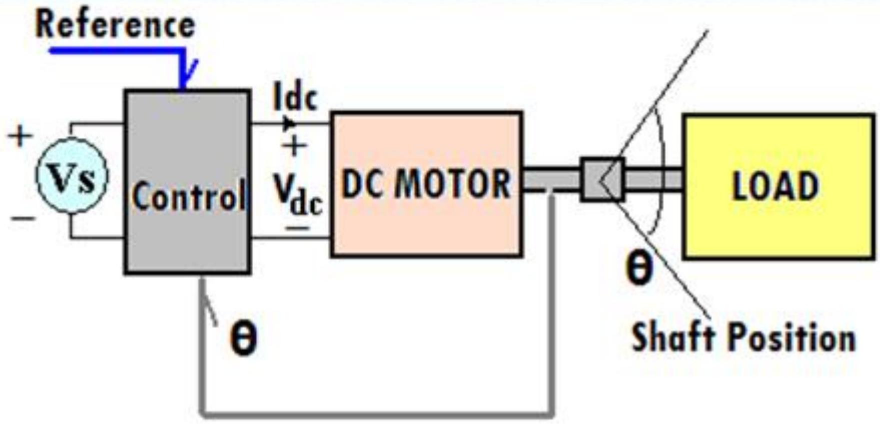
Physically, We Want:

- A 2nd Order SISO System with Input to Control Shaft Position:



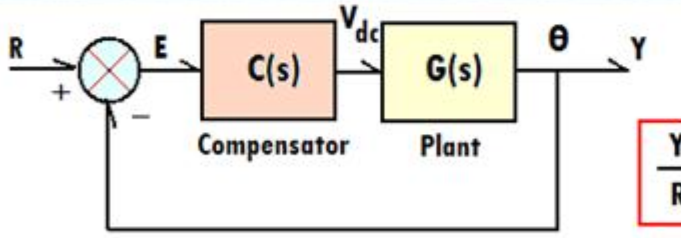
Physically, We Want:

- A 2nd Order SISO System with Input to Control Shaft Position:



Adding the PID:

- Consider the block diagram shown:

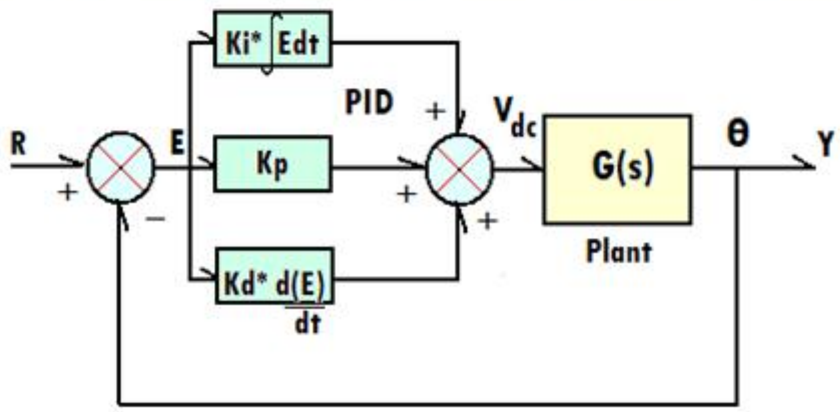


$$\frac{Y}{R} = \frac{C\,G}{1 + C\,G}$$

Closed Loop Transfer Function

- C(s) could also be second order….(PID)

PID Block Diagram:



PID Mathematically:

- Consider the input error variable, $e(t)$:

  – Let $p(t) = Kp * e(t)$ {p proportional to e (mag)}
  – Let $i(t) = Ki * \int e(t)dt$ {i integral of e (area)}
  – Let $d(t) = Kd * de(t)/dt$ {d derivative of e (slope)}

  AND let $Vdc(t) = p(t) + i(t) + d(t)$

  Then in Laplace Domain:

  $$Vdc(s) = [Kp + 1/s\ Ki + s\ Kd]\ E(s)$$

PID Implemented:

Let $C(s) = Vdc(s) / E(s)$ (transfer function)

$$C(s) = [Kp + 1/s\ Ki + s\ Kd]$$

$$= [Kp\ s + Ki + Kd\ s^2] / s \quad (2^{nd}\text{ Order})$$

THEN

$$C(s)G(s) = \frac{K\ [Kd\ s^2 + Kp\ s + Ki]}{s(s^2 + 2as + b^2)}$$

AND

$$Y/R = \frac{Kd\ s^2 + Kp\ s + Ki}{s^3 + (2a+Kd)s^2 + (b^2+Kp)\ s + Ki}$$

Implications:

- Kd has direct impact on damping
- Kp has direct impact on resonant frequency

In General the effects of *increasing* parameters is:

Parameter:	Rise Time	Overshoot	Settling Time	S.S.Error
Kp	Decrease	Increase	Small Change	Decrease
Ki	Decrease	Increase	Increase	Eliminate
Kd	Small Change	Decrease	Decrease	None

Implications:

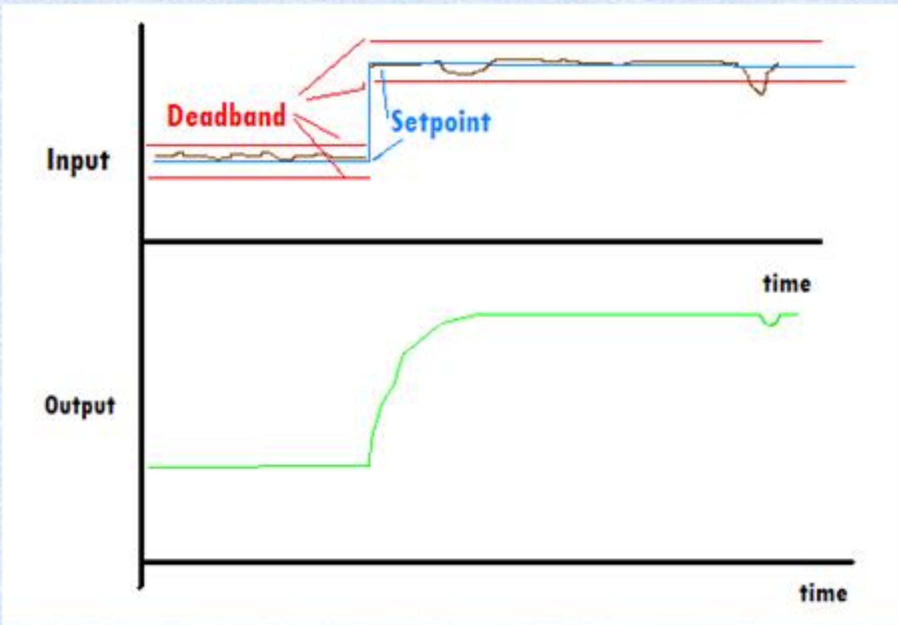
- Kd has direct impact on damping
- Kp has direct impact on resonant frequency

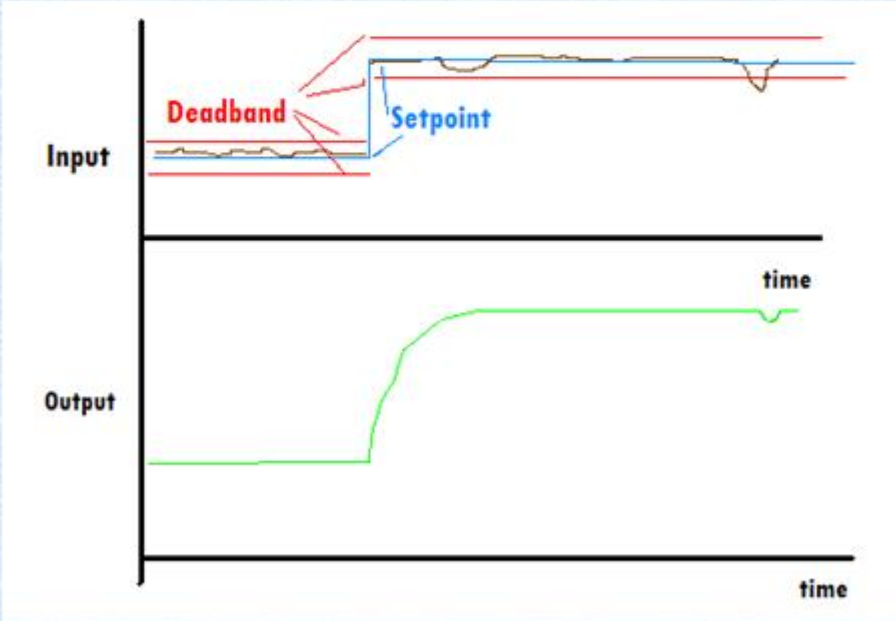
In General the effects of *increasing* parameters is:

Parameter:	Rise Time	Overshoot	Settling Time	S.S.Error
Kp	Decrease	Increase	Small Change	Decrease
Ki	Decrease	Increase	Increase	Eliminate
Kd	Small Change	Decrease	Decrease	None

Deadband

- In noisy environments or with energy intensive processes it may be desirable to make the controller unresponsive to small changes in input or feedback signals
- A deadband is an area around the input signal set point, wherein no control action will occur





From Differential Equation to Difference Equation:

- Definition of Derivative:

$$\frac{dU}{dt} = \lim_{\Delta t \to 0} \frac{U(t + \Delta t) - U(t)}{\Delta t}$$

- Algebraically Manipulate to Difference Eq:

$$U(t + \Delta t) = U(t) + \Delta t * \frac{dU}{dt}$$

(for sufficiently small Δt)

- Apply this to Iteratively Solve First Order Linear Differential Equations (hold for applause)

From Differential Equation to Difference Equation:

- Definition of Derivative:

$$\frac{dU}{dt} = \lim_{\Delta t \to 0} \frac{U(t + \Delta t) - U(t)}{\Delta t}$$

- Algebraically Manipulate to Difference Eq:

$$U(t + \Delta t) = U(t) + \Delta t^* \frac{dU}{dt}$$

(for sufficiently small Δt)

- Apply this to Iteratively Solve First Order Linear Differential Equations (hold for applause)

Differential to Difference with Time-Step, T:

- Differential Equation:

$$dVc/dt = (Vs - Vc)/RC$$

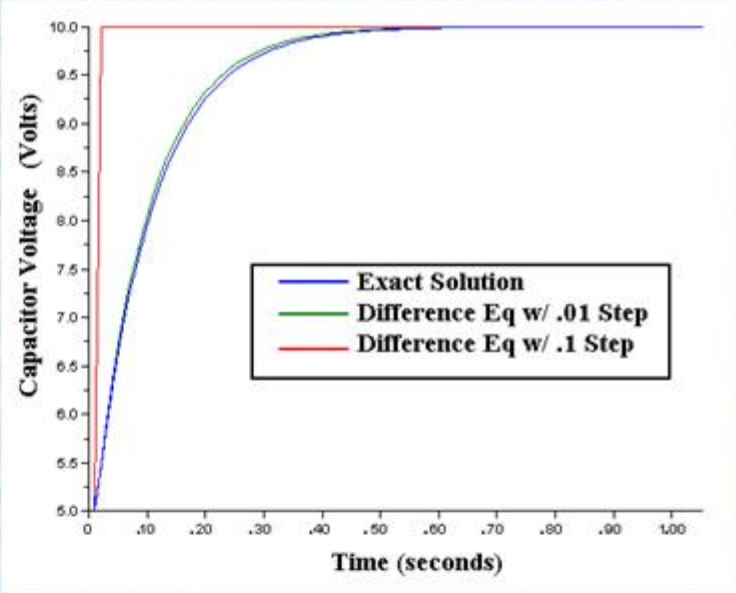
- Difference Equation by Definition:

$$Vc(kT+T) = Vc(kT) + T^*dVc/dt$$

- Substituting:

$$Vc(kT+T) = Vc(kT) + T^*(Vs - Vc(kT))/RC$$

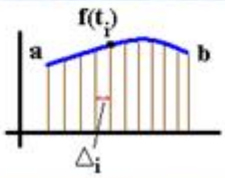
Results:



Integration by Trapezoidal Approximation:

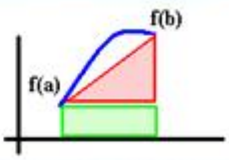
- Definition of Integration (area under curve):

$$F(b) = \int_a^b f(t)dt = \sum_{i=1}^{n} f(t_i)\triangle_i$$

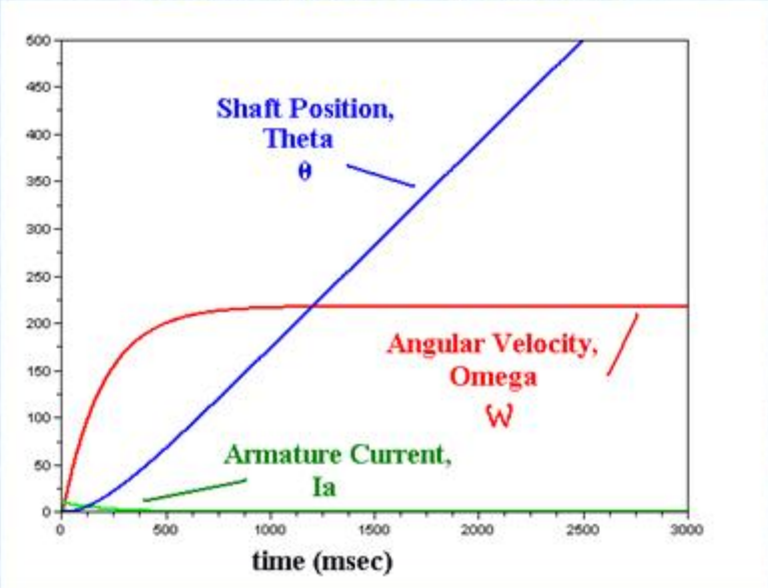


- Approximation by Trapezoidal Areas

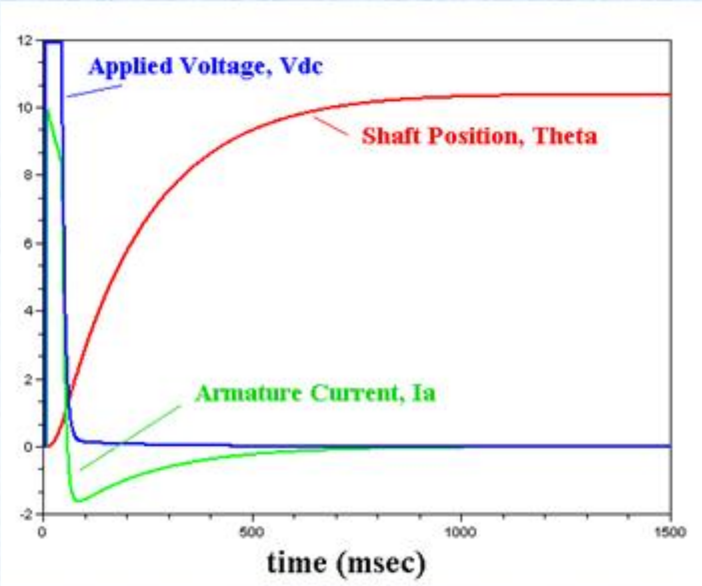
$$F(b) = (b - a)f(a) + \frac{1}{2}(b-a)(f(b) - f(a))$$



Scilab Emulation of PM DC Motor using State Space Equations



Results:



CONTROL THEORY

Dr. Ahmed A. Oglah

References:

1- modern control engineering ,5th edition,ogata.

2-modern control system.11th edition ,richard c. dorf

3-Automatic control systems,9th edition,kuo

1st lecture

Second Year   Theoretical: 2   hrs./week.

CONTROL THEROY (I)

1. Introduction of Control System:

Control theories commonly used today are classical control theory (also called conventional control theory), modern control theory, and robust control theory. This is lecture presents comprehensive treatments of the analysis and design of control systems based on the classical control theory and modern control theory. Automatic control is essential in any field of engineering and science. Automatic control is an important and integral part of space-vehicle systems, robotic systems, modern manufacturing systems, and any industrial operations involving control of temperature, pressure, humidity, flow, etc. It is desirable that most engineers and scientists are familiar with theory and practice of automatic

2. Brief Review of Historical Developments of Control Theories and Practices .

The first significant work in automatic control was James Watt's centrifugal governor for the speed control of a steam engine in the eighteenth century.

Minorsky, Hazen, and Nyquist, among many others. In 1922, Minorsky worked on automatic controllers for steering ships and showed how stability could be determined from the differential equations describing the system .

In 1932, Nyquist developed a relatively simple procedure for determining the stability of closed-loop systems on the basis of open-loop response to steady-state sinusoidal inputs.

In 1934 Hazen, who introduced the term servomechanisms for position control systems discussed the design of relay servomechanisms capable of closely following a changing input.

During the decade of the 1940s, frequency-response methods (especially the Bode diagram methods due to Bode) made it possible for engineers to design linear close loop control systems that satisfied performance requirements.

Many industrial control systems in 1940s and 1950s used PID controllers to control pressure, temperature, etc.

In the early 1940s Ziegler and Nichols suggested rules for tuning PID controllers, called Ziegler–Nichols tuning rules. From the end of the 1940s to the 1950s, the root-locus method due to Evans was fully developed

The frequency-response and root-locus methods, which are the core of classical control theory, lead to systems that are stable and satisfy a set of more or less arbitrary performance requirements. Such systems are, in general, acceptable but not optimal in any meaningful sense. Since the late 1950s, the emphasis in control design problems has been shifted from the design of one of many systems that work to the design of one optimal system in some meaningful sense.

As modern plants with many inputs and outputs become more and more complex the description of a modern control system requires a large number of equations.

Classical control theory, which deals only with single-input, single-output systems, becomes powerless for multiple-input, multiple-output systems. Since about 1960, because the availability of digital computers made possible time-domain analysis of complex systems, modern control theory, based on time-domain analysis and synthesis using state has been developed to cope with the increased complexity of modern plants and the stringent requirements on accuracy, weight, and cost in military, space, and industrial applications .

During the years from 1960 to 1980, optimal control of both deterministic and stochastic systems, as well as adaptive and learning control of complex systems, were fully investigated. From 1980s to 1990s, developments in modern control theory were centered around robust control and associated topics.

Modern control theory is based on time-domain analysis of differential equation systems. Modern control theory made the design of control systems simpler because the theory is based on a model of an actual control system. However, the system’s stability is sensitive to

the error between the actual system and its model. This means that when the designed controller based on a model is applied to the actual system, the system may not be stable. To avoid this situation, we design the control system by first setting up the range of possible errors and then designing the controller in such a way that, if the error of the system stays within the assumed range, the designed control system will stay stable. The design method based on this principle is called robust control theory. This theory incorporates both the frequency response approach and the time-domain approach. The theory is mathematically very complex because this theory requires mathematical background.

3. Definitions:

Before we can discuss control systems, some basic terminologies must be defined Controlled Variable and Control Signal or Manipulated Variable. The controlled variable is the quantity or condition that is measured and controlled. The control signal or manipulated variable is the quantity or condition that is varied by the controller so as to affect the value of the controlled variable. Normally, the controlled variable is the output of the system. Control means measuring the value of the controlled variable of the system and applying the control signal to the system to correct or limit deviation of the measured value from a desired value .

In studying control engineering, we need to define additional terms that are necessary to describe control systems:

Plants: A plant may be a piece of equipment, perhaps just a set of machine parts functioning together, the purpose of which is to perform a particular operation. In this book, we shall call any physical object to be controlled (such as a mechanical device, a heating furnace, a chemical reactor, or a spacecraft) a plant.

Processes: The Merriam–Webster Dictionary defines a process to be a natural, progressively continuing operation or development marked by a series of gradual changes that succeed one another in a relatively fixed way and lead toward a particular result or end; or an artificial or voluntary, progressively continuing operation that consists of a series of controlled actions or movements systematically directed toward a particular result or end. In this book we shall call any operation to be controlled a process. Examples are chemical, economic, and biological processes .

Systems: A system is a combination of components that act together and perform a certain objective. A system need not be physical. The concept of the system can be applied to abstract, dynamic phenomena such as those encountered in economics. The word system should, therefore, be interpreted to imply physical, biological, economic, and the like, systems.

Disturbances: A disturbance is a signal that tends to adversely affect the value of the output of a system. If a disturbance is generated within the system, it is called internal, while an external disturbance is generated outside the system and is an input. **Feedback Control**: Feedback control refers to an operation that, in the presence of disturbances, tends to reduce the difference between the output of a system and some reference input and does so on the basis of this difference. Here only unpredictable disturbances are so specified, since predictable or known disturbances can always be compensated for within the system

CLOSED-LOOP CONTROL VERSUS OPEN-LOOP CONTROL

Feedback Control Systems. A system that maintains a prescribed relationship between the output and the reference input by comparing them and using the difference as a means of control is called a feedback control system. An example would be a room temperature control system. By measuring the actual room temperature and comparing it with the reference temperature (desired temperature), the thermostat turns the heating or cooling equipment on or off in such a way as to ensure that the room temperature remains at a comfortable level regardless of outside conditions.

Feedback control systems are not limited to engineering but can be found in various non-engineering fields as well. The human body, for instance, is a highly advanced feedback control system. Both body temperature and blood pressure are kept constant by means of physiological feedback. In fact, feedback performs a vital function: It makes the human body relatively insensitive to external disturbances, thus enabling it to function properly in a changing environment.

Closed-Loop Control Systems: Feedback control systems are often referred to as closed-loop control systems. In practice, the terms feedback control and closed-loop control are used interchangeably. In a closed-loop control system the actuating error signal, which is the difference between the input signal and the feedback signal (which) may be the output signal itself or a function of the output signal and its derivatives and/or integrals), is fed to the controller so as to reduce the error and bring the output of the system to a desired value. The term closed-loop control always implies the use of feedback control action in order to reduce system error.

Open-Loop Control Systems: Those systems in which the output has no effect on the control action are called open-loop control systems. In other words, in an open loop control system the output is neither measured nor fed back for comparison with the input. One practical example is a washing machine. Soaking, washing, and rinsing in the washer operate on a time basis. The machine does not measure the output signal, that is, the cleanliness of the clothe

In any open-loop control system the output is not compared with the reference input, thus, to each reference input there corresponds a fixed operating condition; as a result the accuracy of the system depends on calibration. In the presence of disturbances, an open-loop control system will not perform the desired task. Open-loop control can be used, in practice, only if the relationship between the input and output is known and if there are neither internal nor external disturbances. Clearly, such systems are not feedback control systems. Note that any control system that operates on a time basis is open loop. For instance, traffic control by means of signals operated on a time basis is another example of open-loop control.

Closed-Loop versus Open-Loop Control Systems: An advantage of the closed-loop control system is the fact that the use of feedback makes the system response relatively insensitive to external disturbances and internal variations in system parameters. It is thus possible to use relatively inaccurate and inexpensive components to obtain the accurate control of a given plant, whereas doing so is impossible in the open-loop case.

From the point of view of stability, the open-loop control system is easier to build because system stability is not a major problem. On the other hand, stability is a major problem in the

closed-loop control system, which may tend to overcorrect errors and thereby can cause oscillations of constant or changing amplitude.

It should be emphasized that for systems in which the inputs are known ahead of time and in which there are no disturbances it is advisable to use open-loop control. Closed-loop control systems have advantages only when unpredictable disturbances and/or unpredictable variations in system components are present. Note that the output power rating partially determines the cost, weight, and size of a control system.

The number of components used in a closed-loop control system is more than that for a corresponding open-loop control system. Thus, the closed-loop control system is generally higher in cost and power. To decrease the required power of a system, open-loop control may be used where applicable. A proper combination of open-loop and closed-loop controls is usually less expensive and will give satisfactory overall system performance.

Most analyses and designs of control systems presented in this book are concerned with closed-loop control systems. Under certain circumstances (such as where no disturbances

exist or the output is hard to measure) open-loop control systems may be desired. Therefore, it is worthwhile to summarize the advantages and disadvantages of using open-loop control systems.

The major advantages of open-loop control systems are as follows:

1. Simple construction and ease of maintenance.
2. Less expensive than a corresponding closed-loop system.
3. There is no stability problem.
4. Convenient when output is hard to measure or measuring the output precisely is economically not feasible. (For example, in the washer system, it would be quite expensive to provide a device to measure the quality of the washer’s output, clean lines of the clothes.)

The major disadvantages of open-loop control systems are as follows:

1. Disturbances and changes in calibration cause errors, and the output may be different from what is desired.
2. To maintain the required quality in the output, recalibration is necessary from time to time.

4.Example of Open-Loop Control Systems (Non feedback Systems):

The idle-speed control system illustrated in Fig. 1, shown previously, is rather unsophisticated and is called an open-loop control system. It is not difficult to see that the system as shown would not satisfactorily fulfill critical performance requirements. For instance, if the throttle angle a is set at a certain initial value that corresponds to a certain engine speed, then when a load torque TL is applied, there is no way to prevent a drop in the engine speed. The only way to make the system work is to have a means of adjusting a in response to a change in the load torque in order to maintain m at the desired level. The conventional electric washing machine is another example of an open-loop control system because, typically, the amount of machine wash time is entirely determined by the judgment and estimation of the human operator.

The elements of an open-loop control system can usually be divided into two parts: the controller and the controlled process, as shown by the block diagram of Fig. 2. An input signal, or command, r, is applied to the controller, whose output acts as the actuating signal u; the actuating signal then controls the controlled process so that the controlled variable y will perform according to some prescribed standards. In simple cases, the controller can be an amplifier, a mechanical linkage, a filter, or other control elements, depending on the nature of the system. In more sophisticated cases, the controller can be a computer such as a microprocessor. Because of the simplicity and economy of open-loop control systems, we find this type of system in many noncritical applications

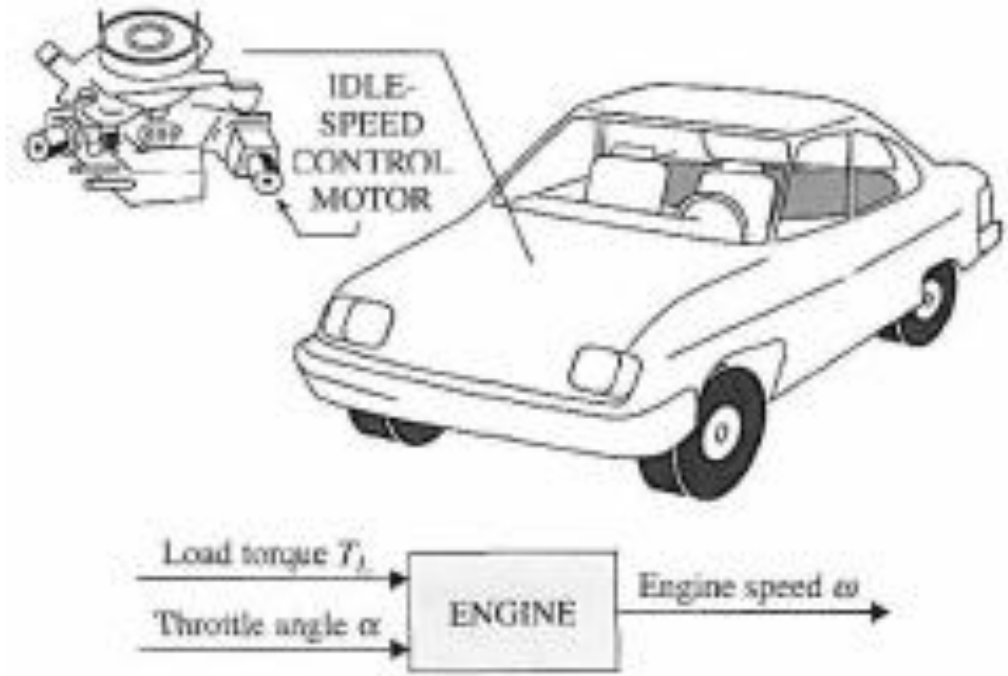


Figure 1. Idle-speed control system

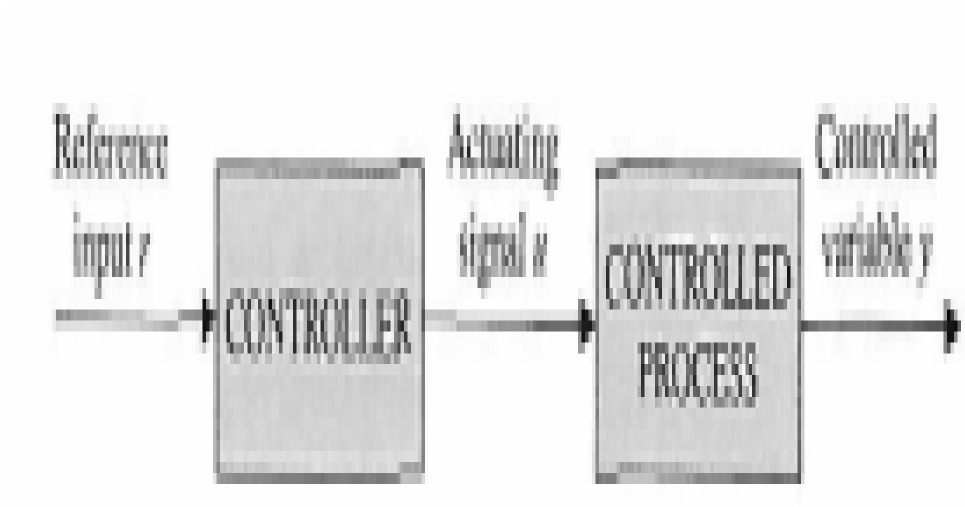


Figure 2. Elements of an open-loop control system.

5.Example of Closed-Loop Control Systems (Feedback Control Systems:

What is missing in the open-loop control system for more accurate and more adaptive control is a link or feedback from the output to the input of the system. To obtain more accurate control, the controlled signal y should be fed back and compared with the reference input, and an actuating signal proportional to the difference of the input and the output must be sent through the system to correct the error. A system with one or more feedback paths such as that just described is called a closed-loop system Closed-loop systems have many advantages over open- systems. A closed-loop idle-speed control system is shown in Fig. 3. The reference input w*r* sets the desired idling speed. The engine speed at idle should agree with the reference value loop systems. w*r*, and any difference such as the load torque TL is sensed by the speed transducer and the error detector. The controller will operate on the difference and provide a signal to adjust the throttle angle α a to correct the error. Fig. 4 compares the typical performances of open loop and closed-loop idle-

speed control systems. In Fig. 4(a), the idle speed of the open loop system will drop and settle at a lower value after a load torque is applied. In Fig. 4 (b), the idle speed of the closed-loop system is shown to recover quickly to the preset value after the application of TLThe objective of the idle-speed control system illustrated, also known as a regulator system, is to maintain the system output at a prescribed level.

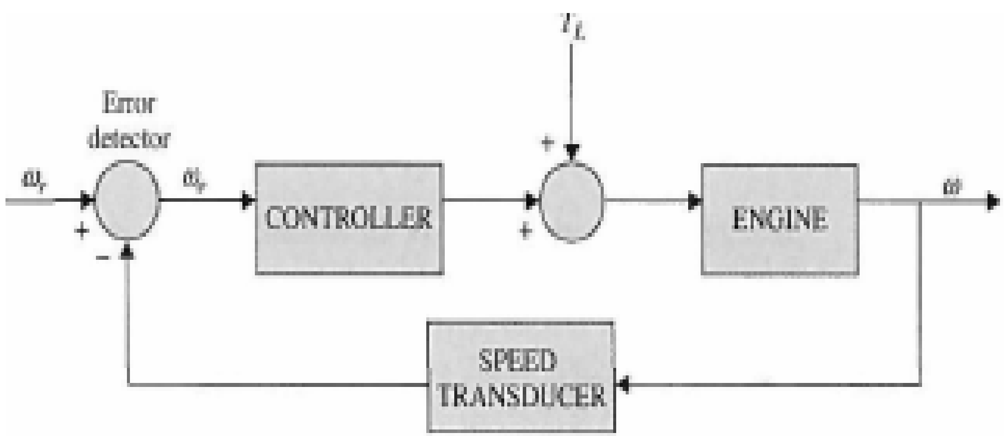


Figure 3. Block diagram of a closed-loop idle-speed control system

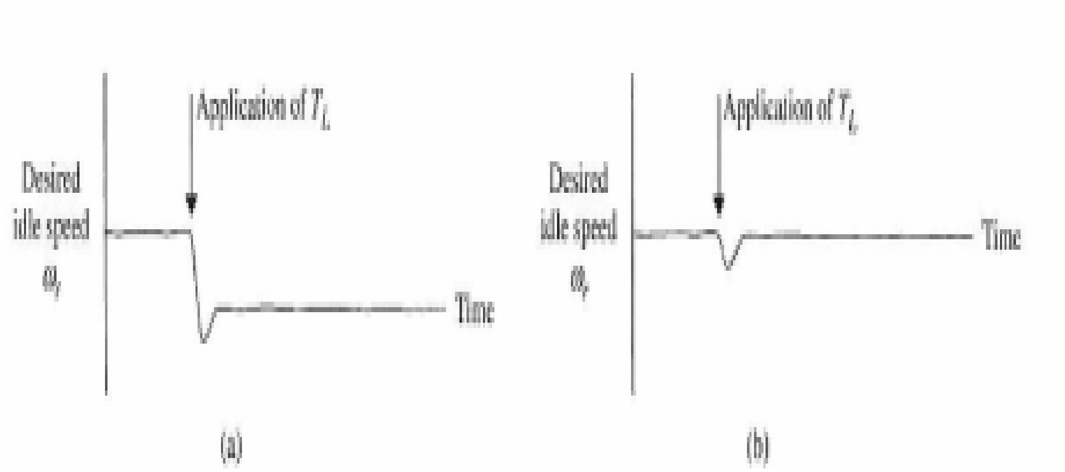


Figure 4. (a) Typical response of the open-loop idle-speed control system,
(b) Typical response of the closed-loop idle-speed control system.

6.Linear Systems: A system is called linear if the principle of superposition applies. The principle of superposition states that the response produced by the simultaneous application of two different forcing functions is the sum of the two individual responses. Hence, for the linear system, the response to several inputs can be calculated by treating one input at a time and adding the results. It is this principle that allows one to build up complicated solutions to the linear differential equation from simple solutions. In an experimental investigation of a dynamic system, if cause and effect are proportional, thus implying that the principle of superposition holds, then the system can be considered linear.

Linear Time-Invariant Systems and Linear Time-Varying Systems: A differ-ential equation is linear if the coefficients are constants or functions only of the independent variable. Dynamic systems that are composed of linear time-invariant lumped-parameter components may be described by linear time-invariant differential equations—that is, constant-coefficient differential equations. Such systems are called linear time-invariant (or linear constant-coefficient) systems. Systems that are represented by differential equations whose coefficients are functions of time are called linear time-varying systems. An example of a time-varying control system is a spacecraft control system. (The mass of a spacecraft changes due to fuel (consumption.).

Nonlinear Systems: A system is nonlinear if the principle of superposition doesnot apply. Thus, for a nonlinear system the response to two inputs cannot be calculated by treating one input at a time and adding the results.

7.Automatic Controllers: An automatic controller compares the actual value of the plant output with the reference input (desired value), determines the deviation, and produces a control signal that will reduce the deviation to zero or to a small value. The manner in which the automatic controller produces the control signal is called the control action. Figure 5 is a block diagram of an industrial control system,

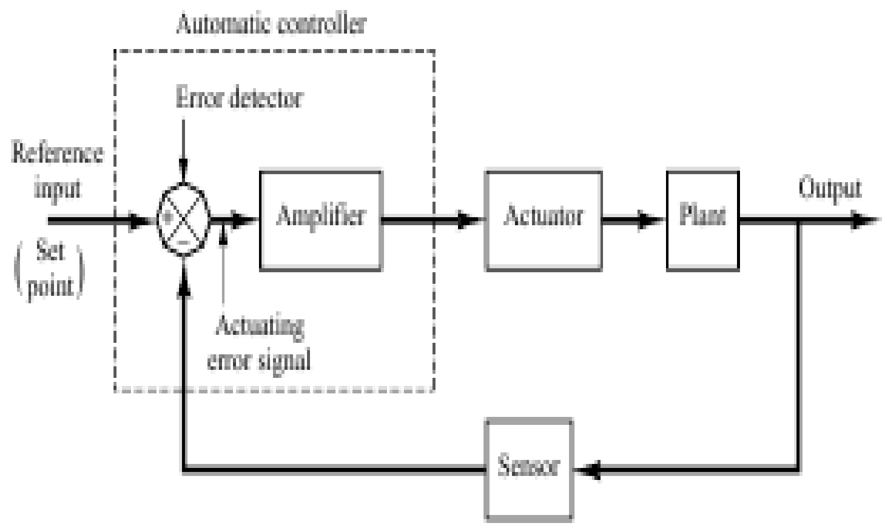


Figure 5 Block diagram of an industrial control system, which consists of measuring)an automatic controller an actuator, a plant and a sensor. (element

which consists of an automatic controller, an actuator, a plant, and a sensor (measuring element). The controller detects the actuating error signal, which is usually at a very low level, and amplifies it to a sufficiently high level. The output of an automatic controller is fed to an actuator, such as an electric motor, a hydraulic motor, or a pneumatic motor or valve. (The actuator is a power device that produces the input to the plant according to the control signal so that the output signal will approach the reference input signal) .The sensor or measuring element is a device that converts the output variable into another suitable variable, such as a displacement, pressure, voltage, etc., that can be used to compare the output to the reference input signal. This element is in the feedback path of the closed-loop system. The set point of the controller must be converted to a reference input with the same units as the feedback signal from the sensor or measuring element power

CONTROL THEROY (I)

Mathematical Model of Physical System:

Before studying the mathematical tools for system investigation, we should be able to a make mathematical model for dynamic systems, electrical, mechanical, thermal, ..etc**.**

The mathematical model can be define as a description of the dynamics characteristic. Models are often obtained by applying physical laws such as Newton law.

The systems are :

1. Electrical System

Basic laws governing electrical circuits are Kirchhoff's current law and voltage law Kirchhoff's current law (node law) states that the algebraic sum of all currents entering and leaving a node is zero. (This law can also be stated as follows: The sum of currents entering a loop law) states that at any given instant the)node is equal to the sum of currents leaving the same node.) Kirchhoff's voltage law algebraic sum of the voltages around any loop in an electrical circuit is zero. (This law can also be stated as follows: The sum of the voltage drops is equal to the sum of the voltage rises around a loop.) A mathematical model of an electrical circuit can be obtained by applying one or both of Kirchhoff's laws to it.

This section first deals with simple electrical circuits and then treats mathematical modeling of operational amplifier systems LRC Circuit. Consider the electrical circuit shown in Figure 1. The circuit consists of an inductance L (henry), a resistance R (ohm), and a capacitance C (farad) Applying Kirchhoff's voltage law to the system, we obtain the following equations:

$$L \frac{di}{dt} + Ri + \frac{1}{C} \int i \, dt = e_i$$

$$\frac{1}{C} \int i \, dt = e_o$$

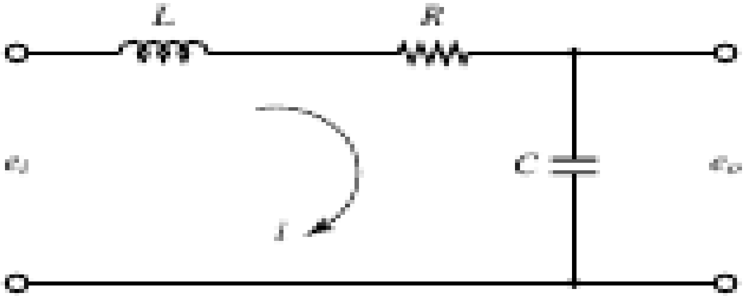


Figure 1. Electrical circuit.

Equations (1) and (2) give a mathematical model of the circuit. A transfer-function model of the circuit can also be obtained as follows: Taking the Laplace transforms of Equations (1) and (2), assuming zero initial conditions, we obtain If ei is assumed to be the input and eo the output, then the transfer function of this system is found to be

$$LsI(s) + RI(s) + \frac{1}{C}\frac{1}{s}I(s) = E_i(s)$$

$$\frac{1}{C}\frac{1}{s}I(s) = E_o(s)$$

$$\frac{E_o(s)}{E_i(s)} = \frac{1}{LCs^2 + RCs + 1}$$

Transfer Functions of Cascaded Elements:

Many feedback systems have components that load each other. Consider the system shown in Figure 2.Assume that ei is the input and eo is the output. The capacitances C1 and C2 are not charged initially

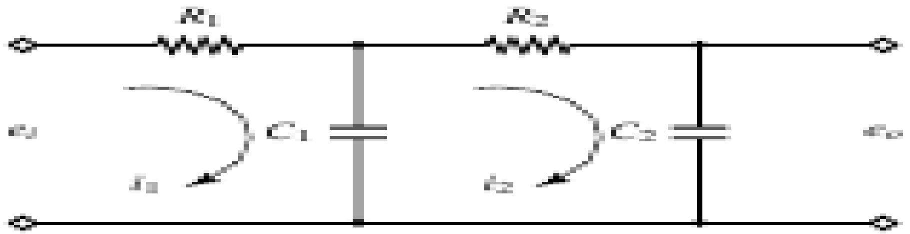


Figure 2

It will be shown that the second stage of the circuit (R2C2 portion) produces a loading effect on the first stage (R1C1 portion).The equations for this system are

$$\frac{1}{C_1}\int (i_1 - i_2)\,dt + R_1 i_1 = e_i \qquad (6)$$

$$\frac{1}{C_1}\int (i_2 - i_1)\,dt + R_2 i_2 + \frac{1}{C_2}\int i_2\,dt = 0 \qquad (7)$$

(5)

$$\frac{1}{C_2}\int i_2\,dt = e_o \qquad (8)$$

Taking the Laplace transforms of Equations (3) through (5), respectively, using zero initial conditions, we obtain

$$\frac{1}{C_1 s}\left[I_1(s) - I_2(s)\right] + R_1 I_1(s) = E_i(s) \qquad (3)$$

$$\frac{1}{C_1 s}\left[I_2(s) - I_1(s)\right] + R_2 I_2(s) + \frac{1}{C_2 s} I_2(s) = 0 \qquad (4)$$

$$\frac{1}{C_2 s} I_2(s) = E_o(s) \qquad (5)$$

Eliminating I1(s) from Equations (7) and (8) and writing Ei(s) in terms of I2(s),we find the transfer function between Eo(s) and Ei(s) to be The term R1C2s in the denominator of the transfer function represents the interaction of two simple RC circuits. Since the two roots of the denominator of Equation (9) are real. The present analysis shows that, if two RC circuits are connected in cascade so that the output from the first circuit is the input to the second, the overall transfer function is not the product of and The reason for this is that, when we derive the transfer function for an isolated circuit, we implicitly assume that the output is unloaded. In other words, the load impedance is assumed to be infinite, which means that no power is being withdrawn at the output. When the second circuit is connected to the output of the first, however, a certain amount of power is withdrawn, and thus the assumption of no loading is violated. Therefore, if the transfer function of this system is obtained under the assumption of no loading, then it is not valid. The degree of the loading effect determines the amount of modification of the transfer function.

$$\begin{aligned}\frac{E_o(s)}{E_i(s)} &= \frac{1}{(R_1C_1s+1)(R_2C_2s+1)+R_1C_2s} \\ &= \frac{1}{R_1C_1R_2C_2s^2+(R_1C_1+R_2C_2+R_1C_2)s+1}\end{aligned} \tag{9}$$

2. Mechanical Systems:

This section first discusses simple spring systems and simple damper systems. Then we derive transfer-function models and state-space models of various mechanical systems

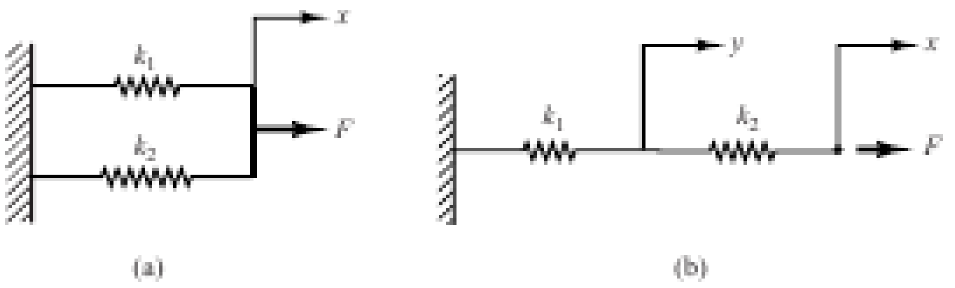


Figure 3 (a) System consisting of two springs in parallel

(b) system consisting of two springs in series.

EXAMPLE 1. Let us obtain the equivalent spring constants for the systems shown in Figures 3(a) and (b) respectively. For the springs in parallel [Figure 3(a)] the equivalent spring constant keq is obtained from

$$k_1 x + k_2 x = F = k_{eq} x$$

$$k_{eq} = k_1 + k_2$$

For the springs in series [Figure–3–1(b)], the force in each spring is the same.Thus

$$k_1 y = F, \qquad k_2(x - y) = F$$

Elimination of y from these two equations results in

$$k_2\left(x - \frac{F}{k_1}\right) = F$$

$$k_2 x = F + \frac{k_2}{k_1} F = \frac{k_1 + k_2}{k_1} F$$

The equivalent spring constant keq for this case is then found as

$$k_{eq} = \frac{F}{x} = \frac{k_1 k_2}{k_1 + k_2} = \frac{1}{\frac{1}{k_1} + \frac{1}{k_2}}$$

EXAMPLE 3–2 Let us obtain the equivalent viscous-friction coefficient for each of the damper systems shown in Figures 3–2(a) and (b).An oil-filled damper is often called a dashpot.A dashpot is a device that provides viscous friction, or damping. It consists of a piston and oil-filled cylinder. Any relative motion between the piston rod and the cylinder is resisted by the oil because the oil must flow around the piston (or through orifices provided in the piston) from one side of the piston to the other. The dashpot essentially absorbs energy. This absorbed energy is dissipated as heat, and the dashpot does not store any kinetic or potential energy

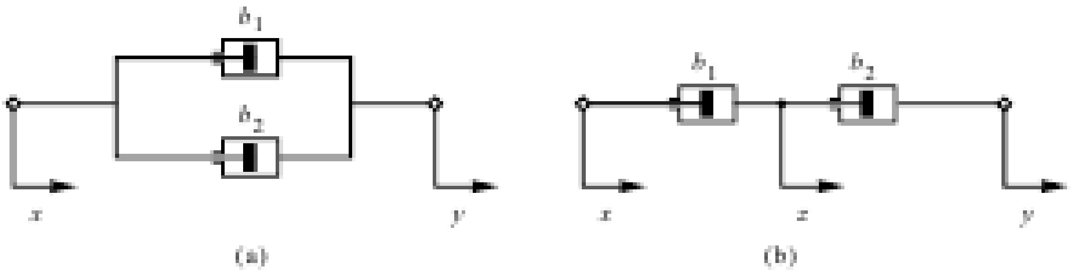


Figure (a) Two dampers connected in parallel ,(b) Two dampers connected in series.

(a) The force f due to the dampers is

$$f = b_1(\dot{y} - \dot{x}) + b_2(\dot{y} - \dot{x}) = (b_1 + b_2)(\dot{y} - \dot{x})$$

In terms of the equivalent viscous-friction coefficient beq, force f is given by

$$f = b_{eq}(\dot{y} - \dot{x})$$

$$b_{eq} = b_1 + b_2$$

b) The force f due to the dampers is

$$f = b_1(\dot{z} - \dot{x}) = b_2(\dot{y} - \dot{z})$$

where z is the displacement of a point between damper b1 and damper b2. (Note that the same force is transmitted through the shaft.) From Equation (10), we have

$$(b_1 + b_2)\dot{z} = b_2\dot{y} + b_1\dot{x}$$

$$\dot{z} = \frac{1}{b_1 + b_2}(b_2\dot{y} + b_1\dot{x})$$

In terms of the equivalent viscous-friction coefficient beq, force f is given by

$$f = b_{eq}(\dot{y} - \dot{x})$$

By substituting Equation (11) into Equation (10), we have

$$f = b_2(\dot{y} - \dot{z}) = b_2\left[\dot{y} - \frac{1}{b_1 + b_2}(b_2\dot{y} + b_1\dot{x})\right]$$

$$= \frac{b_1 b_2}{b_1 + b_2}(\dot{y} - \dot{x})$$

$$f = b_{eq}(\dot{y} - \dot{x}) = \frac{b_1 b_2}{b_1 + b_2}(\dot{y} - \dot{x})$$

$$b_{eq} = \frac{b_1 b_2}{b_1 + b_2} = \frac{1}{\frac{1}{b_1} + \frac{1}{b_2}}$$

Example 2. Consider the spring-mass-dashpot system mounted on a massless cart as shown in Figure 3. Let us obtain mathematical models of this system by assuming that the cart is standing still for t<0 and the spring-mass-dashpot system on the cart is also standing still for t<0. In this system, u(t) is the displacement of the cart and is the input to the system. At t=0, the cart is moved at a constant speed or constant. The displacement y(t) of the mass is the output. (The displacement is relative to the ground.) In this system, m denotes the mass, b denotes the viscous-friction coefficient, and k denotes the spring constant. We assume that the friction force of the dashpot is proportional to and that the spring is a linear spring; that is, the spring force is proportional to y-u

.

For translational systems, Newton's second law states that

$$ma = \sum F$$

where m is a mass, a is the acceleration of the mass, and is the sum of the forces acting on the mass in the direction of the acceleration a. Applying Newton’s second law to the present system and noting that the cart is massless, we obtain

$$m\frac{d^2y}{dt^2} = -b\left(\frac{dy}{dt} - \frac{du}{dt}\right) - k(y - u)$$

$$m\frac{d^2y}{dt^2} + b\frac{dy}{dt} + ky = b\frac{du}{dt} + ku$$

This equation represents a mathematical model of the system considered. Taking the Laplace transform of this last equation, assuming zero initial condition, gives

$$(ms^2 + bs + k)Y(s) = (bs + k)U(s)$$

Taking the ratio of Y(s) to U(s), we find the transfer function of the system to be

$$\text{Transfer function} = G(s) = \frac{Y(s)}{U(s)} = \frac{bs + k}{ms^2 + bs + k}$$

Such a transfer-function representation of a mathematical model is used very frequently in control engineering.

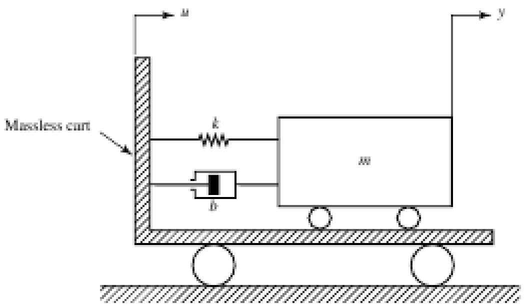


Figure 3.Spring –mass-dashpot system mounted on a car.

3. Thermal Systems

Thermal systems are those that involve the transfer of heat from one substance to another. Thermal systems may be analyzed in terms of resistance and capacitance although the thermal capacitance and thermal resistance may not be represented accurately as lumped parameters, since they are usually distributed throughout the sub stance. For precise analysis, distributed-parameter models must be used. Here, however to simplify the analysis we shall assume that a thermal system can be represented by a lumped-parameter model, that substances that are characterized by resistance to heat flow have negligible heat capacitance, and that substances that are characterized by heat capacitance have negligible resistance to heat flow. There are three different ways heat can flow from one substance to another: conduction, convection, and radiation. Here we consider only conduction and convection. (Radiation heat transfer is appreciable only if the temperature of the emitter is very high compared to that of the receiver. Most thermal processes in process control systems do not involve radiation heat transfer.)For conduction or convection heat transfer,

$q = K\,\Delta\theta$

where q=heat flow rate, kcal/sec

$\Delta\theta$ =temperature difference, °C

The coefficient K is given by

K = kA / X , for conduction

= HA, for convection

Where Δ k=thermal conductivity, kcal/m sec °C

A=area normal to heat flow, m.m

ΔX=thickness of conductor, m

H=convection coefficient, kcal/m2 sec °C

Thermal Resistance and Thermal Capacitance. The thermal resistance R for heat transfer between two substances may be defined as follows:

$$R = \frac{\text{change in temperature difference, °C}}{\text{change in heat flow rate, kcal/sec}}$$

The thermal resistance for conduction or convection heat transfer is given by

$$R = d(\Delta\theta) / dq = 1/ K$$

Since the thermal conductivity and convection coefficients are almost constant, the thermal resistance for either conduction or convection is constant.

The thermal capacitance C is defined by

$$C = \frac{\text{change in heat stored, kcal}}{\text{change in temperature, °C}} \qquad \text{Or} \quad C = mc$$

Where m=mass of substance considered, kg

c=specific heat of substance, kcal / kg °C

Thermal System. Consider the system shown in Figure 4–26(a). It is assumed that the tank is insulated to eliminate heat loss to the surrounding air. It is also assumed that there is no heat storage in the insulation and that the liquid in the tank is perfectly mixed so that it is at a uniform temperature. Thus, a single temperature is used to describe the temperature of the liquid in the tank and of the outflowing liquid.

Let us define

$\bar{\Theta}_i =$ steady-state temperature of inflowing liquid, °C.

$\bar{\Theta}_{0=}$ steady-state temperature of outflowing liquid, °C.

G= steady-state liquid flow rate, kg/sec.

M=mass of liquid in tank, kg.

c=specific heat of liquid, kcal/kg °C

R=thermal resistance, °C sec/kcal

C= thermal capacitance, kcal/°C

H= steady-state heat input rate, kcal/sec.

Assume that the temperature of the inflowing liquid is kept constant and that the heat input rate to the system (heat supplied by the heater) is suddenly changed from $\bar{H}$ to $\bar{H} + h_i$ where hi represents a small change in the heat input rate. The heat outflow rate will then change gradually from $\bar{H}$ To $\bar{H} + h_i$. The temperature of the outflowing liquid will also be changed from $\bar{\Theta}_0$ to $\bar{\Theta}_0 + \theta$. For this case, ho, C, and R are obtained respectively, as

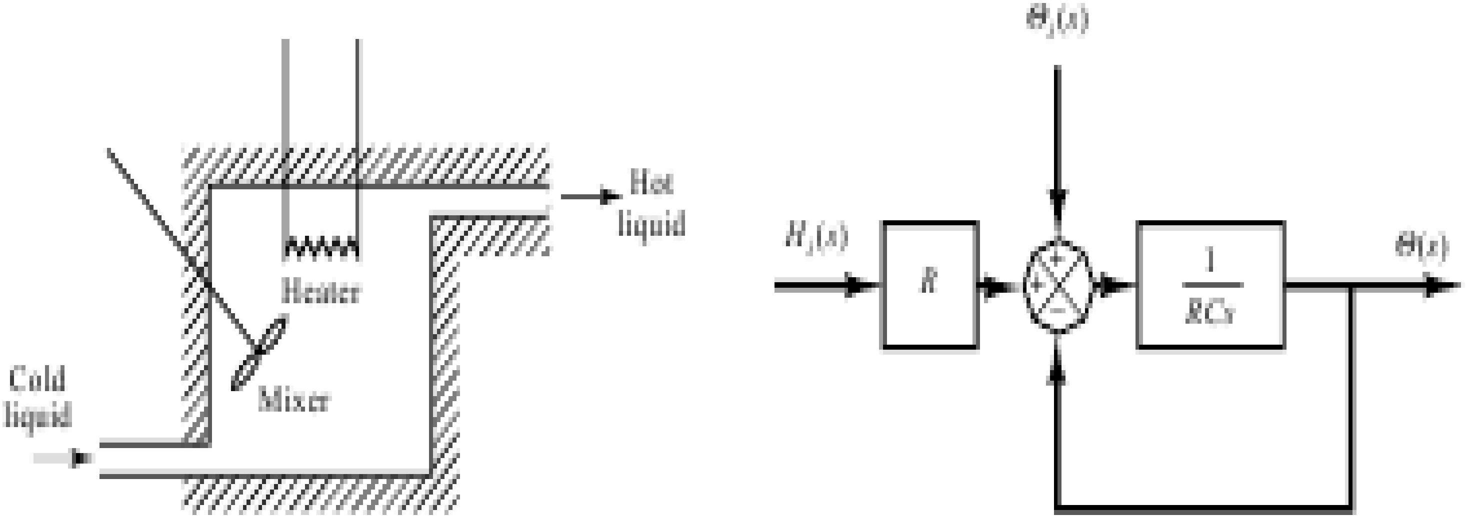
ho = Gc θ

C = Mc

R=θ / ho =1 / Gc

The heat-balance equation for this system is

C dθ = (hi – ho) dt



a) Thermal system , b) block diagram of the system.

C (dθ/dt) = (hi – ho)

which may be rewritten as

RC (dθ/dt) + θ =Rh

Note that the time constant of the system is equal to RC or M/G seconds. The transfer

function relating θ and hi is given by

Θ (s) / Hi(s) = R/ (RCs + 1)

where Θ (s) = Laplace θ(t) and Hi(s) =Laplace hi(t)

The past decades have seen a great development in low pressure pneumatic controllers for industrial control systems, and today they are used extensively in industrial processes. Reasons for their broad appeal include an explosion proof character, simplicity, and ease of maintenance.

Resistance and Capacitance of Pressure Systems. Many industrial processes and pneumatic controllers involve the flow of a gas or air through connected pipelines and pressure vessels.

Consider the pressure system shown in Figure 4–4(a). The gas flow through the restriction is a function of the gas pressure difference pi-po. Such a pressure system may be characterized in terms of a resistance and a capacitance.

The gas flow resistance R may be defined as follows:

$$C = \frac{\text{change in gas stored, lb}}{\text{change in gas pressure, lb}_f/\text{ft}^2}$$

or $R = d(\Delta P)/dq$ (12)

where $d(\Delta P)$ is a small change in the gas pressure difference and dq is a small change in the gas flow rate. Computation of the value of the gas flow resistance R may be quite time consuming. Experimentally, however, it can be easily determined from a plot of the pressure difference versus flow rate by calculating the slope of the curve at a given operating condition, as shown in Figure 4(b).

The capacitance of the pressure vessel may be defined by

$$C = \frac{\text{change in gas stored, lb}}{\text{change in gas pressure, lb}_f/\text{ft}^2}$$

Or $C = (dm / dP) = v\,(d\rho / dP)$ (13)

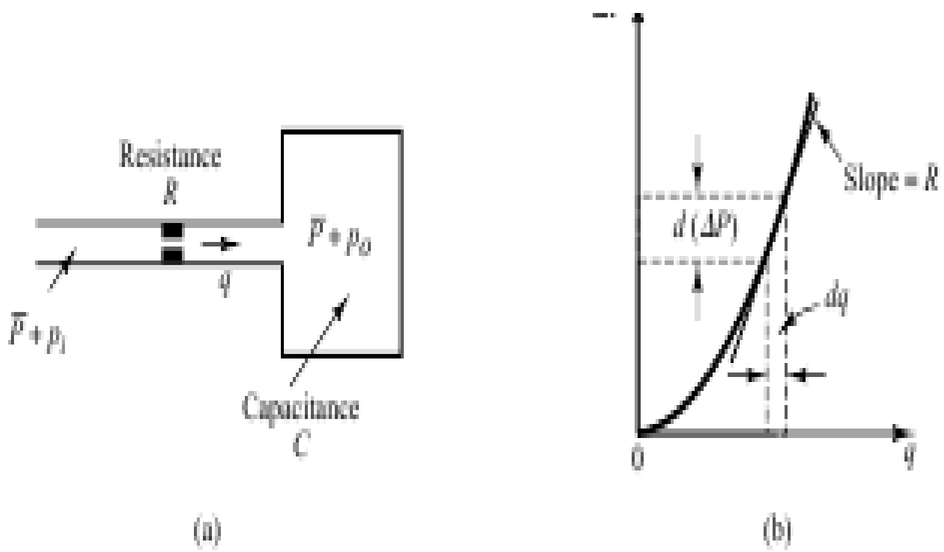


Figure 4. (a) Schematic diagram of a pressure system; (b) pressure-difference-versus-flow-rate curve .

where C = capacitance, lb-ft2/lbf

m = mass of gas in vessel, lb

P = gas pressure, lb/ft.ft

V = volume of vessel, ft.ft.ft.

P = density, lb/ft.ft.ft.

The capacitance of the pressure system depends on the type of expansion process involved. The capacitance can be calculated by use of the ideal gas law. If the gas expansion process is polytropic and the change of state of the gas is between isothermal and adiabatic, then

$$\rho \left(\frac{V}{m}\right)^{n} = \frac{\rho}{\rho^{n}} = \text{constant} = K \qquad (14)$$

where n=polytropic exponent. For ideal gases,

$$P\bar{v} = \bar{R}\,T \quad or \quad Pv = \frac{\bar{R}}{M}\,T$$

where P=absolute pressure, lb_f/ft^2

$\bar{v}$ = volume occupied by 1 mole of a gas, ft^3 / lb-mole

R = universal gas constant, ft-lb_f / lb-mole °R

T = absolute temperature, °R

v = specific volume of gas, ft^3/lb

M = molecular weight of gas per mole, lb/lb-mole

Thus

$$Pv = \frac{P}{\rho} = \frac{\bar{R}}{M}\,T = R_{gas}\,T \qquad (15)$$

where Rgas = gas constant, ft-lb_f/lb °R.

The polytropic exponent n is unity for isothermal expansion. For adiabatic expansion

n is equal to the ratio of specific heats c_P/c_v, where c_P is the specific heat at constant pressure and cv is the specific heat at constant volume. In many practical cases, the value of n is approximately constant, and thus the capacitance may be considered constant.

The value of dr/dp is obtained from Equations (14) and (15). From

Equation (14) we have

$$d_P \; = \mathrm{Kn}\rho^{n-1}\, d_\rho$$

Or